Seller revenue

- N bidders
- Values are independent random values between 0 and 1
- Expected i^{th} largest utility is $(N+1-i)/(N+1)$
- Expected seller revenue is $(N-1)/(N+1)$

1st price auction

- Player with the highest bid gets the good and pays a price equal to her/his bid
- Being truthful is not a dominant strategy anymore!
  - Consider for example if I knew other players' utilities
- How to study it?

1^{st} price auction

- Assumption: for each player the other values are i.i.d. random variables between 0 and 1
  - to overcome the fact that utilities are unknown
- Player i's strategy is a function s() mapping value v_i to a bid b_i
  - s() strictly increasing, differentiable function
  - $0 \leq s(v) \leq v$ ➔ $s(0)=0$
- We investigate if there is a strategy s() common to all the players that leads to a Nash equilibrium

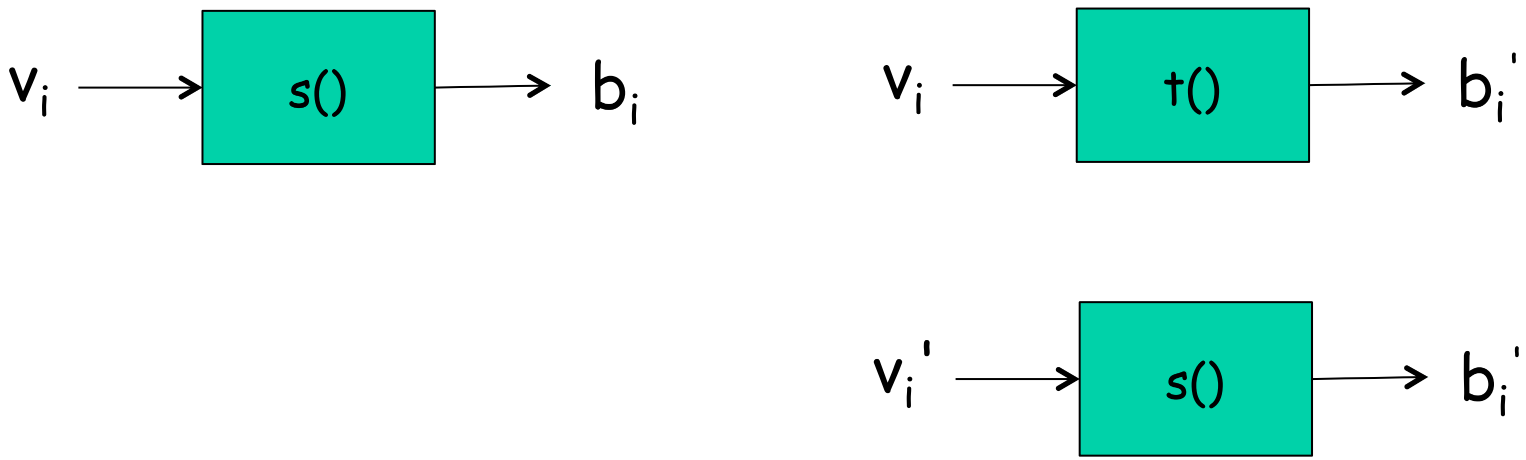
1^{st} price auction

- Assumption: for each player the other values are i.i.d. random variables between 0 and 1
- Player i's strategy is a function s() mapping value v_i to a bid b_i
- Expected payoff of player i if all the players plays s():
  - $U_i(s(v_1),\ldots s(v_i),\ldots s(v_N)) = \underbrace{v_i^{N-1}}_{\text{prob. i wins}} \underbrace{(v_i - s(v_i))}_{\text{i's payoff if he/she wins}}$

