Queue length estimation on urban corridors

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Traffic control strategies

[Source: TRI Old Dominion University website]

Main control schemes:

- Highways
  - Variable speed limits
  - Ramp metering
  - Dynamic lane management
- Arterial streets
  - Adaptative traffic signal timings

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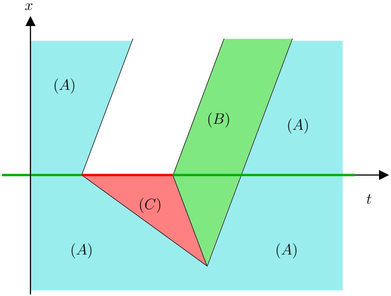
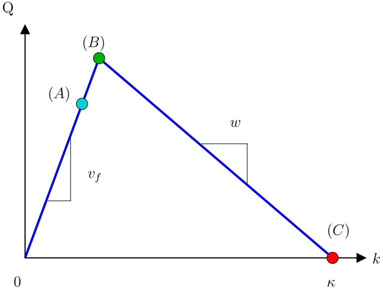
Why introducing bounded acceleration?

Traffic light: What scalar conservation laws theory teaches us

$$\partial_t k + \partial_x Q(k) = 0,$$
$$Q(k) = \min\{v_f k\ ,\ w\,(k-\kappa)\}$$

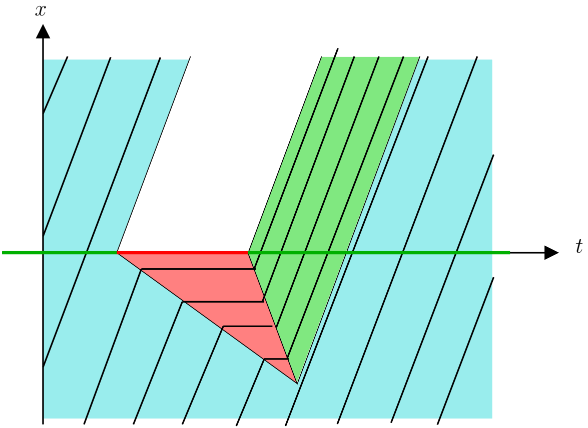




Why introducing bounded acceleration?

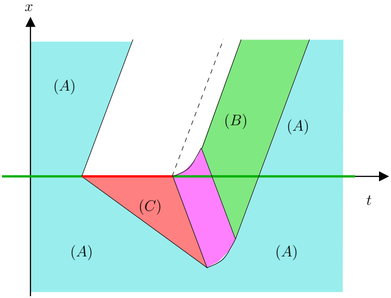
Car trajectories

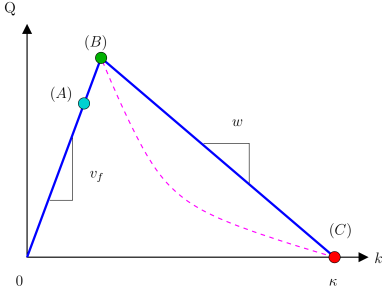
(Assuming no Italian taxi drivers...)



Why introducing bounded acceleration?

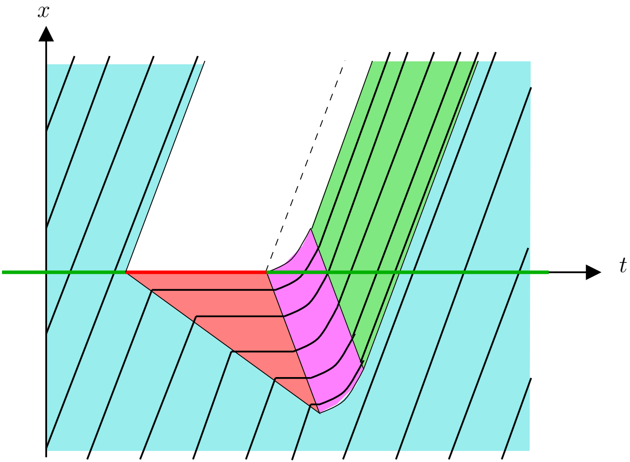
Bounded acceleration phase [Lebacque, 2003, Leclercq, 2007]





Why introducing bounded acceleration?

