Performance Evaluation

Second Part

Lecture 4

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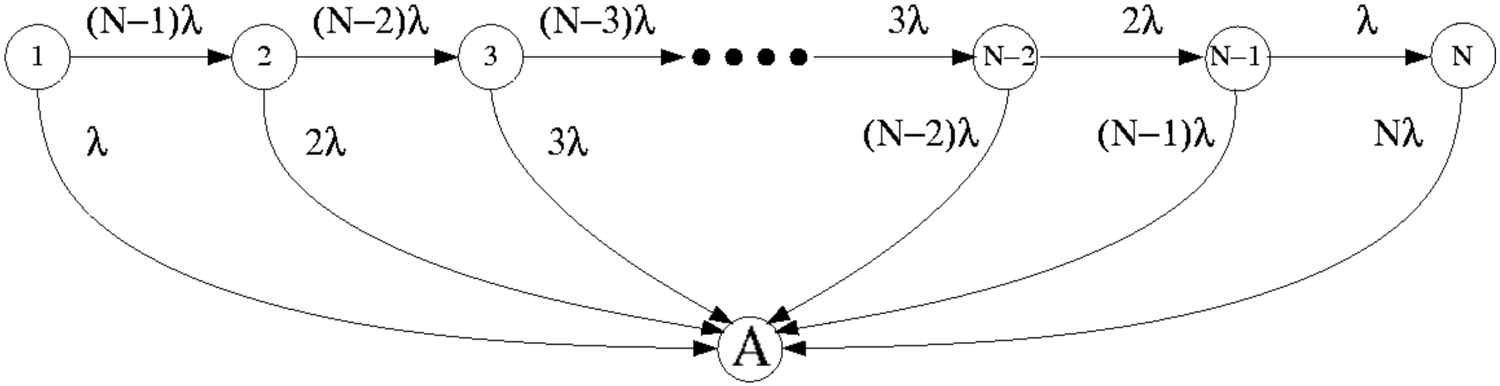
30 January 2012

2-hop routing

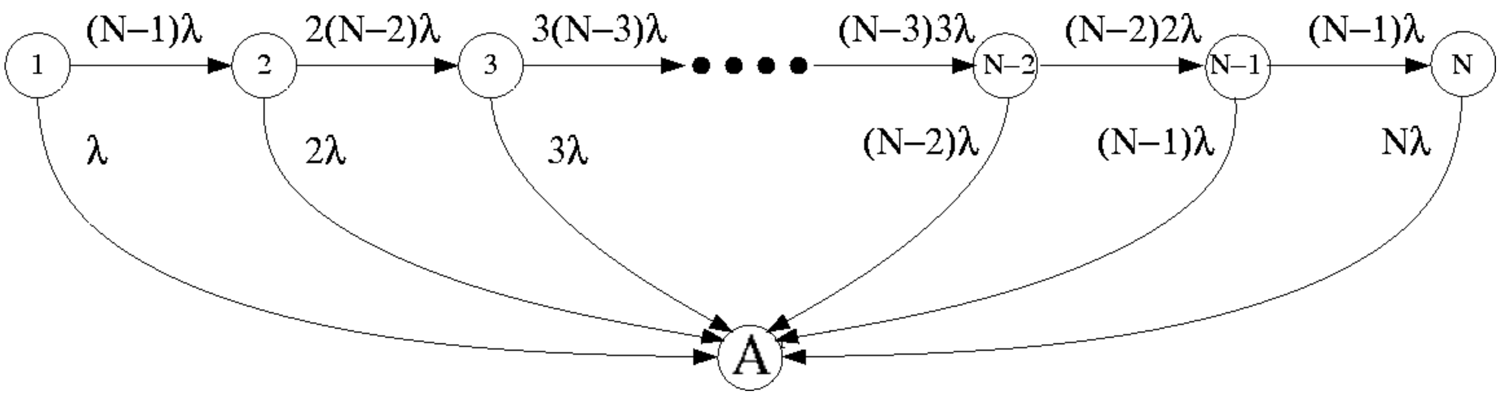
Model the number of occurrences of the message as an absorbing Continuous Time Markov Chain (C-MC):



- State $i \in \{1,\ldots,N\}$ represents the number of occurrences of the message in the network.
- State *A* represents the destination node receiving (a copy of) the message.

Epidemic routing

Model the number of occurrences of the message as an absorbing C-MC:



- State i∈{1,…,N} represents the number of occurrences of the message in the network.
- State *A* represents the destination node receiving (a copy of) the message.

Message delay

Proposition: The Laplace transform of the message delay under the two-hop multicopy protocol is:

$$T_2^*(\theta) = \sum_{i=1}^{N} i \frac{(N-1)!}{(N-i)!} \left(\frac{\lambda}{\lambda N + \theta} \right)^i,$$

and

$$P(N_2 = i) = \frac{i}{N^i} \frac{(N-1)!}{(N-i)!}, \qquad i = 1, \ldots, N$$

Message delay

Proposition: The Laplace transform of the message delay under epidemic routing is:

$$T_E^*(\theta) = \frac{1}{N}\sum_{i=1}^{N}\prod_{j=1}^{i}\frac{j\lambda(N+1-j)}{j\lambda(N+1-j)+\theta},$$

and

$$P(N_E = i) = \frac{1}{N}, \qquad i = 1,\ldots,N$$

Expected message delay

Corollary: The expected message delay under the two-hop multicopy protocol is

$$E[T_2] = \frac{1}{\lambda N}\sum_{i=1}^{N}\frac{i^2(N-1)!}{(N-i)!N^i} = \frac{1}{\lambda}\left(\sqrt{\frac{\pi}{2N}} + O\left(\frac{1}{N}\right)\right),$$

and under the epidemic routing is

$$E[T_E] = \frac{1}{\lambda N}\sum_{i=1}^{N}\frac{1}{i} = \frac{1}{\lambda N}\left(\log(N) + \gamma + O\left(\frac{1}{N}\right)\right),$$

Where $\gamma \approx 0.57721$ is Euler's constant.

