Nonlinear Filter Design for Pose and IMU Bias Estimation

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Introduction

- Context:
  - High quality pose estimation $\rightarrow$ orientation and position.
  - Data fusion between different sources... maximizes virtues and minimizes drawbacks of each sensor!
  - Extreme case of fast dynamics, e.g. MUAVs.
- IMU (Inertial Measurement Unit):
  - **High frequency**: 50 to $1\mathrm{k}$ $[\mathrm{Hz}]$.
  - Incremental measurements.
  - angular rate **gyroscopes**, **accelerometers**.
  - Drawbacks: measurement high frequency **noise** and **additive offset**.
  - Recover pose by IMU integration: **drifts quickly**.
- Pose measurements:
  - Advantages: no drift.
  - **Low frequency**: generally from 1 up to 25 $[\mathrm{Hz}]$.

Nonlinear observer design

Orientation and gyro bias dynamics

$$\begin{cases} \frac{d}{dt}R = RS(\omega - \omega_b), \\ \frac{d}{dt}\omega_b = 0. \end{cases} \quad (1)$$

Position and accelerometer bias dynamics

$$\begin{cases} \frac{d}{dt}p = v, \\ \frac{d}{dt}v = R(a - a_b) + g_{\mathcal{I}}, \\ \frac{d}{dt}a_b = 0. \end{cases} \quad (2)$$

Measurements

$$y = (\omega, a, R, p). \quad (3)$$

Orientation and gyro bias observer

$$\begin{cases} \frac{d}{dt}\widehat{R} = \widehat{R}S(\omega - \widehat{\omega}_b + \alpha_R), \\ \frac{d}{dt}\widehat{\omega}_b = \alpha_\omega. \end{cases} \quad (4)$$

Position and accelerometer bias observer

$$\begin{cases} \frac{d}{dt}\widehat{p} = \widehat{v} + \alpha_p, \\ \frac{d}{dt}\widehat{v} = R(a - \widehat{a}_b) + g_{\mathcal{I}} + \alpha_v, \\ \frac{d}{dt}\widehat{a}_b = \alpha_a. \end{cases} \quad (5)$$

Nonlinear observer design

Error definition

$$\tilde{R} \triangleq R\hat{R}^T,\ \tilde{\omega}_b \triangleq \omega_b - \hat{\omega}_b,$$
$$\tilde{p} \triangleq p - \hat{p},\ \tilde{v} \triangleq v - \hat{v},\ \tilde{a}_b \triangleq a_b - \hat{a}_b.$$

- The **innovation terms** should make $(\tilde{R},\tilde{\omega}_b,\tilde{p},\tilde{v},\tilde{a}_b) = (I_3, 0, 0, 0, 0)$ an **asymptotically stable equilibrium point** of:

Error dynamics

$$\begin{cases} \frac{d}{dt}\tilde{R} = \tilde{R}S(-\hat{R}\tilde{\omega}_b - \hat{R}\alpha_R), \\ \frac{d}{dt}\tilde{\omega}_b = -\alpha_\omega, \end{cases} \tag{6}$$

$$\begin{cases} \frac{d}{dt}\tilde{p} = \tilde{v} - \alpha_p, \\ \frac{d}{dt}\tilde{v} = -R\tilde{a}_b - \alpha_v, \\ \frac{d}{dt}\tilde{a}_b = -\alpha_a. \end{cases} \tag{7}$$

Nonlinear observer design

Orientation and gyro bias estimation

- Orientation and gyro bias dynamics is **independent** from position and accelerometer bias.
- Several solutions with *semi*-global stability...
- Passive complementary filter on $\mathbf{SO}(3)$ (Mahony, Hamel & Pflimlin 2008).