1^{st} price auction

- Expected payoff of player i if all the players play s():
  - $U_i(s(v_1),\dots s(v_i),\dots s(v_N)) = v_i^{N-1} (v_i - s(v_i))$
- What if i plays a different strategy t()?
  - If all players playing s() is a NE, then :
  - $U_i(s(v_1),\dots s(v_i),\dots s(v_N)) = v_i^{N-1} (v_i - s(v_i))$
    $\geq s^{-1}(t(v_i))^{N-1} (v_i - t(v_i)) = U_i(s(v_1),\dots t(v_i),\dots s(v_N))$
- Difficult to check for all the possible functions t() different from s()
- Help from the **revelation principle**

The Revelation Principle



- All the strategies are equivalent to bidder i supplying to s() a different value of v_i

1^{st} price auction

- Expected payoff of player i if all the players plays s():
  - $U_i(s(v_1),\ldots s(v_i),\ldots s(v_N)) = v_i^{N-1} (v_i - s(v_i))$
- What if i plays a different strategy t()?
- By the revelation principle:
  - $U_i(s(v_1),\ldots t(v_i),\ldots s(v_N)) =_{eq} U_i(s(v_1),\ldots s(v),\ldots s(v_N)) = v^{N-1} (v_i - s(v))$
- If $v_i^{N-1} (v_i - s(v_i)) \geq v^{N-1} (v_i - s(v))$ for each v (and for each v_i)
  - Then all players playing s() is a NE

1^{st} price auction

- If $v_i^{N-1}(v_i - s(v_i)) \geq v^{N-1}(v_i - s(v))$ for each v (and for each v_i)
  - Then all players playing s() is a NE
- $f(v) = v_i^{N-1}(v_i - s(v_i)) - v^{N-1}(v_i - s(v))$ is minimized for $v = v_i$
- $f'(v) = 0$ for $v = v_i$,
  - i.e. $(N-1)\, v_i^{N-2}(v_i - s(v_i)) - v_i^{N-1} s'(v_i) = 0$ for each v_i
  - $s'(v_i) = (N-1)(1 - s(v_i)/v_i)$, $s(0) = 0$
  - Solution: $s(v_i) = (N-1)/N\ v_i$

1^{st} price auction

- All players bidding according to
  $s(v) = (N-1)/N\ v$ is a NE
- Remarks
  - They are not truthful
  - The more they are, the higher they should bid
- Expected seller revenue
  - $((N-1)/N)\ E[v_{max}] = ((N-1)/N)\ (N/(N+1)) = (N-1)/(N+1)$
  - Identical to 2^{nd} price auction!
  - A general revenue equivalence principle

Outline

- Preliminaries
  - Auctions
  - Matching markets
- Possible approaches to ads pricing
- Google mechanism

- References
  - Easley, Kleinberg, "Networks, Crowds and Markets", ch.9,**10**,15

Matching Markets

v_{ij}: value that buyer j gives to good i

v_{1a}, v_{2a}, v_{3a}

v_{1b}, v_{2b}, v_{3b}

v_{1c}, v_{2c}, v_{3c}

How to match a set of different goods to a set of buyers with different evaluations

Matching Markets

v_{ij}: value that buyer j gives to good i

goods	buyers	
1	a	v_{1a}, v_{2a}, v_{3a}
2	b	v_{1b}, v_{2b}, v_{3b}
3	c	v_{1c}, v_{2c}, v_{3c}

