Back-engineering in spiking neural networks parameters

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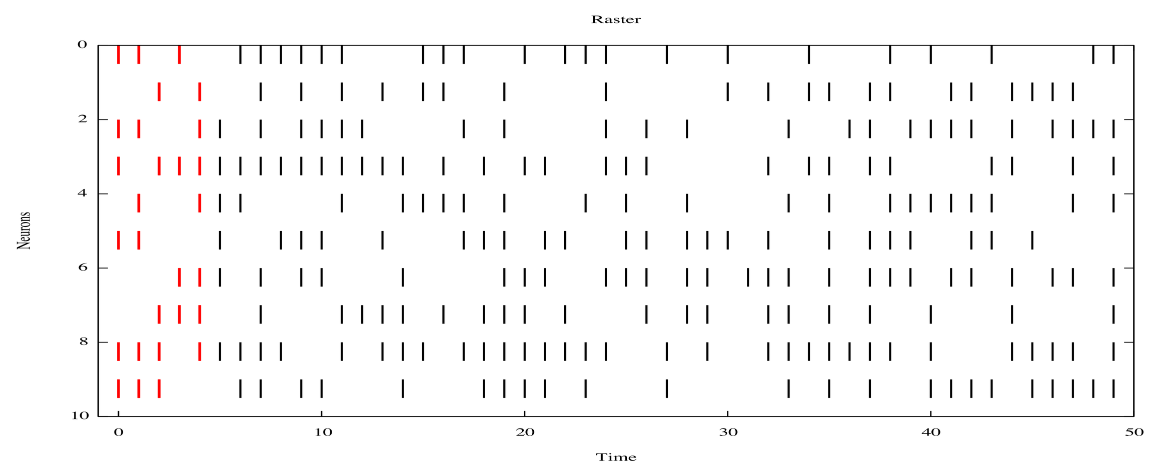
We consider the deterministic evolution of a time-discretized spiking network of neurons with connections weights having delays, modeled as a discretized neural network of the generalizaed integrate and fire (gIF) type. The purpose is to study a class of algorithmic methods allowing to calculate the proper parameters to reproduce exactly a given spike train generated by an hidden (unknown) neural network.

Problem position

Given a spiking neural network, to which extends observing the spike raster allows to infer the network parameters (delayed Weights)?



Problem formulation

We consider a discrete time model of spiking neurons deduced from the LIF model. (**Cessac, 2008**)

$$V_i[k] = \gamma(1 - Z_i[k])V_i[k-1] + \sum_{j=1}^{N}\sum_{d=1}^{D} W_{ijd} Z_j[k - d_{ij}] + I_i^{ext}$$

$$V_i[k] = \sum_{j=1}^{N}\sum_{d=1}^{D} W_{ijd} \sum_{\tau=\tau_{jk}}^{0} \gamma^\tau Z_j[k - \tau - d_{ij}] + I_i^{ext} \qquad (1)$$

$$Z_i[k] = (V < \theta \quad ? \quad 0 : 1)$$

Where:

$d_{ij} \in \{1...D\} \rightarrow$ delays

$W_{ijd} \in R \rightarrow$ synaptic weights

$\gamma \in [0, 1[\rightarrow$ leak rate

$I_i^{ext} \rightarrow$ external current

$V \rightarrow$ membrane potential

$N > 0 \rightarrow$ dimension of the NN

$$Z(x) = \chi(x \geq \theta) \rightarrow \text{indicatrix function}$$

Initial Conditions

$$V_i[0] = 0 \qquad Z_i[k], k \in \{1..D\}$$

Methods

- The L problem

From (1) we can deduce a Linear problem where the easier solution consists in the Singular Value Decomposition. (V is known)

$$\mathbf{A}_i \mathbf{w}_i = \mathbf{b}_i$$

$$\mathbf{A}_i = \begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \sum_{\tau=\tau_{jt}}^{0} \gamma^\tau Z_j(k-\tau-d_{ij}) & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \in \mathcal{R}^{T-D \times N}$$

$$\mathbf{w}_i = (\cdots W_{ijd} \cdots)^T \in \mathcal{R}^N$$

$$\mathbf{b}_i = \left(\cdots V_i[k] - I_i^{ext} \cdots\right)^T \in \mathcal{R}^{T-D}$$