Relative performance

The relative performance of the two relay protocols:

$$\frac{E[T_E]}{E[T_2]} \approx \frac{\log(N)}{\sqrt{N}} \sqrt{\frac{2}{\pi}}$$

and

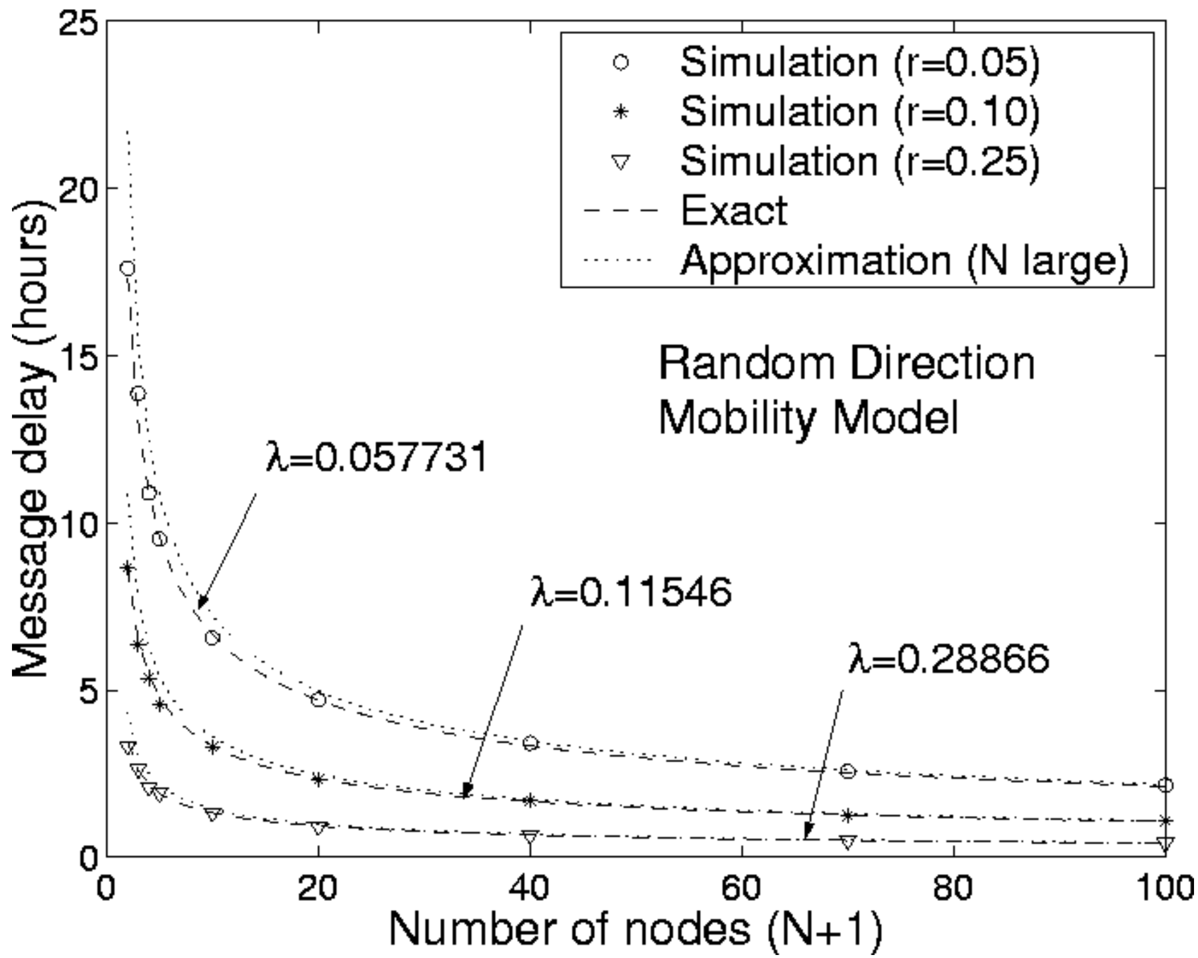
$$\frac{E[N_E]}{E[N_2]} \approx \sqrt{\frac{N}{2\pi}}$$

Note that these are independent of λ!

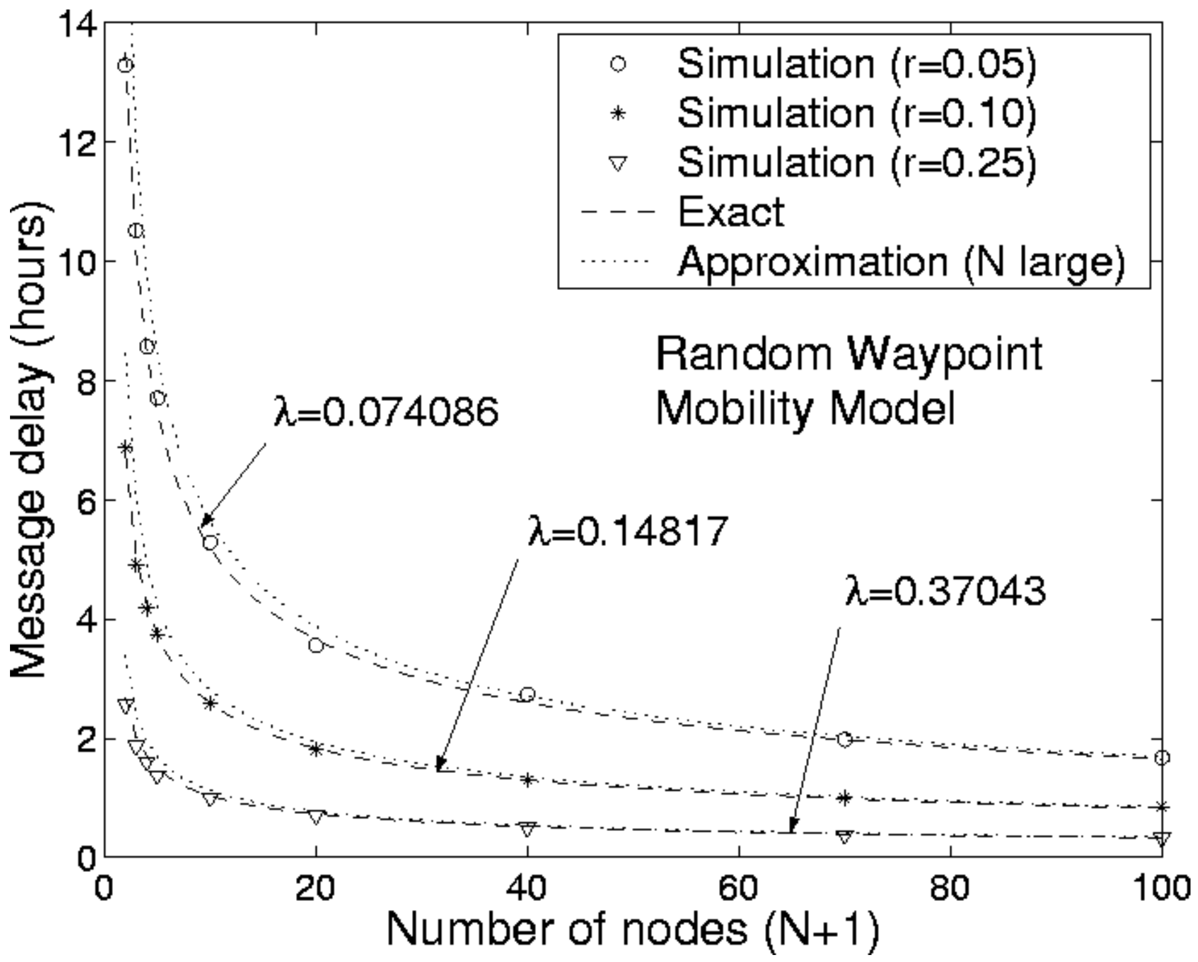
Some remarks

- These expressions hold for any mobility model which has exponential meeting times.
- Two mobility models which give the same λ also have the same message delay for both relay protocols! (mobility pattern is "hidden" in λ)
- Mean message delay scales with mean first-meeting times.
- λ depends on:
  - mobility pattern
  - surface area
  - transmission radius

