How Gibbs Distributions may naturally arise from synaptic adaptation mechanisms

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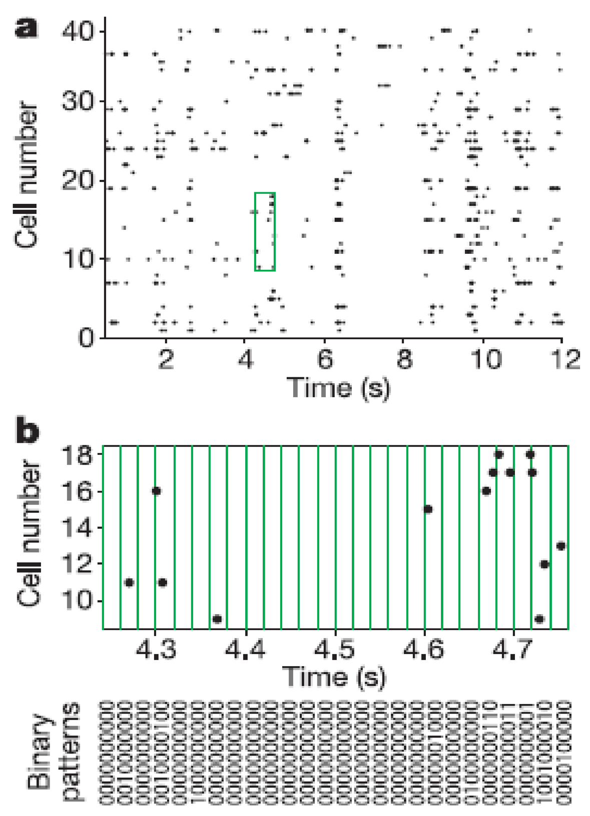
We use the thermodynamic formalism from ergodic theory to show how Gibbs distributions are natural probability measures to describe the statistics of spike trains, given the data of known empirical averages. This framework allows us to characterize miscellaneous forms of neural code (firing rates, synchronizations, correlations of different orders etc). We also show that Gibbs distributions naturally arise when considering slow synaptic plasticity rules (i.e. when the characteristic time for synapse adaptation is longer than for neuron dynamics). We include some simulations results applying this framework on recurrent neural networks with discrete time current based dynamics under the action of an STDP rule. Numerical results are in agreement with our theory establishing that the Topological pressure is a variation quantity with a minimum given by a Gibbs distribution.

Introduction



- Neural activity/response are variable:
  What is the underlying neural code?
  Not a single answer!
  (Riehle et al. 2008, Scheneidman et al. 2006)

$$P[R|S] \Longrightarrow P[S|R]$$

Modeling and Theory

- Infinite number of Candidates for a probability distribution that agrees with a finite set of data observables (rates, correlations etc).
- Entropy : degree of indeterminacy in a spikes sequence given its past.
- (Jaynes 1957) : Minimally structured distribution (consistent with given data observables)= Maximum Entropy distribution.

Examples

$$p_i = \frac{1}{Z} e^{-E_i/(kT)} = e^{-(E_i - A)/(kT)}$$

- Boltzmann distribution (yields to a Bernouilli distrib.)

$$P_2(\sigma_1, \sigma_2, \dots, \sigma_N) = \frac{1}{Z} \exp\left[\sum_i h_i \sigma_i + \frac{1}{2}\sum_{i \neq j} J_{ij}\sigma_i\sigma_j\right]$$

- Ising Model used as a statistical model to analyse spike train from retina

Crash introduction to theory

Data

$$R = [\omega(1) \dots \omega(\tau)]$$
$$\omega(t) = [\omega_i(t)]_{i=1}^{N}$$

Empirical Averaging

$$\pi_{\tilde\omega}^{(T)}(\phi) = \frac{1}{T}\sum_{t=1}^{T} \phi(\sigma_\gamma^t \tilde\omega)$$

Example of Observables

$$\phi_l(\tilde\omega) = \omega_l(0), \text{ then } \pi_{\tilde\omega}^{(T)}(\phi_l) = r_l$$
$$\phi_l(\tilde\omega) \equiv \phi_{(i,j)}(\tilde\omega) = \omega_i(0)\omega_j(\tau)$$

