Mesoscopic multiclass traffic flow modeling on multi-lane sections

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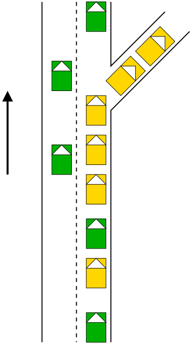
Example: congested off-ramp

*



Example: congested off-ramp

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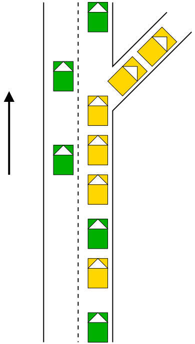


Requirements for modeling the upstream section:

1. multiclass
2. non-FIFO

* [Richmond Bridge, ©Bay Area Council]

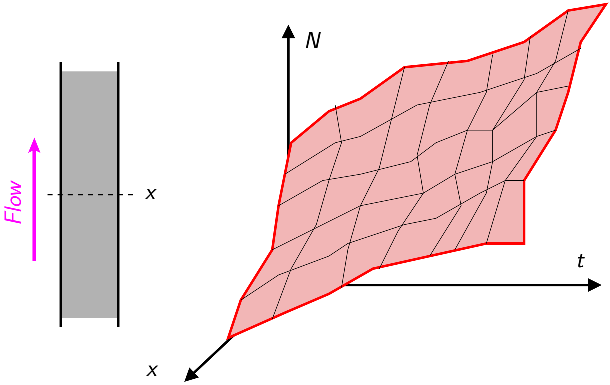
Outline

1. Theoretical background
2. Mesoscopic formulation of multiclass multilane models
3. Numerical scheme
4. Conclusion and perspectives

Outline

Three representations of traffic flow

Moskowitz' surface



See also [Moskowitz(1959), Makigami et al(1971), Laval and Leclercq(2013)]

Mesoscopic resolution of the LWR model

		Lagrangian-Space Meso $n - x$	Eulerian Macro $t - x$
CL	Variables	Pace $p := \frac{1}{v}$ Headway $h := \frac{1}{q} = H(p)$	Density k Flow $q = Q(k)$
	Equation	$\boxed{\partial_n p - \partial_x H(p) = 0}$	$\boxed{\partial_t k + \partial_x Q(k) = 0}$
HJ	Variable	Passing time T $T(n,x) = \int_{-\infty}^{x} p(n,\xi)d\xi$	Label N $N(t,x) = \int_{x}^{+\infty} k(t,\xi)d\xi$
	Equation	$\boxed{\partial_n T - H(\partial_x T) = 0}$	$\boxed{\partial_t N - Q(-\partial_x N) = 0}$

Mesoscopic: what for?

- Strengths
  1. Consistent with micro and macro representations
  2. Large scale networks // spatial discontinuities OK
  3. Data assimilation (from Eulerian and Lagrangian sensors)
- Weakness
  1. Single pipe
  2. Mono class
  3. No capacity drop at junctions
- Developments
  1. Multilane and multiclass approach
  2. Relaxed FIFO assumption

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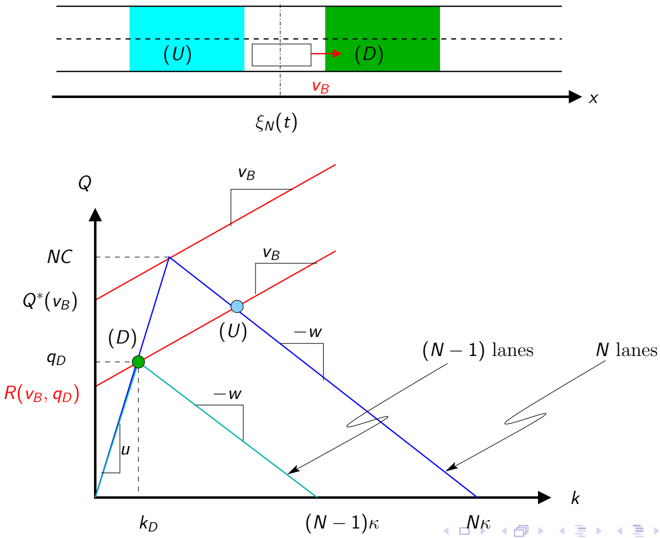
$\longrightarrow$ Moving bottleneck theory

