Performance Evaluation – Master UBINET
Assignment 2

Solutions have to be sent by February 3rd 2013 to `giovanni.neglia@inria.fr`.

Ex. 1 — In an ecological habitat with surface S rabbits and hawks live interacting. The maximum density of rabbits that is sustainable by this habitat is equal to $\rho = 10^{-1}\mathrm{m}^{-2}$. Then the maximum number of rabbits is equal to $R_{max} = \rho S$. Rabbits are the main food of hawks, so that we cannot have more than $H_{max} = 1/10 R_{max}$ hawks in the habitat. The following model has been proposed for the two populations. Given $R(k)$ and $H(k)$ respectively the numbers of rabbits and hawks in the k-th week, at the $(k+1)$-th week a rabbit is born with probability $R(k)/R_{max}\,(1-R(k)/R_{max})$ and a rabbit can die because of some disease or can be killed by a hawk with probability $\beta R(k)/S^2 + \gamma R(k)/S H(k)/S$. Note that the rabbit population can increase of one unit, decrease of one unit or stay stable in case there is no birth and no death or one birth and one death. Similarly at the week $(k+1)$-th a hawk is born with probability $\delta R(k)/S\ H(k)/S$ and a hawk dies with probability $\epsilon H(k)/S$.

1. Show that the model can be represented as a Discrete Markov Chain, and calculate its transition probabilities.
2. Show that a Mean-Field Limit can be correctly derived when the surface of the habitat diverges (or equivalently the maximum total number of animals diverges). Write the corresponding Ordinary Differential Equation (ODE) for the density of rabbits (R/S) and for the density of hawks (H/S).
3. Given the surface S and the population size at a given moment, how would you use the fluid model to approximate the size of the two populations 10 weeks later? (No need to perform the actual calculations.)
4. Determine the equilibrium points for the ODE, i.e. the density vectors that are constant solutions of the ODE (for a specific initial condition) for the following sets of parameters: $\beta = 1\mathrm{m}^4$, $\gamma = 1000\mathrm{m}^4$, $\delta = 1\mathrm{m}^4$ and $\epsilon = 0.05\mathrm{m}^2$. Do these equilibrium points correspond to stationary

distributions for the Markov Chain?

5. How does the set of equilibrium points change if $\epsilon = 2\mathrm{m}^2$? Does this difference reveal something about the population dynamics?

Ex. 2 — Solve the following games:

	A	B	C	D
A	5	-2	2	7
B	1	2	0	-3

	A	B
A	(2,3)	(4,1)
B	(-2,-3)	(5,0)