Lemma

Let

$$\alpha_R = k_1 \widehat{R}^T \mathbf{vex}(\mathbf{P_a}(\widetilde{R})), \; \alpha_\omega = -k_2 \widehat{R}^T \mathbf{vex}(\mathbf{P_a}(\widetilde{R})),$$

with $k_1, k_2 > 0$. *Then, concerning the dynamics* (6)*:*
1) All solutions converge to $\mathbb{E}_s \cup \mathbb{E}_u$ *with* $\mathbb{E}_s = (I_3, 0)$, *and* $\mathbb{E}_u = \left\{ (\widetilde{R}, \widetilde{\omega}_b) \mid \mathbf{tr}(\widetilde{R}) = -1 \right\}$.
2) $(\widetilde{R}, \widetilde{\omega}_b) = (I_3, 0)$ *is a locally exponentially stable equilibrium.*

Nonlinear observer design

Position and accelerometer bias estimation

- First: R and $\omega_{\mathcal{B}}$ are available.
- Position and acceleration estimation:

Theorem

Let

$$\alpha_p = k_3\tilde{p}, \;\; \alpha_v = k_4\tilde{p}, \;\; \alpha_a = -k_5(I_3 + \tfrac{1}{k_3}S(\omega_{\mathcal{B}}))R^T\tilde{p}$$

with $k_3, k_4, k_5 > 0$ *such that* $k_5 < k_3 k_4$. *Then,* $(\tilde{p}, \tilde{v}, \tilde{a}_b) = (0, 0, 0)$ *is a globally exponentially stable equilibrium point of the position estimation error dynamics* (7).

- This theorem yields a **globally asymptotically stable estimator**.
- Stability is achieved **for any angular velocity** $\omega_{\mathcal{B}}$.
- The initial assumption can be relaxed.

Nonlinear observer design

Full pose observer

Corollary

Let

$$\begin{cases} \frac{d}{dt}\widehat{R} &= \widehat{R}S(\omega - \widehat{\omega}_b + \alpha_R), \\ \frac{d}{dt}\widehat{\omega}_b &= \alpha_\omega, \end{cases}$$

$$\begin{cases} \frac{d}{dt}\widehat{p} &= \widehat{v} + \alpha_p, \\ \frac{d}{dt}\widehat{v} &= g_{\mathcal{I}} + \widehat{R}(a - \widehat{a}_b) + \alpha_v, \\ \frac{d}{dt}\widehat{a}_b &= \alpha_a, \end{cases}$$

$$\text{with } \alpha_R = k_1 \widehat{R}^T \mathbf{vex}(\mathbf{P_a}(\widetilde{R})), \;\; \alpha_\omega = -k_2 \widehat{R}^T \mathbf{vex}(\mathbf{P_a}(\widetilde{R})),$$

$$\alpha_p = k_3 \widetilde{p}, \;\; \alpha_v = k_4 \widetilde{p}, \;\; \alpha_a = -k_5 (I_3 + \tfrac{1}{k_3} S(\omega - \widehat{\omega}_b)) \widehat{R}^T \widetilde{p}.$$

Assume that $k_1, \cdots, k_5 > 0$ *and* $k_5 < k_3 k_4$. *Then,*

1) The origin $(\widetilde{R}, \widetilde{\omega}_b, \widetilde{p}, \widetilde{v}, \widetilde{a}_b) = (I_3, 0, 0, 0, 0)$ *is a locally exponentially stable equilibrium.*

2) If $\widetilde{R}$ *converges asymptotically to* I_3, *then* $(\widetilde{\omega}_b, \widetilde{p}, \widetilde{v}, \widetilde{a}_b)$ *converges asymptotically to zero.*

Nonlinear observer design

Tuning the innovation gains

- **How to tune the innovation gains?**
- Rationale: analyze the error dynamics and each gain's effect.
- Using this procedure, one can define 5 settling times $\tau_i > 0$,

