On Unstructured File Sharing Networks

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***Abstract*— We study the interaction among users of unstructured file sharing applications, who compete for available network resources (link bandwidth or capacity) by opening multiple connections on multiple paths so as to accelerate data transfer. We model this interaction with an *unstructured file sharing game*. Users are players and their strategies are the numbers of sessions on available paths. We consider a general bandwidth sharing framework proposed by Kelly [1] and Mo and Walrand [2], with TCP as a special case. Furthermore, we incorporate the Tit-for-Tat strategy (adopted by BitTorrent [3] networks) into the unstructured file sharing game to model the competition in which a connection can be set up only when both users find this connection beneficial. We refer to this as an *overlay formation game*. We prove the existence of Nash equilibrium in several variants of both games, and quantify the losses of efficiency of Nash equilibria. We find that the loss of efficiency due to selfish behavior is still unbounded even when the Tit-for-Tat strategy is believed to prevent selfish behavior.**

I. Introduction

Recently peer-to-peer applications (e.g., BitTorrent [3], Kazaa, eDonkey, and Gnutella [4]) have become very popular. They can be major contributors of the Internet traffic. For example, Sprint's IP Monitoring Project [5] shows that in April 2003, $20-40\%$ of total bytes corresponded to peer-to-peer traffic on one backbone link. CacheLogic [6] estimates that peer-to-peer generated 60% of all US Internet traffic at the end of 2004.

We refer to the networks for these peer-to-peer applications as unstructured file sharing overlay networks. These networks are overlay networks since users forward or relay traffic for each other and each relay hop can be thought of as a virtual link. These networks are also *unstructured* because there are no well-defined network topologies, and users are not under the control of some central entity. For comparison, Resilient Overlay Network [7] is a *structured* overlay network. Given the increasingly large share of Internet traffic from unstructured file sharing networks, it is important to understand the behavior and performance of such networks, and such a fundamental understanding will certainly help ISPs and aid in the design of future Internet architecture.

In this paper, we investigate the strategic behavior of self-interested peers/users of such unstructured file sharing overlay networks. Our work differs from previous works on peer-to-peer applications, whose focuses are on file searching and replication [8], and topology discovery [9]. Specifically, our investigations are from two different angles.

First, we study the interaction among users of unstructured file sharing applications, who compete for available network resources (link bandwidth or capacity) by opening multiple connections or sessions on multiple paths so as to accelerate data transfer. We introduce an *unstructured file sharing game* to model this interaction. In this game, users are players and their strategies are the numbers of sessions on available paths. The data rate allocated to connections are determined by the network. The mechanism of rate allocation considered by us is a general bandwidth sharing framework proposed by Kelly [1] with TCP networks as special cases [10][2]. Our focus is on TCP networks in which all connections/sessions are TCP connections. The unstructured file sharing game generalizes the *TCP connection game* introduced in [11] where the competition for a single bottleneck link capacity is investigated.

Second, we incorporate the Tit-for-Tat strategy into the unstructured file sharing game. This strategy is widely known and built into BitTorrent [3] networks. With this strategy, peers set up a connection between themselves only when they both find it beneficial. We model this interaction scenario as an *overlay formation game*. In order to make our model tractable, we restrict users to open either zero or one connection to another peer.

In both games, users are interested in maximizing their benefits, a combination of some utility function and the cost associated with maintaining data transfer sessions. We assume that utility functions are increasing and concave functions of the data throughput in bits per second. Throughput is defined as the successful packet delivery rate. The cost incurred to users includes memory cost and CPU cost. As in [11], we consider a cost that is proportional to the total number of connections opened by a user. We also consider another type of cost which is proportional to a user's packet sending rate.

We are interested in the following questions. First, does there exist a stable network state (i.e., Nash equilibrium (NE) [12]) in both games? If so, what is the system performance at a NE? Specifically, we are interested in the loss of efficiency of a NE and the price of anarchy[1] [13] of NEs. These metrics capture how bad the competition can be among self-interested users. Here we focus on pure strategy NE.

We make the following contributions.

First, we give a formal formulation of unstructured file sharing game, and show by examples that multiple NEs exist on general network topologies. We then focus on parallel link networks and star networks, which are used to model peer-to-peer applications (similar topologies were also studied in [14][15]). We prove the existence of NE of unstructured file sharing games on both networks, and find that, if users

[1]The loss of efficiency of a NE is defined as the ratio of the optimal system performance over the system performance at the NE, and the worst loss of efficiency of all NEs is referred to as the price of anarchy.

are not resource constrained, the efficiency losses of NEs can be unbounded (i.e., price of anarchy is arbitrarily large). Fortunately, if there are resource constraints for users, the efficiency losses are upper bounded. We also demonstrate the stability of NE in best-response dynamics in several variants of the game.

Second, we model the Tit-for-Tat strategy in unstructured file sharing networks by an overlay formation game. We show analytically the existence of equilibrium overlay networks and that the loss of efficiency can be arbitrarily large. Tit-for-Tat is believed to prevent selfish behavior. However, our results show that the loss of efficiency due to selfish behavior can still be unbounded.

The rest of this paper is organized as follows. Related work is presented in Section II. The problem formulation for unstructured file sharing game is given in Section III. In Sections IV and V, we focus on unstructured file sharing game on a parallel link network and star network. We address the overlay formation game in Section VI. Conclusions are given in Section VII.

II. Related Work

Johari *et al* [16] study a congestion game where users of a congested resource anticipate the effect of their actions on the price of the resource. In [16] users compete for each link independently from other links in the network. But this independence characteristic is not true for our model, because if a user opens a connection on a path, then all links of this path must carry this connection. [17] and [11] study the interactions among selfish TCP users competing for a single bottleneck link. The unstructured file sharing game in this paper can be thought of as a generalized version of the game in [11].

[18][19] propose multi-path congestion controllers by which users can coordinate the data transfer sessions on several different paths to improve data throughput. A multi-path congestion controller chooses rates at which to send data on all of the paths available to it. In our models, all sessions controlled by a single user are independent congestion controllers. [14] studies how Tit-for-Tat affects selfish peers who are able to set their uploading bandwidth. Our work differs from [14] in that we assume that a user can benefit by changing the number of connections to open. The analytical framework for our overlay formation game is in [20].

