Back-engineering of spiking neural networks parameters.

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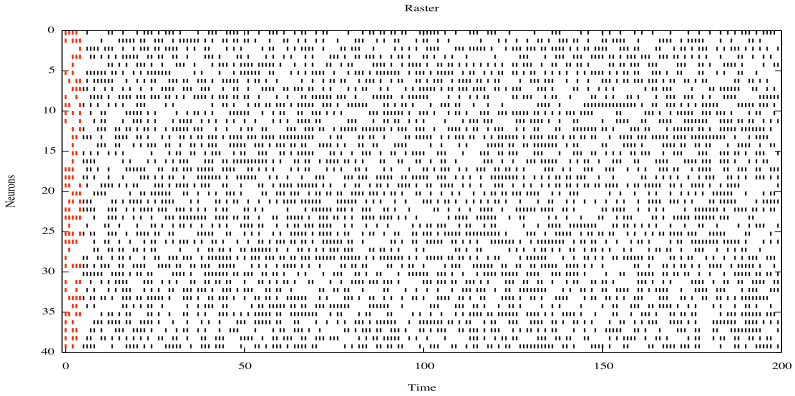
Introduction

Neurons in the brain communicate by short electrical pulses, the so-called action potentials or spikes.

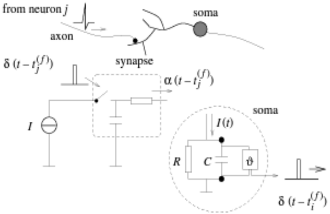
- How can we understand the process of spike generation?
- How can we understand information transmission by neurons?
- What happens if thousands of neurons are coupled together in a seemingly random network?
- How does the network connectivity determine the activity patterns?
- And, vice versa, how does the spike activity influence the connectivity pattern?

Problem position

Given a spiking neural network to which extends observing the spike raster allows to infer the networks parameters ?



Model - From the Leaky Integrate and Fire model (LIF) to BMS model (Cessac, 2008)



$$I_i(t) = I_R + I_C$$

$$I_i(t) = \frac{V_i(t)}{R} + C\frac{dV_i(t)}{dt}$$

$$\frac{dV_i(t)}{dt} = -\frac{V_i(t)}{\tau} + \frac{I_i(t)}{C}$$

Time discretization of the LIF model using the Euler method:

$$\frac{V_i(t+dt) - V_i(t)}{dt} = -\frac{V_i(t)}{\tau} + \frac{I_i(t)}{C}$$

Setting $dt = 1$ (sampling time scale) and $\gamma = 1 - \frac{1}{\tau}$ we have:

$$V_i(t+1) = V_i(t) - \frac{V_i(t)}{\tau} + \frac{I_i(t)}{C}$$

$$V_i(t+1) = V_i(t)\left(1 - \frac{1}{\tau}\right) + \frac{I_i(t)}{C}$$

$$V_i(t+1) = \gamma V_i(t) + \frac{I_i(t)}{C}$$

The discretization imposes that $\tau \geq 1$, thus $\gamma \in [0, 1[$. Considering that $C = 1$ we have:

$$V_i(t+1) = \gamma V_i(t) + I_i(t)$$

$$I_i(t) = I_i^S + I_i^{ext}$$

$$I_i^S(t) = \sum_{j=1}^{N} W_{ij} Z[V_j]$$

$$V_i(t+1) = \gamma V_i(t) + \sum_{j=1}^{N} W_{ij} Z[V_j] + I_i^{ext}$$

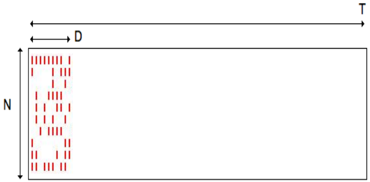
$$V_i(t+1) = \gamma V_i(t)(1 - Z[V_i]) + \sum_{j=1}^{N} W_{ij} Z[V_j] + I_i^{ext}$$

$$V_i[k] = \gamma V_i[k-1](1 - Z_i[k-1]) + \sum_{j=1}^{N} W_{ij} Z_j[k - d_{ij}] + I_i^{ext}$$

$$Z_i[k] = (V < \theta \quad ? \quad 0 : 1)$$

Master-Slave paradigm

For the master:



- $Z_i[k], k \in \{1..D\}$
- $V_i[k] = 0, k \leq D$
- Weights $W_{ij} \in \mathcal{R}$
- Delays $d_{ij} \in \{1..D\}$

$$V_i[k] = \gamma V_i[k-1](1 - Z_i[k-1]) + \sum_{j=1}^{N} W_{ij} Z_j[k - d_{ij}] + I_i^{ext}$$

For the slave we have 3 possible solutions:

- L Linear problem, if we observe raster and potential and known delays.
- LP Linear programming problem, if we observe raster and known delays.
- NP NP-complete problem, in any general case. Unknown delays and weights.

