Performance Evaluation

Second Part
Lecture 3

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Outline

- Limit of Markovian models
- Mean Field (or Fluid) models
  - exact results
  - extensions
  - Applications
    - Bianchi’s model
  - (Heterogeneous networks)
  - Applications
    - Epidemic routing
Heterogeneous Networks

- Denote $P(d)$ the probability that a node has degree $d$
- If the degree does not change much, we can replace $d$ with $<d>$
  - what we have done for ER graphs $(N,p)$
    - Binomial with parameters $(N-1,p)$
- How should we proceed (more) correctly?
  - Split the nodes in degree classes
  - Write an equation for each class
- Remark: following derivation will not be as rigorous as previous ones
Heterogeneous Networks

- \( N_d \): number of nodes with degree \( d \) (=\( N \cdot P(d) \))
- \( I_d \): number of infected nodes with degree \( d \)
- Given node \( i \) with degree \( d \) and a link \( e_{ij} \), what is the prob. that \( j \) has degree \( d' \)?
  - \( P(d') \)? NO

- and if degrees are uncorrelated? i.e. \( \text{Prob(}	ext{neighbour has degree } d' | \text{node has a degree } d\text{)} \) independent from \( d \),
  - \( P(d') \)? NO
  - Is equal to \( d' / \langle d \rangle \cdot P(d') \)
Heterogeneous Networks

- Given node $i$ with degree $d$ and a link $e_{ij}$
- Prob. that $j$ has degree $d'$ is
  \[ \frac{d'}{\langle d \rangle} P(d') \]
- Prob. that $j$ has degree $d'$ and is infected
  \[ \frac{d'}{\langle d \rangle} P(d') \frac{I_{d'}}{N_{d'}} \]
  \[ \text{more correct } \frac{(d'-1)}{\langle d \rangle} P(d') \frac{I_{d'}}{N_{d'}} \]
- Prob. that $i$ is infected through link $e_{ij}$ is
  \[ p = p_g \sum_{d'} (d'-1) \frac{P(d')}{\langle d \rangle} \frac{I_{d'}}{N_{d'}} \]
- Prob. that $i$ is infected through one link
  \[ 1 - (1-p)^d \]
Heterogeneous Networks

\[ E[(I_d(k+1)-I_d(k)|I(k) = I)] = (N_d-I_d)(1-(1-p)^d) \]

- \[ p = p_g \sum_{d'} (d'-1)/\langle d \rangle \ P(d') \ I_{d'}/N_{d'} \]

\[ f_d^{(N)}(i) = (1-i_d)(1-(1-p)^d) \]

- \[ i_d = I_d/N_d \]

- if we choose \( p_g = p_{g0} / N \)

\[ f_d(i) = (1-i_d) \ p_{g0} \ d \sum_{d'} (d'-1)/\langle d \rangle \ P(d') \ i_{d'} \]

\[ di_d(t)/dt = f_d(i(t)) = p_{g0} \ (1-i_d(t)) \ \Theta(t) \]
Heterogeneous Networks

- \( \frac{di_d(t)}{dt} = f_d(i(t)) = p g_0 (1-i_d(t)) \ d \ \Theta(t), \)
  - for \( d=1,2... \)
  - \( \Theta(t) = \sum_{d'}(d'-1)/<d> \ P(d') \ i_{d'}(t) \)
  - \( i_d(0) = i_{d0}, \ \text{for} \ d=1,2... \)

If \( i_d(0) << 1, \ \text{for small} \ t \)
- \( \frac{di_d(t)}{dt} \approx p g_0 \ d \ \Theta(t) \)
- \( \frac{d\Theta(t)}{dt} = \sum_{d'}(d'-1)/<d> \ P(d') \ \frac{di_{d'}(t)}{dt} \)
  \( \approx p g_0 \ \sum_{d'}(d'-1)/<d> \ P(d') \ d' \ \Theta(t) = \)
  \( = p g_0 \ (\langle d^2 \rangle - \langle d \rangle)/\langle d \rangle \ \Theta(t) \)
Heterogeneous Networks