III. Unstructured File Sharing Game

A. Formulations

Consider a network consisting of J links, numbered $1, ..., J$. Link j has a capacity given by $C_j > 0$; we let $\mathbf{C} = (C_1, C_2, ..., C_J)$ denote the vector of capacities. A set of users $\{1, ..., R\}$ share this network. We assume that there exists a set of paths through the network, numbered $1, ..., P$. By an abuse of notation, we will use J, R, P to also denote the sets of links, users, and paths, respectively. Each path $p \in P$ uses a subset of the set of links J; if link j is used by path p, we will denote this by writing $j \in p$. Each user $r \in R$ has a collection of paths available through the network; if path p serves user r, we will denote this by writing $p \in r$.

Each user can open a number of concurrent connections n_{rp} on each path p with $p \in r$. This defines a strategy vector for user r as $\mathbf{n}_r = (n_{rp})$ with $p \in P$ and $p \in r$. Then a composite strategy vector of all users is given by $\mathbf{n} = (\mathbf{n}_1, ..., \mathbf{n}_R)$. For a given $\mathbf{n}$, a certain rate allocation mechanism allocates a traffic rate y_p to each connection on path p. We will discuss rate allocation mechanisms in the following section. For now, we simply state that, $\forall p \in P$, y_p is a function of $\mathbf{n}$. We use vector $\mathbf{y} = (y_p, p \in P)$ to represent a rate allocation on all paths.

The total date rate or throughput G_r obtained by a user r is: $G_r(\mathbf{n}_r) = \sum_{p \in r} n_{rp} y_p$, where n_{rp} is the number of connections opened by user r on path p. As y_p ($\forall p \in P$) is a function of $\mathbf{n}$, the throughput of user r is a function of the number of connections of all users, namely, $G_r = f(\mathbf{n})$. Any feasible rate allocation $\mathbf{y}$ must satisfy the capacity constraint: $\sum_{r \in R} \sum_{p: j \in p} n_{rp} y_p \leq C_j, j \in J$.

We assume that user r receives a utility $U_r(G_r)$ when obtaining throughput G_r. We assume that U_r is a continuous, concave, and non-decreasing function of G_r, with domain $G_r \geq 0$. A user r has some cost $\Phi_r(\mathbf{n}_r)$ associated with opened connections. We assume that this cost is proportional to the total number of connections opened by this user on all its available paths[2]: $\Phi_r(\mathbf{n}_r) = \beta \sum_{p \in r} n_{rp}$. Note that $\beta \in [0, 1]$, and it is interpreted as the aggressiveness coefficient. Smaller β corresponds to more powerful computation resources. This type of cost is also considered in [11]. In general, we can assume that Φ_r is a continuous, convex, and non-decreasing function of $\mathbf{n}_r$.

The payoff or benefit of a user r is a linear combination of utility U_r and cost Φ_r, defined as:

$$B_r(\mathbf{n}_r) = U_r(\mathbf{n}_r) - \Phi_r(\mathbf{n}_r). \quad (1)$$

B. Rate Allocation Mechanism

We assume that the network allocates data rates to connections based on the α-bandwidth allocation scheme [10][1][2]:

$$\text{maximize}_{\mathbf{y}} \quad \sum_p w_p n_p^{\alpha} \frac{(y_p n_p)^{(1-\alpha)}}{1-\alpha} \quad (2)$$

$$\text{subject to} \sum_{r \in R} \sum_{p: j \in p} n_{rp} y_p \leq C_j, j \in J \quad (3)$$

$$n_p = \sum_{r: p \in r} n_{rp} \forall p \in P. \quad (4)$$

where w_p is the weight of path p. n_p is the number of connections or sessions on path p. Different values of α give different rate allocations. For example, as $\alpha \to \infty$, this allocation mechanism corresponds to Max-Min fairness. Rate allocation in a TCP network is well approximated with $\alpha = 2$ and $w_p = 1/(RTT_p)^2$. Here, RTT_p is the Round Trip Time (RTT) of path p.

In a single link case and where all paths have the same RTT, this α-bandwidth allocation is simplified to a *simple rate allocation mechanism* [11]. That is, for a link shared by n

[2]We assume that Φ_r is linear increasing in $\mathbf{n}_r$. There is a roughly linear relationship between allocated memory and the number of connections/sockets based on our simple experiments. However, it is not clear if computation cost (CPU usage) is linear in the number of opened sockets. This is our future research topic. Nevertheless, linear cost functions give us important insight into the game and provide us with a foundation for future research.

flows with the same RTT, each flow or connection gets an equal share of the bandwidth of the link, namely, $y = C/n$. Thus if a user r has n_r flows, then its throughput G_r is:

$$G_r(\mathbf{n}_r) = \begin{cases} Cn_r / \sum_{w \in R} n_w, & \text{if } n_r > 0 \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

Remarks. Note that this *simple rate allocation mechanism* cannot be extended to a network setting. Please see [21] for more details. Note that the authors of [16] can use this rate allocation mechanism because in their case, users compete for each link independently from other links. However, in our case, links cannot be treated independently, as all links of a path must carry the connections opened on this path. As shown in the following section, this requirement makes the throughput of a user neither concave nor convex in the numbers of connections opened by this user. Thus, it is difficult to apply the existing game-theoretic results (which requires concavity of utility functions) to the unstructured file sharing on general network topology. Thus, in this paper we focus on two specific networks: parallel links and a star.

