Analyzing the neural Code using Gibbs Distributions

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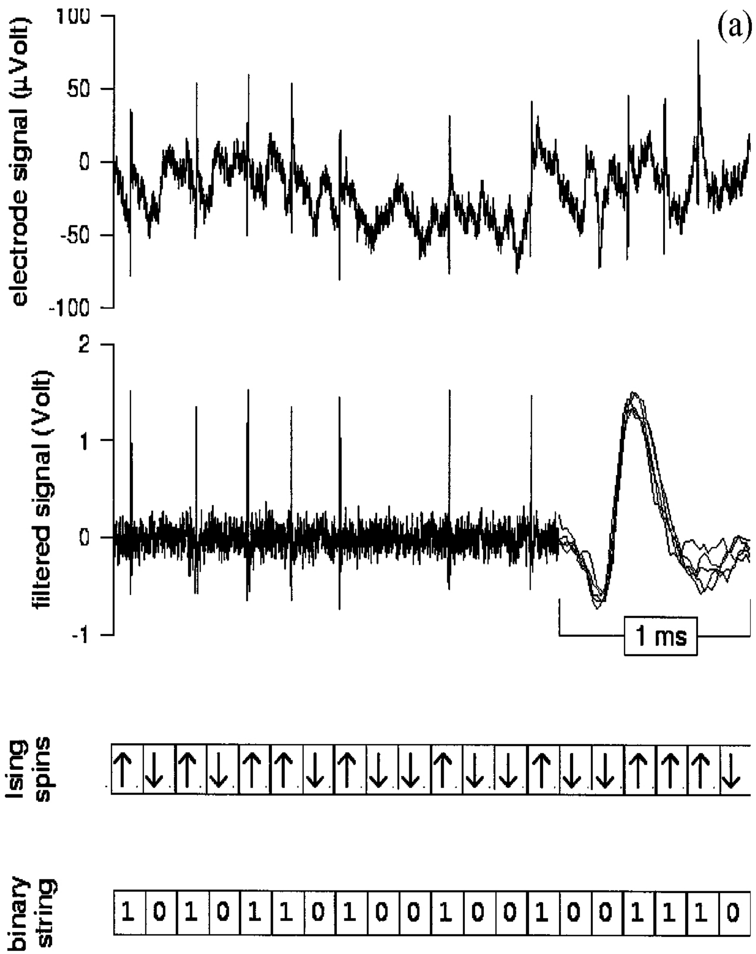
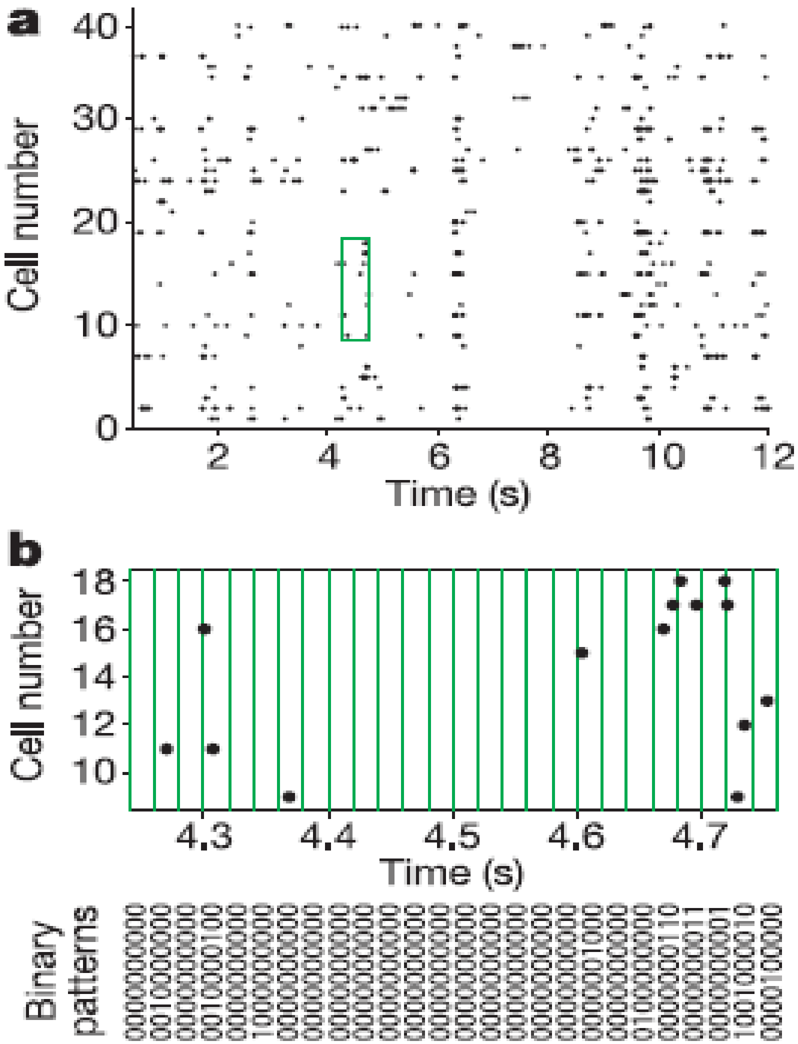
Introduction to neural activity



- Action Potentials or Spikes are the basic process at neuron scale.
- Most of processing or communication is spike-based.
- But Neural activity/response are variable so

What is the underlying neural code? Not a single answer!!

