Multi-anticipative car-following behaviour: macroscopic modeling

G. Costeseque[1,2] and J-P. Lebacque[2]

[1] Université Paris-Est/Ecole des Ponts ParisTech/CERMICS, France
costeseg@cermics.enpc.fr
[2] Université Paris-Est/IFSTTAR/GRETTIA, France

Abstract. In this work we will deal with a macroscopic model of multi-anticipative car-following behaviour i.e. driving behaviour taking into account several vehicles ahead. Some empirical studies have suggested that drivers not only react to the closest leader vehicle but also anticipate on traffic conditions further ahead. Using a recent mathematical result of homogenization for a general class of car-following models (and also available for multi-anticipative models), we will deeply investigate the effects of multi-anticipation at the microscopic level on the macroscopic traffic flow. To investigate multi-anticipation behaviour may be fundamental to understand better cooperative traffic flow dynamics.

1 Introduction

1.1 Motivation

Our motivation comes from the sky-rocketing development of new technologies in transportation leading to the multiplication of Intelligent Transportation Systems (ITS). More precisely we would like to assess the impact of cooperative systems on general traffic flow. Cooperative systems include vehicle-to-vehicle (V2V) or vehicle-to-infrastructure (V2I) communications, generally designated as V2X technologies. There is a fast growing literature about cooperative systems. The interested reader can refer for instance to [14] and references therein.

The fact that the observed headway between two consecutive vehicles is often strictly less than the reaction time of the drivers, suggests that drivers anticipate on more than one leader. Indeed, if not, the proportion of accidents should be dramatically increased. The multi-anticipation has been shown as a key element for the stabilization of traffic flow, above all in dense traffic situations.

However multi-anticipation behaviour has only been taken into consideration at a microscopic scale. Indeed such macroscopic models as the Payne-Whitham one only account for anticipation in time. The macroscopic issues encompass multi-lane traffic with lane changing and assignment but also multi-anticipation on each lane or the combination of those both processes. In classical approaches, the whole traffic is projected on a single line to simplify the problem in a one-dimensional framework.

1.2 Notations

Let x denotes the position and $t > 0$ the time. $x_i(t)$ refers to the trajectory of the vehicle $i \in \mathbb{Z}$. The speed and the acceleration of vehicle i are described by the first and second derivative of x_i w.r.t. time. Notice that we also introduce a time delay $T \geq 0$.

We assume that the vehicles are labelled according to a snapshot of a section of road from upstream to downstream (see Figure 1). Vehicle labels increase with x. Thus $(x_{i+1} - x_i)(t)$ is the spacing and $(\dot{x}_{i+1} - \dot{x}_i)(t)$ the relative speed at time t between vehicle i and its leader $(i+1)$. We also denote by $m \geq 1$ the total number of leaders that are considered by vehicle $i \in \mathbb{N}$.

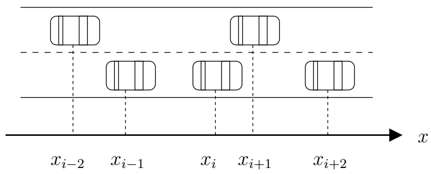

Fig. 1. Notations for the microscopic car-following models

At the macroscopic level, we denote respectively the density and the flow of vehicles at location x and time t as $\rho(x,t)$ and $Q(x,t)$.

1.3 Main results and organization of the paper

As a main result we describe a new macroscopic model that encompass multi-anticipative car-following behaviour that are classically taken into account only at the microscopic scale. Moreover our model is able to consider multi-lane dynamics.

The rest of the paper is organized as follows: we first recall some existing multi-anticipative car-following models in Section 2. We particularly highlight what we think to be the seminal form of such a model. In Section 3, we describe the formal mathematical result that allows us to pass from microscopic models to equivalent macroscopic ones. Our macro model is also described in Section 4. Finally we provide some numerical considerations in Section 5 before to discuss the results and to conclude.

2 Multi-anticipative traffic modelling: an introduction

Multi-anticipative models. The main existing multi-anticipative car-following models come from adaptations of classical car-following ones with a single leader vehicle. We recall below some examples. The interested reader could refer to [4] for references. We can quote for instance

- the model of Bexelius extends the car-following model of Chandler *et al.*.