C. Unstructured File Sharing Game

Based on the previous formulations, we now introduce an *unstructured file sharing game*. In this game, each user r tries to maximize its aggregate benefit B_r by adjusting $\mathbf{n}_r$, its number of connections on its available paths. Namely, a user r tries to solve the following optimization problem:

$$\max_{\mathbf{n}_r} B_r(\mathbf{n}_r, \mathbf{y}^*(\mathbf{n}_r)) \quad (6)$$

$$\text{s.t.} n_{rp} \in (0, n_{rp}^{max}], \forall p \in P_r \quad (7)$$

$$\mathbf{y}^* = \text{argmax}_{\mathbf{y}} \sum_p w_p n_p^\alpha \frac{(y_p n_p)^{(1-\alpha)}}{1-\alpha} \quad (8)$$

$$\text{s.t.} \quad \sum_{r \in R} \sum_{p: j \in p} n_{rp} y_p \le C_j, j \in J$$

$$n_p = \sum_{r: p \in r} n_{rp}, \forall p \in P$$

The decision variables of user r is given by vector $\mathbf{n}_r$. As indicated by (7), this game is a continuous kernel game [12] as we assume that a user's strategy is a real-valued vector. The set of available paths of user r is represented by P_r. (8) indicates that the throughput of each connection on a path is the solution of the optimization problem defined in (2). If $\alpha = 2$ and $w_p = 1/(RTT_p)^2$ and the network is a single bottleneck link, this game becomes the TCP connection game in [11].

For a general network, we cannot obtain an explicit form of function $B_r(\mathbf{n}_r)$ because there is no closed form solution for the rate allocation problem (8). However, as shown later, we can obtain an explicit form of $B_r(\mathbf{n}_r)$ for some specific networks such as grid network, parallel link, and star network.

In fact, (6) is a Bi-level Programming problem which in general is NP-hard [22]. In this paper, we do not try to obtain a general solution for (6) for each user. Instead, we focus on some special network topologies for which there exist analytically tractable and closed form solutions to (8), and for these networks, we investigate the existence of Nash equilibrium.

Let $\mathbf{n}_r^*$ represent the solution to user r's optimization problem defined above. Formally, we have $\mathbf{n}_r^* = \text{argmax}_{\mathbf{n}_r} B_r(\mathbf{n})$. A Nash equilibrium (NE) is defined as a composite strategy profile or a vector of connections of all users, and no user can gain by unilaterally deviating from it. We denote a Nash equilibrium by: $\mathbf{n}^* = (\mathbf{n}_1^*, \mathbf{n}_2^*, ..., \mathbf{n}_R^*)$.

The NE of this game represents the stable network state of the interaction among all users. The network performance loss at a NE is described by the loss of efficiency, defined as:

$$L_{eff} = B_{max}/B_{ne} \quad (9)$$

where B_{ne} is the total benefit of all users when the network is at a NE, and B_{max} is the maximum benefit. The worst efficiency loss of all NEs is also known as the *price of anarchy* [13].

Remarks. It is not necessarily true that the throughput $G_r(\mathbf{n}_r)$ is an increasing function of $\mathbf{n}_r$. For example, in the network shown in Figure 1, user r has three paths: p_1, p_2 and p_3. p_1 contains two links j_1 and j_2 with capacity C. p_2 contains link j_1 and p_3 contains link j_2. According to the simple rate allocation mechanism introduced before, if $\mathbf{n}_r = (0, 1, 1)$, then $G_r(\mathbf{n}_r) = 2C$. However, if user r increases its number of connections on path p_1 from zero to one, then $G_r(\mathbf{n}_r) = 3C/2$. Thus, $G_r(\mathbf{n}_r)$ is a decreasing function of n_{p1}.

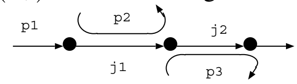

Fig. 1. A case where the throughput of user r is not increasing in $\mathbf{n}_r$.

One interesting special case is that a user can only choose either zero or one connection on a given available path. That is, (7) can be described as $n_{rp} \in \{0, 1\}, \forall p \in P_r$. In this case, each user only has finite number of strategies. This variant of the game is a finite game. According to [23], this game admits a mixed strategy NE. This NE is related to randomly choosing connections to other peers in BitTorrent applications [3]. This is an interesting future research topic.

D. Existence of Multiple Nash Equilibria in Grid Network

In this section, we use a simple example to illustrate the unstructured file sharing game and possible NEs. The network topology in this example is a so called grid network introduced in [10], shown in Figure 2.(a). A possible instance of this grid network is called "fish" network, shown in Figure 2.(b).

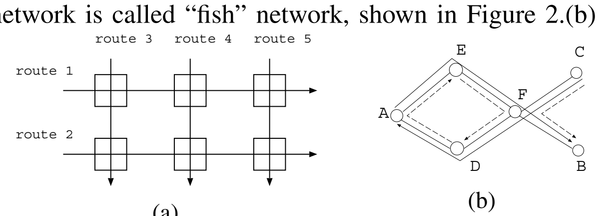

Fig. 2. (a) is a grid network where squares represent links. (b) is an instance of (a). $A \to E \to B$ and $C \to D \to A$ correspond to route 1 and 2 in (a). $D \to A \to E, E \to F \to D, C \to F \to B$ correspond to routes 3, 4, 5.

A closed form rate allocation based on the α-bandwidth sharing mechanism for such a grid network is given in [10]. Specifically, if there are K horizontal and L vertical routes, then the total throughput on horizontal path p is given by

$$n_p y_p = \frac{(\sum_{k=1}^K \frac{1}{RTT_k} n_k^\alpha)^{1/\alpha}}{(\sum_{k=1}^K \frac{1}{RTT_k} n_k^\alpha)^{1/\alpha} + (\sum_{l=1}^L \frac{1}{RTT_l} n_l^\alpha)^{1/\alpha}} \quad (10)$$

where n_p denotes the number of flows on horizontal path p. y_p is the throughput of a single flow on path p.

In the following, we discuss two variants of the game by considering two users playing the game on the grid network. User 1 uses route 1 and user 2 uses route 2. Suppose $\alpha = 2$ in (10), which corresponds to TCP. Suppose that all vertical and horizontal routes have RTT of 50ms, and there are 10 background flows on all vertical routes.

Benefit includes throughput only. When both users are only concerned with total throughput and have no resource limitations, we have identified a situation where there is a unique NE, at which both players open their maximal allowable number of connections. See [21] for details.

Benefit includes both throughput and cost. In this variant of the game, not only is B_r neither a concave nor a convex function of its number of connections $\mathbf{n}_r$, but B_r is not always increasing in $\mathbf{n}_r$. See [21] for details.