Car trajectories with bounded acceleration phase



Outline

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Queue length estimation at signalized intersections:

- **[data-driven]** input-output techniques
  - (-) Need good estimate of the initial queue length

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- **[data-driven]** statistical/probabilistic approaches
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- **[model based]** "shockwaves-based" approach
  - (-) Previous works do not account for bounded acceleration

Our focus

- "Shockwaves-based" approach:
  - optimization-based framework [Anderson et al., 2013]

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- “Shockwaves-based” approach:
  - optimization-based framework [Anderson et al., 2013]
  - + explicit solutions for the macroscopic traffic flow models
- Basic assumptions:
  - triangular fundamental diagram (FD)

$$Q(k) = \min\{v_f k \ , \ w(k-\kappa)\}$$

  - piecewise affine conditions

- LWR model [Lighthill and Whitham, 1955, Richards, 1956]: scalar conservation law

$$\partial_t k + \partial_x Q(k) = 0, \quad \text{on} \quad (0, +\infty) \times \mathbb{R}, \tag{1}$$

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- LWR model with bounded acceleration [Lebacque, 2002, Lebacque, 2003, Leclercq, 2002, Leclercq, 2007]

$$\begin{cases} \partial_t k + \partial_x Q(k) = 0, & \text{if} \quad v = V_e(k), \\ \begin{cases} \partial_t k + \partial_x (kv) = 0 \\ \partial_t v + v \partial_x v = a \end{cases} & \text{if} \quad v < V_e(k), \end{cases} \tag{2}$$

  - a is the maximal acceleration rate
  - $V_e : k \mapsto V_e(k)$ equilibrium speed such that $Q(k) = kV_e(k)$

- Consider the *Moskowitz function*

$$\mathbf{M}(t,x) = \int_x^{+\infty} k(t,y)dy \tag{3}$$

such that

$$\partial_x \mathbf{M} = -k \quad \text{and} \quad \partial_t \mathbf{M} = kv$$

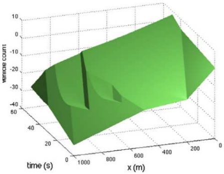
- Then the LWR with bounded acceleration can be recast as

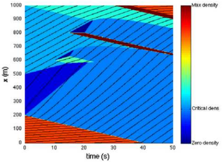
$$\begin{cases} \partial_t \mathbf{M} - Q\left(-\partial_x \mathbf{M}\right) = 0, & \text{if} \quad v = V_e\left(-\partial_x \mathbf{M}\right), \\ \begin{cases} \partial_t \mathbf{M} + v \partial_x \mathbf{M} = 0, \\ \partial_t v + v \partial_x v = a, \end{cases} & \text{if} \quad v < V_e\left(-\partial_x \mathbf{M}\right) \end{cases} \tag{4}$$

Explicit solutions

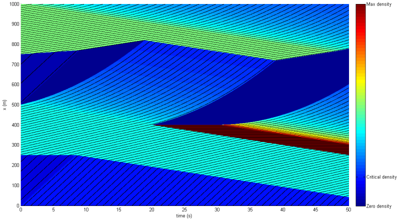
Viability theory + Lax-Hopf formula
[Claudel and Bayen, 2010a, Claudel and Bayen, 2010b]
$\Longrightarrow$ explicit solutions

LWR model
[Mazaré et al., 2011]



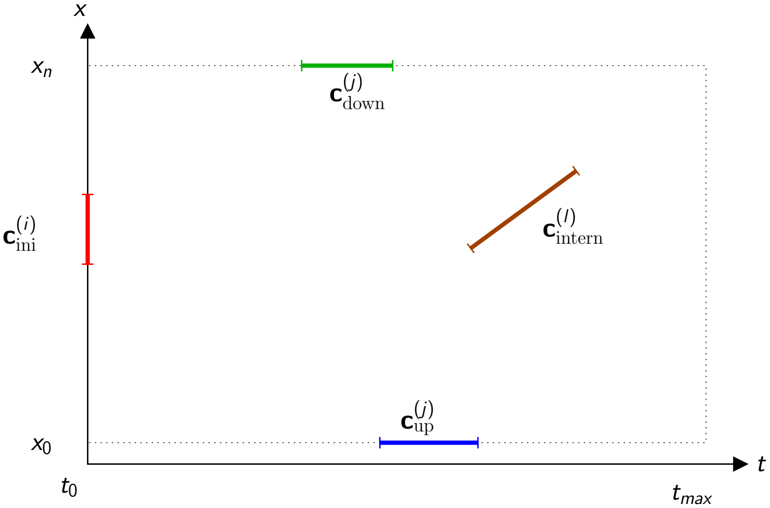


LWR model
with bounded acceleration
[Qiu et al., 2013]