(L) Retrieving weights from the observation of spikes and membrane potential

We assume:

1. $V_i[k] = 0, \; k \in \{0, D\{$, or
2. the neuron i has fired at least one.

From the BMS model we have:

$$V_i[k] = \sum_{j=1}^{N} W_{ij} \sum_{\tau=\tau_{jk}}^{0} \gamma^{\tau} Z_j[k - \tau - d_{ij}] + I_i^{ext}$$

with, $\tau_{jk} = k - arg \; min_{\tau > D}\{Z_i[k - 1 - \tau] = 1\}$

where: τ_{jk} is the delay from the last spiking time, i.e., the last membrane potential reset.

Last equation writes as a matrix:

$$\mathbf{A}_i \mathbf{w}_i = \mathbf{b}_i$$

$$\begin{array}{rcll}
\mathbf{A}_i & = & \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \sum_{\tau=\tau_{jt}}^{0} \gamma^{\tau} \, Z_j(k-\tau-d_{ij}) & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} & \in \; \mathcal{R}^{T-D \times N} \\
\mathbf{w}_i & = & (\cdots W_{ij} \cdots)^T & \in \; \mathcal{R}^{N} \\
\mathbf{b}_i & = & (\cdots V_i[k] - I_i^{ext} \cdots)^T & \in \; \mathcal{R}^{T-D}
\end{array}$$

Solving the (L) problem

The Singular Value Decomposition of $\mathbf{A} \in \mathcal{R}^{M \times N}, M \geq N$:

$$\mathbf{A} = \mathbf{U} \underbrace{\begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \sigma_N \end{pmatrix}}_{\mathbf{S}} \mathbf{V}^T$$

Given $\mathbf{b} = \mathbf{A}\,\mathbf{w}$, it appears that $\mathbf{w} = \mathbf{U}^T\,\mathbf{S}^\dagger\,\mathbf{V}$ is the:

- Solution to $\mathbf{b} = \mathbf{A}\,\mathbf{w}$, if unique
- The smallest solution to $\mathbf{b} = \mathbf{A}\,\mathbf{w}$, i.e. with $|\mathbf{w}|^2$ minimal, if many.
- The least-square solution, i.e. with $|\mathbf{b} - \mathbf{A}\,\mathbf{w}|^2$ minimal, if none.

Here $\mathbf{S}^\dagger$ is the diagonal matrix with σ_i^{-1} or 0 as diagonal term.

(LP) Retrieving weights from the observation of spikes

In this case, the value of $V_i[k]$ is not known but only its sign with respect to the firing threshold, i.e.:

$$Z_i[k] = 0 \Rightarrow V_i[k] < 1 \quad \text{and} \quad Z_i[k] = 1 \Rightarrow V_i[k] > 1,$$

which is equivalent to write:

$$(2Z_i[k] - 1)(V_i[k] - 1) > 0,$$

Expanding the BMS model modified in the previous condition allow us to write the follow:

$$(2Z_i[k] - 1)\left(\sum_{j=1}^{N} W_{ij} \sum_{\tau=\tau_{jk}}^{0} \gamma^{\tau} Z_j[k - \tau - d_{ij}] + I_i^{ext} - 1\right) > 0$$

in matrix form:

$$\mathbf{A}_i \mathbf{w}_i + \mathbf{b}_i > 0$$

writing:

$$\begin{array}{rcll}
\mathbf{A}_i & = & \left(\begin{array}{ccc} \cdots & \cdots & \cdots \\ \cdots & (2Z_i[k]-1) \sum_{\tau=\tau_{jt}}^{0} \gamma^\tau \, Z_j(k-\tau-d_{ij}) & \cdots \\ \cdots & \cdots & \cdots \end{array} \right) & \in \quad \mathcal{R}^{T-D\times N} \\
\mathbf{w}_i & = & \left(\cdots W_{ij} \cdots \right)^T & \in \quad \mathcal{R}^{N} \\
\mathbf{b}_i & = & \left(\cdots (2Z_i[k]-1)(I_i^{ext}-1) \cdots \right)^T & \in \quad \mathcal{R}^{T-D}
\end{array}$$

Solving the (LP) problem

1. Replace the inequalities by equalities as follows:

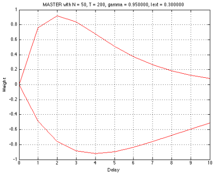
$$\mathbf{A}_i \mathbf{w}_i + \mathbf{b}_i - \mathbf{e}_i = 0$$

2. We have a maximization problem, with: $-1 < W_{ijd} < 1$ and $0 < e_i \leq e^{max}$.
3. Solve it by the Simplex method.