Consider that both players have the same $\beta = 0.0005$. We define the best response $\mathbf{n}_r^*$ of player r as the solution of r's optimization problem given fixed strategies of all other players. In Figure 3, we plot the best response curves of both players. Note that there are three intersecting points. An intersecting point is a NE because at that point, each user's response is the best response to the other user's strategy. Thus, there are three NEs in this game. For comparison, in the single link TCP connection game [11], there is only one unique NE when the cost is proportional to the number of connections.

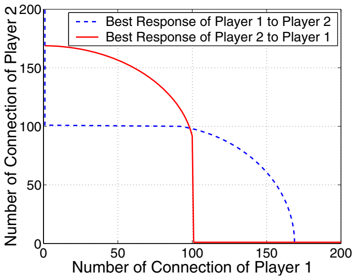

Fig. 3. Best response curves of both player 1 and player 2.

It is also interesting to note that these two players do not share any common link (Figure 2), so, their interaction arises because they share links with other common sessions.

This simple example indicates that the interaction among multiple users on a general network topology can be much more complex than the single link TCP connection game. The existence and uniqueness of Nash equilibrium can depend on network topologies and the utility functions adopted by users.

In the following, we focus on two special networks: a parallel link network and a star network. The latter can be used to model peer-to-peer networks, as shown in Section V.

IV. Parallel Link Network

In this section, we investigate an unstructured file sharing game on a parallel-link network where all users share a common source node and a common destination node interconnected by a number of parallel links. Parallel-link networks can be used as simple models for unstructured file sharing. For example, in eDonkey networks [4], a peer can download a file from multiple other peers providing this file. There are possibly many peers simultaneously downloading the same file, and they can be thought of as associated with a common destination node. Each of the file-providing peers can be thought of as a "link" or "path" connecting the common destination node with a common super virtual file-providing source node. Those downloading peers compete for these parallel links/paths for bandwidth. This scenario can be approximated by a parallel link network.

In this section, we first show the existence of stable network states (NEs) on a parallel-link network. We then present the results on the efficiency loss of NE and the stability of NE in the best-response dynamics.

A. Nash Equilibrium

Suppose that there are L links and R users. By an abuse of notation, we will use L and R to denote the set of links and the set of users respectively. An example of a parallel link network is shown in Figure 4. The throughput G_{rj} obtained by user r on link j is given by the simple rate allocation mechanism introduced in the previous section: $G_{rj}(n_{rj}) = C_j n_{rj}/RTT_{rj}/(\sum_{k=1}^{R} n_{kj}/RTT_{kj})$, where RTT_{rj} is the Round Trip Time of user r on link/path j, C_j is the capacity of link j, and n_{rj} is the number of connections of user r on link j. The strategy of user r is a vector of the number of connections on its available paths or links: $\mathbf{n}_r = (n_{r1}, ..., n_{rL})$ and $n_{rj} \in (0, n_r^{max}], \forall j \in L$. n_r^{max} is the maximum allowable number of connections for user r. As before, this game is a continuous kernel game [12].

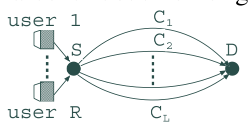

Fig. 4. A parallel-link network topology.

In this section, we only consider the case where $U_r(\mathbf{n}_r) = G_r(\mathbf{n}_r)$. The benefit or payoff obtained by user r is: $B_r(\mathbf{n}_r) = G_r(\mathbf{n}_r) - \Phi_r(\mathbf{n}_r)$.

We consider two scenarios: an unconstrained game and a constrained game. In an unconstrained game, there is no upper limit on the total number of connections a user can open. In a constrained game, each user must choose a certain total number of connections (motivated by BitTorrent [3]). We have shown the existence of a unique NE in both constrained and unconstrained games. More results for the constrained game can be found in [21]. In the following we only present results for the unconstrained game.

In an unconstrained game, users essentially play an independent game on each distinct path/link. Since a NE exists and is unique on a single link game [11], we know that a NE also exists and is unique on this parallel link network. This is summarized in the following theorem.

Theorem 1: There exists a unique interior-point NE in an unstructured file sharing game on a parallel link network.

Social Benefit at Nash equilibrium. As shown in [11], a single bottleneck link TCP connection game admits a symmetric NE when users have the same Round Trip Time (RTT) and their benefit function includes throughput and a cost proportional to the number of connections. This result can be extended to our uncontrained game. That is, when all users have the same RTT, the unique NE is symmetric in the sense that all users have the same number of connections at the NE.

Solving the optimization problem for a user r, we can get the vector of connections of user r at the symmetric NE as: $n^*_{rj} = (R-1)C_j/(R^2\beta)$. Then, user r's benefit at the NE is $B^*_r = \sum_{j=1}^{L} C_j/R - \sum_{j=1}^{L}(R-1)C_j/R^2$. Therefore, the total social benefit of the NE is $B_{ne} = \sum_{j=1}^{L} C_j/R$. Note that B_{ne} is simply a function of the total network capacity and the number of users. As the number of users increases, the total social benefit of the NE goes to zero.

Reaction functions. The reaction function of a user r is defined as the best response of user r as a function (if it exists) of the total number of connections of all other users. A response of user r is $\mathbf{n_r} = (n_{r1}, n_{r2}, ..., n_{rL})$. Since in an unconstrained game users essentially play an independent game on each individual link, we can solve for a user's best response on each link separately. Specifically, for any link j, we have $\bar{n}_{rj} = \text{argmax}_{n_{rj}\in(0,\infty)} B_{rj}(\sum_{k\neq r}^{R} n_{kj})$. For convenience, let n_{-rj} denote $\sum_{k\neq r}^{R} n_{kj}$. It is easy to show that $\bar{n}_{rj} = f(n_{-rj}) = -n_{-rj} + \sqrt{C_j n_{-rj}/\beta}$. $\bar{n}_{rj}$ is a continuous function of n_{-rj}. Please see [21] for more details.

As shown in Section III-D, we can use reaction functions to identify NEs by checking the intersecting point(s) of the reaction function (best response) curves of all players. We can also use reaction functions to investigate the best-response dynamics of the game playing process, as discussed later.