Introduction to Neural code



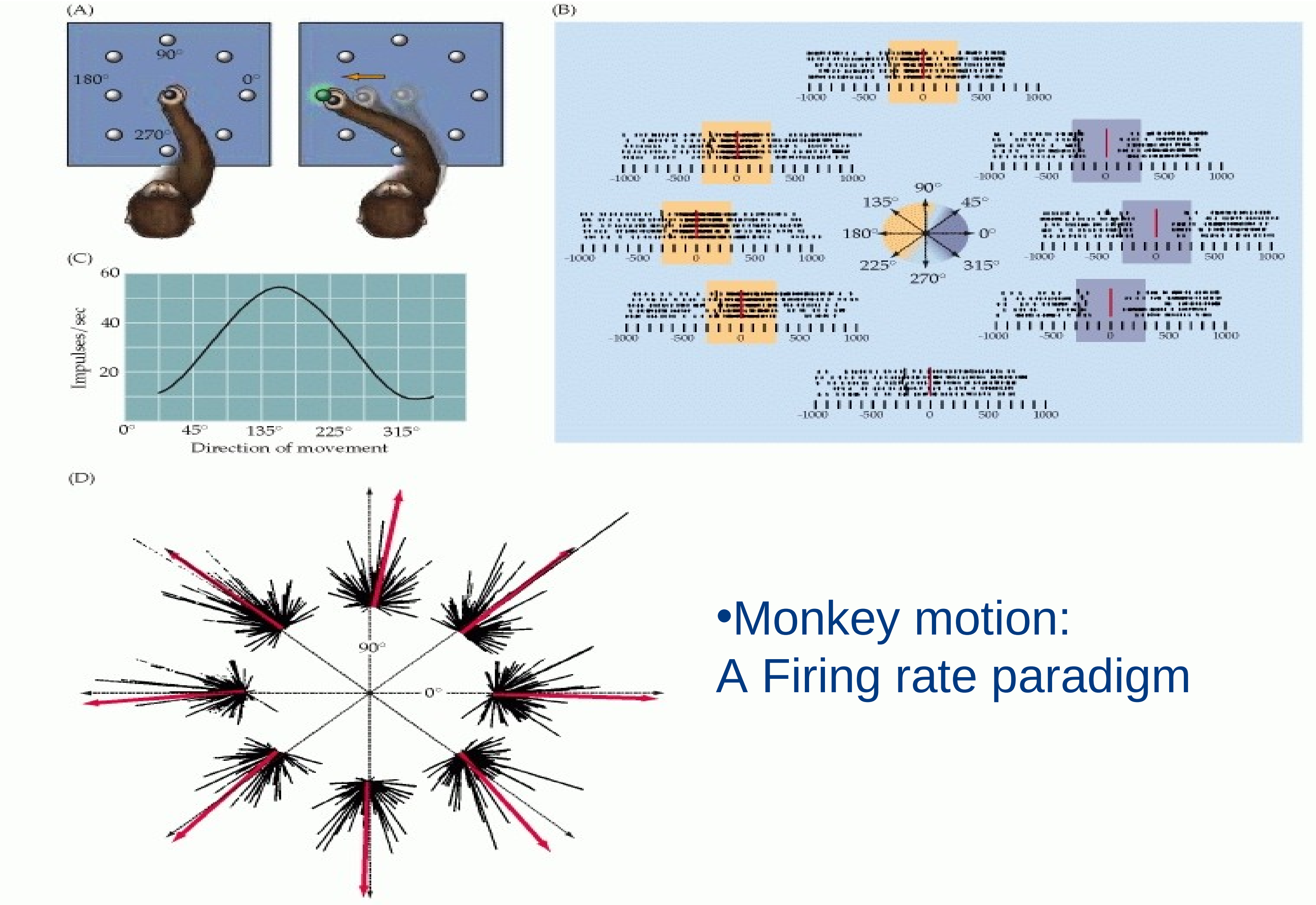
- Statistical characterization of spike trains