- \( \frac{d\Theta(t)}{dt} \approx p_g_0 (<d^2>-<d>) / <d> \Theta(t) \)
  - Outbreak time: \( <d> / ((<d^2>-<d>) p_g_0) \)
    - For ER \( <d^2>=<d>(<d>+1) \), we find the previous result, \( 1/(<d>p_g_0) \)
    - What about for Power-law graphs, \( P(d) \sim d^{-\gamma} \)?
- For the SIS model:
  - \( \frac{d\Theta(t)}{d} \approx p_g_0 (<d^2>-<d>) / <d> \Theta(t) - r_0 \Theta(t) \)
  - Epidemic threshold: \( p_g_0 (<d^2>-<d>) / (<d>r_0) \)
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  - (Heterogeneous networks)
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    - Epidemic routing
Outline and references

Introduction to epidemic routing

Markovian models

Message Delay in Mobile Ad Hoc Networks, R. Groenevelt, G. Koole, and P. Nain, Performance, Juan-les-Pins, October 2005


Fluid models

Standard Epidemic Routing

\[ S \rightarrow 2 \rightarrow 4 \rightarrow 5 \rightarrow 3 \rightarrow D \]

\[ T_D \]

\[ t_1 \]

\[ t_2 \]
Epidemic Style Routing

Epidemic Routing [Vahdat&Becker00]

- Propagation of a pkt -> Disease Spread
- achieve min. delay, at the cost of transm. power, storage

trade-off delay for resources

- K-hop forwarding, probabilistic forwarding, limited-time forwarding, spray and wait...
2-Hop Forwarding

At most 2 hop
Limited Time Forwarding
Epidemic Style Routing

Epidemic Routing [Vahdat&Becker00]

Propagation of a pkt -> Disease Spread
achieve min. delay, at the cost of transm. power, storage

trade-off delay for resources
K-hop forwarding, probabilistic forwarding,
limited-time forwarding, spray and wait...

Recovery: deletion of obsolete copies after delivery to dest., e.g.,

TIMERS: when time expires all the copies are erased

IMMUNE: dest. cures infected nodes

VACCINE: on pkt delivery, dest propagates anti-pkt through network
IMMUNE Recovery

\[ t_0 \]

\[ T_D \]

Time
VACCINE Recovery
Outline

Introduction to Epidemic Routing

Markovian models
  the key to Markov model
  Markovian analysis of epidemic routing

Fluid models
The setting we consider

$N+1$ nodes
moving independently in an finite area $A$
with a fixed transmission range $r$
and no interference
1 source, 1 destination

Performance metrics:
Delivery delay $T_d$
Avg. num. of copies at delivery $C$
Avg. total num. of copies made $G$
Avg. buffer occupancy
Standard random mobility models

Random Waypoint model (RWP)

- Next positions (Xi)s are uniformly distributed
- Speeds (Vi)s are uniformly distributed (Vmin, Vmax)

Random Direction model (RD)

- Directions (αi) are uniformly distributed (0, 2π)
- Speeds (Vi) are uniformly distributed (Vmin, Vmax)
- Travel times (Ti) are exponentially / generally distributed
The key to Markov model

[Groenevelt05]

if nodes move according to standard random mobility model (random waypoint, random direction) with average relative speed \( E[V^*] \), and if \( Nr^2 \) is small in comparison to \( A \) pairwise meeting processes are *almost* independent Poisson processes with rate:

\[
\lambda \approx \frac{2wrv^*}{A}
\]

\( w \): mobility specific constant
Exponential distribution finds its roots in the independence assumptions of each mobility model:

- **Nodes move independently of each other**
- **Random waypoint**: future locations of a node are independent of past locations of that node.
- **Random direction**: future speeds and directions of a node are independent of past speeds and directions of that node.

There is some probability $q$ that two nodes will meet before the next change of direction. At the next change of direction the process repeats itself, almost independently.
Why “almost”? 

