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Time-varying Feedback Stabilization of the Attitude of a Rigid Spacecraft with two controls

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Abstract

Rigid body models with two controls cannot be locally asymptotically stabilized by continuous state feedbacks. Existence of a locally stabilizing smooth time-varying feedback has however been proved. Here, such a feedback is explicitely derived.

1 Introduction

The attitude control of a rigid spacecraft operating in degraded mode, i.e with only one or two controls, has already been much studied in the literature. With respect to other contributions on the subject, the present paper focuses on feedback stabilization . The related but simpler problem consisting of stabilizing the angular velocity of the spacecraft with one or two actuators has been investigated by several authors (see e.g [1]). Smooth stabilization of the attitude seems to have been previously ruled out due to that the resulting system, although controllable, cannot be stabilized via continuous state feedback (as easily proved by application of Brockett's necessary condition [2]). An article by Samson [8] has recently triggered the discovery that many systems of this type can in fact be stabilized by smooth "*time-varying*" feedback. Research on time-varying control has then expanded quickly (see e.g [4], [7]). In particular, results by Kerai [5] and Coron [4] ensure that the attitude of a controllable spacecraft with two actuators can be locally stabilized by continuous time-varying feedbacks. The purpose of this note is to describe a smooth (C^∞) stabilizing time-varying feedback.

2 Main results

The angular velocity vector of the inertial frame F_0 with respect to a fixed frame F_1, expressed in the basis of F_0, is denoted as ω. The matrix representation of the cross product ($x \longmapsto x \wedge \omega$) is denoted as $S(\omega)$. It is common to use Euler angles in order to work with a minimal parametrization of $SO(3)$. We prefer here the parametrization $X = \sin\frac{\theta}{2}\, u$, with $(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\, u)$ being the unitary quaternion associated with the rotation matrix representing the attitude of F_1 with respect to F_0. With this parametrization, the equation of the system may locally be written:

$$\left\{\begin{array}{lcl} \dot{X} & = & \frac{1}{2}(\sqrt{1-|X|^2}Id - S(X))\omega \\ \dot{\omega}_1 & = & c_1\omega_2\omega_3 + u_1 \\ \dot{\omega}_2 & = & c_2\omega_1\omega_3 + u_2 \\ \dot{\omega}_3 & = & c_3\omega_1\omega_2 \end{array}\right. \tag{1}$$

where the u_i ($i = 1,2$) are the torques applied to the rigid body, and the parameters c_j ($j = 1,2,3$) are deduced from the coefficients of the body's inertia matrix. We assume that $c_3 \neq 0$, since otherwise the system would not be controllable. Moreover, by an adequate change of variables, we may also assume that $c_3 > 0$. We first consider the following reduced order system obtained by taking $\omega_1 = v_1$ and $\omega_2 = v_2$ as control variables:

$$\left\{\begin{array}{lcl} \dot{X} & = & \frac{1}{2}(\sqrt{1-|X|^2}Id - S(X))(v_1, v_2, \omega_3)^T \\ \dot{\omega}_3 & = & c_3 v_1 v_2 \end{array}\right. \tag{2}$$

Proposition 1 *The smooth time-varying controls:*

$$\begin{array}{l} v_1(X,\omega_3,t) = 2g_1\dot{h}_1 + h_1\dfrac{\partial g_1}{\partial x_3}\omega_3 - 2k_1(x_1 - g_1h_1) \\ v_2(X,\omega_3,t) = 2g_2\dot{h}_2 - x_3v_1(X,\omega_3,t) + x_1\omega_3 \\ \qquad\qquad +h_2\dfrac{\partial g_2}{\partial x_3}\omega_3 - 2k_2(x_2 - g_2h_2) \end{array} \tag{3}$$

with

$$\left\{\begin{array}{lcl} g_1 & = & \alpha x_3 + \beta\omega_3 \\ g_2 & = & x_3^2 + \omega_3^2 \\ h_1 & = & a_1 \sin t \\ h_2 & = & a_2 \sin t + a_3 \cos t \end{array}\right. \tag{4}$$

and $k_1, k_2, \alpha, \beta, a_1, a_2$ and a_3 being real numbers such that:

$$\left\{\begin{array}{l} k_1 > 0, \quad k_2 > 0, \quad a_1 > 0, \quad a_2 < 0, \\ a_3 > 0, \quad \alpha = -\dfrac{a_3^2}{8a_1a_2}, \quad \beta = \dfrac{a_3}{4a_1} \end{array}\right. \tag{5}$$

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locally asymptotically stabilize the origin of (2).