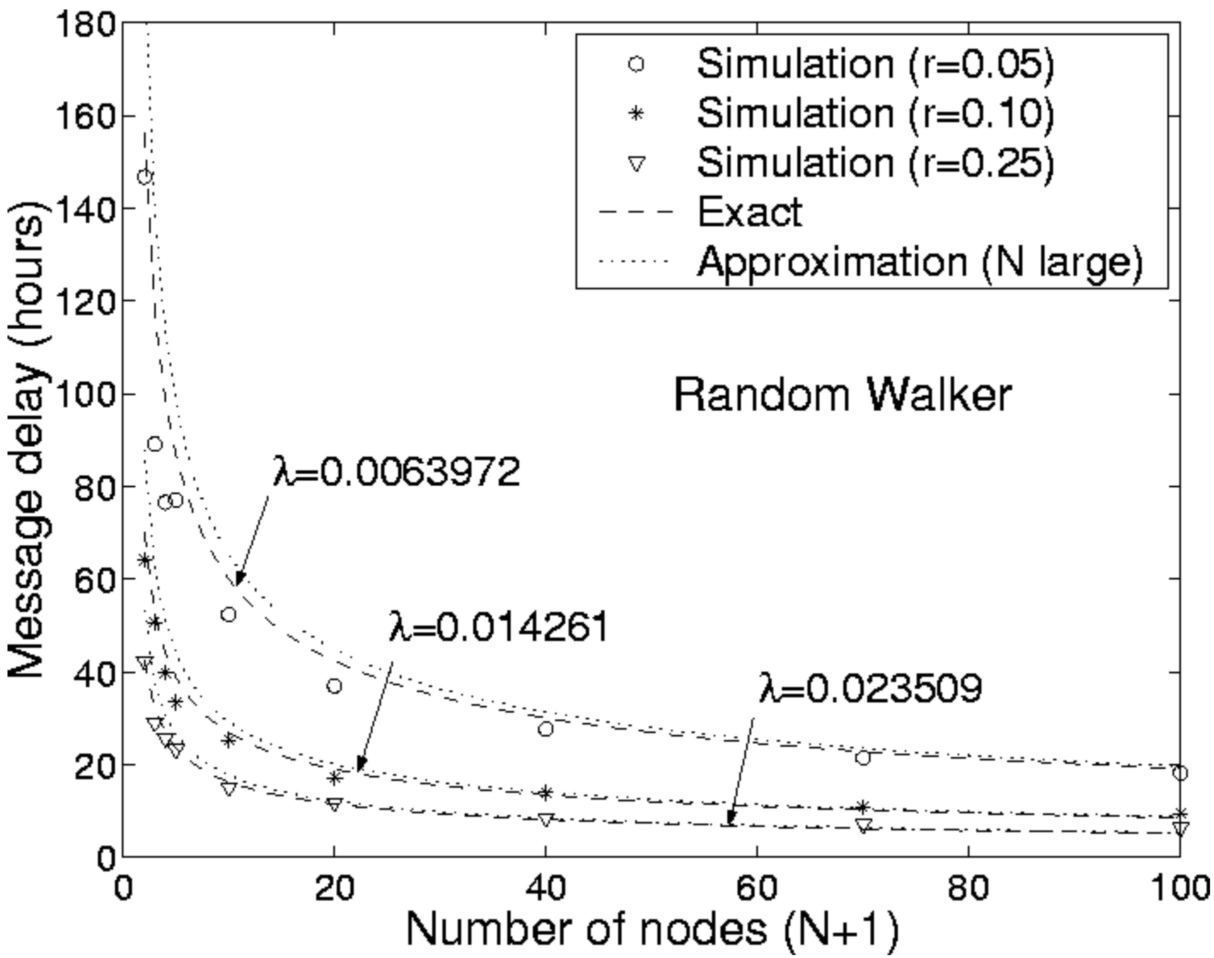
Example: two-hop multicopy



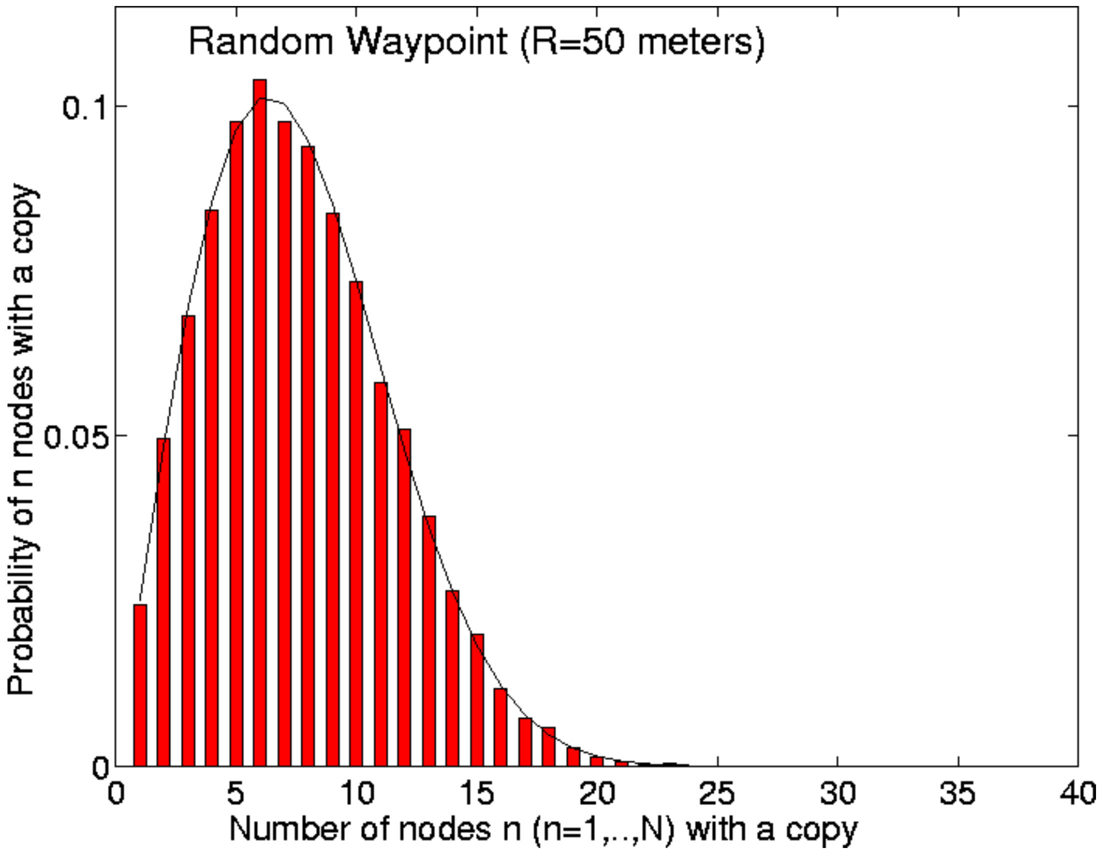
Example: two-hop multicopy



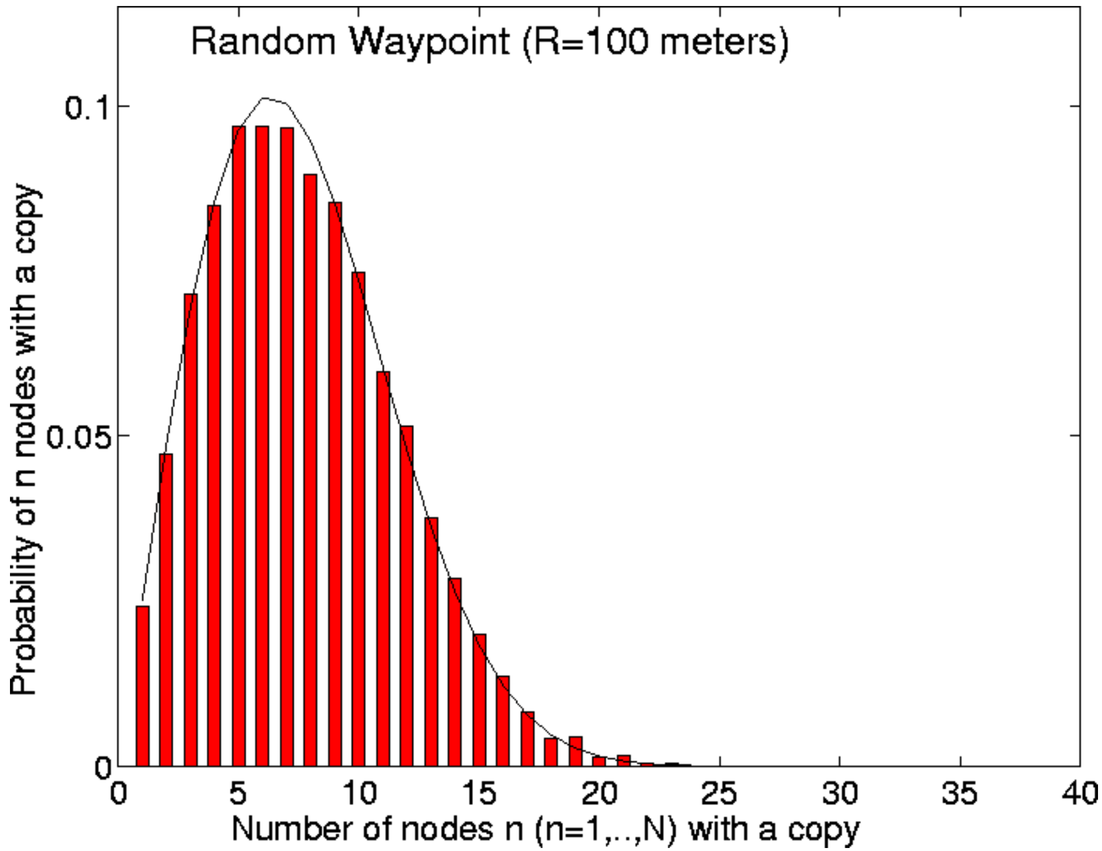
Example: two-hop multicopy



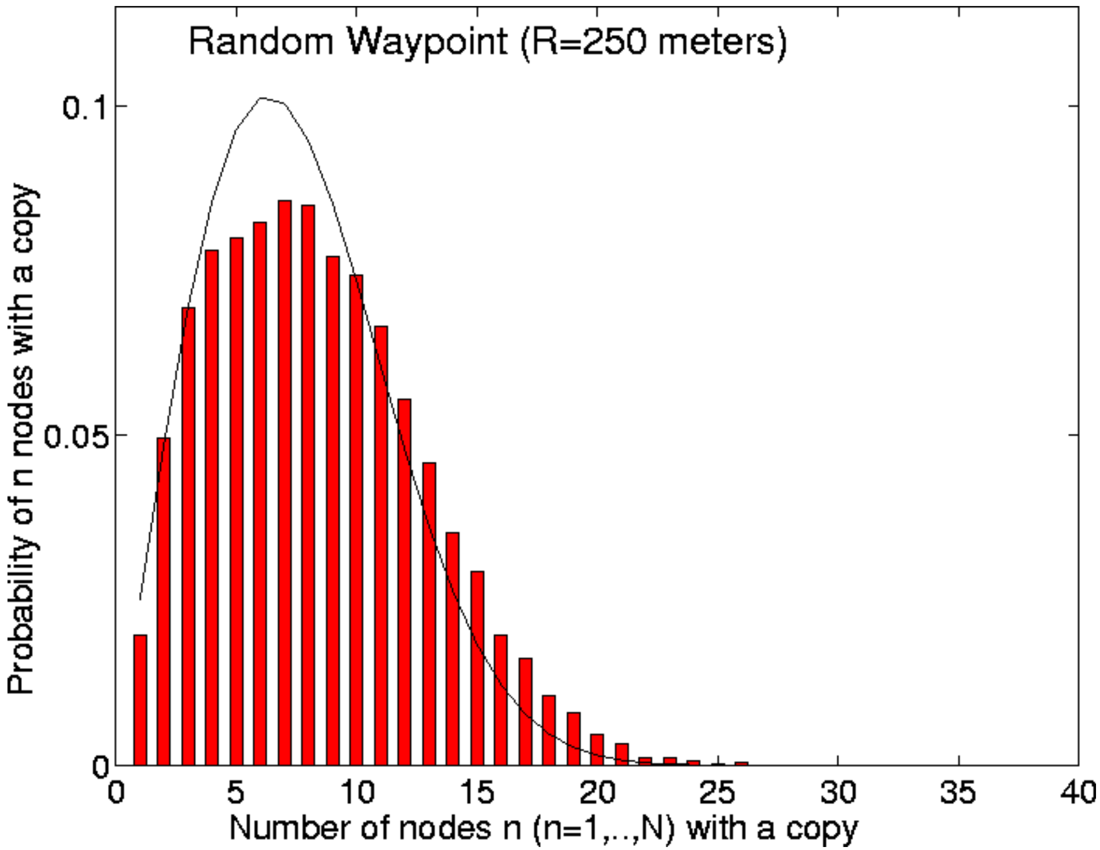
Example: two-hop multicopy



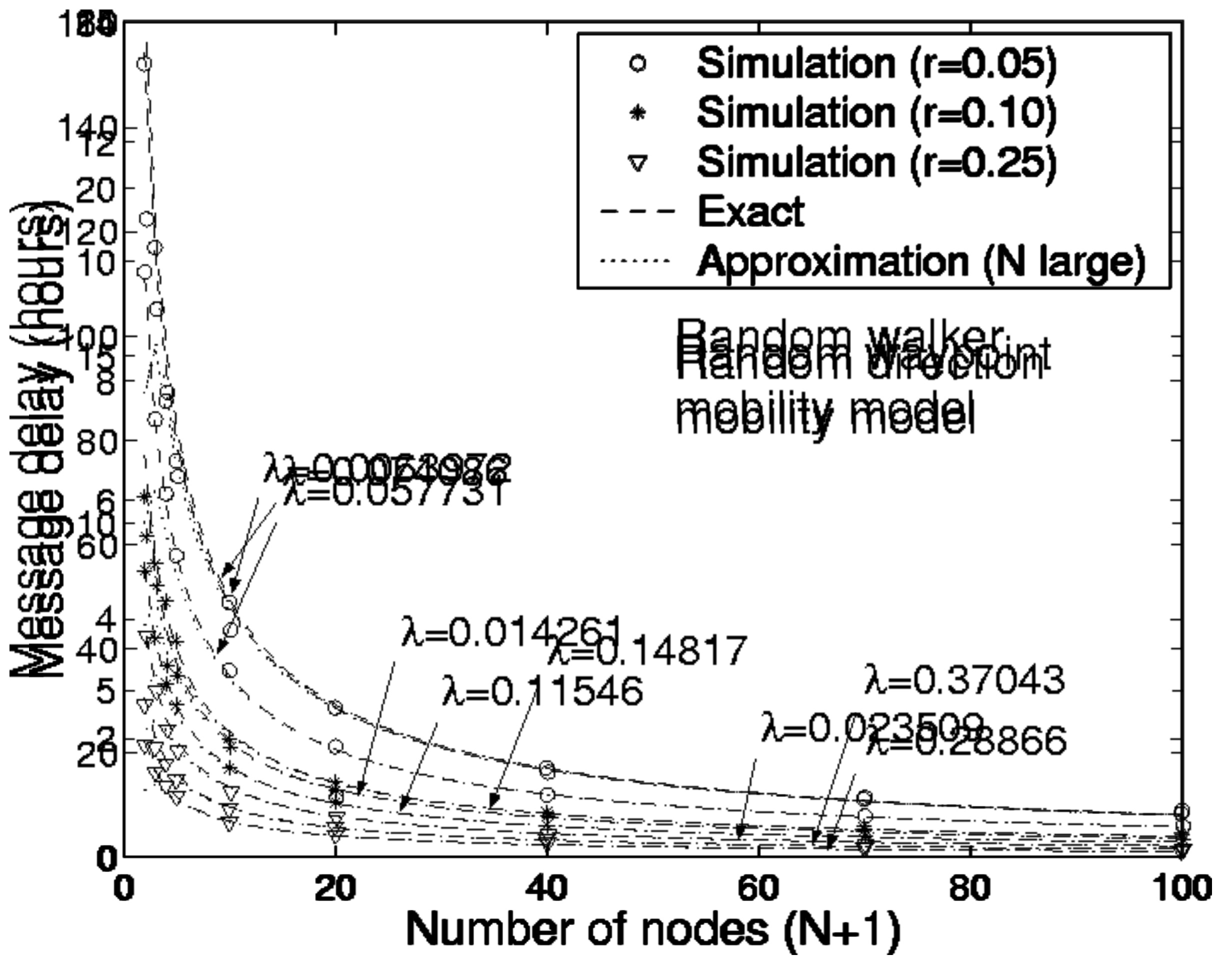
Example: two-hop multicopy



Example: two-hop multicopy



Example: unrestricted multicopy



Outline

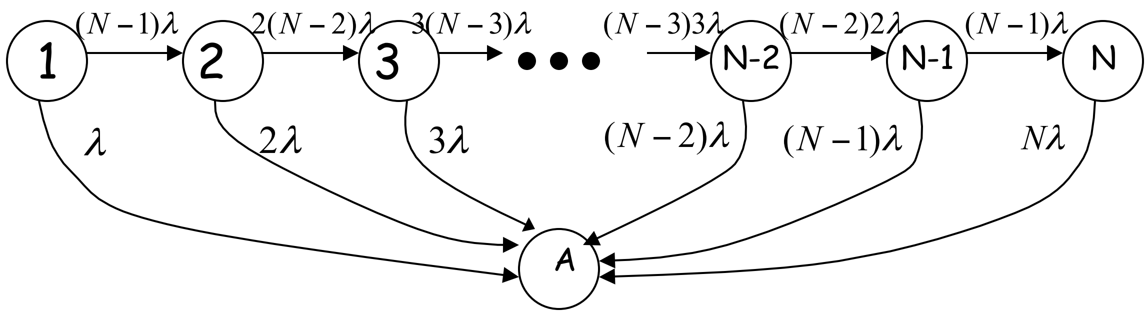
- Introduction on Intermittently Connected Networks (or Delay/Disruption Tolerant Networks)
- Markovian models
