A queue estimation method aware of bounded acceleration: application to arterials

Edward S. Canepa
King Abdullah University of Science and Technology,
4700 KAUST, Thuwal Jeddah, Saudi Arabia.
Email: edward.canepa@kaust.edu.sa

Guillaume Costeseque
Inria Sophia Antipolis - Méditerranée,
2004, route des Lucioles, 06902 Sophia Antipolis, France.

Christian G. Claudel
Department of Civil, Architectural and Environmental Engineering,
The University of Texas at Austin, Austin, TX 78712-0273, USA.

1 Introduction

1.1 Motivation and objectives

Real-time accurate estimation of traffic state on networks is a major issue for efficient traffic management schemes. It is particularly the case for signalized intersections where the majority of responsive management schemes rely on the estimation of the queue lengths.

In this work, we are concerned with Hamilton-Jacobi partial differential equations (HJ PDE) derived from a modified LWR traffic flow model able to reproduce the boundedness of the vehicles acceleration. We intend to develop an optimization framework in order to estimate queue lengths on arterial traffic road networks, taking into account data from conventional sensors and from GPS-enabled sensors. We also aim at comparing our results on real data coming from the NGSIM Lankershim Boulevard dataset, using the LWR model with and without the bounded acceleration.

1.2 Quick review of the literature

A first optimization-based method has been developed and tested in Anderson et al. (2014). The optimal control framework relies on the one proposed in Claudel and Bayen (2011) and explicitly established in Canepa and Claudel (2012) for triangular Hamiltonian and piecewise affine conditions. However, the work of Anderson and co-authors consider the seminal first order LWR model (Lighthill and Whitham, 1955; Richards, 1956)

$$\frac{\partial \rho}{\partial t} + \frac{\partial \psi(\rho)}{\partial x} = 0, \quad \text{on} \quad (0, +\infty) \times \mathbb{R}, \tag{1}$$

where $\rho \in [0, k_m]$ and $q = \psi(\rho)$ denote respectively the density and the flow. While being simple and robust, the LWR model doesn't take into account the boundedness of the vehicles acceleration. This feature can have a significant impact on the dynamics of traffic flow, especially on arterials when a traffic light turns green.

1.3 Recalls on the LWR model with bounded acceleration

Assume a triangular fundamental diagram (FD) $\psi : \rho \mapsto \psi(\rho) = \min\{v_f \rho\ ,\ w(\rho - k_m)\}$ where $v_f > 0$ stands for the maximal speed and $w < 0$ is the congestion wave speed. Under this assumption, the macroscopic traffic flow models taking into account a bounded acceleration proposed in (Lebacque, 2002, 2003) and (Leclercq, 2007) are equivalent to:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \quad \text{with} \quad \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} \begin{cases} < A, & \text{if} \quad v = V_e(\rho), \\ = A, & \text{if} \quad v \neq V_e(\rho), \end{cases} \tag{2}$$

where A is the maximal acceleration assumed to be identical for all the vehicles. $V_e : \rho \mapsto V_e(\rho)$ denotes the equilibrium speed-density FD such that $\psi(\rho) = \rho V_e(\rho)$ for any $\rho \in [0, k_m]$. It is noteworthy that the vehicle trajectories in the bounded acceleration (BA) areas, say when $v \neq V_e(\rho)$, can be explicitly computed as parabolas. This BA phase constitutes the single difference with the original LWR model (1).

1.4 Setting of the optimization framework

Consider a road section $[\xi, \chi]$. The *Moskowitz function* or cumulative vehicle count $\mathbf{M}(t, x)$ at position $x \in [\xi, \chi]$ and time $t > 0$, is defined such that $\dfrac{\partial \mathbf{M}(t,x)}{\partial x} = -\rho(t, x)$. It follows that if ρ solves (2), then $\mathbf{M}$ satisfies the following homogeneous HJ equation

$$\begin{cases} \dfrac{\partial \mathbf{M}(t,x)}{\partial t} - \psi\left(-\dfrac{\partial \mathbf{M}(t,x)}{\partial x}\right) = 0, & \text{on} \quad (0, +\infty) \times [\xi, \chi], \\ \mathbf{M}(t, x) \leq \mathbf{c}(t, x), & \text{on} \quad \text{Dom}(\mathbf{c}), \end{cases} \tag{3}$$

where $\mathbf{c} := \min\limits_{i \in J} \mathbf{c}_i$ denotes the constraints. If one can define partial solutions $\mathbf{M}_{\mathbf{c}_i}$ for any constraint $\mathbf{c}_i$, $i \in J$, then the global solution of (3) is given by

$$\mathbf{M}(t, x) = \min_{i \in J} \mathbf{M}_{\mathbf{c}_i}(t, x) \quad \text{on} \quad (0, +\infty) \times [\xi, \chi]. \tag{4}$$