$$\Updownarrow$$

$$P\left[R|S\right]$$

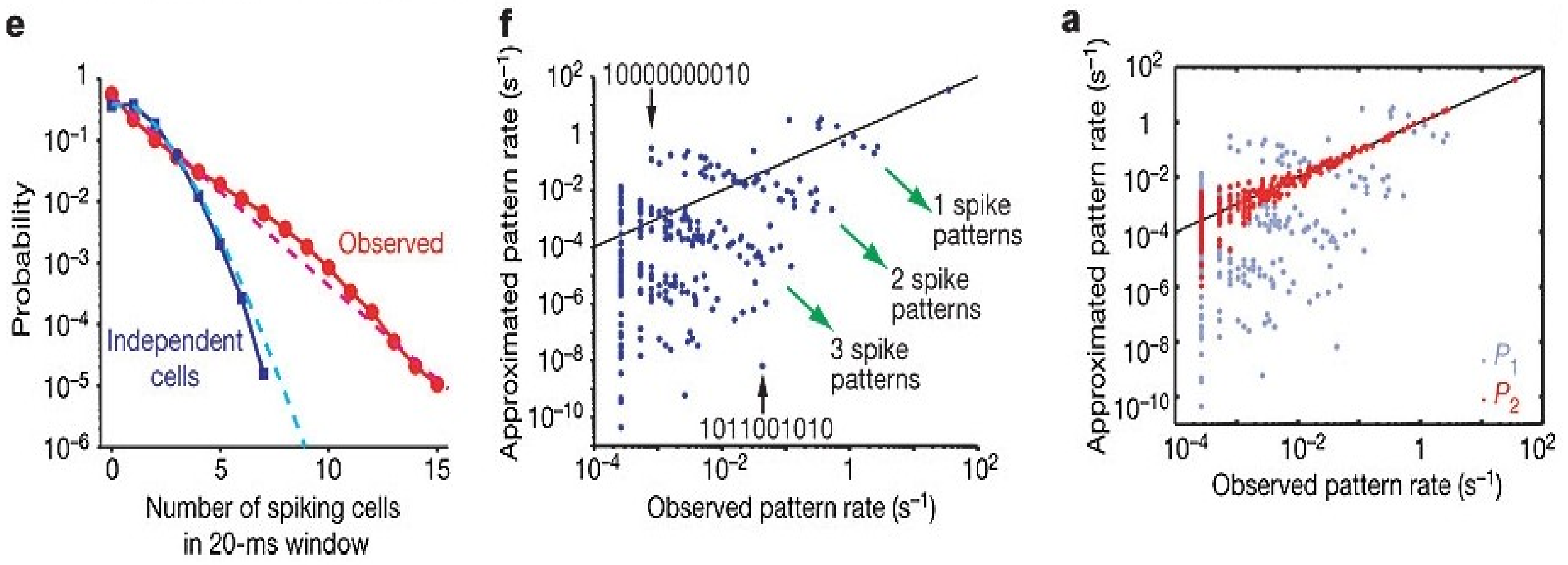
$$P\left[R|S\right] \Longrightarrow P\left[S|R\right]$$

Examples of Neural code(I)



•Monkey motion:
A Firing rate paradigm

Examples of Neural code(II)



- Retinal and cortical “small” networks : A pairwise paradigm
- Strong system correlations as a result of weak pairwise correlations.

Setting the start point

Position of the problem

- To characterize the statistical properties of sequences of spike trains produced by a neural networks.
- What are the effects of synaptic plasticity at this sight?

Select an study case: (Neuron/Network/plasticity)

- ✓ Fully connected Network, Beslon-Mazet-Soula Model
- ✓ Spike-Time Dependent Plasticity (STDP)

Neuron Dynamics basic models

- Generalized Integrate and Fire-Model

$$C\frac{dV_k}{dt} + g_k V_k = i_k$$ + "Reset" phase

$$g_k(t,\tilde{\omega}) = g_L + \sum_{j=1}^{N} G_{kj} \sum_{n=1}^{M_j(t,\tilde{\omega})} \alpha(t - t_j^n)$$

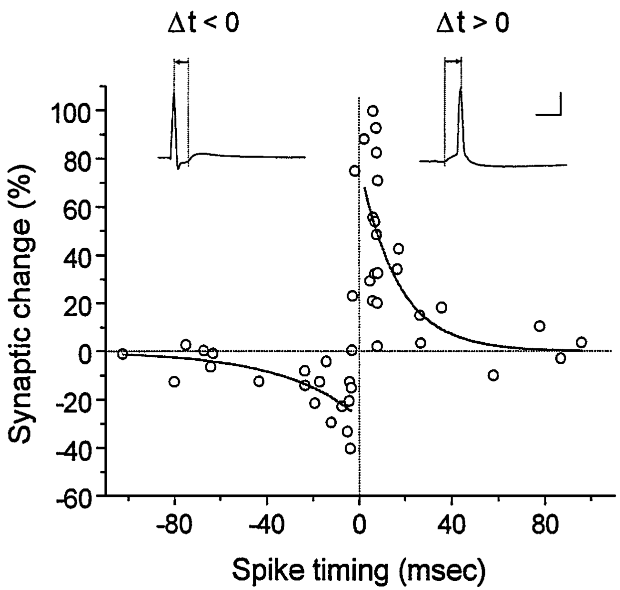
- Discrete time + assumptions= BMS Model

$$\mathbf{F}_{\gamma,i}(\mathbf{V}) = \rho V_i \left(1 - Z[V_i]\right) + \sum_{j=1}^{N} W_{ij} Z[V_j] + I_i^{ext}; \qquad i = 1 \ldots N,$$

$$Z[V_i(t)] = \begin{cases} 0 & \text{if} \quad V_i(t) < \theta \\ 1 & \text{if} \quad V_i(t) \geq \theta \end{cases}$$