Outline

Piecewise affine conditions



Piecewise affine conditions

- Initial conditions

$$\mathbf{c}_{\text{ini}}^{(i)}(x) = \begin{cases} -k_i x + b_i, & \text{if} \quad x \in [x_i, x_{i+1}], \\ +\infty, & \text{else}, \end{cases}$$

- Upstream boundary conditions

$$\mathbf{c}_{\text{up}}^{(j)}(t) = \begin{cases} q_j t + d_j, & \text{if} \quad t \in [t_j, t_{j+1}], \\ +\infty, & \text{else}, \end{cases}$$

- Downstream boundary conditions

$$\mathbf{c}_{\text{down}}^{(j)}(t) = \begin{cases} p_j t + b_j, & \text{if} \quad t \in [t_j, t_{j+1}], \\ +\infty, & \text{else}, \end{cases}$$

- Internal boundary condition

$$\mathbf{c}_{\text{intern}}^{(l)}(t,x) = \begin{cases} M^{(l)} + q_{\text{intern}}^{(l)}(t - t_{\text{min}}^{(l)}), & \text{if} \quad (t,x) \in \mathcal{D}^{(l)}, \\ +\infty, & \text{else} \end{cases}$$

- Decision variable

$$y := \Big(\underbrace{\ldots, k_i, \ldots}_{\text{initial densities}}, \underbrace{\ldots, q_j, \ldots}_{\text{upstream flows}}, \underbrace{\ldots, p_j, \ldots}_{\text{downstream flows}}, \underbrace{\ldots, M^{(l)}, q_{\text{intern}}^{(l)}, \ldots}_{\text{internal conditions}} \Big)$$

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- Optimization problem as a Mixed Integer Linear Programming (MILP)

$$\text{Maximize } g(y)$$
$$\text{subject to } \begin{cases} A_{\text{model}} y \leq b_{\text{model}}, & \text{(model constraints)}, \\ C_{\text{data}} y \leq d_{\text{data}}, & \text{(data constraints)}. \end{cases}$$

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- Objective function: maximize the downstream outflows

$$g(y) = \left(\mathbf{0}_{\mathbb{R}^n}, \mathbf{0}_{\mathbb{R}^m}, \mathbb{1}_{\mathbb{R}^m}, \mathbf{0}_{\mathbb{R}^o \times \mathbb{R}^o}\right) \cdot y^T = \sum_{j=0}^{m-1} p_j$$

Algorithm

1. Compute the optimal solution to the MILP

$$y^* := \Big(\underbrace{\ldots, k_i^*, \ldots}_{\text{initial densities}}, \underbrace{\ldots, q_j^*, \ldots}_{\text{upstream flows}}, \underbrace{\ldots, p_j^*, \ldots}_{\text{downstream flows}}, \underbrace{\ldots, \left(M^{(l)}\right)^*, \left(q_{\text{intern}}^{(l)}\right)^*, \ldots}_{\text{internal conditions}} \Big)$$

$$= \text{argmax}_y g(y)$$

2. Compute the traffic states $\mathbf{M}$ and $k = -\partial_x \mathbf{M}$ thanks to the explicit solutions [Qiu et al., 2013]
3. Deduce queue lengths by computing for any time step the extremal points of

$$\mathcal{Q}_\varepsilon(t) := \left\{ (\alpha, \beta) \;\middle|\; \begin{array}{l} \xi \le \alpha < \beta \le \chi, \\ |k(t,z) - \kappa| \le \varepsilon, \quad \forall z \in [\alpha, \beta] \end{array} \right\}$$

where $\varepsilon > 0$ is a prescribed sensitivity parameter

Outline

Compatibility conditions

Proposition (Compatibility conditions [Claudel and Bayen, 2011])