(NP) Retrieving delayed weights from the observation of spikes

From the reduced BMS model we can rewrite the following:

$$V_i[k] = \sum_{j=1}^{N} \sum_{d=1}^{D} W_{ijd} \sum_{\tau=\tau_{jk}}^{0} \gamma^\tau Z_j[k-\tau-d] + I_i^{ext}$$



The way to solve it is the same that the LP problem

Results

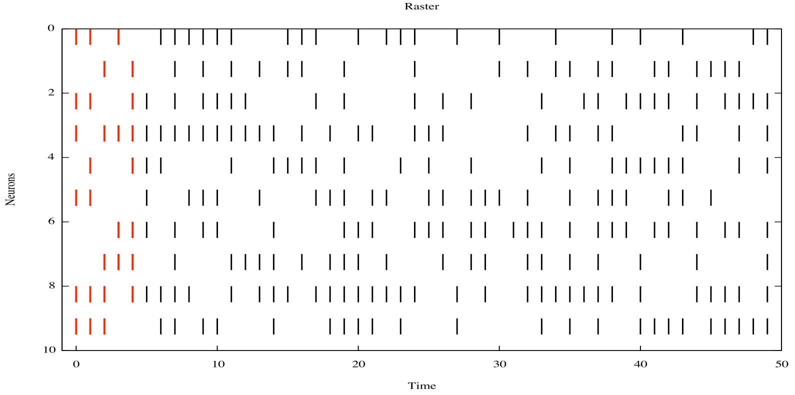


Figure: N = 10, T = 50, D = 5, Iext = 0.6, $\gamma = 0.95$

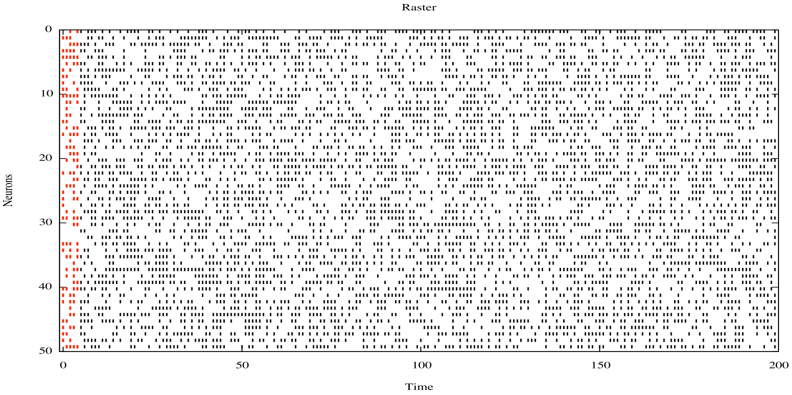


Figure: N = 50, T = 200, D = 5, Iext = 0.6, $\gamma = 0.95$

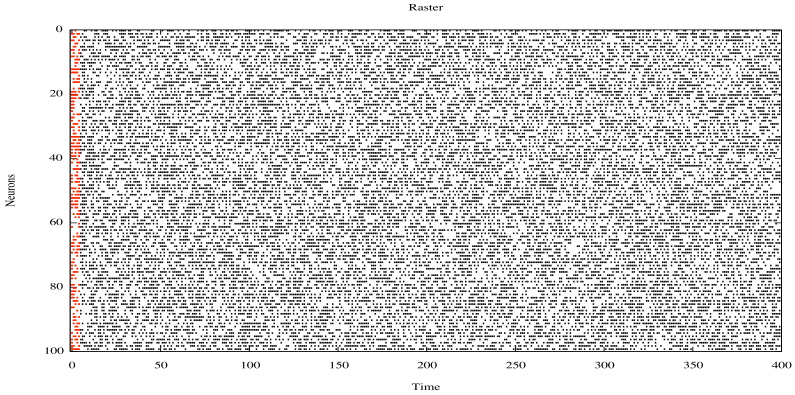


Figure: N $= 100$, T $= 400$, D $= 5$, Iext $= 0.6$, $\gamma = 0.95$

Introducing hidden units to match any raster

In all these cases we have seen a solution always exists if the observation period is small enough i.e., $T < O(ND)$. Let now consider the case where $T >> O(ND)$. The key idea, borrowed from the reservoir computing paradigm reviewed in the introduction, is to add a reservoir of "hidden neurons", i.e., to consider not N but $N + S$ neurons.