  - the key to Markov model
  - Markovian analysis of epidemic routing
- Fluid models

Why a fluid approach?

[Groenevelt05]

Markov models can be developed

States: n_I =1,…, N: num. of infected nodes, different from destination; A: packet delivered to the destination



Infection rate:

$$r_N(I) = \lambda n_I (N - n_I)$$

Delivery rate:

$$\lambda n_I$$

Transient analysis to derive delay, copies made by delivery; hard to obtain closed form, specially for more complex schemes

Modeling Works: Small and Haas

Mobicom 2003 [small03]

- ODE introduced in a *naive* way for simple epidemic scheme $I'(t) = \lambda I(t)(N - I(t))$
  - N is the total number of nodes,
  - I the total number of infected nodes
  - λ is the average pairwise meeting rate
- Average pair-wise meeting rate obtained from simulations

TON 2006 [haas06]

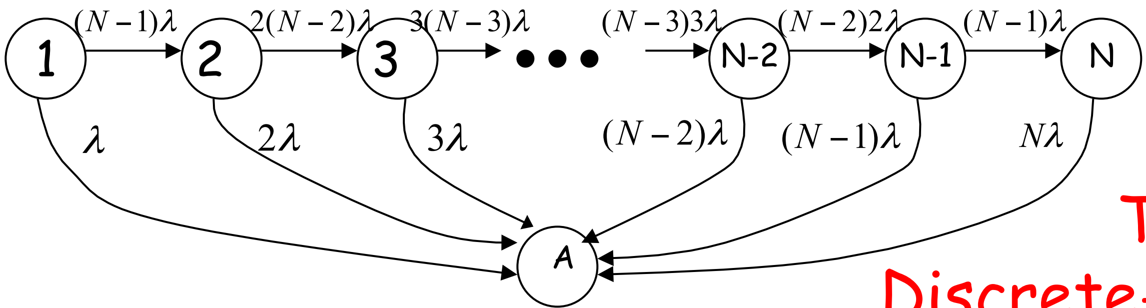
- consider a Markov Chain with N-1 different meeting rates depending on the number of infected nodes (obtained from simulations)
- Numerical solution complexity increases with N

Our contribution [Zhang07]

A unified ODE framework...

- limiting process of Markov processes as N increases [Kurtz 1970]

How do we proceed?