Consider a family of value conditions $\mathbf{c}_j$ *and define their minimum*

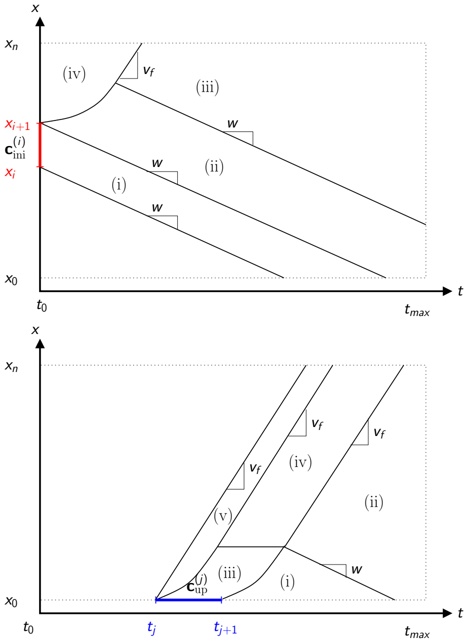
$$\mathbf{c}(t,x) := \min_{j \in \mathbb{J}} \mathbf{c}_j(t,x).$$

Then, the solution $\mathbf{M}$ *of the LWR-BA PDE verifies*

$$\mathbf{M}(t,x) = \mathbf{c}(t,x), \quad \text{for any} \quad (t,x) \in Dom(\mathbf{c}),$$

if and only if

$$\mathbf{M}_{\mathbf{c}_i}(t,x) \geq \mathbf{c}_j(t,x), \quad \text{for all} \quad i,\ j \in \mathbb{J}, \quad \text{and} \quad (t,x) \in Dom(\mathbf{c}_j).$$



Check

$$\mathbf{M}_{\mathbf{c}_{\text{ini}}^{(i)}} \geq \mathbf{c}_{\text{up}}^{(j)}$$

and

$$\mathbf{M}_{\mathbf{c}_{\text{up}}^{(j)}} \geq \mathbf{c}_{\text{ini}}^{(i)}$$

only for crossing points of domains of influence

Data constraints

Assume that the data constraints are linear w.r.t. the decision variable y

$$C_{\text{data}}\, y \le d_{\text{data}}.$$

1. Downstream outflow constraint (red light)

$$p_j = 0, \quad \forall\ j \text{ s.t. } \Omega_{red} \cap [t_j, t_{j+1}] \ne \emptyset,$$

2. **[Loops]** Upstream flow data q^{meas} with errors e^{meas}_{flow}

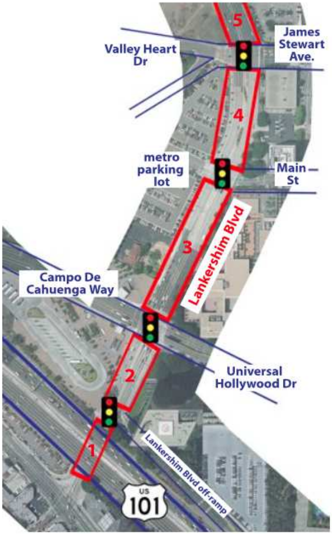
$$(1 - e^{meas}_{flow}) q^{meas}(t) \le q_j \le (1 + e^{meas}_{flow}) q^{meas}(t), \quad \forall\ t \in [t_j, t_{j+1}]$$

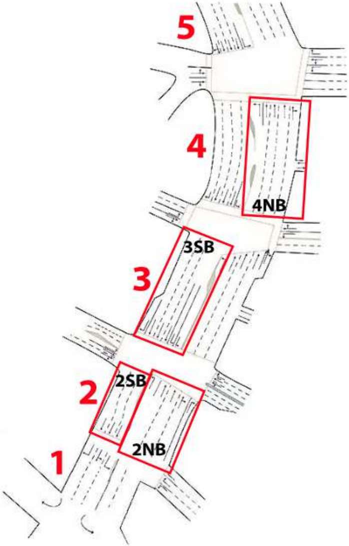
3. **[GPS]** Travel times data d^{meas}_{travel} with errors e^{meas}_{time}

$$\mathbf{M}\left(t^{meas}_{exit} - d^{meas}_{travel} - e^{meas}_{time}, \xi\right) \le \mathbf{M}(t^{meas}_{exit}, \chi) \le \mathbf{M}\left(t^{meas}_{exit} - d^{meas}_{travel} + e^{meas}_{time}, \xi\right).$$

Outline

NGSIM dataset (2006)