Neural Spike Time Synaptic Plasticity

$$f(x) = \begin{cases} A_- e^{\frac{x}{\tau_-}}, \ x < 0; \\ A_+ e^{-\frac{x}{\tau_+}}, \ x > 0; \\ 0, \ x = 0; \end{cases}$$



Usually protocol/organism dependent

$$\delta W_{ij}^{(\tau)} = \epsilon \left[r_d W_{ij}^{(\tau)} + \frac{1}{T} \sum_{t=T_s}^{T+T_s} \omega_j^{(\tau)}(t) \sum_{u=-T_s}^{T_s} f(u)\, \omega_i^{(\tau)}(t+u) \right]$$

"Offline": An epoch has fixed connections during a transient time and a computation time T.

$-1 < r_d < 0$, corresponding to passive LTD. $\quad T_s \stackrel{\text{def}}{=} 2 \max(\tau_+, \tau_-)$.

Base for Modeling ideas

- Infinite number of Candidates for a probability distribution that agree with finite given data observables (firing rate, correlations etc.)

⇒ Entropy : randomness or lack of interactions among variables...

- (Jaynes 1957) :Minimally structured distribution (consistent with given data observables)= Maximum Entropy distribution.

Moreover : It exist an energy function for the system.

Crash introduction to theory(I)

$$p_i = \frac{1}{Z} e^{-E_i/(kT)} = e^{-(E_i - A)/(kT)}$$

- Boltzmann distribution (yields to a Poisson distrib.)

$$P_2(\sigma_1, \sigma_2, \ldots, \sigma_N) = \frac{1}{Z} \exp\left[\sum_i h_i \sigma_i + \frac{1}{2} \sum_{i \neq j} J_{ij} \sigma_i \sigma_j\right]$$

(•Ising Model distribution (used for retina pairwise paradigm)

$$p(\mathrm{x}) = \frac{1}{\sum_{\mathrm{x} \in \mathcal{X}} \exp\left(\sum_{i=1}^{k} \lambda_i f_i(\mathrm{x})\right)} \cdot \exp\left(\sum_{i=1}^{k} \lambda_i f_i(\mathrm{x})\right)$$

•Generally it is a Lagrange multipliers problem

•Gibbs Distribution(Ergodic Theory)

Crash introduction to theory (II)

Data

$$R = [\omega(1) \ldots \omega(\tau)]$$

$$\omega(\mathbf{t}) = [\omega_{\mathbf{i}}(\mathbf{t})]_{\mathbf{i}=1}^{\mathbf{N}}$$

Empirical Averaging

$$\pi_{\tilde{\omega}}^{(T)}(\phi) = \frac{1}{T} \sum_{t=1}^{T} \phi(\sigma_\gamma^t \tilde{\omega})$$

Example of Observables

$$\phi_l(\tilde{\omega}) = \omega_l(0), \text{ then } \pi_{\tilde{\omega}}^{(T)}(\phi_l) = r_l$$

$$\phi_l(\tilde{\omega}) \equiv \phi_{(i,j)}(\tilde{\omega}) = \omega_i(0)\omega_j(\tau)$$

Topological Pressure (Maximum Entropy Estimator)

$$P\left[\vec{\lambda}\right] = \sup_{\mu \in M_{inv}} \left\{ h(\mu) + \sum_{\alpha=1}^{K} \lambda_\alpha \mu(\phi_\alpha) \right\}$$

$$P[\lambda^*] = \lim_{n\to\infty} \frac{1}{n} \log\left(Z_n[\lambda^*]\right).$$

$$\frac{\partial P}{\partial \lambda_l}[\lambda^*] = \nu_{\lambda^*}(\phi_l) = C_l.$$

