Performance Evaluation – Master UBINET
Assignment 1

Solutions have to be sent by January 30th 2011 at 12.00 to `giovanni.neglia@inria.fr`. Explain carefully your reasoning. The code has to be easily readable (and then well commented).

Ex. 1 — (P2P backup system)

A company offers a P2P-assisted backup system. For simplicity we describe its operation for a single file. There is a server that is always online and where the file is always present, additional copies can be stored at the computers of the N customers. Each of the customers has in fact installed a P2P client that can store locally a copy of the file and replicate it to other clients. The customer can be online or not.

The system works according to the following time-slotted model. During one slot each online node with the file (there is no difference between the server and the peers in this respect) selects at random one peer. If a connection can be established (i.e. the remote peer is online), and the file is not present at the remote peer, then it is replicated with probability p_c. At the end of a slot each online peer can go offline with probability p_{off}, in this case the local copy of the file is lost (if the peer has one). At the end of a slot each offline peer can come online with probability p_{on}. Peer dynamics and replication activities are independent.

1. Show that the system can be modeled as a Discrete Markov Chain.
2. Show that a Mean-Field limit can be correctly derived when N diverges under an opportune parameter scaling. Write the corresponding system of Ordinary Differential Equations (ODE).
3. Determine the equilibrium points for the ODE system, i.e. the occupancy measure vectors that are constant solutions of the system (for a specific initial condition). Do these equilibrium points correspond to stationary distributions for the Markov Chain?
4. Consider $N = 1000$, $p_c = 10^{-3}$, $p_{on} = 3 * 10^{-4}$, $p_{off} = 10^{-4}$. If you can only simulate the system up to a size $N = 100$, describe which experiment you could carry on to evaluate if the Mean-Field approximation is satisfactory for $N = 1000$.
5. Perform the experiment above. This requires to develop a simulator for the Markov Chain and to solve numerically the ODE system. You can use Matlab or write your code in C or Java. Provide a figure comparing simulation results and the model for $N = 100$ and $N = 1000$.