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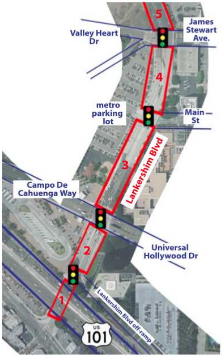
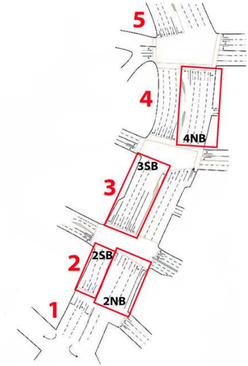
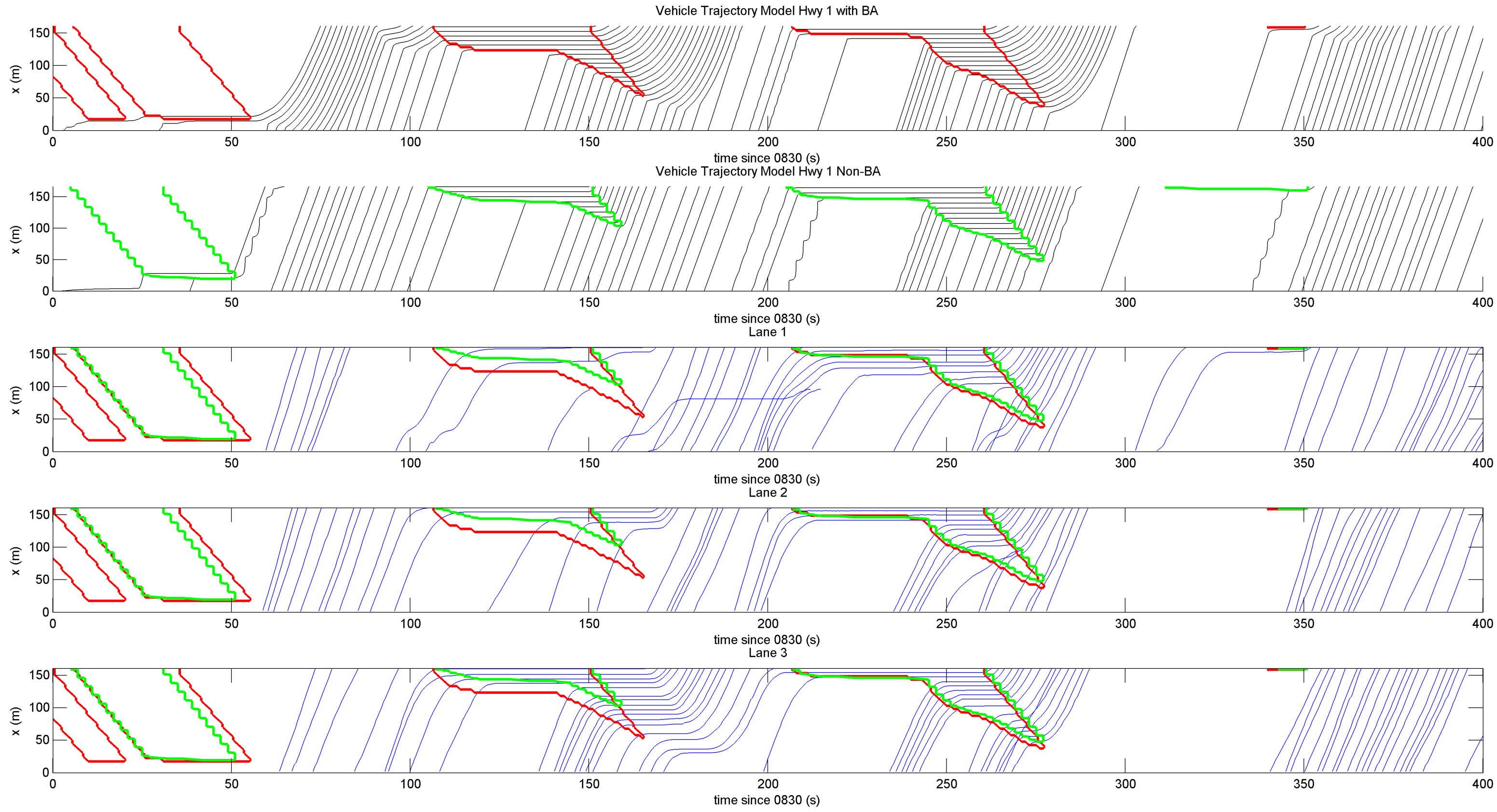




- monitored section = 5 blocks and 4 signalized intersections
- individual trajectories for each vehicle (+2,400) over 30 min