Probability (Gibbs) Distribution

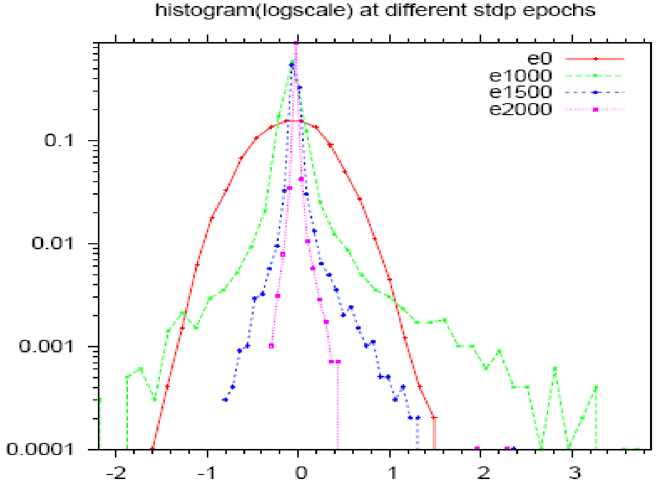
$$\nu[R] \sim \frac{1}{Z_n(\vec{\lambda})} \sum_{\tilde{\omega} \subset [R]} \exp\left[\sum_{\alpha=1}^{K} \lambda_\alpha \sum_{t=1}^{n} \phi_\alpha(\sigma^t \tilde{\omega}) \right]$$

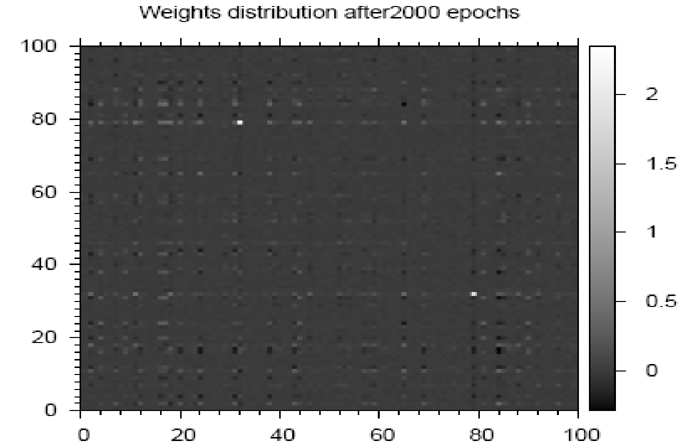
Gibbs Dist. evolution (e.g STDP)

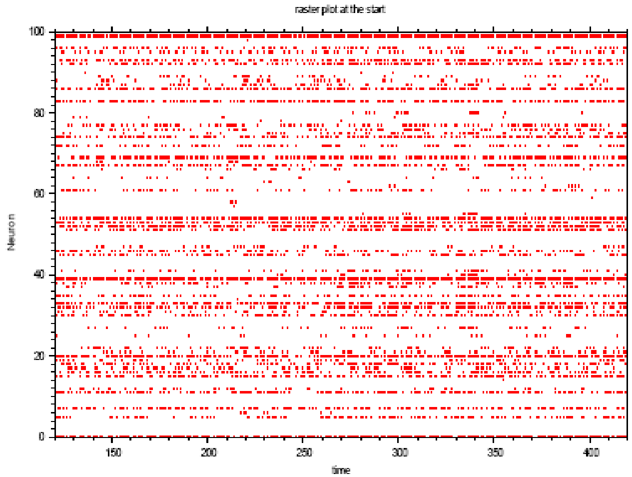
$$\mathcal{F}_l(u) = P[\lambda^* + u e_l] - P[\lambda^*],$$

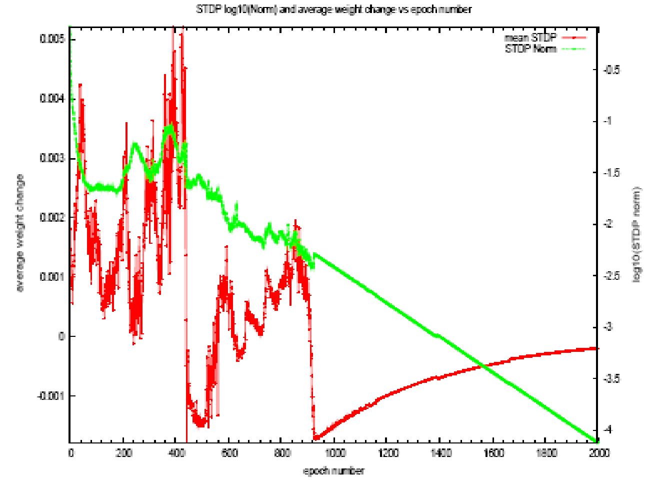
$$\delta W^{(\tau)} = \epsilon \nabla_{W = W^{(\tau)}} \mathcal{F}_\phi^{(\tau)}(W).$$