Assume now that the constraints $(\mathbf{c}_i)_{i \in J}$ are linear functions for a *decision variable* y correctly defined. Then, the model constraints which ensure that (4) holds true, can be rewritten as linear constraints for y (see Claudel and Bayen (2011); Canepa and Claudel

(2012)), say $C_{model}\ y \leq b_{model}$. Thus, the optimal control problem (following Anderson et al. (2014)) writes as a Mixed Integer Linear Program (MILP)

$$\begin{aligned} &\text{Maximize } g(y) \\ &\text{such that } \begin{cases} C_{model}\ y \leq b_{model}, & \text{(model constraints)}, \\ C_{data}\ y \leq b_{data}, & \text{(data constraints)}, \end{cases} \end{aligned} \tag{5}$$

where g denotes the cost function (maximization of the outflows for instance). The data constraints (C_{data}, b_{data}) could come from (i) estimations of error on the sensors or (ii) travel times estimates on $[\xi, \chi]$.

Thanks to the explicit solutions for the HJ PDE (3) given in Qiu et al. (2013), one can compute the traffic states $\mathbf{M}(t, x)$ (or $\rho(t, x)$). This method is exact and it allows fast computations in time. Notice that the boundary flows and the initial conditions are given by $y^* := \text{argmax}_y g(y)$. The queues are finally deduced by delimiting the zones where the maximal density k_m is computed.

2 Results and Discussions

2.1 Numerical example

The sample we use (NGSIM, 2006) was collected on a 1,600-foot stretch of the Lankershim Boulevard in Los Angeles, CA, encompassing 4 intersections equipped with traffic signals. The detailed trajectory data of more than 2,440 vehicles were recorded from 08:30 a.m. to 09:00 a.m. on June 16, 2005.

On the Figure 1 below, we compare the vehicle trajectories and position of queues (in red) with the LWR model with bounded acceleration (top), with the real trajectories data (bottom). All lanes of the link are summed up. We have considered 10 initial condition and 360 boundary condition blocks. The estimates correspond to the scenario that minimizes the total number of vehicles at the initial time, subject to sensor measurement data and model constraints.

2.2 Discussion

The LWR model with bounded acceleration is of great interest for queue estimation on arterials. The optimization-based method we develop here allow to take into account this modified LWR model for computing queue lengths on urban networks. However, the NGSIM dataset we use is not totally well suited to highlight the benefit of introducing the bounded acceleration since one can notice that the trajectories of cars at the entry of the section do not seem to be slow down by the upstream traffic light (while one can expect that in reality this is the case for re-starting traffic).

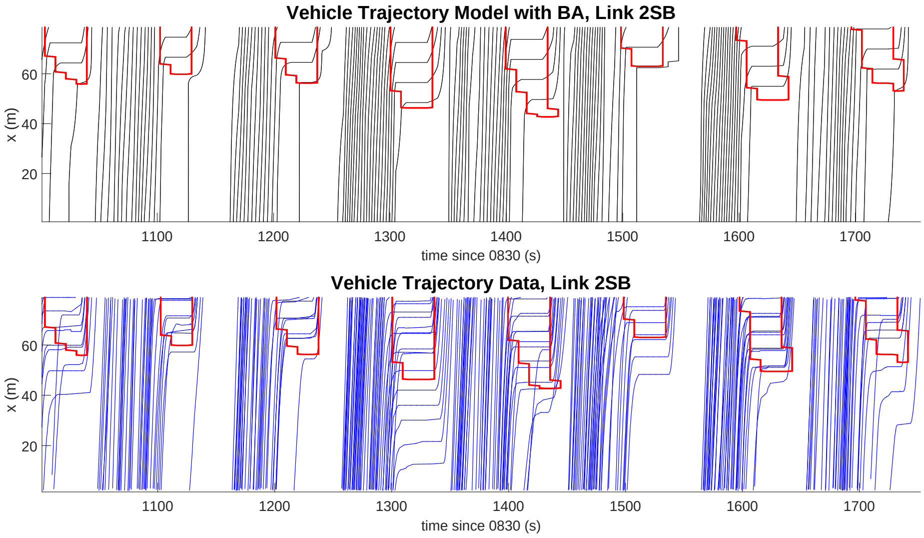

Figure 1: Traffic state estimation using the LWR model with bounded acceleration.

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