- Lenz *et al.* extend the model of Bando *et al.*, yielding a second order multi-anticipative model.
- Hoogendoorn *et al.* [7] propose an extension of the model of Helly

$$\ddot{x}_i(t+T) = \sum_{j=1}^{m_1} \alpha_j \left(\dot{x}_{i+j} - \dot{x}_i\right)(t) + \sum_{j=1}^{m_2} \beta_j \left[\left(x_{i+j} - x_i\right)(t) - S_0 - jT\dot{x}_i(t)\right], \quad (1)$$

  with two different number of considered leaders $m_1 \geq 1$ and $m_2 \geq 1$ according to either speed variations or headway variations.
- Treiber *et al.* introduces the Human Driver Model (HDM) as an extension of his well-known Intelligent Driver Model (IDM).
- Farhi *et al.* [4] describes a first order model that extends the Min-Plus car-following model. This model is described in Section 3. As it is based on algebra Min-Plus, it is easy to check its global properties.

Remark 1. The additive form in the multi-anticipative models yields models which are easier to study analytically. But the minimum form expresses the fact that a driver will adapt its velocity (or equivalently its acceleration) according to the worst behaviour of all the considered leaders and thus offers more physical interpretation.

Experimental results. In [6, 15], the model (1) is calibrated on real data and it fits best for $m_1 = 3$ and $m_2 = 1$, meaning that the drivers are more sensitive to speed variations than headway variations. It is also shown that the multi-anticipative models improves the representation of driving behaviour. However there is a high variance in driving behaviour which is not totally accounted for.

In many studies (for instance [14, 17] and references therein), platoon stability (on a single lane) is shown to decrease when the reaction time increases, and to increase when the spatial and/or temporal anticipation are increased.

3 Macroscopic model for multi-anticipative traffic.

3.1 First result in the Min-Plus algebra

First order multi-anticipative models can be viewed as high-viscosity approximation of second order models (such as the Frenkel-Kontorova model studied in [5]). For instance, in [4] the model is a first order and based on a piecewise linear fundamental diagram (FD). The velocity is computed by taking the minimum of all constraints imposed by preceding vehicles. The model is expressed in the Min-Plus algebra as follows

$$x_i(t+1) = x_i(t) + \min_{1\leq j\leq m} (1+\lambda)^{m-1} \min_{v\in\mathcal{U}} \max_{w\in\mathcal{W}} \left[\alpha_{vw}\left(\frac{x_{i+j}(t) - x_i(t)}{j}\right) + \beta_{vw}\right] \quad (2)$$

where $\mathcal{U}$ and $\mathcal{W}$ are two finite sets of indices. The $\lambda \geq 0$ is a discount parameter favouring closer leaders over the farther ones.

The authors obtain semi-analytical results concerning the stability of the model and the existence of fixed points. These fixed points match invariant states for the macroscopic traffic flow.

Notice moreover that the simulation results in [4] show the smoothing effects of multi-anticipative driving on the macroscopic traffic flow.

3.2 Multi-anticipative first order models and Hamilton-Jacobi equation

One approach to micro-macro passage relies on the mathematical homogenization of car-following models into Hamilton-Jacobi equation.

Let us first consider the following first order multi-anticipative model

$$\dot{x}_i(t+T) = \max\left[0, V_{max} - \sum_{j=1}^{m} f\left(x_{i+j}(t) - x_i(t)\right)\right] \tag{3}$$

with $T \geq 0$ and with $f : \mathbb{R}_+^* \mapsto \mathbb{R}_+$ which needs to be a non-negative and non-decreasing function describing the speed-spacing relationship. Let us choose

$$f(r) = \beta \exp(-\gamma r), \quad \text{for any} \quad r > 0, \tag{4}$$

with β, $\gamma > 0$. This choice is mathematically convenient because if we set

$$F\left(\{x_k\}_k\right) := V_{max} - \sum_{j=1}^{m} f\left(x_{i+j}(t) - x_i(t)\right),$$

then we can check that $\dfrac{\partial F}{\partial x_k} \geq 0$, for any $k = \{i, ..., i+m\}$. Thus, it is possible to recover (at least formally) homogenization results.