Numerical results N=100

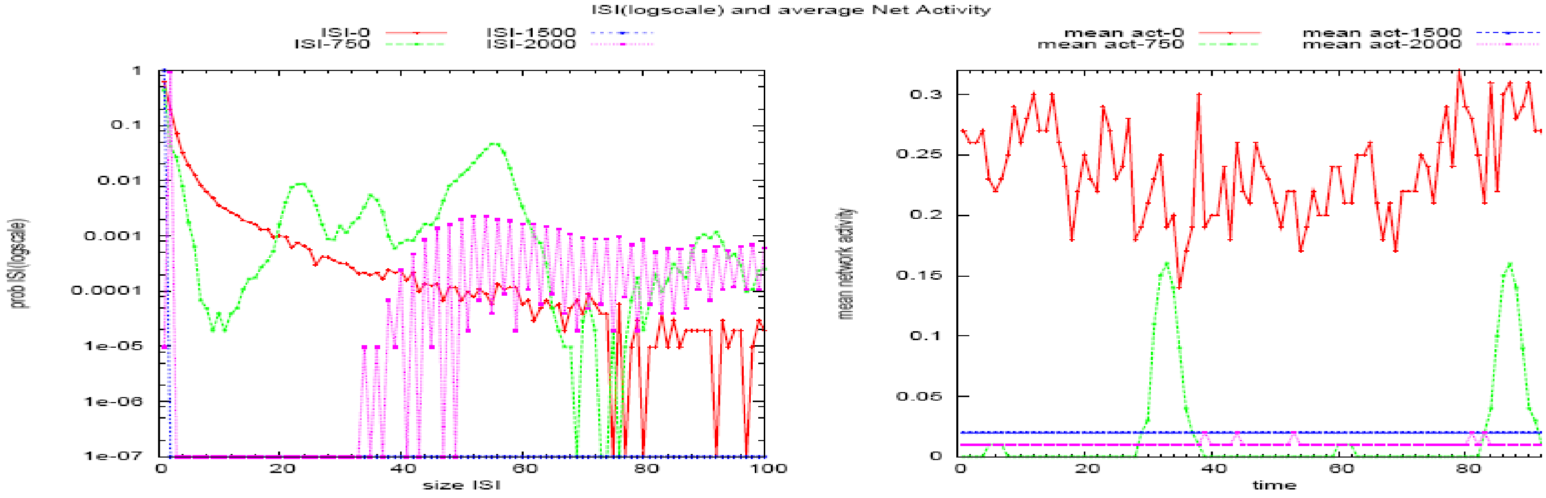


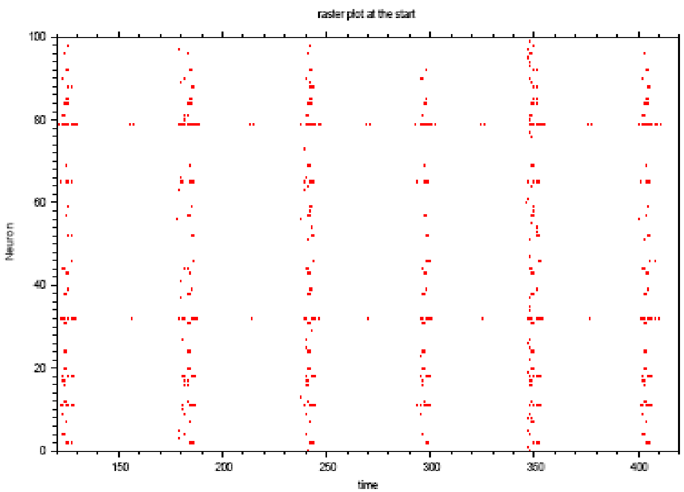


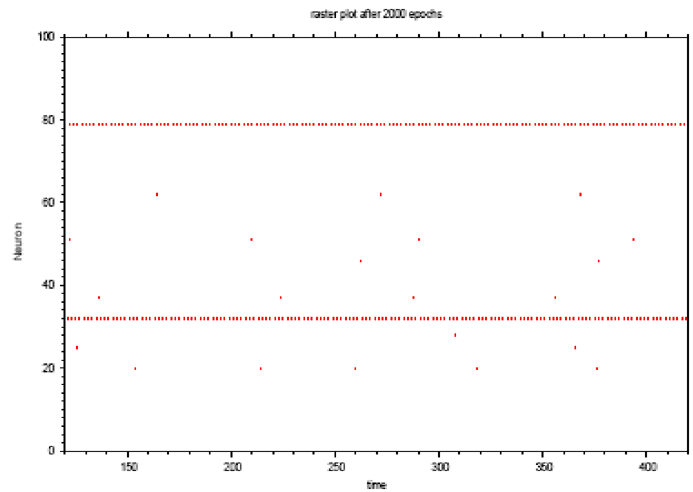




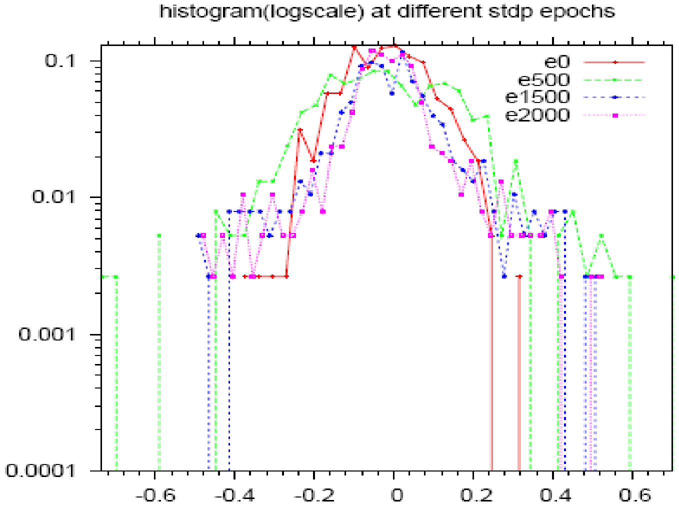
Numerical results N=100

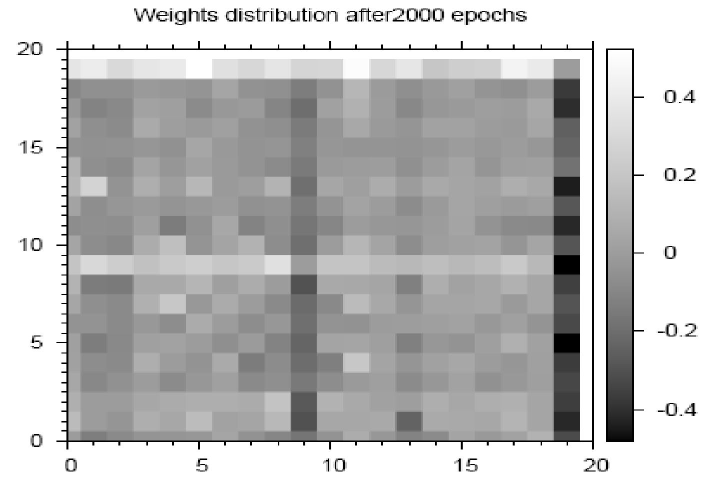


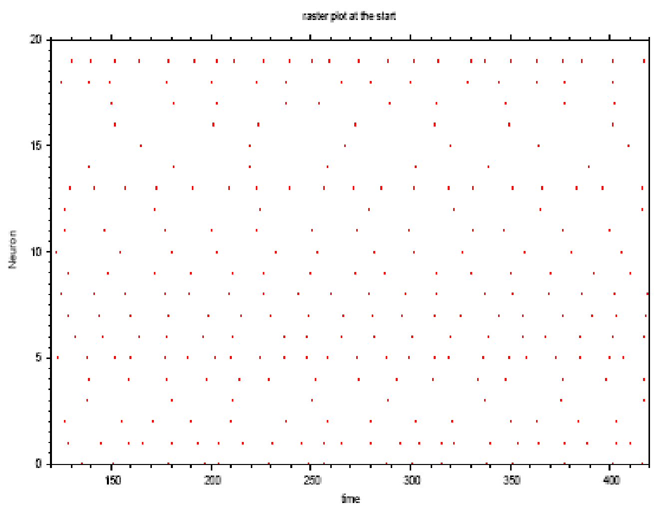


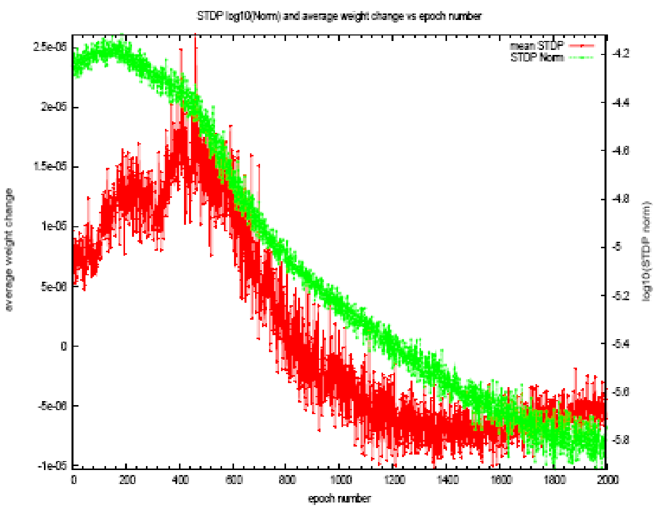


Numerical results N=20