Our second result concerns the original system controlled by the torques u_1 and u_2.

Proposition 2 *The control laws:*

$$\left\{\begin{array}{rcll} u_1(X,\omega,t) & = & -c_1\omega_2\omega_3 + s_1(X,\omega,t) & \\ & & -k_3(\omega_1 - v_1(X,\omega_3,t)) & k_3 > 0 \\ u_2(X,\omega,t) & = & -c_2\omega_1\omega_3 + s_2(X,\omega,t) & \\ & & -k_4(\omega_2 - v_2(X,\omega_3,t)) & k_4 > 0 \end{array}\right. \quad (6)$$

with v_1 and v_2 given by (3), (4) and (5) and, s_1 and s_2 their time derivatives along the trajectories:

$$s_i = \frac{1}{2}\frac{\partial v_i}{\partial X}(\sqrt{1-X^2}Id - S(X))\omega + \frac{\partial v_i}{\partial \omega_3}c_3\omega_1\omega_2 + \frac{\partial v_i}{\partial t}$$

locally asymptotically stabilize the origin of (1).

The proof of Proposition 1 is based upon the two following lemmas, with $O_t^q(X)$ denoting any continuous function of t and X such that,

$$\exists\delta > 0, \exists K : |X| \leq \delta \Longrightarrow |O_t^q(X)| \leq K|X|^q.$$

Lemma 1 is an adaptation of Center Manifold Theory [3] to periodic time varying systems, while Lemma 2 is an original averaging result which takes advantage of the particular structure of the closed-loop system obtained via an adequate choice of the controls.

Lemma 1 *Consider the system*

$$\left\{\begin{array}{rcll} \dot{x} & = & Ax + f(x,z,t) & \\ \dot{z}_1 & = & -k_1 z_1 + l_1(x,z,t)\,, & k_1 > 0 \\ \dot{z}_2 & = & -k_2 z_2 + l_2(x,z,t)\,, & k_2 > 0 \end{array}\right. \quad (7)$$

with $x \in \mathbb{R}^n, z = (z_1, z_2)$, A a matrix with eigenvalues having zero real parts, f, l_1 and l_2 C^2 T periodic functions such that, for all t, $f(0,0,t) = 0, f'_{(x,z)}(0,0,t) = 0$, $l_i(0,0,t) = 0$ and $l'_{i\,(x,z)}(0,0,t) = 0$ for $i = 1,2$.

It is assumed that $l_1(x,0,0,t)$ is a $O_t^{q_1}$ function and that $l_2(x, O_t^{q_1}(x), 0, t)$ is a $O_t^{q_2}$ function with $2 \leq q_1 \leq q_2$. Assume further that the origin of the time-varying system:

$$\dot{x} = Ax + f(x, \pi_1(t,x), \pi_2(t,x), t) \quad (8)$$

is locally asymptotically stable when $\pi_1(t,x)$ (resp $\pi_2(t,x)$) is any $O_t^{q_1}$ (resp $O_t^{q_2}$) function. Then, the origin of (7) is locally asymptotically stable.

Lemma 2 *Consider the system*

$$\dot{x} = Ax + D(t)d(x) + O_t^{2k-1}(x) \quad (9)$$

with $x \in \mathbb{R}^n$, A a $n \times n$ (strictly) upper triangular matrix ($j \leq i \Rightarrow a_{ij} = 0$), $D(t)$ a $n \times p$ matrix where $d_{ij}(t)$ is a T periodic $C^r (r \geq 1)$ function for all $(i,j) \in \{1,..,n\} \times \{1,..,p\}$ and $d(x) = (d_1(x),...,d_p(x))^T$ where the d_i are polynomials in $(x_1,...,x_n)$ of degree $k \geq 2$. Then there exists a neighborhood Ω of 0 in $\mathbb{R}^n$ and a C^r local change of coordinates: $(x,t) \mapsto (y,t)$ defined on $\Omega \times \mathbb{R}$ such that: $y - x = O_t^k(x)$ and, for any solution x of (9), y is solution of:

$$\dot{y} = Ay + \overline{D}d(y) + O_t^{2k-1}(y) \quad (10)$$

where $\overline{D}$ is the time average of $D(t)$.

Proposition 2 is obtained from Proposition 1 by a standard method used for cascaded systems.

Details concerning the way the aforementionned lemmas are derived and applied to prove Propositions 1 and 2 can be found in [6].

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