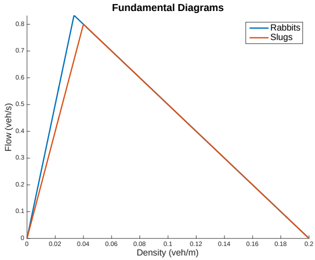
Notations

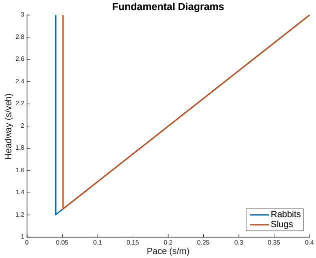
(Eulerian)



Outline

Settings

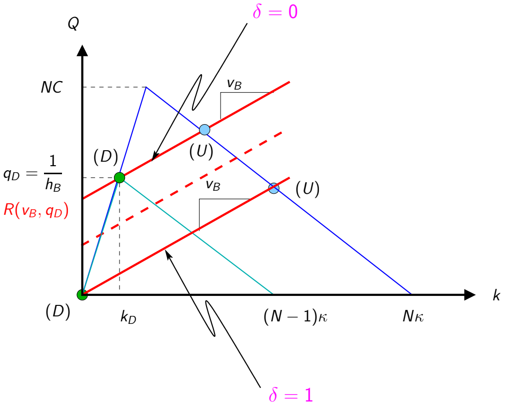




- Stretch of road $[x_0, x_1]$ (diverge at x_1) composed by N separate lanes
- Two classes of users: "rabbits" ($I = I_1$) and "slugs" ($I = I_2$).
- Triangular class-dependent headway-pace FD

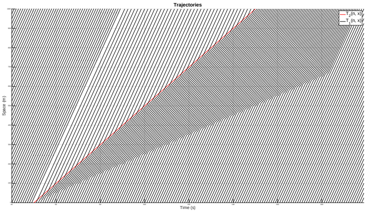
$$H : (p, I) \mapsto H(p, I)$$

Capacity drop parameter

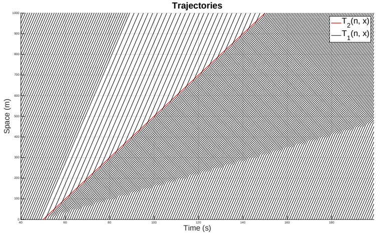


Introduce parameter $\delta \in [0, 1]$

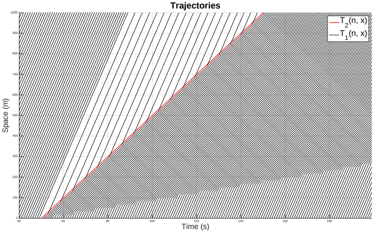
- If $\delta = 0$, strictly non-FIFO
- If $0 < \delta < 1$, reduction of the passing rate
- If $\delta = 1$, strictly FIFO



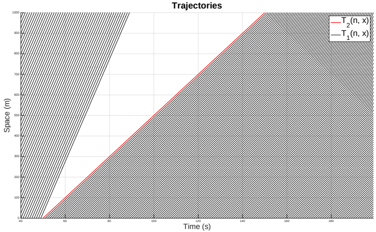
$\delta = 0$



$\delta = 0.4$



$\delta = 0.6$



$\delta = 1$ (FIFO case)

System of coupled HJ PDEs

$$\begin{cases} \partial_n T_1 - H\left(\partial_x T_1, l_1\right) = 0, & \text{(rabbits)} \\ \partial_n T_2 - H\left(\partial_x T_2, l_2\right) = 0, & \text{(slugs)} \end{cases}$$

