Exercises for the course of Performance Evaluation
Master UBINET

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Ex. 1 — (Straffin, chapter 3., ex. 3 and 5)
Solve the following games:

	A	B
A	-3	5
B	-1	3
C	2	-2
D	3	-6

	A	B	C
A	5	2	1
B	4	1	3
C	3	4	3
D	1	6	2

Ex. 2 — Missile shield, (Straffin, chapter 5)
In a far, far, far away land, two countries, Red and Blues have not very pacific relations. Red wishes to destroy a Blue military base. Red has four missiles which will be fired in sequence. Two of the missiles have real warheads, while two are dummies. For defense, Blue has two anti-missiles. Each anti-missile can scan two Red missiles and destroy the first one it sees which has a real warhead. Red must choose the order in which to send the live warheads and the dummies, while Blue's choice is when to fire anti-missiles.
Model this situation as a zero-sum game, assuming that Blue wins (payoff +1) if it destroys both Red warheads, while Blue loses (payoff 0) if even one Red warhead gets through. Study the equilibria of the game.

Ex. 3 — Mirror selection
There are n users that can download a content from m identical servers with uploading bandwidth C. If different users select the same server, they equally share its bandwidth.

1. Consider $n = 2$ and $m = 2$ and model user interactions as a game. Study this game, in particular
   - determine if it is zero-sum or not,
   - determine equilibria in pure and mixed strategies and Pareto optimal outcomes.
2. For n and m larger than 2, what do you expect? Are you able to characterize pure strategies equilibria? How many equilibria in mixed strategies are you able to find?

Ex. 4 — Hawks, Doves, Bullies and Retaliators, (Straffin chapter 15)
Extend the Hawk-Dove game considering two different possible behaviours:

bully: in any contest, a bully shows initial fight. It continues to fight if its opponent does not fight back, otherwise it runs away;

retaliator: in any contest, a retaliator starts as a dove and does not attack. If it is attacked, then it fights back until it does not get the resources or gets injured.

Determine Evolutionary Stable Strategies and write the replicator dynamics equations.

Ex. 5 — Selfish WiFi nodes, [1]
Consider n IEEE 802.11 wireless stations (say it n) transmitting to a single Access Point. When stations work in saturation conditions, i.e. they always have a packet available in the transmission buffer, Distributed Coordination Function can be modeled as a slotted access protocol, while station behavior can be summarized in terms of per-slot access probability [2]. Let τ_i be the per-slot channel access probability representing the access strategy of station i. It is quite natural to consider that a selfish mobile station would like to maximize its uploading rate. For this purpose we assume that each station i can set its access probability τ_i. Station i experiences collisions during its own transmission, when at least one of the competing nodes transmits in the same slot. This happens with probability $p_i = 1 - \prod_{j \neq i}(1 - \tau_j)$. Note that the product $P_{idle} = (1-\tau_i)(1-p_i)$ is the probability that no station accesses the channel in a given slot and does not depend on the index $i = 1, 2, \cdots n$. Following [2], we can express the uplink throughput S_u^i for the i-th station as:

$$S_u^i = \frac{\tau_i(1-p_i)P}{P_{idle}\sigma + [1-P_{idle}]T} \tag{1}$$

where P is the frame payload which is assumed to be fixed, while σ and T are, respectively, the empty and the busy slot duration[1], so that the denominator is the expected slot duration ($E[T_{slot}]$). We consider the case of 802.11b physical layer, $P = 1500$ bytes, a data rate equal to 11 Mb/s, and an acknowledgment rate of 1 Mb/s. In such a scenario, by including physical preambles, acknowledgment transmissions, MAC headers, and interframe times, T and σ durations are equal to 1667 μs and 20 μs respectively.

1. We want to model the interaction of these stations as a single stage game where each station decides once for all its channel access probability. For simplicity let us assume that each station can only select 3 possible access probability values: 0, 1/2, 1[2]. Study the game for $n = 2$. In particular
   - determine if this game is zero-sum or not,
   - determine equilibria in pure and mixed strategies and Pareto optimal outcomes.

[1] We are implicitly considering a basic access scheme, with EIFS=ACK_Timeout +DIFS, which corresponds to have a fixed busy slot duration in both the cases of successful transmission and collision.

