Performance Evaluation

Lecture 2: Complex Networks

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Configuration model

- A family of random graphs with given degree distribution
Configuration model

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  - Uniform random matching of stubs
Configuration model

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![Graph Diagram]
What can we do in networks without a geographical structure?

Random walks
How much time is needed in order to reach a given node?
Random Walks: stationary distribution

- \( \pi_i = \sum_{j \in N_i} \frac{1}{k_j} \pi_j \)
- \( \pi_i = \frac{k_i}{\sum_{i=1}^{N} k_j} = \frac{k_i}{2M} \)

- Avg time to come back to node \( i \) starting from node \( i \): \( 1 = \frac{2M}{\pi_i k_i} \)
- Avg time to reach node \( i \)
  - Intuitively \( \approx \Theta(M/k_i) \)
Another justification

- Random walk as random edge sampling
  - Prob. to pick an edge (and a direction) leading to a node of degree \( k \) is \( \frac{kp_k}{< k >} \)
  - Prob. to arrive to a given node of degree \( k \):
    \[
    \frac{kp_k}{p_kN < k >} = \frac{k}{2M}
    \]
  - Avg. time to arrive to this node \( 2M/k \)
- ...equivalent to a RW where at each step we sample a configuration model
Distributed navigation (speed up random walks)

- Every node knows its neighbors
Distributed navigation
(speed up random walks)

- Every node knows its neighbors
- If a random walk looking for $i$ arrives in $a$ the message is directly forwarded to $i$

\{a,b,c,d\}
Distributed navigation reasoning 1

- We discover $i$ when we sample one of the links of $i$'s neighbors.
- Avg # of these links: $k_i \sum_k ((k-1) \frac{kp_k}{<k>}) = k_i \left( \frac{<k^2>}{<k>} - 1 \right)$
- Prob. to arrive at one of them: $\frac{k_i}{2M} \left( \frac{<k^2>}{<k>} - 1 \right)$
Distributed navigation reasoning 2

- Prob that a node of degree $k$ is neighbor of node $i$ given that RW arrives to this node from a node different from $i$

\[
1 - \left(1 - \frac{k_i}{2M}\right)^{k-1} \approx \frac{k_i(k - 1)}{2M}
\]

- Prob that the next edge brings to a node that is neighbor of node $i$:

\[
\sum_k \frac{k_i(k - 1)}{2M} \frac{kp_k}{<k>} = \frac{k_i}{2M} \left(\frac{<k^2>}{<k>} - 1\right)
\]
Distributed navigation

- **Avg. Hop#** \[
\frac{2M}{k_i} \left( \frac{<k^2>}{<k^2>} - \frac{<k>}{<k>} \right)
\]

- Regular graph with degree d: \[
\frac{2M}{d(d-1)}
\]

- ER with \(<k>\): \[
\frac{2M}{k_i(<k>-1)}
\]

- Pareto distribution \[
P(k) \approx \frac{\alpha x_m^\alpha}{x^\alpha - 1}
\]

\[
\approx \frac{2M}{k_i} \frac{(\alpha - 2)(\alpha - 1)}{x_m - (\alpha - 2)(\alpha - 1)} \quad \text{If } \alpha \to 2\ldots
\]
Distributed navigation

- Application example:
  - File search in unstructured P2P networks through RWs