Remark 2. Notice that qualitatively this choice of an exponential speed-spacing fundamental diagram (FD) implies that the more the vehicles anticipate on their leaders, the lower their speeds and the higher their spacings. However one would expect that multi-anticipation allows shorter spacing and with high speeds.

Remark 3 (Equivalence result). One can check that if we consider a piecewise linear speed-spacing relationship, then the model (3) can be approximated by the Min-Plus model (2).

Let us consider the model (3). If we apply an unzooming procedure by introducing the rescaled position of vehicles as follows

$$X^{\varepsilon}(y,t) = \varepsilon x_{\lfloor \frac{y}{\varepsilon} \rfloor}\left(\frac{t}{\varepsilon}\right), \quad \text{for} \quad y \in \mathbb{R}, \quad t \in [0, +\infty) \tag{5}$$

where $\lfloor . \rfloor$ denotes the floor integer, then we can recover a Hamilton-Jacobi equation by homogenization when the scale factor ε goes down to zero:

$$\frac{\partial X^0}{\partial t} = \bar{V}\left(\frac{\partial X^0}{\partial n}, m\right) \tag{6}$$

with m the number of considered leaders and the (macroscopic) flow speed as follows

$$\bar{V}(r,m) = \max\left[0, V_{max} - \sum_{j=1}^{m} f(jr)\right].$$

The unknown $X^0(n,t)$ denotes the position of the vehicle labelled n at time t:

$$\frac{\partial X^0}{\partial t} = v \quad \text{and} \quad \frac{\partial X^0}{\partial n} = r,$$

where v and r describe respectively the speed and the spacing.

We recover the classical LWR model (standing for Lighthill, Whitham [13] and Richards [16]) in Lagrangian coordinates (n, t) that is

$$\begin{cases} \partial_t r + \partial_n v = 0, \\ v = \bar{V}(r, m). \end{cases} \tag{7}$$

We recall that the LWR model in Eulerian coordinates (t, x) expresses the conservation of vehicles on a section

$$\begin{cases} \partial_t \rho + \partial_x (\rho v) = 0, \\ v = V(\rho, m) := \bar{V}(1/\rho, m). \end{cases}$$

with ρ the density of vehicles and the *modified* speed-density fundamental diagram (FD) $V : (\rho, m) \mapsto V(\rho, m)$ which is non-negative and non-increasing w.r.t. ρ.

This homogenization result is fully described in [1, 9]. Homogenization is a general technique which has been used for several different models involving interactions with a finite number of particles. The interested reader is referred to [5] and references therein.

3.3 General multi-lane traffic flow model with multi-anticipation

We consider a multi-lane road section and we consider the projection of vehicles on the spatial axis as in Figure 1. Assume that the traffic flow on such a section is composed of a mixture of multi-anticipative vehicles. The model (7) implies that low anticipation vehicles will be stuck behind high anticipation vehicles. In the case of multi-lane traffic such behaviour is precluded by the fact that vehicles can overtake each other.

Therefore let us denote by χ_j the fraction of j-anticipative vehicles. Then the traffic flow is the superposition of traffic of j-anticipative vehicles, i.e.

$$\chi = (\chi_j)_{j=1,\ldots,m},$$

with $0 \leq \chi_j \leq 1$ for any $j = \{1, ..., m\}$ and $\sum_{j=1}^{m} \chi_j = 1$. It is then obvious that the composition is advected with the traffic flow. We can express this concept using a model of the Generic Second Order Modelling (GSOM) familly as it has been introduced by Lebacque *et al.* in [11]. The driver attribute is the composition χ. We get the following expression

$$\begin{cases} \partial_t \rho + \partial_x (\rho v) = 0, \\ v := \displaystyle\sum_{j=1}^{m} \chi_j V(\rho, j) = \sum_{j=1}^{m} \chi_j \bar{V}(1/\rho, j), \\ \partial_t(\rho\chi) + \partial_x(\rho\chi v) = 0. \end{cases} \tag{8}$$

Let us set

$$\bar{W}(\rho,\chi) := \sum_{j=1}^{m} \chi_j \bar{V}(1/\rho, j), \quad \text{and} \quad W(r,\chi) = \bar{W}(1/r,\chi) = \sum_{j=1}^{m} \chi_j \bar{V}(r,j)).$$

We can check that the third line in equation (8) could be rewritten as a simple advection equation

$$\partial_t \chi + v \partial_x \chi = 0.$$

As it has been already shown (see for instance [11]), the system (8) admits only two kinds of waves:

- kinematic waves (rarefaction or shock) for the vehicles density, similar to kinematic waves for the LWR model. Through such a wave, the composition of traffic χ is preserved but not the speed;
- contact discontinuities for the composition of traffic. In this case, the wave velocity is equal to the speed of traffic v which is conserved through the wave.

