Mesoscopic multiclass traffic flow modeling on multi-lane sections

Guillaume Costeseque*, Aurélien Duret

Inria Sophia-Antipolis Méditerranée & Université de Lyon-IFSTTAR-ENTPE, LICIT

TRB Annual Meeting 2016, Washington DC
January 12, 2016
Motivations

Example: congested off-ramp
Example: congested off-ramp

Requirements for modeling the upstream section:

1. multiclass
2. non-FIFO
Outline

1. Theoretical background
2. Mesoscopic formulation of multiclass multilane models
3. Numerical scheme
4. Conclusion and perspectives
Theoretical background

Outline

1. Theoretical background
2. Mesoscopic formulation of multiclass multilane models
3. Numerical scheme
4. Conclusion and perspectives
Three representations of traffic flow

Moskowitz’ surface

See also [Moskowitz(1959), Makigami et al(1971), Laval and Leclercq(2013)]
## Mesoscopic resolution of the LWR model

<table>
<thead>
<tr>
<th>Lagrangian-Space</th>
<th>Eulerian Space</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Meso</strong></td>
<td><strong>Macro</strong></td>
</tr>
<tr>
<td>( n - x )</td>
<td>( t - x )</td>
</tr>
</tbody>
</table>

### Variables

- **CL**
  - **Pace**: \( p := \frac{1}{v} \)
  - **Headway**: \( h := \frac{1}{q} = H(p) \)

- **HJ**
  - **Passing time**: \( T \)
    - \( T(n, x) = \int_{-\infty}^{x} p(n, \xi) d\xi \)
  - **Label**: \( N \)
    - \( N(t, x) = \int_{x}^{+\infty} k(t, \xi) d\xi \)

### Equation

- **CL**
  - \( \partial_n p - \partial_x H(p) = 0 \)
  - \( \partial_t k + \partial_x Q(k) = 0 \)

- **HJ**
  - \( \partial_n T - H(\partial_x T) = 0 \)
  - \( \partial_t N - Q(-\partial_x N) = 0 \)
Mesoscopic: what for?

**Strengths**
1. Consistent with micro and macro representations
2. Large scale networks // spatial discontinuities OK
3. Data assimilation (from Eulerian and Lagrangian sensors)

**Weakness**
1. Single pipe
2. Mono class
3. No capacity drop at junctions

**Developments**
1. Multilane and multiclass approach
2. Relaxed FIFO assumption
Mesoscopic: what for?

- **Strengths**
  1. Consistent with micro and macro representations
  2. Large scale networks // spatial discontinuities OK
  3. Data assimilation (from Eulerian and Lagrangian sensors)

- **Weakness**
  1. Single pipe
  2. Mono class
  3. No capacity drop at junctions

- **Developments**
  1. Multilane and multiclass approach
  2. Relaxed FIFO assumption

→ Moving bottleneck theory
Notations
(Eulerian)

\( \xi_N(t) \)

\[ Q \]

\( Q^*(v_B) \)

\( NC \)

\( q_D \)

\( R(v_B, q_D) \)

\( (U) \)

\( (D) \)

\( k_D \)

\( (N - 1) \kappa \)

\( N \kappa \)

\( v_B \)

\(-w\)

\( N \) lanes

\( (N - 1) \) lanes
Outline

1. Theoretical background

2. Mesoscopic formulation of multiclass multilane models

3. Numerical scheme

4. Conclusion and perspectives
Stretch of road \([x_0, x_1]\) (diverge at \(x_1\)) composed by \(N\) separate lanes

- Two classes of users: “rabbits” \((I = I_1)\) and “slugs” \((I = I_2)\).

- Triangular class-dependent headway-pace FD

\[ H : (p, I) \mapsto H(p, I) \]
Capacity drop parameter

Introduce parameter $\delta \in [0, 1]$

- If $\delta = 0$, strictly non-FIFO
- If $0 < \delta < 1$, reduction of the passing rate
- If $\delta = 1$, strictly FIFO
\( \delta = 0 \)  

\( \delta = 0.4 \)  

\( \delta = 0.6 \)  

\( \delta = 1 \) (FIFO case)
System of coupled HJ PDEs

\begin{equation*}
\begin{cases}
\partial_n T_1 - H(\partial_x T_1, l_1) = 0, & \text{(rabbits)} \\
\partial_n T_2 - H(\partial_x T_2, l_2) = 0, & \text{(slugs)}
\end{cases}
\end{equation*}
System of coupled HJ PDEs

\[
\begin{aligned}
\partial_n T_1 - H (\partial_x T_1, l_1) &= 0, \\
\partial_n T_2 - H (\partial_x T_2, l_2) &= 0, \\
H (\partial_x T_1(n, \xi(n)), l_1) - (1 - \delta) \dot{\xi} (n_2^*) \partial_x T_1(n, \xi(n)) &
\geq \frac{N}{N - 1} H^\Box \left((1 - \delta) \dot{\xi} (n_2^*), l_1\right), \\
\end{aligned}
\]