Gain tuning for the nonlinear observers:

$$k_1 = 3\frac{\tau_1+\tau_2}{\tau_1\tau_2},\ k_2 = 9\frac{1}{\tau_1\tau_2},$$

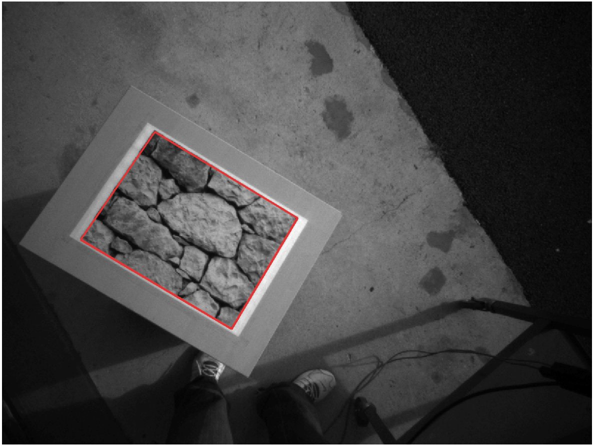
$$k_3 = 3\frac{\tau_3\tau_4+\tau_3\tau_5+\tau_4\tau_5}{\tau_3\tau_4\tau_5},\ k_4 = 9\frac{\tau_3+\tau_4+\tau_5}{\tau_3\tau_4\tau_5},\ k_5 = \frac{27}{\tau_3\tau_4\tau_5}.$$

- This definition of k_3, k_4, and k_5 satisfies $k_5 < k_3 k_4$.

Results

Enhancing visual pose estimation using data fusion – Visual estimation at 40 [Hz].

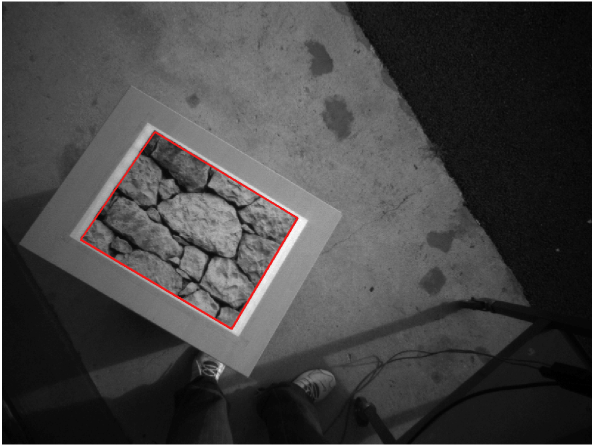
Pose estimation using (Benhimane & Malis 2007).
Visual update at 40 [Hz].



Results

Enhancing visual pose estimation using data fusion – Visual–IMU fusion at 200 [Hz].

The same visual pose estimation using the proposed filter.
IMU at 200 [Hz], visual update at 10 [Hz].



Results

Enhancing visual pose estimation using data fusion – Visual–IMU fusion at 200 [Hz].

The same visual pose estimation using the proposed filter.
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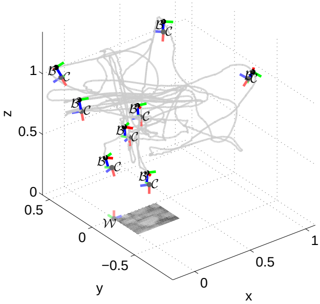

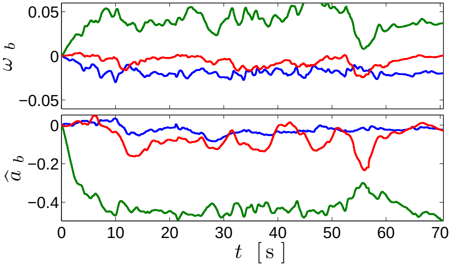

Last Remarks

Conclusion

- Design of **nonlinear observers** to estimate pose with online calibration of IMU bias.
- **Semi-global stability** is achieved.
- **Gain tuning** method based on errors settling times.

Future work

- Evaluation of **time varying innovation gains**, and the use of gain matrices, also relating with **measurement and estimate uncertainties**.
- Extension for **coordinate system parameter estimation**, e.g. camera-to-IMU orientation and translation.

Bibliography

Benhimane, S. & Malis, E. (2007). Homography-based 2D visual tracking and servoing, *Intl. Journal of Robotics Research* **26**: 661–676.

Mahony, R., Hamel, T. & Pflimlin, J.-M. (2008). Nonlinear complementary filters on the special orthogonal group, *IEEE Trans. on Automatic Control* **53**: 1203–1218.