Stability of NE in Best-response Dynamics. Suppose that users interact with each other using best-response in a discrete time process, a so called *best-response dynamics* [12][11]. This process proceeds in discrete time steps or rounds, and only one randomly chosen user makes a move at each round. Whenever a user makes a move, it calculates its best response to other users' numbers of connections which are determined in previous steps. That is, the user who makes a move solves its optimization problem to maximize its benefit. If all users' strategies converge to or stabilize at some point $\mathbf{n}_s$ as time goes to infinity, then $\mathbf{n}_s$ is a NE, and it is *globally stable*. Regarding an unstructured file sharing game on a parallel link network, we have the following stability result. Please refer to [21] for a detailed proof.

Theorem 2: The unique NE is globally stable in the two-player version of the game on a parallel link network when both players use best-response to play the game.

B. Efficiency Loss of Nash Equilibrium

First note that the maximal system benefit is the solution of a straightforward optimization problem. The system benefit can be represented as: $B = \sum_{r=1}^{R} B_r = \sum_{r=1}^{R}\sum_{j=1}^{L} G_{rj} - \beta\sum_{r=1}^{R}\sum_{j=1}^{L} n_{rj}$. We find that the maximal value of B is $B_{max} = \sum_{j=1}^{L} C_j - \beta N_{min}$.

Consider a homogeneous network where all links have the same capacity. Then we have $B_{max} = LC - \beta L$, as we need at least one connection for each link in order to get the bandwidth of each link. The efficiency loss of a NE is given by $L_{eff} = \frac{B_{max}}{B_{ne}} = \frac{LC-\beta L}{LC/R}$. This result essentially suggests that the efficiency loss of the unique NE is bounded. However, if L, C are fixed, and let $R \to \infty$, then $L_{eff} \to \infty$. This suggests that the system performance at NE can degrade arbitrarily if the number of users becomes large.

C. Socially Responsible Users

Note that we can think of users as data senders in the game discussed above. Let the packet loss rate associated with each link/path j be p_j. Suppose that the packet sending rate of a TCP connection of user r on path/link j is T_{rj}. The throughput of this connection is given by $G_{rj} = T_{rj}(1-p_j)$. Not all packets coming to bottleneck link j are delivered. The network resources before link j are partially wasted because they carry data at a higher rate than the actual delivery rate of link j. Therefore we can think of this extra traffic as a cost to the network and that is proportional to the packet sending rate T_{rj}. A user is considered as socially responsible if his/her benefit function includes this cost term. That is, we have $B_r(\mathbf{n}_r) = \sum_{j=1}^{L} G_{rj} - \gamma\sum_{j=1}^{L} n_{rj}T_{rj}$, where $\gamma \in (0,1)$. Based on [11], we can show that there exists a pure strategy unique NE because users actually play a game on each link independently from other links. It also follows that the loss of efficiency of the NE is bounded as the unique NE is an interior point in the strategy space. See [21] for more details.

V. Star Network

In this section, we use a star network to approximately model a peer-to-peer file sharing overlay network, and investigate the unstructured file sharing game on such a star network. Figure 5 presents one such example.

In the star network, we assume that a user has two asymmetric access links to the Internet: one downstream link and one upstream link. This assumption is supported in a measurement study in [24], where it is found that most users in current peer-to-peer networks use cable modem or ADSL to get connected to the Internet. Usually the downstream link has higher capacity than the upstream link [24].

A user r uses its downstream link to get data from other peers. The downstream link of user r is a "private" link in the sense that this link is only used by user r itself. On the other hand, the upstream link of user r is shared by all other peers or users who are downloading files from user r. We can think of the upstream link of user r as a "public" link.

In addition, similar to [14][15], we assume that in a peer-to-peer file sharing network, bottlenecks can occur at access links, not in the core Internet. This assumption is a reasonable approximation of the current peer-to-peer file sharing networks such as Gnutella and BitTorrent, where usually the data throughput is limited by the "last mile" (cable or ADSL or modem) of a connection. Thus, in the star network shown in Figure 5, the Internet cloud can be represented simply as a central node. Note that a star network can be thought of as a generalization of parallel link network. One such transformation is given in our technical report [21].

In the following, we first prove the existence of NE in unstructured file sharing game on a star network. We then use examples to illustrate the best response dynamics of this game playing process, and finally we present our results on the loss of efficiency of NE.

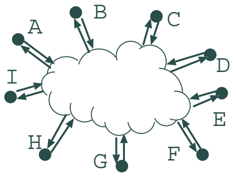

Fig. 5. An example of star network.

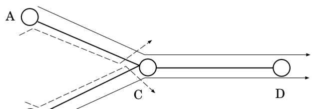

Fig. 6. A three node topology.

A. Nash Equilibrium

Recall that the benefit of user r is given by (1). In the following, we first present a lemma (Lemma 1) and later use it to prove that utility function $U_r(G_r(\mathbf{n_r}))$ is a non-decreasing, continuous, and concave function of user r's number of connections $\mathbf{n_r} = (n_{r1}, ..., n_{r|P_r|})$, where P_r represents the set of available paths of user r. Recall that $U_r(x)$ is a continuous, nondecreasing, and concave function. Since we assume that cost $\Phi_r(\mathbf{n}_r)$ is an increasing and convex function of $\mathbf{n}_r$, it then follows that the benefit B_r is a non-decreasing, continuous, and concave function of $\mathbf{n_r}$.

Lemma 1 is introduced for the simple network in Figure 6, where a user r has two paths ($A \to C \to D$ and $B \to C \to D$) to transfer data to destination node D. Both paths share a common link CD. Suppose that the number of connections user r opens on path $A \to C \to D$ is n_{p1}, and on path $B \to C \to D$ is n_{p2}. Then we have $\mathbf{n}_r = (n_{p1}, n_{p2})$.

We assume that link CD is a private link of user r, i.e., no other users use this link. This private link corresponds to the downstream link of user r in a star network. On the other hand, links AC and BC are shared by user r and other users. AC and BC correspond to two public links in a star network.

Recall that throughput G_r obtained by user r is a function of $\mathbf{n}_r$. Lemma 1 shows that G_r is a concave function of $\mathbf{n}_r$. A detailed proof and an illustrative example are given in [21].

Lemma 1: Throughput G_r of user r in Figure 6 is a concave function of $\mathbf{n}_r = (n_{p1}, n_{p2})$.