This is not a
Discrete-Time Markov chain

1. We show in an intuitive way why we expect to be able to derive the same results
2. We present rigorous results for fluid models for C-MC

C-MC

- N states, $q_{n,m}$ is the transition rate from state n to state m
- if C-MC is in state n, the exit time is the minimum of at most N-1 independent exponential r.v. with rates $q_{n,m}$ for m<>n
- An equivalent description is that the C-MC abandons state n with rate $q_n=\Sigma_{m<>n}q_{n,m}$ and then jumps to state m with probability $q_{n,m}/\Sigma_{m<>n}\, q_{n,m}$
- It follows that the sequence of states of a C-MC is a D-MC (the *embedded* MC).

C-MC: Uniformization

- Let q be a rate such that $q \geq q_n$ for every n
- Assume that all transitions occur at rate q, but that in state n, they lead only with prob q_n/q to a different state, and with prob $1-q_n/q$ they leave the system in state n
- This is an equivalent description

From a C-MC to a D-MC

- Consider a uniformized C-MC with rate q and state $X_C(t)$
- Consider its embedded D-MC with state $X_D(k)$, k=0,1,2...
- We can consider that

  $X_D(k) \approx X_C(k/q)$
- Then from a MF result for $X_D(k)$, we can derive an analogous one for $X_C(t)$

MF for Epidemic Routing

- Let's ignore at the moment state D
- Transition Rates (under MF approximation)
  - λ n (N-n) to state n+1 (with 1 infected node more)

Then $q_n = \lambda n (N-n)$

- Let's choose $q = \lambda N^2$
- in the embedded MC the probabilities of moving to n+1 and staying in n are

  1. n/N (1-n/N) and 2. 1-n/N (1-n/N)

MF for Epidemic Routing

- Measure occupancy: $(i_D^{(N)}(t), s_D^{(N)}(t))$
- Drift:
  - Only one component is necessary, e.g. that for the infected nodes
  - $f_1^{(N)}(m)=1/N\ n/N\ (1-n/N)$
- All the conditions are satisfied!
  - and $\varepsilon(N)=1/N$

MF for Epidemic Routing

- Let $i_D(t)$ be the solution of the ODE
  $di_D(t)/dt=i_D(t)(1-i_D(t))$,
  with $i_D(0)=i_0$
- If $i_D^{(N)}(0)=i(0)$, then $i_D^{(N)}(k)\approx i_D(t\ \varepsilon(N))$
- But $i_C(k/q)\approx i_D^{(N)}(k)$, then $i_C(t)\approx i_D(tq/N)=i_D(\lambda t)$
- $di_C(t)/dt=\lambda N i_C(t)(1-i_C(t))$
  - We need the pairwise meeting rate to go to zero $\lambda=\lambda_0/N$

[Kurtz1970]

$\{X_N(t), N \text{ natural}\}$

a family of Markov process in Z^m

with rates $r_N(k,k+h)$, k,h in Z^m

It is called density dependent if it exists a continuous function f() in R^m such that

$r_N(k,k+h) = N f(1/N\, k, h)$, $h \neq 0$

Define $F(x)=\Sigma_h h f(x,h)$

Kurtz's theorem determines when $\{X_N(t)\}$ are ***close*** to the solution of the differential equation:

$$\frac{\partial x(s)}{\partial s} = F(x(s)),$$

The formal result [Kurtz1970]

Theorem. Suppose there is an open set E in R^m and a constant M such that

$|F(x)-F(y)|<M|x-y|$, x,y in E

$\sup_{x \text{ in } E}\Sigma_h|h|\ f(x,h) <\infty$,

$\lim_{d\to\infty}\sup_{x \text{ in } E}\Sigma_{|h|>d}|h|\ f(x,h) = 0$

Consider the set of processes in $\{X_N(t)\}$ such that

$\lim_{N\to\infty} 1/N\ X_N(0) = x_0$ in E

and a solution of the differential equation

$$\frac{\partial x(s)}{\partial s} = F(x(s)), \quad x(0) = x_0$$

such that x(s) is in E for 0<=s<=t, then for each δ>0

$$\lim_{N\to\infty} \Pr\left\{ \sup_{0\le s\le t} \left| \frac{1}{N} X_N(s) - X(s) \right| > \delta \right\} = 0$$

Application to epidemic routing

$r_N(n_I)=\lambda\, n_I\, (N-n_I) = N\, (\lambda N)\, (n_I/N)\, (1-n_I/N)$

assuming $\beta = \lambda N$ keeps constant (e.g. node density is constant)

$f(x,h)=f(x)=x(1-x)$, $F(x)=f(x)$

as $N\rightarrow\infty$, $n_I/N \rightarrow i(t)$, s.t.

$$i'(t) = \beta i(t)(1-i(t))$$

with initial condition

$$i(0) = \lim_{N\rightarrow\infty} n_I(0)/N$$

multiplying by N

$$I'(t) = \lambda I(t)(N-I(t))$$

What can we do with the fluid model?

- Derive an estimation of the number of infected nodes at time t
  - e.g. if $I(0)=1 \rightarrow I(t)=N/(1+(N-1)e^{-N\lambda t})$

What can we do with the fluid model?

Delivery delay T_d: time from pkt generation at the src until the dst receives the pkt

CDF of T_d, $P(t) := Pr(T_d < t)$ given by:

$$P'(t) = \lambda I(t)(1 - P(t))$$

delivery rate at time t

infected nodes-dst meeting rate

prob. that pkt is not delivered yet

Average delay

$$E[T_d] = \int_0^{\infty} (1 - P(t))dt$$

Avg. num. of copies sent at delivery

$$E[C] = \int_0^{\infty} I(t)P'(t)dt - 1$$

What can we do with the fluid model?

Consider recovery process, eg IMMUNE (dest. node cures infected node):

$$I'(t) = \lambda I(N - I - R) - \lambda I$$

$$R'(t) = \lambda I$$

R(t): num. of recovered nodes

Num. of susceptible nodes

Total num. of copies made: $\lim_{t \to \infty} R(t)$

Total buffer usage $\int_0^\infty I(t)\,dt$

More flexible than Markov models

to model all the different variants,

e.g. limited-time forwarding

$$I_r'(t) = \lambda(I_r(t)+1)(N - I_r(t) - T(t) - 1) - \mu I_r(t),$$
$$T'(t) = \mu I_r(t)$$

or probabilistic forwarding, K-hop forwarding...

under different recovery schemes (VACCINE, IMMUNE,...)