Topological Pressure (Maximum Entropy Estimator)

$$P[\lambda] = \sup_{\mu \in M_{inv}} \left\{ h(\mu) + \sum_{a=1}^{K} \lambda_a \mu(\phi_a) \right\}$$
$$P[\lambda^*] = \lim_{n\to\infty} \frac{1}{n} \log(Z_n[\lambda^*]).$$
$$\frac{\partial P}{\partial \lambda_l}[\lambda^*] = \nu_{\lambda^*}(\phi_l) = C_l.$$

Probability (Gibbs) Distribution

$$\nu[R] \sim \frac{1}{Z_n(\lambda)} \sum_{\tilde\omega \subset [R]} \exp\left[\sum_{a=1}^{K} \lambda_a \sum_{t=1}^{n} \phi_a(\sigma^t \tilde\omega)\right]$$

Gibbs Dist. evolution (e.g STDP)

$$F_l(u) = P[\lambda^* + u e_l] - P[\lambda^*],$$
$$\delta W^{(\tau)} = \epsilon \nabla_{W = W^{(\tau)}} F_\phi^{(\tau)}(W).$$

Simulation Methods

C++ libraries in enas.gforge.inria.fr

Neuron model

Integrate-and-Fire

$$V(t) = \begin{cases} \tau_m \frac{dV}{dt} = -V(t) + R_m I(t) \\ V(t+\epsilon) = V_{reset}, \quad \text{if } V(t) \geq \theta \end{cases}$$

Beslon-Mazet-Soula (2006)

$$V(t+1) \equiv F[V(t)] = \begin{cases} \gamma V(t) + I(t) \\ V_{reset} + I(t), \quad \text{if } V(t) \geq \theta \end{cases}$$

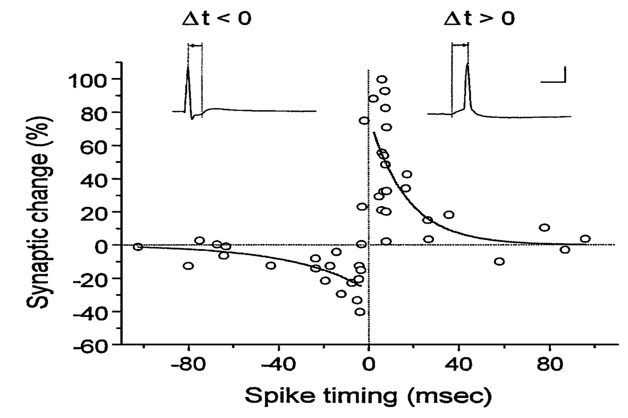
Network model

Random Weights (Gaussian distribution)

$$V_i(t+1) = F_i(V(t)) = \gamma V_i(t)(1 - Z[V_i(t)]) + \sum_{j=1}^{N} W_{ij} Z[V_j(t)] + I_i^{ext}(t) \quad \text{where } Z[x] = 1 \text{ if } x \geq \theta \text{ or } 0 \text{ otherwise}$$

STDP model

$$f(x) = \begin{cases} A_- e^{\frac{x}{\tau_-}}, & x < 0; \\ A_+ e^{-\frac{x}{\tau_+}}, & x > 0; \\ 0, & x = 0; \end{cases}$$



$$\delta W_{ij}^{(\tau)} = \epsilon \left[r_d W_{ij}^{(\tau)} + \frac{1}{T} \sum_{t=T_s}^{T+T_s} \omega_j^{(\tau)}(t) \sum_{u=-T_s}^{T_s} f(u)\, \omega_i^{(\tau)}(t+u) \right]$$

$-1 < r_d < 0$, corresponding to passive LTD. $T_s \stackrel{\text{def}}{=} 2\max(\tau_+, \tau_-)$.