- The LP problem

From (1) we can also deduce a Linear Programming problem where several solution methods have been proposed, in this case we are using the simplex method. (V is not known)

$$Z_i[k] = 0 \Rightarrow V_i[k] < 1 \quad \text{and} \quad Z_i[k] = 1 \Rightarrow V_i[k] > 1$$

$$(2Z_i[k] - 1)(V_i[k] - 1) > 0$$

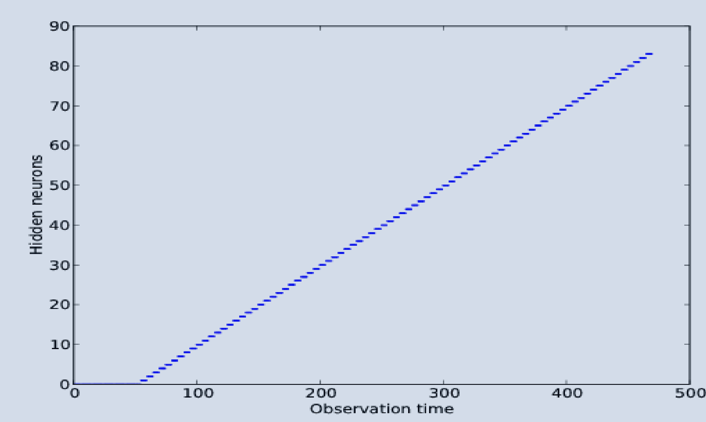
$$\mathbf{A}_i \mathbf{w}_i + \mathbf{b}_i > 0$$

NOTE: is important to know that in order to have an exact matching we need to be sure that the observation time is enough small i.e., $T < O(ND)$.

- Solving the constraint $T >> O(ND)$

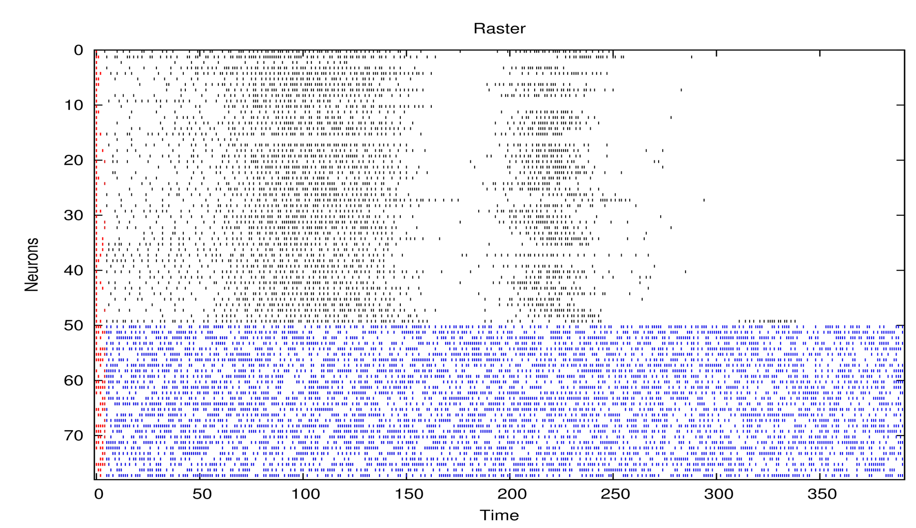
In order to control the relashionship between the observation time (T) and the number of neurons (N) and the initial condtions (D), we propose to include a hidden neurons (Nh) in order to compensate the number of neurons necessary to match any raster.