System of coupled HJ PDEs

$$\begin{cases} \partial_n T_1 - H\left(\partial_x T_1, l_1\right) = 0, & \text{(rabbits)} \\ \partial_n T_2 - H\left(\partial_x T_2, l_2\right) = 0, & \text{(slugs)} \\ H\left(\partial_x T_1(n,\xi(n)), l_1\right) - (1-\delta)\dot{\xi}\left(n_2^*\right)\partial_x T_1(n,\xi(n)) & \\ \qquad \geq \dfrac{N}{N-1} H^{\boxtimes}\left(\left(1-\delta\right)\dot{\xi}\left(n_2^*\right), l_1\right), & {\color{red}(2 \to 1)} \end{cases}$$

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where

$$H^{\boxtimes}(s,l) = \inf_{p \in \mathrm{Dom}(H(\cdot,l))} \left\{ H(p,l) - sp \right\}$$

System of coupled HJ PDEs

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$$H^{\boxtimes}(s, l) = \inf_{p \in \mathrm{Dom}(H(\cdot, l))} \left\{ H(p, l) - sp \right\}$$

and $n_i^* =$ the nearest leader from class i for vehicle n of class $j \neq i$

System of coupled HJ PDEs

$$
\begin{cases}
\partial_n T_1 - H\left(\partial_x T_1, I_1\right) = 0, & \text{(rabbits)} \\
\partial_n T_2 - H\left(\partial_x T_2, I_2\right) = 0, & \text{(slugs)} \\
H\left(\partial_x T_1(n,\xi(n)), I_1\right) - (1-\delta)\dot{\xi}\left(n_2^*\right)\partial_x T_1(n,\xi(n)) & \\
\qquad \geq \dfrac{N}{N-1} H^{\boxtimes}\left(\left(1-\delta\right)\dot{\xi}\left(n_2^*\right), I_1\right), & (2 \to 1) \\
T_2\left(n,\xi(n)\right) \geq T_1(n_1^*,\xi(n)) + H\left(\partial_x T_1(n_1^*,\xi(n)), I_2\right), & (1 \to 2)
\end{cases}
$$

where

$$
H^{\boxtimes}(s, I) = \inf_{p \in \mathrm{Dom}(H(\cdot, I))} \{H(p, I) - sp\}
$$

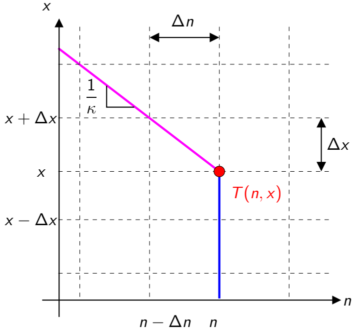
and n_i^* = the nearest leader from class i for vehicle n of class $j \neq i$

Outline

Lax-Hopf formula & Dynamic Programming

Finite steps $(\Delta n, \Delta x)$

$$\Delta n = \kappa\ \Delta x.$$

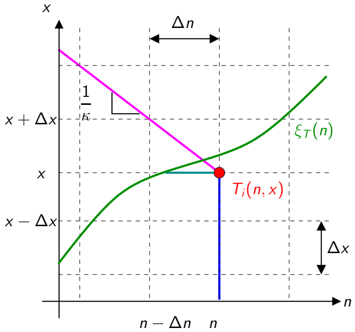


Solution reads:

$$T(n,x) = \max\left\{ \overbrace{T(n, x-\Delta x) + \frac{\Delta x}{u}}^{=\text{ free flow}},\ \underbrace{T\left(n - \Delta n, x + \Delta x\right) + \frac{\Delta x}{w}}_{=\text{ congested}} \right\}.$$

See [Laval and Leclercq(2013)]

Representation formulæ



New supply constraint:

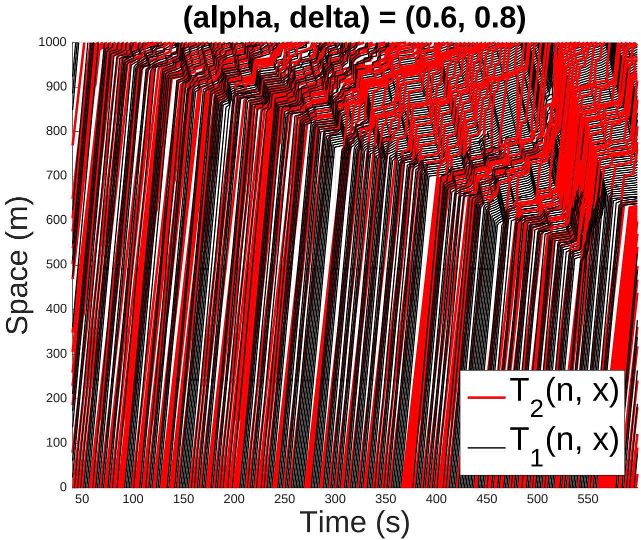
$$T_i(n,x) = \max\left\{ \overbrace{T_i(n, x-\Delta x) + \frac{\Delta x}{u_i}}^{=\text{ free flow}},\ \underbrace{T_i\left(n-\Delta n, x+\Delta x\right) + \frac{\Delta x}{w}}_{=\text{ congested}},\ \textbf{coupling condition} \right\}.$$

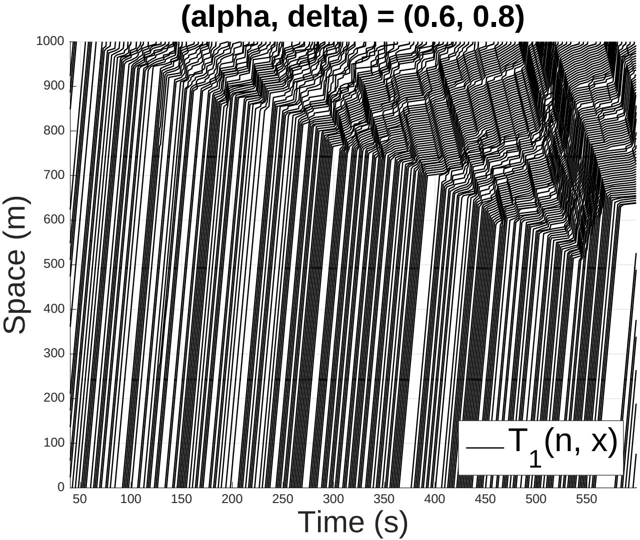
Representation formulæ

(Coupling conditions)

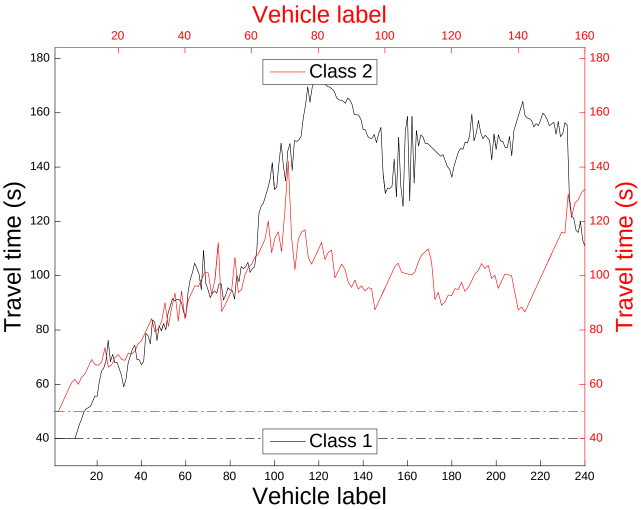
$$\begin{cases} T_1(n,x) = \max \Big\{ T_1(n, x-\Delta x) + \dfrac{\Delta x}{u_1}, T_1(n-\Delta n, x+\Delta x) + \dfrac{\Delta x}{w}, \\ \qquad T_2(n_2^*, x) + \dfrac{1}{1-\delta} h_B \Big\} \\ T_2(n,x) = \max \Big\{ T_2(n, x-\Delta x) + \dfrac{\Delta x}{u_2}, \ T_2(n-\Delta n, x+\Delta x) + \dfrac{\Delta x}{w}, \\ \qquad T_1(n_1^*, x) + H\left(\dfrac{T_1(n_1^*, x) - T_1(n_1^*, x-\Delta x)}{\Delta x}, l_2 \right) \Big\} \end{cases} \tag{1}$$