Consider the network in Figure 7, a generalized version of the network in Figure 6. In Figure 7, there are M (multiple) paths along which user r can get data from the sender. All paths share a common link BA. A strategy vector of user r is $\mathbf{n}_r = (n_{r1}, n_{r2}, ..., n_{rM})$ with $M \geq 2$. We can extend the result in Lemma 1 to show that a user r's throughput is also a concave function of $\mathbf{n}_r$. This is summarized in Lemma 2.

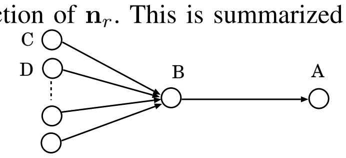

Fig. 7. A network where a user has multiple paths(or peers) to get data.

Lemma 2: Suppose that user r has M ($M \geq 2$) paths in the network shown in Figure 7, then the throughput of user r is a concave function of its strategy vector $\mathbf{n}_r = (n_{r1}, n_{r2}, ..., n_{rM})$.

Based on Lemma 2, we can show in the following theorem the existence of NE on a star network. See [21] for a detailed proof. One example of star network is shown in Figure 5.

Theorem 3: There exists a Nash equilibrium of unstructured file sharing game on a star network (shown in Figure 5).

An illustrative example.

We use a simple star network shown in Figure 8 to illustrate the existence of NE proved in Theorem 3. On this star network, there are 6 links AD, DA, BD, DB, CD, and DC. The capacities of all links are $C_{AD} = 10, C_{DA} = 20, C_{BD} = 30, C_{DB} = 40, C_{CD} = 50$, and $C_{DC} = 60$. There are three users associated with nodes A, B and C respectively. For convenience, we refer to the user at node A as user A. Note that each user has two download paths with each path consisting of two links. For example, user A has two download paths $B \to D \to A$ and $C \to D \to A$. For any given download path, one link is shared with other users, and the other link is a private link. For example, for user A, path $B \to D \to A$ has two links: BD and DA. Link BD is a link shared with user C. Link DA is a private link of user A, which is shared by both of its paths $B \to D \to A$ and $C \to D \to A$.

User A's strategy is a vector of number of connections on two available paths, i.e., $\mathbf{n}_A = (n_{BA}, n_{CA})$. Similarly, strategies of user B and C are: $\mathbf{n}_B = (n_{AB}, n_{CB})$ and $\mathbf{n}_C = (n_{AC}, n_{BC})$.

Consider the unstructured file sharing game played by users A, B, and C. Each user tries to maximize its benefit B_r ($r = A, B, C$). We use best response dynamics to demonstrate the existence of a NE in this game. At the first step, each user opens a random number of connections on two available paths. In the following steps, only one player is randomly chosen to compute its best response at each step. As shown in Figure 8, the best response dynamics converges to a NE, which can be verified by checking the optimality of benefits of all three users.

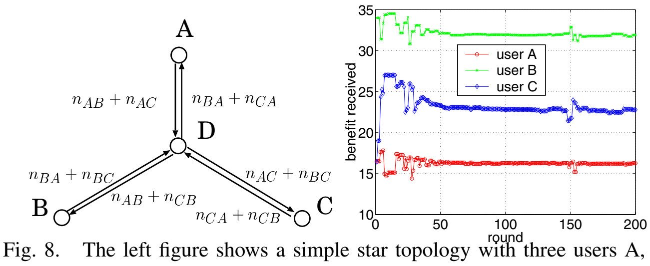

Fig. 8. The left figure shows a simple star topology with three users A, B, and C. The right figure shows the best response dynamics. All three users' benefits converge to those at the Nash equilibrium.

Loss of Efficiency. We can show that the loss of efficiency of any NE in this game is bounded when all downstream links have higher capacity than upstream links and users are homogeneous. However, if users are aggressive in that their benefit functions do not contain cost terms, then a unique NE is a point where all users open their maximum allowable number of connections. Clearly, the loss of efficiency of the NE is unbounded if users can open arbitrarily large numbers of connections. For more information, please refer to our technical report [21].

Network Resource Utilization. Suppose that all downstream links have higher capacities than upstream links. Then the capacities of all upstream links will be fully utilized at the NE. This is a good situation in terms of the network resource utilization because the total throughput can be supported by the network is just the aggregate capacity of these upstream links. Note that this is not always true for general network topologies. For counter examples, please see our technical report [21].

VI. Overlay Formation Game

In this section, we introduce an *overlay formation game* to study the Tit-for-Tat strategy adopted by BitTorrent (BT) [3], one of the most popular peer-to-peer applications.

As before, we assume that the physical network is a star network where each peer is attached to a physical node, and the center node models the Internet, and peers connect to the center node via access links. However, unlike last section, here we assume that bottlenecks only occur at upstream access links. As before, we assume that peers always have demands that can be satisfied by each other, and that connections are always allowed.

A connection between a pair of peers can be thought of as a virtual link. Through setting up connections between themselves, peers form an overlay network, in which each node represents a unique peer, and virtual links are connections between peers. A peer i can get a share of the upload bandwidth (BW) of peer j through the connection (or virtual link) between i and j. In the mean time, other peers may want to get some share of peer j's upload BW by setting up connections with j. The upload BW of j is equally shared among all connections with other peers. Note that a peer may want to get shares of all other peers' upload BW and want to maximize its received total BW. If all peers behave this way, we have a game among peers, and any stable point of this game is an overlay network consisting of a set of virtual links among peer nodes. We call this game an *overlay formation game*.

We can think of the *overlay formation game* as a variant of an *unstructured file sharing game* with two major unique characteristics: 1) two peers set up a connection between themselves only when they both find it beneficial; 2) there can only be zero or one connection between a pair of peers. The first characteristic represents a variant of the so called *Tit-for-Tat strategy* in BitTorrent [3]. Note that we ignore optimistic unchoking in BT [3].

The authors of [14] study how Tit-for-Tat can affect selfish peers who are able to set their upload bandwidth in a BT network. Under several assumptions, they show that there is a *good* NE at which each peer uploads at the maximum rate. Note that in [14], for a given peer, the total number of other peers to set up a connection with is fixed. However, we observe that BT clients can change the number of connections to open in order to gain advantage or to improve their performance. We illustrate this observation in the following example.