Simulation Results

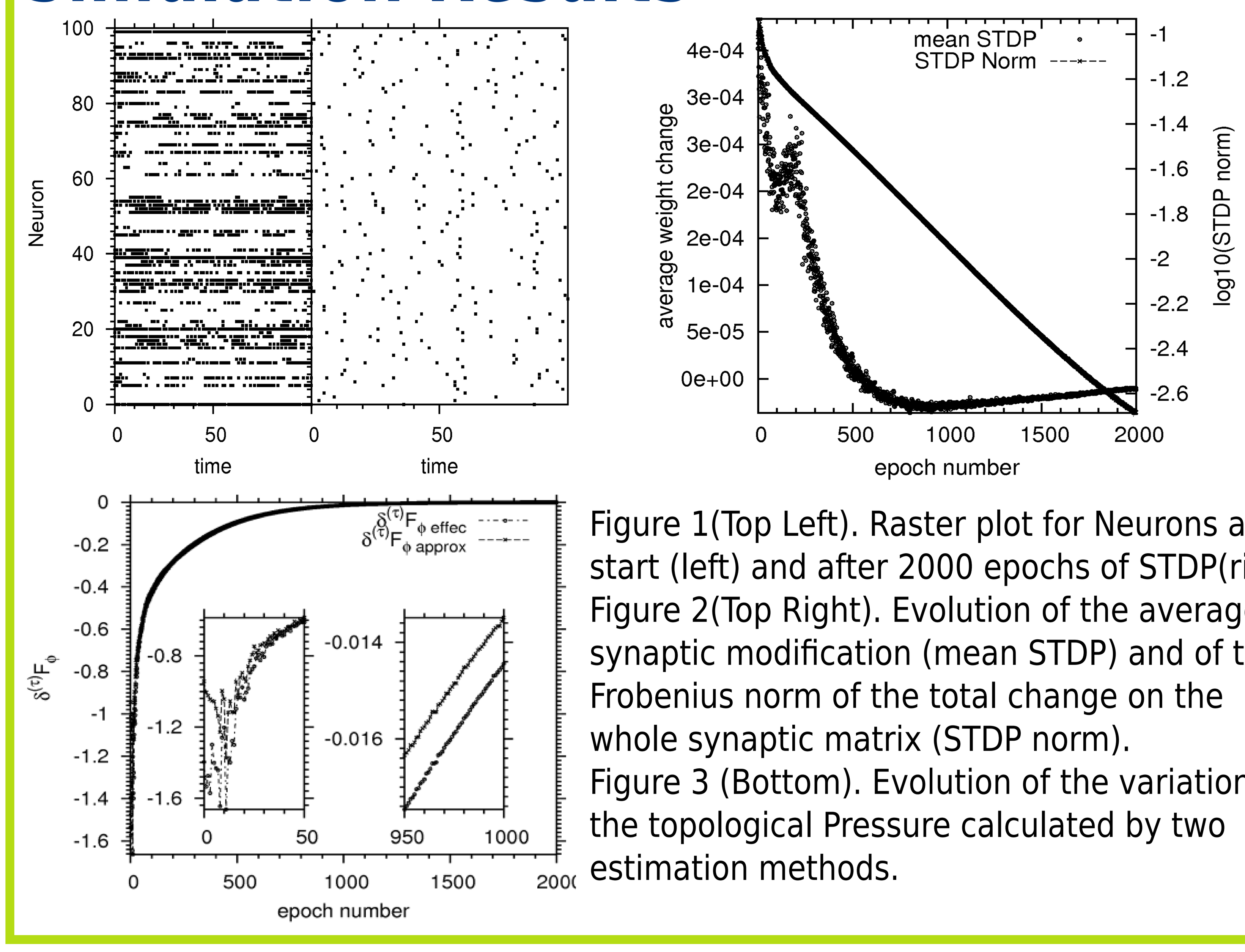


Figure 1(Top Left). Raster plot for Neurons at start (left) and after 2000 epochs of STDP(right).
Figure 2(Top Right). Evolution of the average synaptic modification (mean STDP) and of the Frobenius norm of the total change on the whole synaptic matrix (STDP norm).
Figure 3 (Bottom). Evolution of the variation of the topological Pressure calculated by two estimation methods.

Conclusions

- Our framework allows us to formally express arbitrary observables and then to compare between statistical models including those with non-zero time order interactions.
- The variation of the topological pressure of the simulated system decreases along STDP process.
- Slow plasticity rules can be expressed as gradient systems of the variation of the topological pressure.

Perspectives

- MODELING: Comparison between statistical models of different time order, especially with respect to time-zero order models (rates, Ising) on both simulated neural networks/experimental data.
- LEARNING: After STDP, is the final distribution a Gibbs distribution? Which is its Potential (order of intrinsic interactions)?
- CONTROL: How to control the probability distribution/spike statistics by using STDP in not trivial cases ? What can be the effects on dynamics and statistical properties when using other plasticity/ dynamical rules (e.g. Dale's principle).?

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Bibliography

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>B. Cessac et al. , *CNS09 Poster 113-*