(rabbits)

(slugs)

(2 \rightarrow 1)
System of coupled HJ PDEs

\[
\begin{align*}
\partial_n T_1 - H (\partial_x T_1, l_1) &= 0, & \text{(rabbits)} \\
\partial_n T_2 - H (\partial_x T_2, l_2) &= 0, & \text{(slugs)} \\
H (\partial_x T_1(n, \xi(n)), l_1) - (1 - \delta)\dot{\xi}(n_2^*) \partial_x T_1(n, \xi(n)) & \geq \frac{N}{N-1} H^{\boxplus} \left( (1 - \delta)\dot{\xi}(n_2^*), l_1 \right), \quad (2 \rightarrow 1)
\end{align*}
\]

where

\[
H^{\boxplus}(s, l) = \inf_{p \in \text{Dom}(H(\cdot, l))} \{ H(p, l) - sp \}
\]
System of coupled HJ PDEs

\[
\begin{align*}
\partial_n T_1 - H(\partial_x T_1, l_1) &= 0, \\
\partial_n T_2 - H(\partial_x T_2, l_2) &= 0, \\
H(\partial_x T_1(n, \xi(n)), l_1) - (1 - \delta)\dot{\xi}(n_2^*) \partial_x T_1(n, \xi(n)) &\geq \frac{N}{N-1} H(\cdot, l_1), \quad \text{(2} \to 1)
\end{align*}
\]

where

\[
H(\cdot, l) = \inf_{p \in \text{Dom}(H(\cdot, l))} \{H(p, l) - sp\}
\]

and \(n_i^* = \text{the nearest leader}\) from class \(i\) for vehicle \(n\) of class \(j \neq i\)
System of coupled HJ PDEs

\[
\begin{align*}
\partial_n T_1 - H(\partial_x T_1, l_1) &= 0, \quad \text{(rabbits)} \\
\partial_n T_2 - H(\partial_x T_2, l_2) &= 0, \quad \text{(slugs)} \\
H(\partial_x T_1(n, \xi(n)), l_1) - (1 - \delta) \dot{\xi}(n_2^*) \partial_x T_1(n, \xi(n)) &\geq \frac{N}{N-1} H^\boxdot \left( (1 - \delta) \dot{\xi}(n_2^*), l_1 \right), \quad (2 \to 1) \\
T_2(n, \xi(n)) &\geq T_1(n_1^*, \xi(n)) + H(\partial_x T_1(n_1^*, \xi(n)), l_2), \quad (1 \to 2)
\end{align*}
\]

where

\[
H^\boxdot(s, l) = \inf_{p \in \text{Dom}(H(\cdot, l))} \{ H(p, l) - sp \}
\]

and \( n_i^* = \) the nearest leader from class \( i \) for vehicle \( n \) of class \( j \neq i \).
Outline

1. Theoretical background
2. Mesoscopic formulation of multiclass multilane models
3. Numerical scheme
4. Conclusion and perspectives
Lax-Hopf formula & Dynamic Programming

Finite steps \((\Delta n, \Delta x)\)

\[ \Delta n = \kappa \Delta x. \]

Solution reads:

\[ T(n, x) = \max \left\{ T(n, x - \Delta x) + \frac{\Delta x}{u}, \quad T(n - \Delta n, x + \Delta x) + \frac{\Delta x}{w} \right\}. \]

See [Laval and Leclercq(2013)]
New supply constraint:

\[ T_i(n, x) = \max \left\{ \begin{array}{l} T_i(n, x - \Delta x) + \frac{\Delta x}{u_i}, \\ T_i(n - \Delta n, x + \Delta x) + \frac{\Delta x}{w}, \end{array} \right\} \]

= free flow

= congested

coupling condition
Representation formulæ
(Coupling conditions)

\begin{align*}
T_1(n, x) &= \max \left\{ T_1(n, x - \Delta x) + \frac{\Delta x}{u_1}, T_1(n - \Delta n, x + \Delta x) + \frac{\Delta x}{w}, \\
&\hspace{1cm} T_2(n^*_2, x) + \frac{1}{1 - \delta} h_B \right\} \\
T_2(n, x) &= \max \left\{ T_2(n, x - \Delta x) + \frac{\Delta x}{u_2}, T_2(n - \Delta n, x + \Delta x) + \frac{\Delta x}{w}, \\
&\hspace{1cm} T_1(n^*_1, x) + H \left( \frac{T_1(n^*_1, x) - T_1(n^*_1, x - \Delta x)}{\Delta x}, l_2 \right) \right\}
\end{align*}

(1)
- Distribution per class: class 1 = 60% and class 2 = 40%
- Capacity drop: $\delta = 0.8$

(alpha, delta) = (0.6, 0.8)
Numerical scheme

Simulation with a mixed traffic

$(\alpha, \delta) = (0.6, 0.8)$
Individual travel times

![Graph showing travel times for Class 1 and Class 2 vehicles over time.](image)
Outline