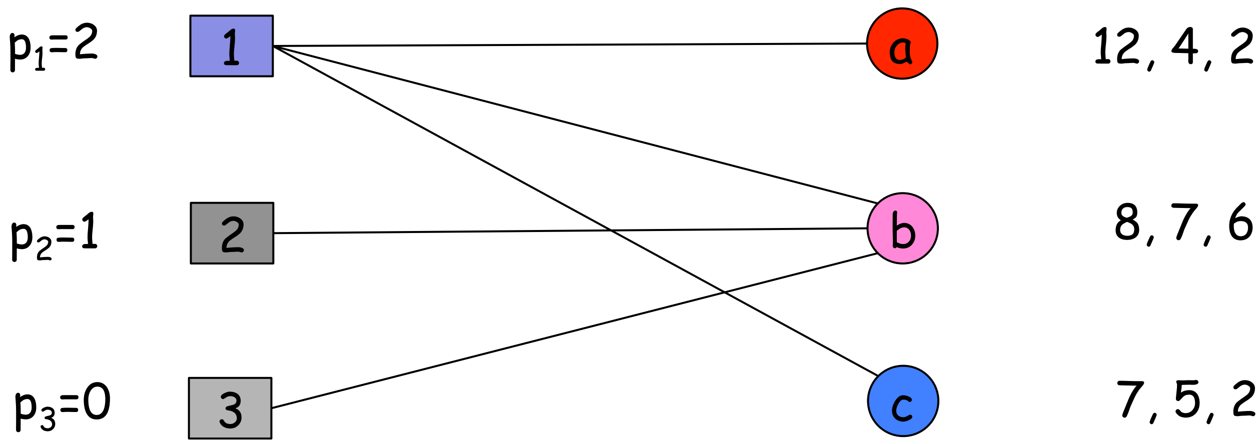
How to match a set of different goods to a set of buyers with different evaluations

maximize $$\sum_{i,j=1}^{N} x_{ij} v_{ij}$$

subject to $$\sum_{j=1}^{N} x_{ij} = 1, \qquad \sum_{i=1}^{N} x_{ij} = 1,$$

over $$x_{ij} \in \{0,1\}$$

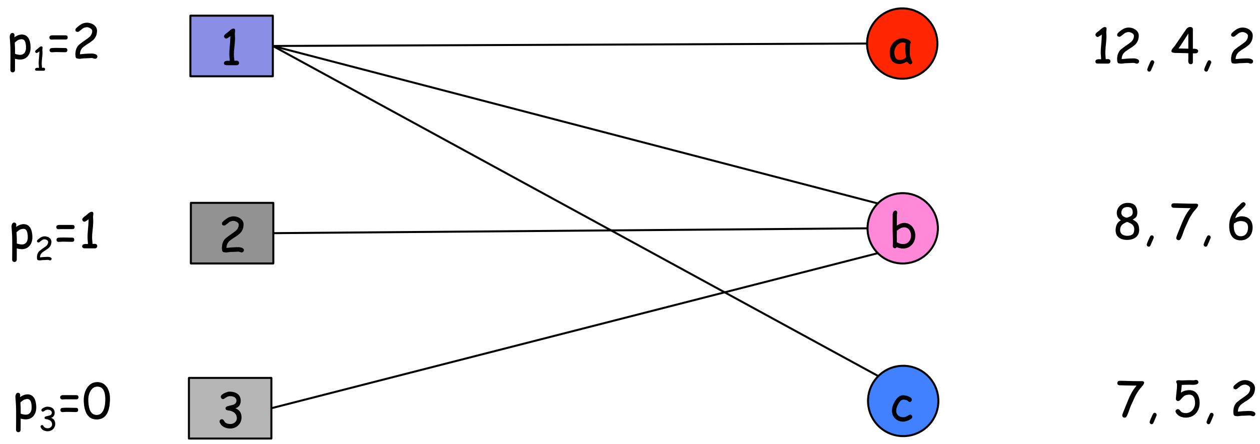
Matching Markets



Which goods buyers like most? Preferred seller graph

How to match a set of different goods to a set of buyers with different evaluations