In Lagrangian coordinates (t, n), with n the label of cars, the flux variable is v and the stock variable is the spacing $r = 1/\rho$. The model can be recast as:

$$\begin{cases} \partial_t r + \partial_n v = 0 \\ v = W(r,\chi) \\ \partial_t \chi = 0 \end{cases} \tag{9}$$

The model (8), (9) is new in the sense that there already exist some multilanes models such as the model of Greenberg, Klar and Rascle (see [11] and references therein) which belongs to the GSOM family. However to the best authors knowledge, there does not exists any macroscopic PDE model taking into consideration multi-anticipative behaviour in a multi-lane context.

4 Numerical approaches

To numerically solve the system (8), we can use classical GSOM methodologies [11, 12] that encompass:

- Godunov-like schemes for which we need to introduce finite time and space steps Δt, Δx_k that need to satisfy a CFL condition. Consider the following scheme

$$\begin{cases} \rho_k^{t+1} = \rho_k^t + \dfrac{\Delta t}{\Delta x_k}\left[q_k^t - q_{k+1}^t\right], \\ q_k^t := \min\left\{\Delta_k\left(\rho_k^t, \chi_k^t\right), \Sigma_{k+1}\left(\rho_{k+1}^t, \chi_{k+1}^t\right)\right\}, \\ \rho_k^{t+1}\chi_k^{t+1} = \rho_k^t \chi_k^t + \dfrac{\Delta t}{\Delta x_k}\left[q_k^t \chi_k^t - q_{k+1}^t \chi_{k+1}^t\right]. \end{cases} \tag{10}$$

  We also need to define the supply and demand functions as in [10]

$$\Delta_k(\rho,\chi) = \max_{0 \le \xi \le \rho}\left[\xi \bar{W}_k(\xi,\chi)\right], \quad \text{and} \quad \Sigma_k(\rho,\chi) = \max_{\xi \ge \rho}\left[\xi \bar{W}_k(\xi,\chi)\right].$$

- variational formulation and dynamic programming techniques [2].

- particle methods in the Lagrangian framework (t,n). A standard way of obtaining these (refer to [11], [12]) is to apply a Godunov scheme to (9). This is easy: the supply is simply v_{max}, the demand is W, because $r \mapsto W(r,\chi)$ is increasing. Since a cell can be associated to a packet of Δn vehicles having a total spacing r_n^t and tail position x_n^t, a simple car-following like model (11) is derived. Considering Lagrangian finite difference methods, we can either deal with a vectorial attribute χ or with an integer $j = 1, ..., m$ with randomization of probability χ_j. We opt here for the first option. The resulting model is described hereafter:

$$\left| \begin{array}{l} x_n^{t+1} = x_n^t + \Delta t W\left(r_n^t, \chi_n^t\right) \\ r_n^t = \left(x_{n+1}^t - x_n^t\right)/\Delta n \\ \chi_n^{t+1} = \chi_n^t \end{array} \right. \tag{11}$$

The Lagrangian method (11) is more precise (less smoothing of waves) than (10) and easier to calculate (the demand being the speed).

4.1 Choice of the fundamental diagram

For this numerical example, we have used the speed-spacing function described in (4) that is

$$\bar{V}(r,m) = \max\left[0, V_{max} - \sum_{j=1}^{m} \beta \exp(-\gamma j r)\right]$$

with β, $\gamma > 0$. As we consider that those coefficients are strictly independent of the number of considered leaders $j \in \{1, ..., m\}$, one can easily check that

$$\beta = V_{max} \exp\left(\gamma r_{min}\right)$$

where r_{min} is the minimal spacing between two consecutive vehicles. The maximal speed V_{max} is equal to 25 m/s and γr_{min} is fixed to 0.18 in order to ensure a proper critical density. The maximal number of considered leaders m is equal to 5.