1. Theoretical background
2. Mesoscopic formulation of multiclass multilane models
3. Numerical scheme
4. Conclusion and perspectives
A new event-based mesoscopic model for multi-class traffic flow on multi-lane sections
Use of theory of moving bottlenecks

Among the perspectives:

- Sensitivity analysis w.r.t. $\delta$
- Validation with real traffic data
- Data assimilation for real-time applications
  (→ Aurélien’s presentation)
Thanks for your attention

Any question?

guillaume.costeseque@inria.fr
Some references I


Mesoscopic resolution of the LWR model

Introduce

- the pace $p := \frac{1}{v}$
- the headway $h = H(p)$
- the passing time $T(n, x) := \int_{-\infty}^{x} p(n, \xi) d\xi$.

$$\begin{cases} \partial_n T = h, & \text{(headway)} \\ \partial_x T = p, & \text{(pace)} \end{cases}$$

[Leclercq and Bécarie(2012), Laval and Leclercq(2013)]
Lax-Hopf formula

Assume

\[ H(p) = \begin{cases} \frac{1}{\kappa} p + \frac{1}{w}\kappa, & \text{if } p \geq \frac{1}{u}, \\ +\infty, & \text{otherwise,} \end{cases} \]

**Proposition (Representation formula (Lax-Hopf))**

The solution under smooth boundary conditions is given by

\[ T(n, x) = \max \left\{ T(n, 0) + \frac{x}{u}, \quad T \left(0, x + \frac{n}{\kappa}\right) + \frac{n}{w}\kappa \right\}. \]  \hspace{1cm} (2)

= free flow \hspace{2cm} = congested

See [Laval and Leclercq(2013)]
Lax-Hopf formula

\[ T(0, x) \quad \frac{1}{\kappa} \quad T(n, x) \]

\[ T(n, 0) \]
Math problem in Eulerian framework

Coupled ODE-PDE problem

\[
\begin{aligned}
\partial_t k + \partial_x \left( Q(k) \right) &= 0, \\
Q(k(t, \xi_N(t))) - \dot{\xi}_N(t) k(t, \xi_N(t)) &\leq \frac{N - 1}{N} Q^* \left( \dot{\xi}_N(t) \right), \\
\dot{\xi}_N(t) &= \min \left\{ \nu_b, \ V \left( k(t, \xi_N(t)') \right) \right\}, 
\end{aligned}
\]  

with

\[
\begin{aligned}
k(0, x) &= k_0(x), \quad \text{on } \mathbb{R}, \\
\xi_N(0) &= \xi_0.
\end{aligned}
\]
Math problem in Eulerian framework

Coupled ODE-PDE problem

\[
\begin{aligned}
&\partial_t k + \partial_x \left(Q(k)\right) = 0, \\
&Q(k(t, \xi_N(t))) - \dot{\xi}_N(t)k(t, \xi_N(t)) \leq \frac{N - 1}{N} Q^* \left(\dot{\xi}_N(t)\right), \\
&\dot{\xi}_N(t) = \min \left\{ \nu_b, \ V \left(k(t, \xi_N(t)^{+})\right) \right\},
\end{aligned}
\]

with

\[
\begin{aligned}
k(0, x) &= k_0(x), \quad \text{on } \mathbb{R}, \\
\xi_N(0) &= \xi_0.
\end{aligned}
\]

and \(Q^*\) is the Legendre-Fenchel transform of \(Q\)

\[
Q^*(\nu) := \sup_{k\in\text{Dom}(Q)} \left\{ Q(k) - \nu k \right\}.
\]
Capacity drop parameter

\[ \xi_N(t) = x(U)(D) \]

\[ v_B \]

\[ (U) \rightarrow (D) \]

\[ (U) \rightarrow (MB) \]

Case non-FIFO
\[ \delta = 0 \]

Case FIFO
\[ \delta = 1 \]

\[ p_B = 1 \]

\[ v_B \]

\[ h_B = \frac{1}{q_D} \]

\[ 1 \]

\[ \frac{1}{R(v_B, q_D)} \]

\[ \frac{1}{w} \]

\[ \frac{1}{u} \]

\[ \kappa \]

\[ \kappa_N \]

\[ N \]

\[ k_D \]

\[ (D) \]

\[ (U) \]

\[ (N - 1) \kappa \]

\[ N \kappa \]

\[ Q \]

\[ NC \]

\[ q_D = \frac{1}{h_B} \]

\[ R(v_B, q_D) \]
Mixed Neumann-Dirichlet boundary conditions

\[
\begin{aligned}
\partial_n T_i(n, x_0) &= \mathring{g}_i(n), & \text{on} & \quad [n_0, +\infty), \\
\partial_n T_i(n, x_1) &= \hat{g}_i(n), & \text{on} & \quad [n_0, +\infty), \\
T_i(n_0, x) &= G_i(x), & \text{on} & \quad [x_0, x_1],
\end{aligned}
\]

for \( i \in \{1, 2\} \).

\[(5)\]