An Illustrative Example. Consider 10 peers divided into two groups. Five peers have physical upload bandwidth $C_1 = 3$ and the other five have bandwidth $C_2 = 2$. Suppose that the default number of connections is $n_u = 3$. According to [14], peers would use all their upload bandwidth and would create the overlay shown in Figure 9, where big circles and small circle respectively represent high-bandwidth and low bandwidth peers. Note that the peers do not receive the same download rate, even if they belong to the same group. Four high-bandwidth peers receive a download rate of 3 ($= 3C_1/3$), while the peer connecting the two groups (peer **S** in the figure) receives only $8/3$ ($= 2C_1/3 + C_2/3$). Similarly four low-bandwidth peers receive rates of 2, while the other receives $7/3$. According to [14] the formed overlay network is stable in the sense that no pair of peers want to change a link.

Let us now remove the constraint on the number of connections. For example, peer **S** decides to increase its number of connections to 5. If all other peers keep $n_u = 3$, the new equilibrium is presented in Figure 10. Note that peer **S** improves its performance, because its download rate increases from $8/3$ to $10/3$ ($= 5C_2/3$).

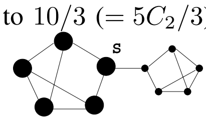

Fig. 9. Regular Graph.

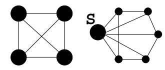

Fig. 10. Peers can change numbers of connections.

This example shows that peers can benefit by changing their numbers of connections. This is formally supported by a result in Section VI-B.1 regarding a homogeneous network where all peers have the same capacity. In the rest of this section, we first formally introduce the overlay formation game in which peers act selfishly as player **S**. We then study the network equilibria arising in this game and quantify the loss of efficiency using the analytical framework of network formation games [20].

A. Model of Overlay Formation Game

We formally introduce the overlay formation game in this section. Assumptions are given in the above. We refer to peers as players and connections as links. As before, let $R = \{1, 2, \cdots, R\}$ denote the set of players. The strategy of a player i is the set of intended connections player i wants to establish, which is denoted by $s_i = \{s_{i,j} | \, j \in R\backslash\{i\}\}$, where $s_{i,j} = 1$ means that player i intends to create a link (open a connection) with player j and $s_{i,j} = 0$ means that player i does not intend to create such a link. With the *Tit-for-Tat strategy*, both players have to agree in order to create a link, hence a link between players i and j is formed if and only if $s_{i,j} = s_{j,i} = 1$. A strategy profile $s = \{s_1, s_2, \cdots, s_r\}$ therefore induces a network $g(s) = \{g_{i,j}, i, j \in R\}$, where $g_{i,j} = 1$ denotes the existence of link ij and $g_{i,j} = 0$ denotes the absence of link ij. Given a network g, we use $g + g_{i,j}$ or $g - g_{i,j}$ to denote the network obtained by adding or severing link ij. We also let $N_i(g) = \{j \in R : j \neq i, g_{i,j} = 1\}$ be the set of player i's neighbors in graph g, and let $n_i(g) = |N_i(g)|$. A network is symmetric if $n_i(g) = n, \forall i \in R$, i.e. all players have the same number of connections. This kind of network is also known as a regular graph.

The payoff or benefit of player i is given by its download rate minus the cost of opening connections: $B_i = G_i - \Phi_i(n_i) = \sum_{j \in N_i(g)} C_j/n_j - \Phi_i(n_i)$. As before, we assume that Φ_i is a convex function of n_i. The marginal benefit for player i to open a new connection with player j is:

$$b_i(n_i(g), n_j(g)) = \frac{C_j}{n_j(g)+1} - \Phi_i(n_i(g)+1) + \Phi_i(n_i(g)).$$

A connection between two players can be set up only when both of them find this connection beneficial. This coordination requirement makes the concept of Nash equilibrium (NE) *partially inadequate*. To address this issue, the idea of NE has been supplemented with the requirement of pairwise stability [25], described below.

Definition 1: A network g is a *pairwise equilibrium network* (PEN) if the following conditions hold: 1) There is a NE strategy profile which supports g; 2) For $g_{i,j} = 0$, $B_i(g + g_{i,j}) > B_i(g) \Rightarrow B_j(g + g_{i,j}) < B_j(g)$.

B. Equilibria in Homogeneous Networks

In this section we consider homogeneous networks in which all peers have the same upload capacity and payoff function.

1) Overlay Network Characterization: Based on the previous assumptions, our game is the local spillovers game with strategic substitutes properties studied in [26]. The following results can be derived from [26] (see [21] for details).

Theorem 4: If the number of players is even, a symmetric PEN always exists. Specifically, if $b(0,0) \leq 0$, the empty network is a PEN; if $b(r-2, r-2) \geq 0$ where $r = |R|$, the complete network is a PEN; if $b(k,k) \leq 0 \leq b(k-1, k-1)$, the regular graph with degree k is a PEN. When the previous inequalities are strict, the degree of the PEN is unique.

Remarks. *First*, note that for a set of R players or nodes, if R is even, we can expect a PEN to be a symmetric or regular graph of any possible degree from 0 to $R-1$ (recall that all nodes have the same degree k in a regular graph of degree k); this is not true when the number of players is odd. *Second*, this theorem states that the degree of a PEN can be determined by considering only the marginal benefit $b(k,k)$ for a pair of nodes with the same number of connections k, and in particular this degree is the smallest value k that makes $b(k,k)$ negative. *Third*, the symmetric network at equilibrium is not necessarily connected. Figure 11 shows two possible equilibria with $|R| = 8$ players and degree $k = 2$. *Finally*, even when a symmetric network can arise from player interaction according to Theorem 4, the degree of the network (k) is in general different from the default value used in current BitTorrent implementation ($n_u = 4$). This implies that the symmetric network created by compliant peers in BitTorrent networks is not in general a PEN.

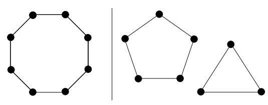

Fig. 11. Different Pairwise Symmetric Equilibria.

Besides symmetric PENs discussed in the above, we have the following theorem addressing asymmetric PENs.