Results

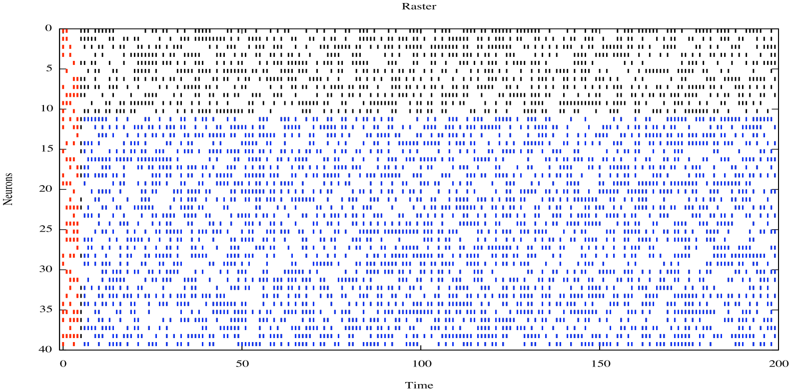


Figure: N = 10, T = 200, D = 5, Iext = 0.6, $\gamma = 0.95$

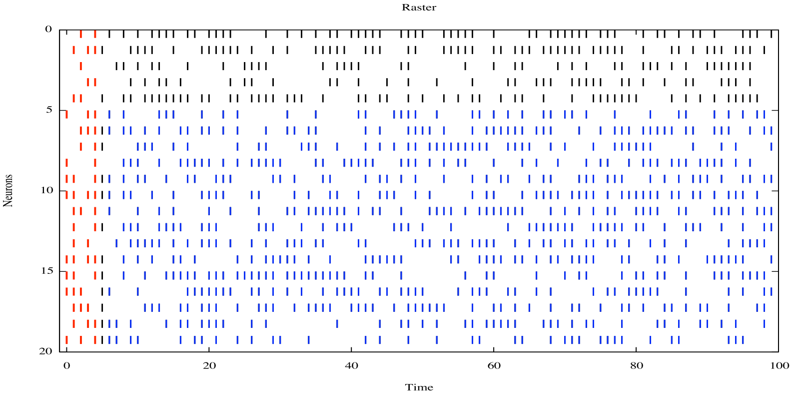


Figure: N = 5, T = 100, D = 5, Iext = 0.6, $\gamma = 0.95$

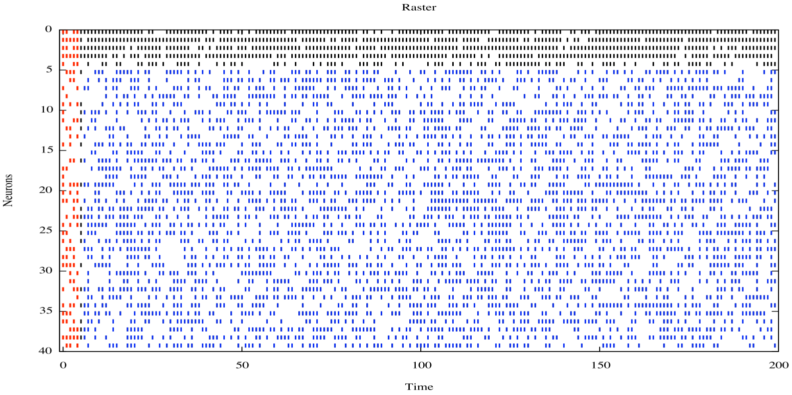


Figure: N = 5, T = 200, D = 5, Iext = 0.6, $\gamma = 0.95$

Results from spiking activity in monkey cortex during movement preparation(courtesy of Alexa Riehle et al.)

slideConclusions and Perspectives

* Considering a deterministic time-discretized spiking network of neurons with connection weights having delays, we have been able to investigate in details to which extend it is possible to back-engineer the networks parameters, i.e., the connection weights.
* The method proposes here can produce any rasters produced by more realistic models such Hodgkin-Huxley.
* We have an useful tool for match raster using Linear Solver and Linear Programming Software. ENAS.

- optimal number of hidden units.
- approximate raster matching.
- application to unsupervised or reinforcement learning.