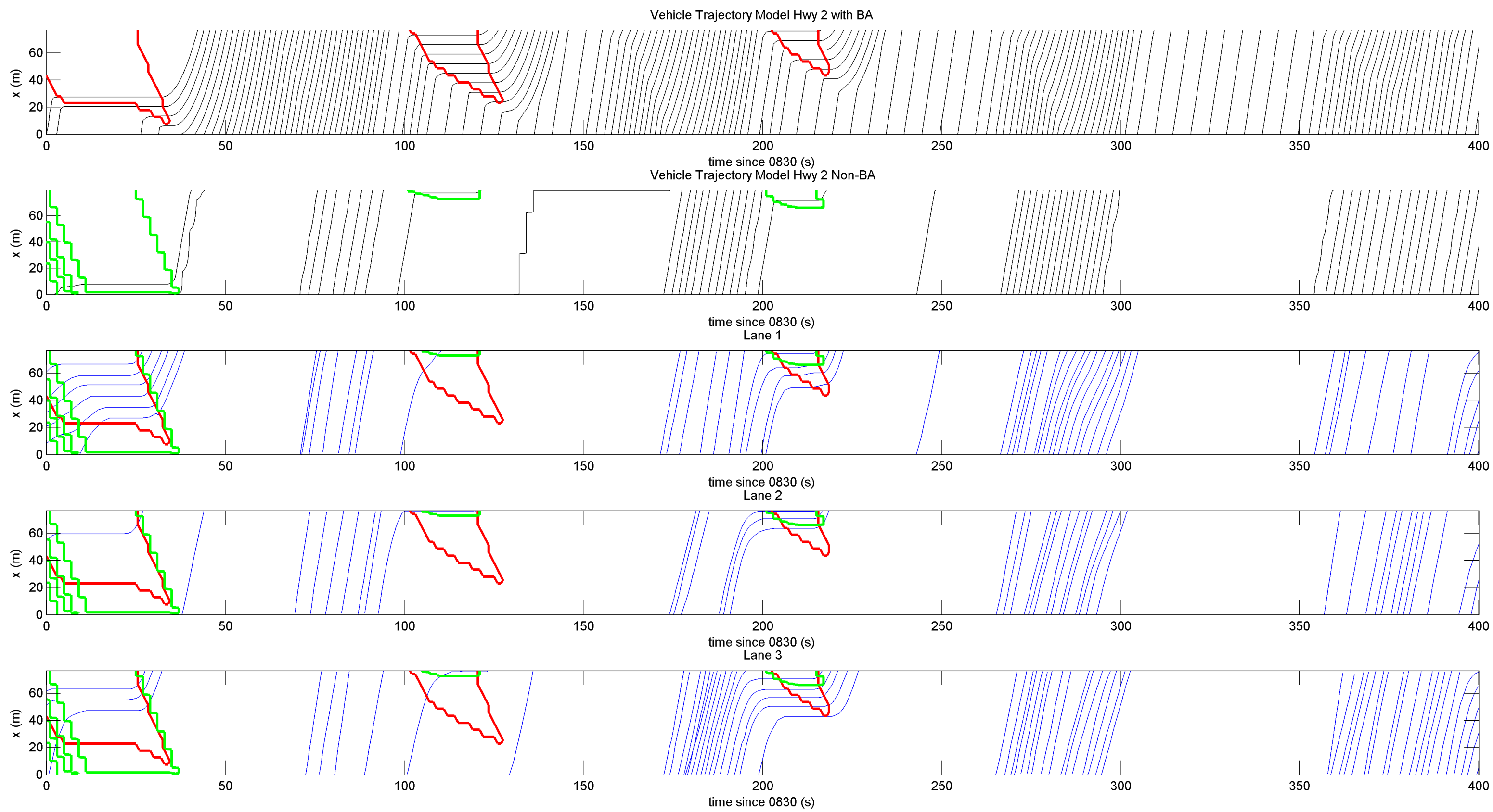
Queue Estimation on Networks

Link 1



Queue Estimation on Networks

Link 2



Thanks for your attention

Any question?