Theorem 5: There can be at most one player or node not connected to any other players in a PEN and the rest of the network is a symmetric network of a unique degree. In asymmetric networks with a single component, if two players with the same number of connections k (i.e. two nodes with the same degree k) are connected to each other, then any two players with fewer number of links than k (or two nodes with lower degrees than k) must be mutually connected.

Remarks. *First*, this property rules out two or more isolated players and interlinked stars with two or more central players, but does allow a star to arise in equilibrium. Please see [21] for more details. Note that for file sharing purposes, an overlay with a star topology is very inefficient: the operation falls back to the server-client paradigm with the center of the star acting as the server. *Second*, in some cases symmetric and asymmetric networks can be pairwise equilibria for a given set of link capacities and cost functions (see [26] for examples).

The following result shows that players having more connections can gain higher payoffs than other players, supporting the example introduced at the beginning of this section.

Theorem 6: Let g be a pairwise equilibrium network in which $n_i(g) < n_j(g)$. If $\forall u \in N_i(g)$, $\exists v \in N_j(g)$ s.t. $n_u = n_v$, then $B_i(g) < B_j(g)$.

Note that if player i's neighborhood is included in player j's neighborhood ($N_i \subset N_j$), the condition, "$\forall u \in N_i(g)$, $\exists v \in N_j(g)$, s.t. $n_u = n_v$", is satisfied.

2) Loss of Efficiency of Symmetric Equilibria: In our game, given the number of players, the number of possible overlays players can create is finite. Hence there is one network g_{opt} with the highest total payoff $\sum_{i \in R} B_i(g_{opt})$. We define the efficiency loss of a PEN g as the ratio of the highest total payoff over the total payoff of the PEN: $L_{eff}(r, C, \Phi) = \frac{\sum_{i \in R} B_i(g_{opt})}{\sum_{i \in R} B_i(g)}$. We note that L depends in general on the number of players, and the upload capacities and cost functions of those players. The following theorem states that L_{eff} is unbounded even for the class of linear connection cost functions ($\Phi(n) = \alpha n$). Therefore, the price of anarchy (the worst efficiency loss of all NEs) is infinite.

Theorem 7: For the class of linear connection cost functions, the loss of efficiency is unbounded. In particular, given an even number of players and an upload capacity C, $\forall M \in \mathbb{R}, \exists \alpha^* \in \mathbb{R}^+$ s.t. $L_{eff}(r, C, \Phi^*) > M$, where $\Phi^*(n) = \alpha^* n$.

3) Dynamic Models: We investigate in this section how peers can dynamically reach a PEN. Here we consider linear costs ($\Phi(n_i) = \alpha n_i$ with $\alpha \in \mathbb{R}^+$) and cases where there is a single symmetric PEN. Some of the results here are extended to other settings in [21]. We consider the following dynamic discrete-time process. Starting from an empty network, at each time step a player pair (i,j) is randomly chosen. Link (i, j) is created (or kept) if both players find it beneficial. An existing link is removed if at least one of the two players of that link does not find it useful. If there is no topology change for some time, this process stops if the network is a PEN, otherwise it will continue.

We have following results from our analysis and simulations. First, the dynamic process always reach a PEN. Second, for a given cost function, as R increases, the achieved PENs *converge* to the symmetric pairwise equilibrium. Finally, the average number of iterations (estimated by simulation) to reach a PEN is of the order of R^2. Consider this number of iterations in the context of BitTorrent (BT) [3]. Each peer in a BT network tries to replace an existing connection with a new, better connection in every 10 seconds. All peers do such replacement simultaneously, unlike the sequential replacement in our simulations. So R^2 iterations in our simulations corresponds to $10R$ seconds in a BT network. For a population of 100 peers, the time needed to reach a PEN is of the order of about 17 minutes, which is shorter than the population change which is normally at least a few hours. See [21] for details.

Simulations. We simulated 5000 runs of the above dynamical process for $\alpha = 0.245$, for which the degree of a symmetric PEN is 4. Each run identifies a PEN and the average degree (d_{avg}) over all players. Figure 12 shows the minimum and the mean of d_{avg} of all the runs. We see that as R increases both the mean and the minimum converge to 4, which verifies the above second result. In Figure 13, the mean and the minimum of the total benefit are compared with the highest total benefit (described in [21]). This figure shows the convergence of the payoffs of all PENs to the payoff of the symmetric PEN when R increases.

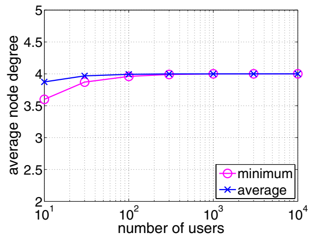

Fig. 12. Average node degree.

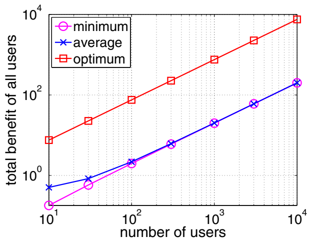

Fig. 13. Total benefit.

VII. Conclusions

Motivated by unstructured file sharing networks such as BitTorrent [3], we introduced an unstructured file sharing game and an overlay formation game to model the interaction among self-interested users who can open multiple connections on multiple paths to accelerate data transfer. Users are modelled as players, and each user adjusts its numbers of connections on its available paths to maximize its benefit.

We demonstrated by examples that there exist multiple stable network states, so called Nash equilibria (NE), in the unstructured file sharing game on general networks. We further restricted our attention to parallel link networks and star networks which were used to model unstructured file sharing networks. We proved the existence of NE in several variants of the game on both networks. We found that the loss of efficiency of NE can be arbitrarily large if users have no cost constraints. However, when there are cost constraints, the loss of efficiency is bounded. In addition, we proved the global stability of NE in some variants of the game. Furthermore, we studied the Tit-for-Tat strategy (built in BitTorrent [3]) through an overlay formation game. We proved the existence of equilibrium overlays, and demonstrated the convergence of the dynamical game-playing process. Although a general belief is that the Tit-for-Tat can prevent selfish behavior, we showed that it can still lead to an unbounded loss of efficiency.

An interesting future research direction is to investigate nonlinear cost models in unstructured file sharing networks.

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