Our contribution

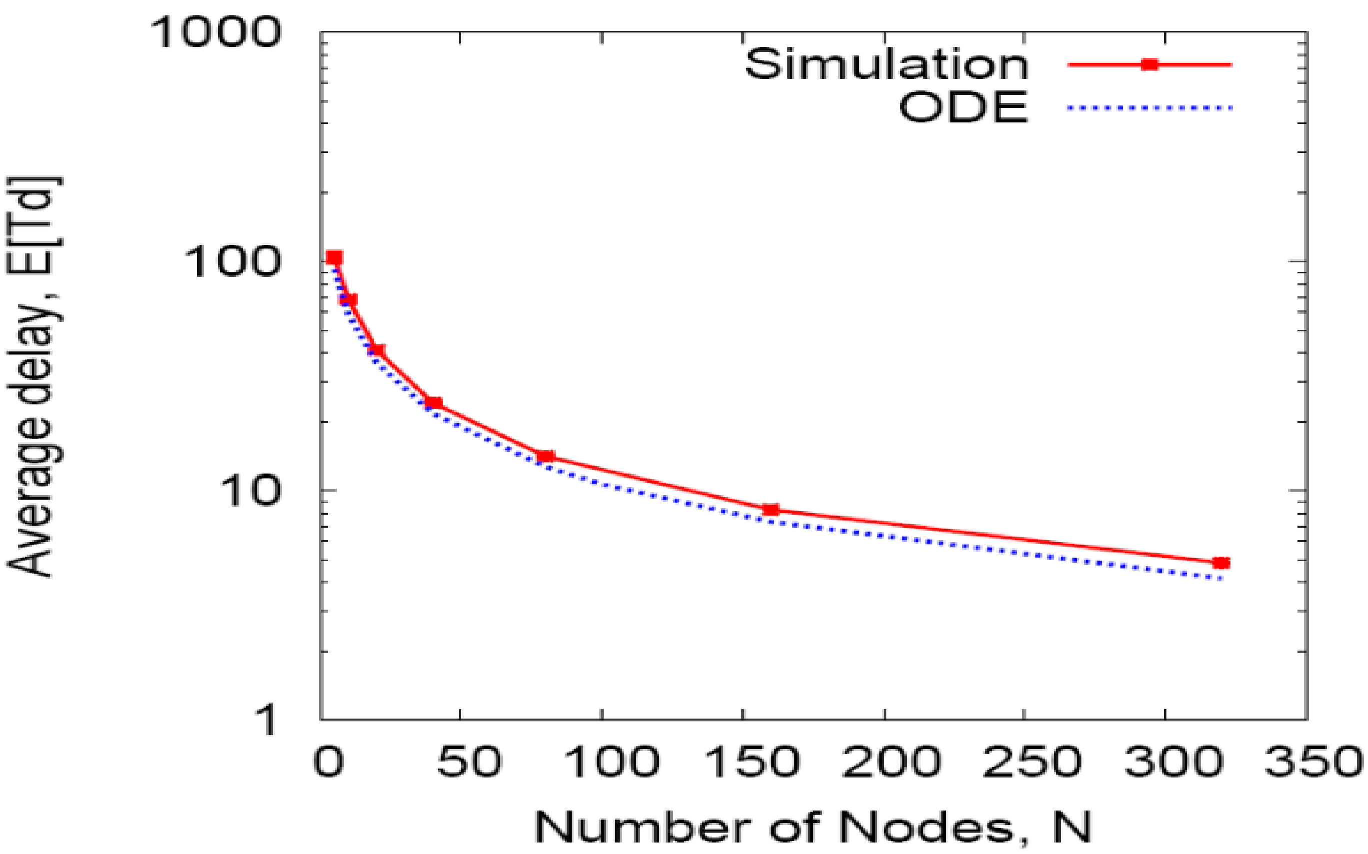
Closed formulas for average delay, number of copies and CDF in many cases

Asymptotic results

Numerical evaluation always possible without scaling problems

Study of delay vs buffer occupancy or delay vs power consumption for different forwarding schemes