- Distribution per class: class 1 = 60% and class 2 = 40%
- Capacity drop: $\delta = 0.8$




(alpha, delta) = (0.6, 0.8)
Space (m)
1000
900
800
700
600
500
400
300
200
100
0
50
100
150
200
250
300
350
400
450
500
550
Time (s)
T1(n, x)

Individual travel times



Outline

- A new event-based mesoscopic model for multi-class traffic flow on multi-lane sections
- Use of theory of moving bottlenecks

Among the perspectives:

- Sensitivity analysis w.r.t. δ
- Validation with real traffic data
- Data assimilation for real-time applications ($\rightarrow$ Aurélien's presentation)

THANKS FOR YOUR ATTENTION

Any question?

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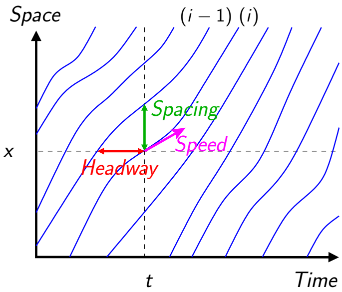
Some references I

Laval, J. A., Leclercq, L., 2013. The Hamilton–Jacobi partial differential equation and the three representations of traffic flow. Transportation Research Part B: Methodological 52, 17–30.

Leclercq, L., Bécarie, C., 2012. A meso LWR model designed for network applications. In: Transportation Research Board 91th Annual Meeting. Vol. 118. p. 238.

Mesoscopic resolution of the LWR model



Introduce

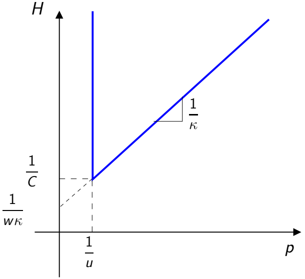
- the pace $p := \frac{1}{v}$
- the headway $h = H(p)$
- the passing time

$$T(n,x) := \int_{-\infty}^{x} p(n,\xi)d\xi.$$

$$\begin{cases} \partial_n T = h, & \text{(headway)} \\ \partial_x T = p, & \text{(pace)} \end{cases}$$

[Leclercq and Bécarie(2012), Laval and Leclercq(2013)]

Lax-Hopf formula



Assume

$$H(p) = \begin{cases} \frac{1}{\kappa} p + \frac{1}{w\kappa}, & \text{if} \quad p \geq \frac{1}{u}, \\ +\infty, & \text{otherwise,} \end{cases}$$

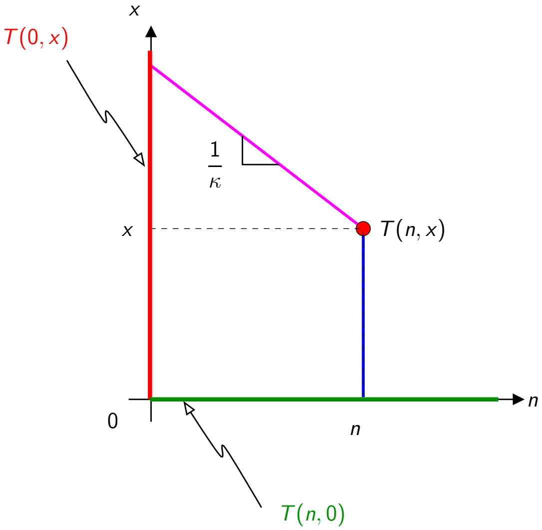
Proposition (Representation formula (Lax-Hopf))