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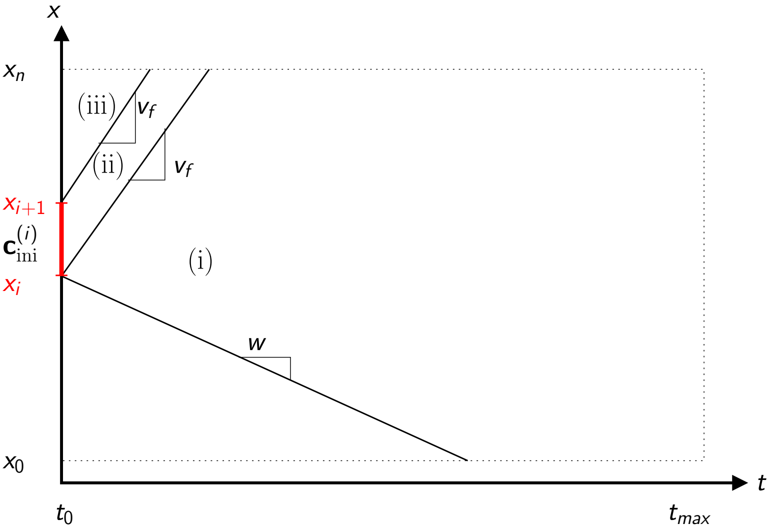
Outline



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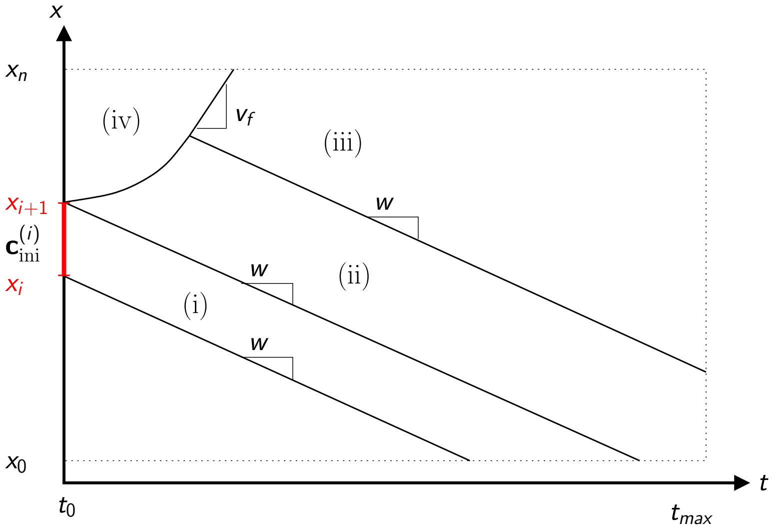