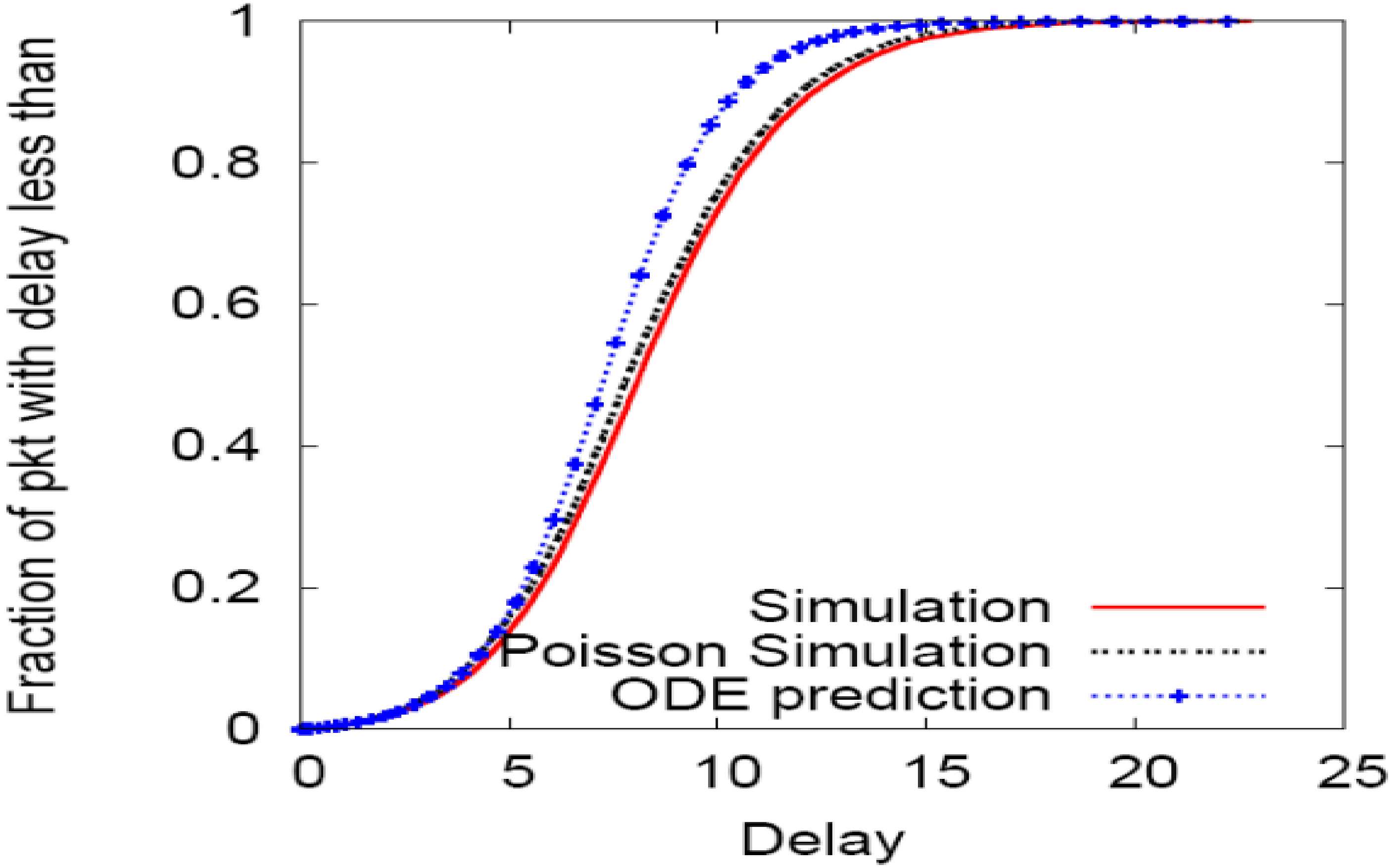
Epidemic Routing Average delay



ODE provides good prediction on average delay

Delay distribution

CDF of delay under epidemic routing, N=160



Modeling error mainly due to approx. of ODE

Some results

Extensible to other schemes

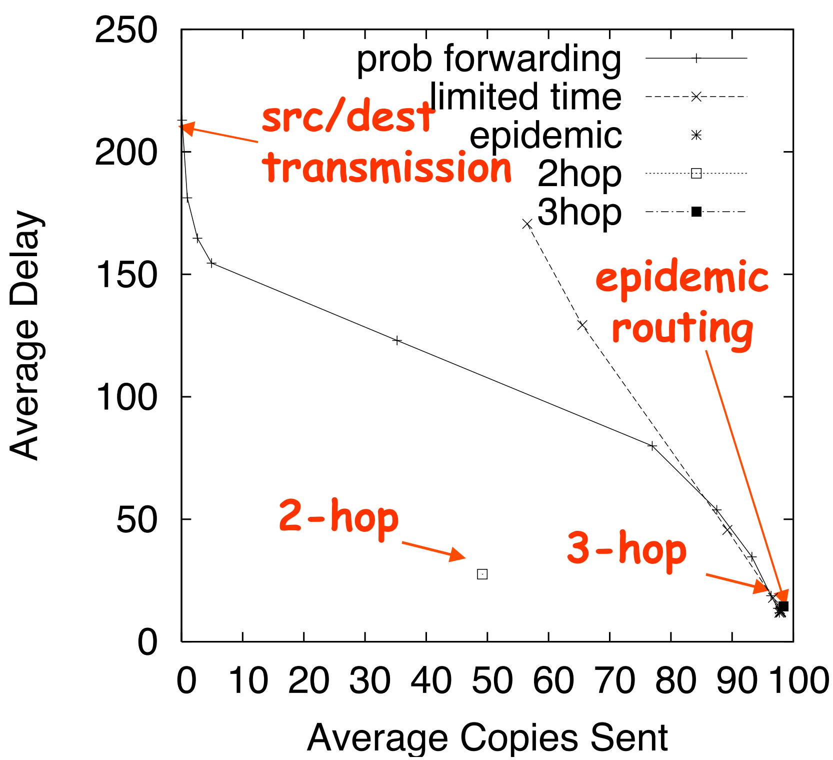
	$I(t), P(t)$	$E[T_d]$	C, G
Epidemic routing	$I(t) = \frac{N}{1+(N-1)e^{-\lambda N t}}$ $P(t) = 1 - \frac{N}{N-1+e^{\lambda N t}}$	$\sim \frac{\ln N}{\lambda(N-1)}$	$C = \frac{N-1}{2}, G \approx N-1(IM)$ $G = \frac{N-3+\sqrt{N^2-2N+5}}{2}(IM\_TX)$
2-hop forwarding	$I(t) = N-(N-1)e^{-\lambda t}$ $P(t) = 1-e^{N-1-\lambda N t-(N-1)e^{-\lambda t}}$	$\sim \frac{1}{\lambda}\sqrt{\frac{\pi}{2}}\frac{1}{\sqrt{N-1}}$	$C \sim \sqrt{\frac{\pi}{2}}\sqrt{N}$, $G = \frac{N-1}{2}$
Prob. forwarding	$I(t) = \frac{N}{1+(N-1)e^{-p\lambda N t}}$ $P(t) = 1-(\frac{N}{N-1+e^{p\lambda N t}})^{1/p}$	$\frac{\ln N}{\lambda(N-1)} \le E[T_d] \le \frac{\ln N}{\lambda p(N-1)}$	$C = \frac{p(N-1)}{1+p}$

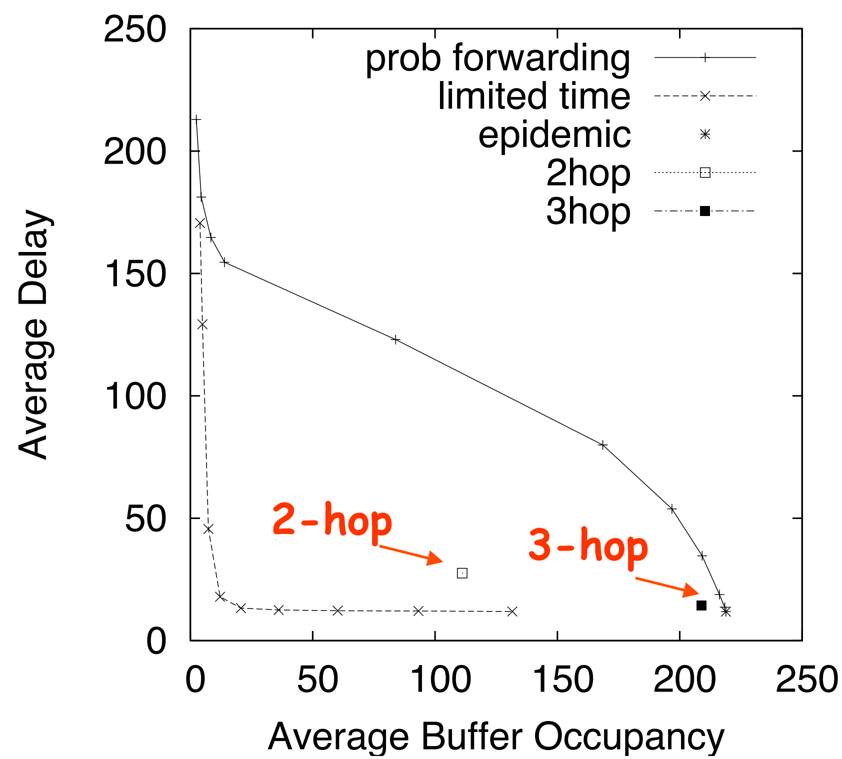
Matching results from Markov chain model, obtained easier

An application: Tradeoffs evaluation

Delay vs Power

Delay vs Buffer





Other issues

Not considered in this presentation

- Effect of different buffer management techniques when the buffer is limited
- ODEs by moment closure technique

References

Papers discussed

- Markovian models
  - Message Delay in Mobile Ad Hoc Networks, R. Groenevelt, G. Koole, and P. Nain, Performance, Juan-les-Pins, October 2005
  - Impact of Mobility on the Performance of Relaying in Ad Hoc Networks, A. Al-Hanbali, A.A. Kherani, R. Groenevelt, P. Nain, and E. Altman, IEEE Infocom 2006, Barcelona, April 2006
- Fluid models
  - Performance Modeling of Epidemic Routing, X. Zhang, G. Neglia, J. Kurose, D. Towsley, Elsevier Computer Networks, Volume 51, Issue 10, July 2007, Pages 2867-2891

Other

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- [Gupta99] The capacity of Wireless Networks, P. Gupta, P.R. Kumar, IEEE Conference on Decision and Control 1999
- [Kurtz70] Solution of ordinary differential equations as limits of pure jump markov processes, T. G. Kurtz, Journal of Applied Probabilities, pages 49-58, 1970