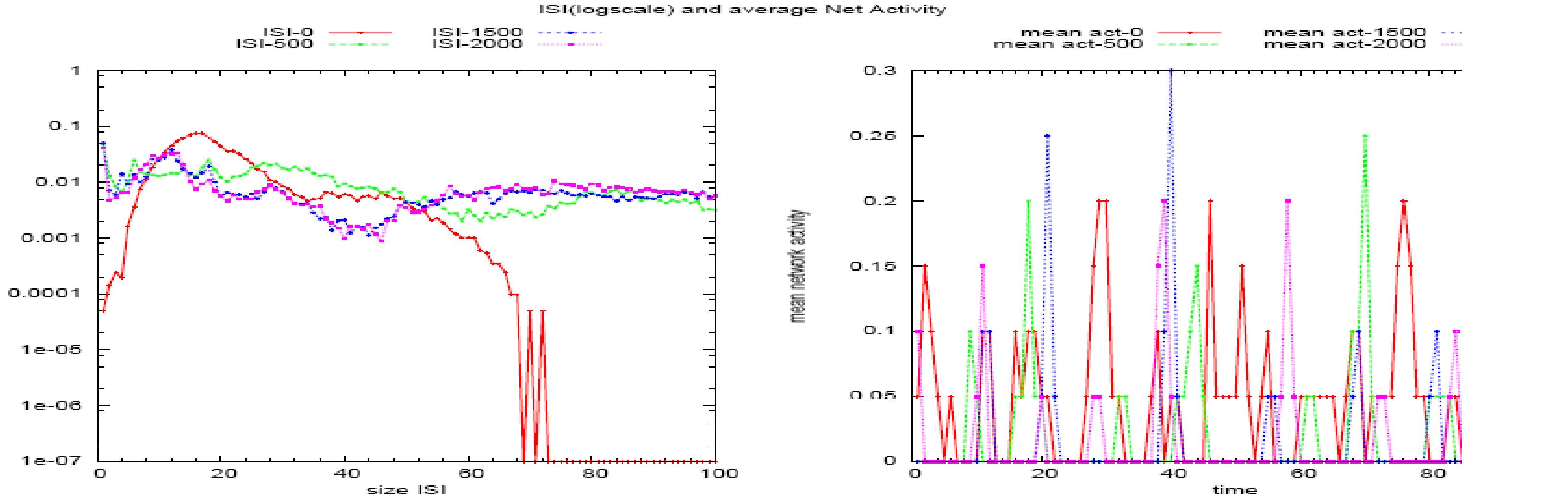


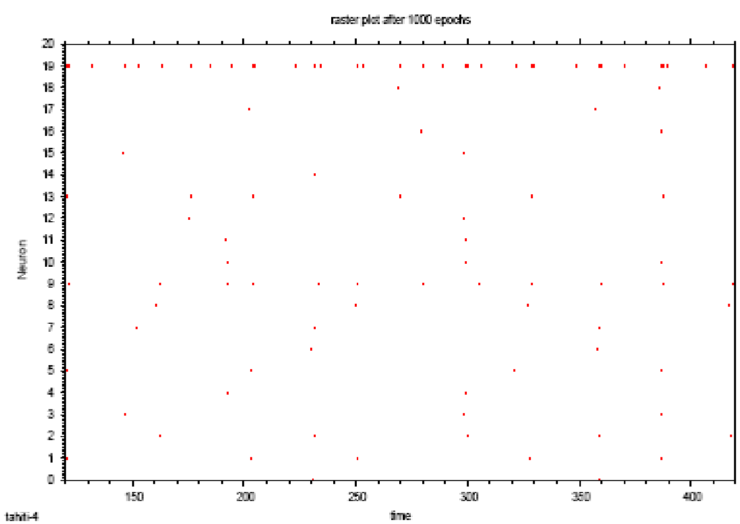


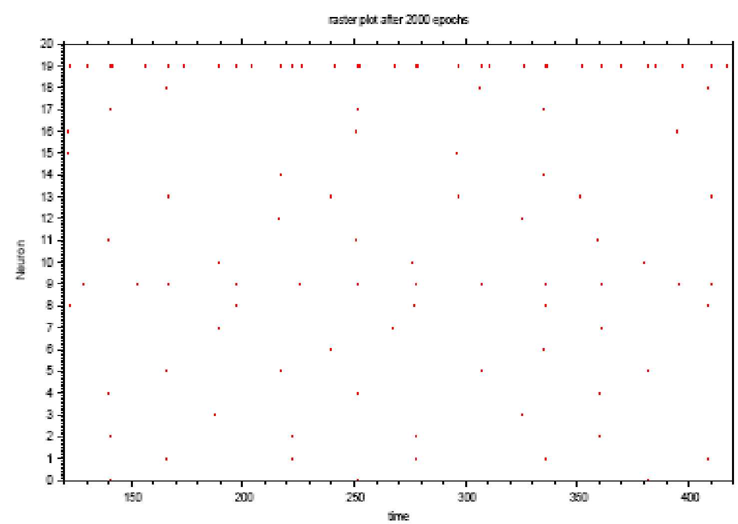




Numerical results N=20







Future/Ongoing work

- NEURAL MODELING: Comparison between statistical models of different time order, specially with respect to time-zero order models (rates, Ising models) on both simulated neural networks/experimental data.

- LEARNING: After STDP, is the final distribution a Gibbs distribution? Which is its Potential (order of intrinsic interactions)?

- CONTROL: How to control the probability distribution/spike statistics by using STDP in not trivial cases?

- CONTROL (naive approach): continue to analyze the effects on dynamics and statistical properties by other plasticity/dynamical rules (e.g. Dale's principle)