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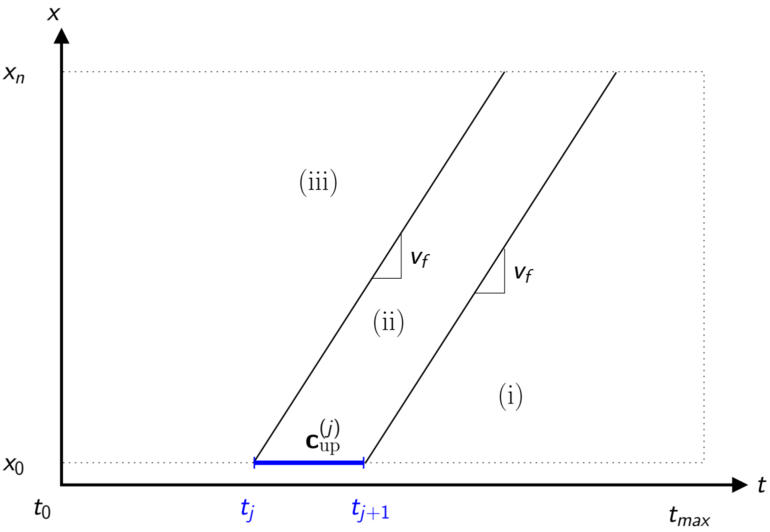
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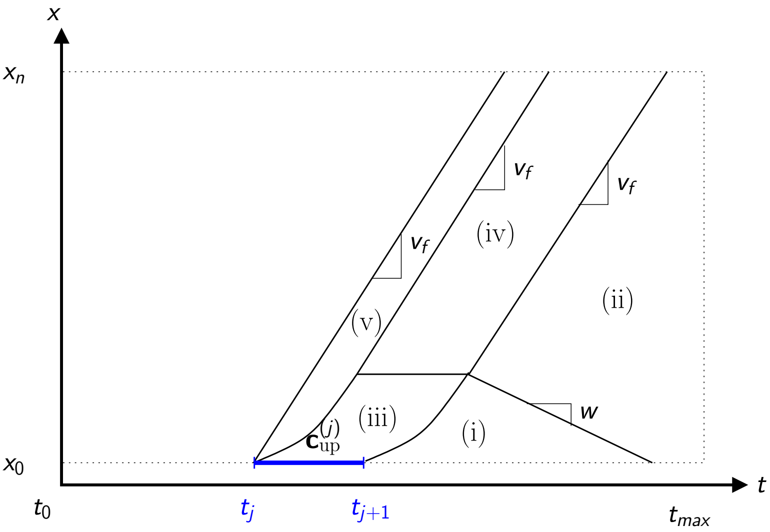
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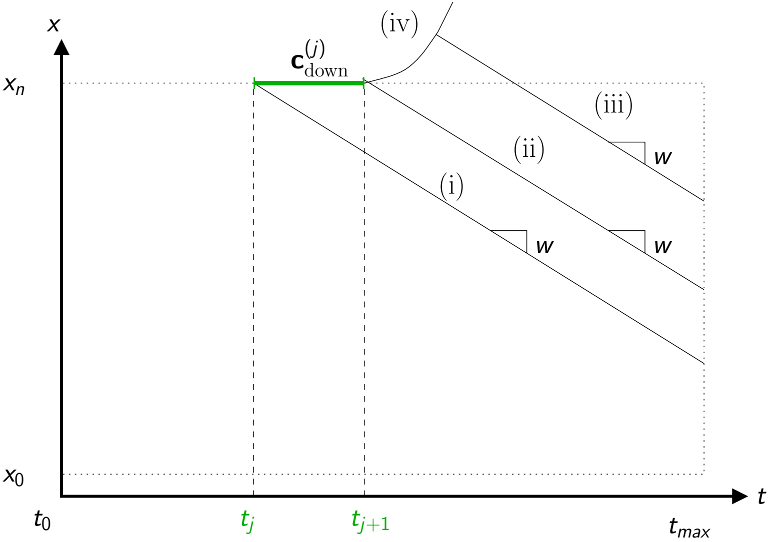
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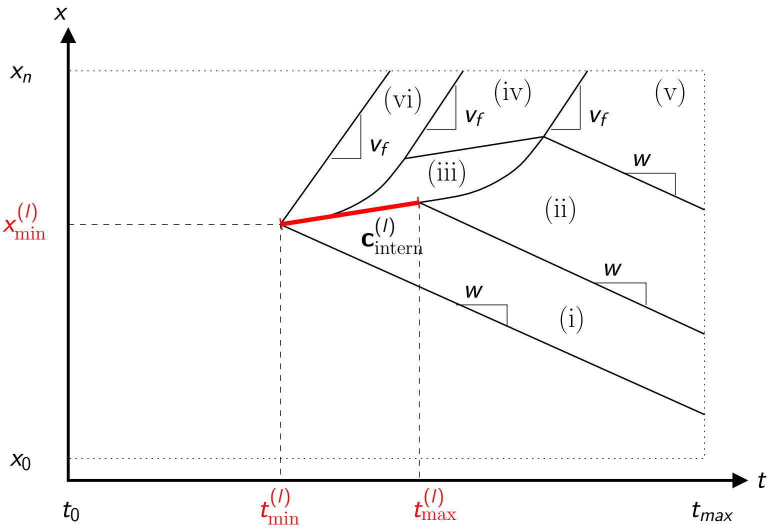

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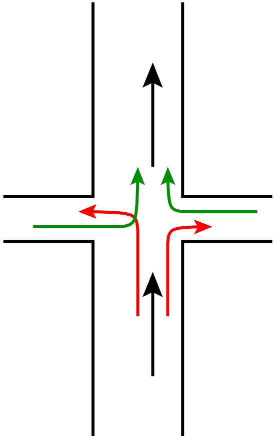

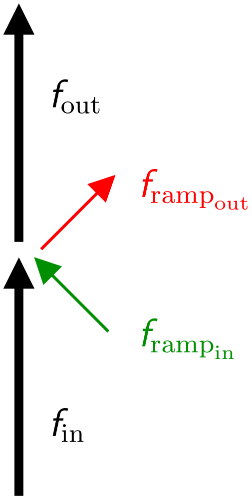
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