The solution under smooth boundary conditions is given by

$$T(n,x) = \max \Big\{ \underbrace{T(n,0) + \frac{x}{u}}_{=\text{ free flow}}, \underbrace{T\left(0, x + \frac{n}{\kappa}\right) + \frac{n}{w\kappa}}_{=\text{ congested}} \Big\}. \quad (2)$$

See [Laval and Leclercq(2013)]

Lax-Hopf formula



Math problem in Eulerian framework

Coupled ODE-PDE problem

$$\begin{cases} \partial_t k + \partial_x \left(Q(k) \right) = 0, \\ Q(k(t,\xi_N(t))) - \dot{\xi}_N(t) k(t,\xi_N(t)) \leq \dfrac{N-1}{N} Q^* \left(\dot{\xi}_N(t) \right), \\ \dot{\xi}_N(t) = \min \left\{ v_b, \; V\left(k(t,\xi_N(t)^+) \right) \right\}, \end{cases} \tag{3}$$

with

$$\begin{cases} k(0,x) = k_0(x), \qquad \text{on} \quad \mathbb{R}, \\ \xi_N(0) = \xi_0. \end{cases} \tag{4}$$

Math problem in Eulerian framework

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$$\begin{cases} \partial_t k + \partial_x \left(Q(k) \right) = 0, \\ Q(k(t, \xi_N(t))) - \dot{\xi}_N(t) k(t, \xi_N(t)) \leq \dfrac{N-1}{N} Q^* \left(\dot{\xi}_N(t) \right), \\ \dot{\xi}_N(t) = \min \left\{ v_b, \; V\left(k(t, \xi_N(t)^+) \right) \right\}, \end{cases} \tag{3}$$

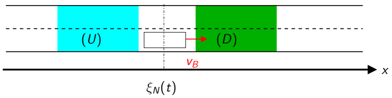
with

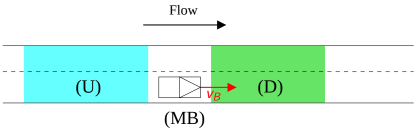
$$\begin{cases} k(0,x) = k_0(x), \qquad \text{on} \quad \mathbb{R}, \\ \xi_N(0) = \xi_0. \end{cases} \tag{4}$$

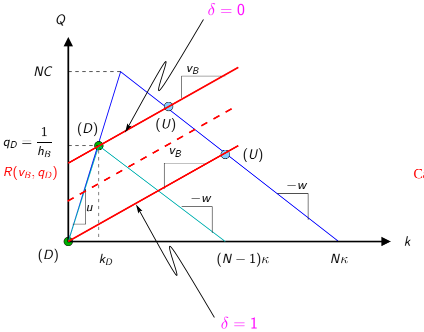
and $Q*$ is the Legendre-Fenchel transform of Q

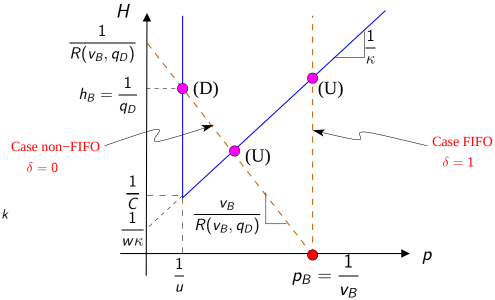
$$Q^*(v) := \sup_{k \in \mathrm{Dom}(Q)} \left\{ Q(k) - vk \right\}.$$

Capacity drop parameter









Mixed Neumann-Dirichlet boundary conditions

$$\begin{cases} \partial_n T_i(n, x_0) = \check{g}_i(n), & \text{on} \quad [n_0, +\infty), \\ \partial_n T_i(n, x_1) = \hat{g}_i(n), & \text{on} \quad [n_0, +\infty), \\ T_i(n_0, x) = G_i(x), & \text{on} \quad [x_0, x_1], \end{cases} \quad \text{for} \quad i \in \{1, 2\}. \tag{5}$$