Pairwise meeting processes are almost independent Poisson processes with rate:

\[ \lambda \approx \frac{2wrV^*}{A} \]

\( w \): mobility specific constant

1. Inter-meeting times are not exponential
   - If N1 and N2 have met in the near past they are more likely to meet (they are close to each other)
   - The more the bigger it is \( r^2 \) in comparison to \( A \)

2. Meeting processes are not independent
   - If in \([t, t+\tau]\) N1 meets N2 and N2 meets N3, it is more likely that N1 meets N3 in the same interval
   - The more the bigger it is \( r^2 \) in comparison to \( A \)
   - Moreover if \( Nr^2 \) is comparable with \( A \) (dense network) a lot of meeting happen at the same time.
Examples

Nodes move on a square of size 4x4 km$^2$ ($L=4$ km)
Different transmission radii ($R=50,100,250$ m)

Random waypoint and random direction:
no pause time
$[v_{\text{min}}, v_{\text{max}}]=[4,10]$ km/hour

Random direction: travel time $\sim \exp(4)$
Pairwise Inter-meeting time

\[ P(T > t) = e^{-\lambda t} \]
\[ \log P(T > t) = -\lambda t \]
Pairwise Inter-meeting time

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Pairwise Inter-meeting time

\[ P(T > t) = e^{-\lambda t} \]
\[ \log P(T > t) = -\lambda t \]
The derivation of $\Lambda$

Assume a node in position $(x_1, y_1)$ moves in a straight line with speed $V_1$. Position of the other node comes from steady-state distribution with pdf $\pi(x, y)$.

Look at the area $A$ covered in $\Delta_t$ time:
The derivation of \( \Lambda \)

Probability that nodes meet given by

\[
p_{x_1,y_1} = \int \int_{A} \pi(x,y) dx dy.
\]

For small \( r \) the points in \( \pi(x,y) \) in \( A \) can be approximated by \( \pi(x_1,y_1) \) to give

\[
p_{x_1,y_1} \approx 2r \cdot V_1^* \cdot \Delta_t \cdot \pi(x_1,y_1).
\]

Unconditioning on \( (x_1,y_1) \) gives

\[
p = \int_{0}^{L} \int_{0}^{L} p_{x_1,y_1} \cdot \pi(x_1,y_1) dx_1 dy_1
\]

\[
\approx 2r \cdot V_1^* \cdot \Delta_t \cdot \int_{0}^{L} \int_{0}^{L} \pi^2(x_1,y_1) dx_1 dy_1
\]
The derivation of $\lambda$

**Proposition:** Let $r \ll L$. The inter-meeting time for the random direction and the random waypoint mobility models is approximately exponentially distributed with parameter

$$\lambda \approx 2 r \cdot E[V^*] \cdot \int_0^L \int_0^L \pi^2(x,y) dx dy,$$

Here $E[V^*]$ is the average relative speed between two nodes and $\pi(x,y)$ is the pdf in the point $(x,y)$. 
The derivation of $\lambda$

**Proposition:** Let $r \ll L$. The inter-meeting time for the random direction and the random waypoint mobility models is approximately exponentially distributed with parameter

\[
\lambda_{RD} \approx \frac{2rE[V^*]}{L^2}, \quad \lambda_{RW} \approx \frac{2r\omega E[V^*]}{L^2}.
\]

Here $E[V^*]$ is the average relative speed between two nodes and $\omega \approx 1.3683$ is the Waypoint constant.

If speeds of the nodes are constant and equal to $v$,

\[
\lambda_{RD} \approx \frac{8rv}{\pi L^2}, \quad \lambda_{RW} \approx \frac{8\omega rv}{\pi L^2}.
\]
Summary up to now

First steps of this research
  a good intuition
  some simulations validating the intuition for a reasonable range of parameters

What could have been done more
  prove that the results is asymptotically ($r \to 0$)
    true “in some sense”

What can be built on top of this?
  Markovian models for routing in DTNs
Model the number of occurrences of the message as an absorbing Continuous Time Markov Chain (C-MC):

- State $i \in \{1, \ldots, N\}$ represents the number of occurrences of the message in the network.
- State $A$ represents the destination node receiving (a copy of) the message.
Model the number of occurrences of the message as an absorbing C-MC:

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