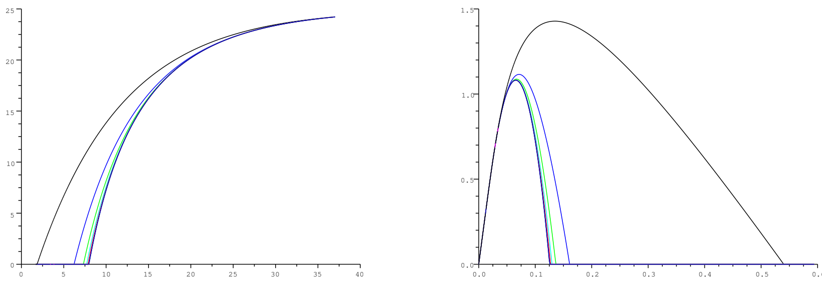

Fig. 2. Speed-spacing fundamental diagram $r \mapsto \bar{V}(r,\kappa)$ (left) and Flow-density fundamental diagram $\rho \mapsto \rho V(\rho,\kappa)$ (right) for different values of κ

Note that the fundamental diagrams plotted on Figure 2 are intended for a single lane. Then the higher the number of considered leaders, the higher the critical spacing (or equivalently the lower the critical density) per single lane.

4.2 Description of the use case

Let consider a traffic flow on a multi-lane road section. Roughly speaking, assume that entering the section we have two distinct compositions of traffic: high anticipatory, then low anticipatory, then high anticipatory again (see Figure 3). The downstream supply is formulated in terms of speed, which is more convenient in the Lagrangian framework. The supply is assumed to drop in the middle of the considered period (from times $t = 250$ to $t = 2200$), generating a high-density wave propagating backwards.

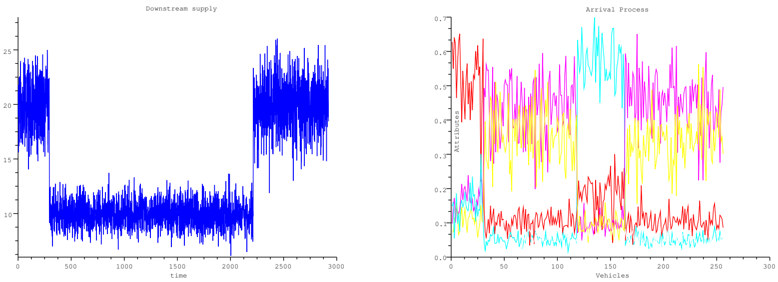

Fig. 3. Downstream supply value (left) and traffic composition attribute χ (right)

This shock wave interacts with the contact discontinuities carried by the incoming traffic (at times $t = 1000$ and $t = 1800$). Note that the increase of downstream supply at time $t = 2200$ generates also a rarefaction wave (see Figure 4).

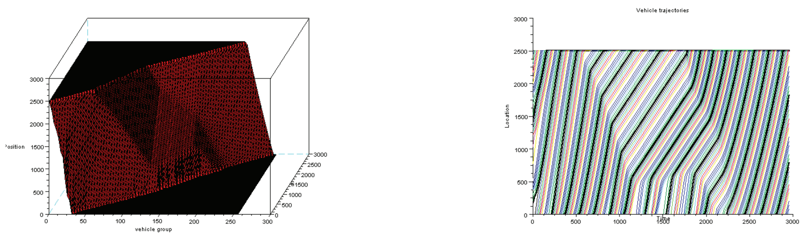

Fig. 4. Positions in Lagrangian framework (left) and Eulerian trajectories (right)

This simple numerical example shows that the low anticipatory fraction of the traffic allows to reduce or annihilate the shock wave because drivers accept lower critical spacings. This effect results is strongly dependent on our choice of the speed-spacing relationship $\bar{V}$ which implies that less anticipative drivers driver faster, take more risks. The inclusion of stochastic effects [8] would show another effect: that multi-anticipation smoothens traffic.