$$Nh = \frac{T}{D} - N$$



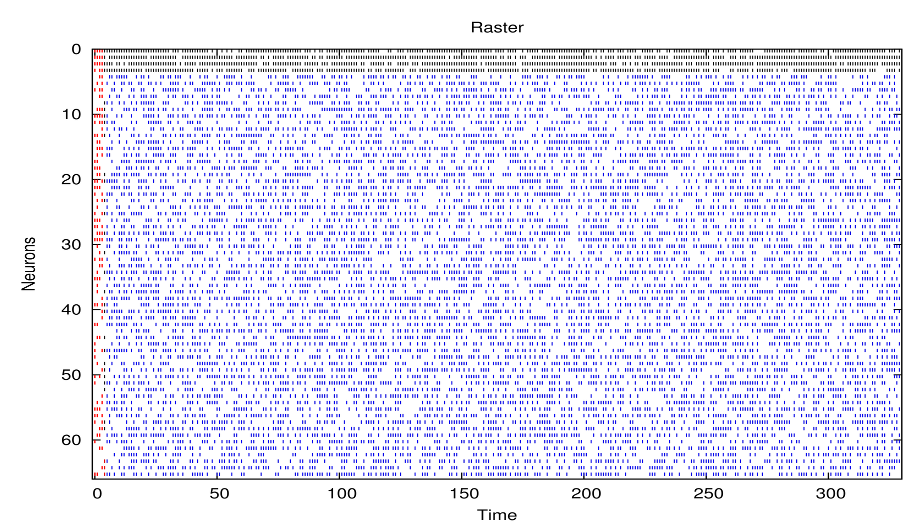
Results

Exact raster reproduction on artificial and biological data with the estimated weights.



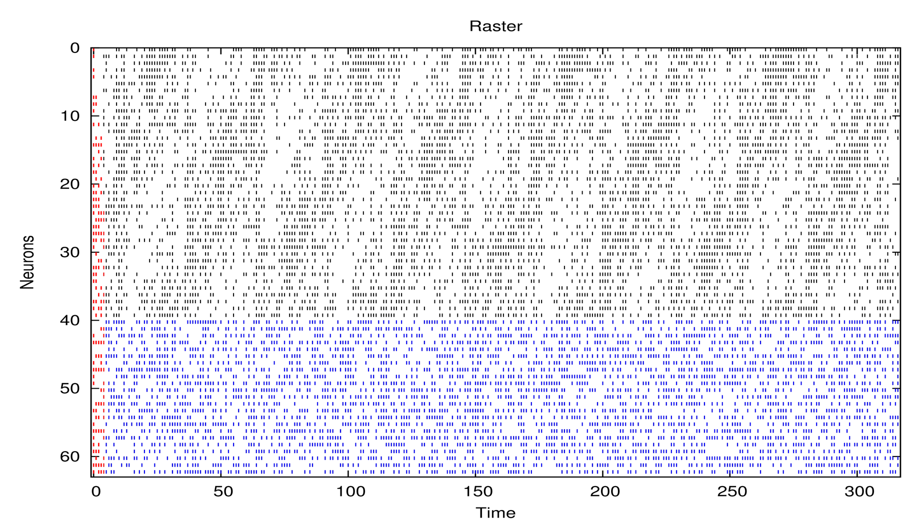
Biological Data

Spiking activity in monkey cortex during movement preparation. (Courtesy of Alexa Riehle et al. 2000)



Artificial Data

Spike-trains generated with a given statistical parameters and maximal entropy (Gibbs distribution with N = 4, T = 200, R = 5, L = 9).



Biological Data

Multicell-recording in retina (Courtesy of M. Berry)

About the figures the red lines correspond to initial conditions (D = 5), the **black ones** represent the input to be calculated (N) and finally the blue ones correspond to hidden neurons.

C++ libraries in enas.gforge.inria.fr

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