[2] Note that "transmit in each slot with probability $\tau_i = x$" and "transmit in each slot with probability $\tau_i = y$" are two *pure* strategies of station i. A mixed strategy using these two pure strategies would be "with probability q transmit in each slot with probability $\tau_i = x$, with probability $1 - q$ transmit in each slot with probability $\tau_i = y$".

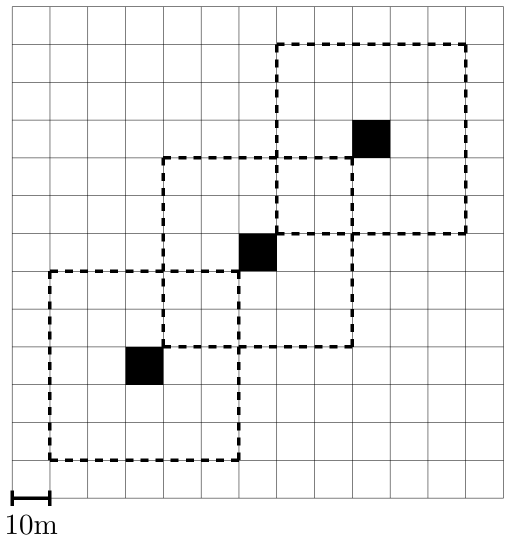
2. Imagine that the one-shot game defined above is the basic stage of a repeated game. Can better outcomes arise as equilibria if the repeated new game has finite horizon? and if it has infinite horizon?
3. Consider the previous game in the framework of Evolutionary Game Theory. Which strategies would be evolutionary stable?
4. How do you think the game would change if we would have n stations (consider a non-cooperative framework)? How if every station would be able to arbitrarily choose its channel access probability τ_i in $[0, 1]$?

Ex. 6 — Consider 2-hop routing and epidemic routing with a timer based recovery process. In particular, assume that whenever a relay node receives a copy of the message, it starts a timer. When the time expires, the copy is erased. The node does not keep track of the erased messages so that it can later receive another copy of the same message. Assume that timer durations are independent and identically distributed exponential random variables with expected value $1/\rho$.

1. Modify the Markovian models presented in the course in order to include also the timer in two following two cases:
   a) when also the source uses the timer and erases the original message,
   b) when the source does not use the timer.

   Discuss qualitatively the different performance in the two cases, for example comparing the corresponding cumulative distribution functions.

Ex. 7 — Neighborhood problem.
There are three houses inside the same property. Each house belongs to a different family and has its own access point connected to the Internet. The figure below represents the houses (black squares) and the coverage of each access point (squares with dashed sides). Two (or more) families could decide to share their wireless access points, in order to be able to connect to the Internet from a larger area of the property. This problem can be modeled as a coalition game.



1. Assume that the utility of each family is equal to the surface S of the area from which it can connect to the Internet, minus a fixed cost C[3] if it shares its connection with another family. Determine:
   a) the characteristic function of the game;
   b) for which range of cost values the game is superadditive;
   c) the core of the game when $C = 100m^2$;
   d) the Shapley value of the game when $C = 100m^2$. Comment this result.
2. Assume now that the value of the coverage is not uniform, but a family values more places where it spends more time. We can the consider the utility equal to $\int_A \pi(x,y)\mathrm{d}x\mathrm{d}y$, where $\pi(x,y)$ is the probability that the family is at location (x,y) minus the fixed cost $C = 100m^2$. Consider that each family moves according to the following mobility model: it spends half of the time at home and half moving inside the property according to random waypoint model. Discuss qualitatively how do you expect the previous results to change.

[3]Note that this cost has to be expressed in surface equivalent units.

Bibliography

[1] Ilenia Tinnirello, Laura Giarré, and Giovanni Neglia. "The Role of the Access Point in Wi-Fi Networks with Selfish Nodes". In *Proc. of the 1st International Conference on Game Theory for Networks (GameNets 2009)*, Instanbul, Turkey, May 2009.

[2] Giuseppe Bianchi, "Performance Analysis of the IEEE 802.11 Distributed Coordination Function", *IEEE Journal of Selected Areas in Communication*, vol. 18, no. 3, pp.535-547, March 2000.