5 Conclusion and future directions

Possible extensions include adding source terms for the equation of advection of the composition. This could account for the spatial variability of multi-anticipatory behaviour. See for instance [12]. Moreover our model should be tested on real measurement data. The main problem is the identification of instantaneous traffic composition χ as well the speed-spacing function parameters as it was done in [4].

Another study should be based on the analysis of individual trajectories to recover the results of previous studies which state that the multi-anticipative car-following models improve the representation of individual driving behaviour. While the existing experiments only take into account already congested situations, these work should extend the results by considering for congested and also fluid traffic flow situations. Such a study could also confirm or infirm the impact of anticipatory traffic on the driving behavior (see Remark 2).

Aknowledgements

The authors are grateful to Prof. Régis Monneau. We also want to thank the organizers of the Conference on Traffic and Granular Flow (TGF'13) in Jülich, Germany.

This work was partially supported by the ANR (Agence Nationale de la Recherche) through HJnet project ANR-12-BS01-0008-01.

References

1. Al Haj, M., Costeseque, G., & Monneau, R. (2014). Homogenization of some time- delayed systems of particles: application to traffic vehicular models, in preparation.
2. Costeseque, G., & Lebacque, J. P. (2013). A variational formulation for higher order macroscopic traffic flow models: numerical investigation, in preparation.
3. Farhi, N. (2012). Piecewise linear car-following modeling. Transportation Research Part C: Emerging Technologies, 25, 100-112.
4. Farhi, N., Haj-Salem, H., & Lebacque, J. P. (2012). Multi-Anticipative Piecewise Linear Car-Following Model. In Transportation Research Board 91st Annual Meeting (No. 12-3823).
5. Forcadel, N., Imbert, C., & Monneau, R. (2012). Homogenization of accelerated Frenkel-Kontorova models with n types of particles. Transactions of the American Mathematical Society, 364(12), 6187-6227.
6. Hoogendoorn, S. P., Ossen, S., & Schreuder, M. (2006). Empirics of multianticipative car-following behavior. Transportation Research Record: Journal of the Transportation Research Board, 1965(1), 112-120.

7. Hoogendoorn, S. P., Ossen, S., & Schreuder, M. (2007). Properties of a microscopic heterogeneous multi-anticipative traffic flow model. IN Allsop, R. E., Bell, M. G. H. & Heydecker, B. G. (Eds.) Transportation and Traffic Theory 2007. Amsterdam, The Netherlands, Elsevier.
8. Khoshyaran, M. M., & Lebacque, J. P. (2009). A stochastic macroscopic traffic model devoid of diffusion. In Traffic and Granular Flow07 (pp. 139-150). Springer Berlin Heidelberg.
9. Khoshyaran, M.M., Lebacque, J.P., & Monneau, R. (2010). Private communications.
10. Lebacque, J. P. (1996). The Godunov scheme and what it means for first order traffic flow models. In International symposium on transportation and traffic theory, 647-677.
11. Lebacque, J. P., Mammar, S., & Haj Salem, H. (2007). Generic second order traffic flow modelling. In Transportation and Traffic Theory 2007. Papers Selected for Presentation at ISTTT17.
12. Lebacque, J. P., & Khoshyaran, M. M. (2014). GSOM traffic flow models with eulerian source terms. In preparation.
13. Lighthill, M. J., & Whitham, G. B. (1955). On kinematic waves. II. A theory of traffic flow on long crowded roads. Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences, 229(1178), 317-345.
14. Monteil, J., Billot, R., & El Faouzi, N. E. (2013). Véhicules coopératifs pour une gestion dynamique du trafic: approche théorique et simulation. Recherche Transports Sécurité, (29).
15. Ossen, S., & Hoogendoorn, S. P. (2006, September). Multi-anticipation and heterogeneity in car-following empirics and a first exploration of their implications. In Intelligent Transportation Systems Conference, 2006. ITSC'06. IEEE (pp. 1615-1620). IEEE.
16. Richards, P. I. (1956). Shock waves on the highway. Operations research, 4(1), 42-51.
17. Treiber, M., Kesting, A., & Helbing, D. (2006). Delays, inaccuracies and anticipation in microscopic traffic models. Physica A: Statistical Mechanics and its Applications, 360(1), 71-88.