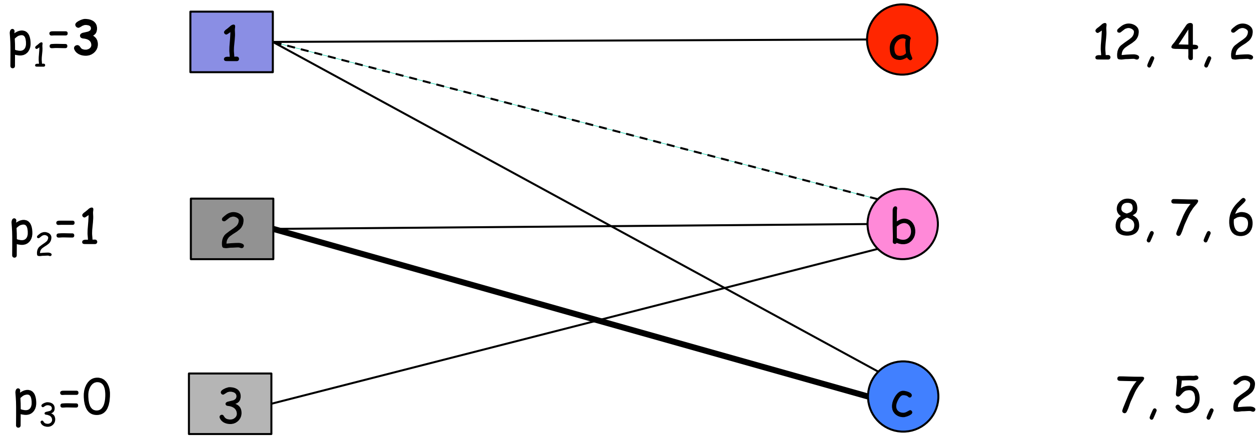
Matching Markets



Which goods buyers like most? Preferred seller graph

- Given the prices, look for a perfect matching on the preferred seller graph
- There is no such matching for this graph

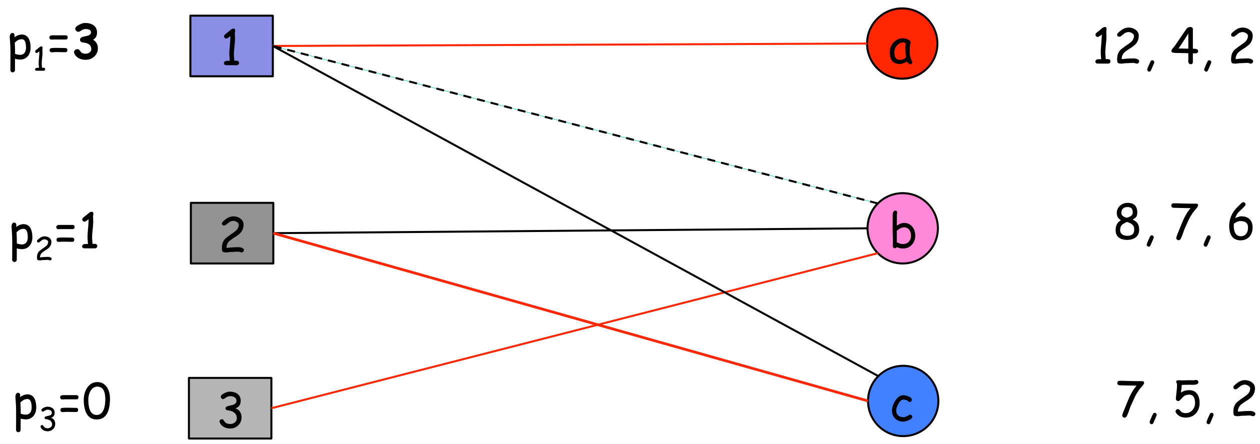
Matching Markets



Which goods buyers like most? Preferred seller graph

- But with different prices, there is

Matching Markets



Which goods buyers like most? Preferred seller graph

- But with different prices, there is
- Such prices are **market clearing prices**

Market Clearing Prices

- They always exist
  - And can be easily calculated if valuations are known
- They are socially optimal in the sense that
  - they achieve the maximum total valuation of any assignment of sellers to buyers
  - Or, equivalently, they maximize the sum of all the payoffs in the network (both sellers and buyers)

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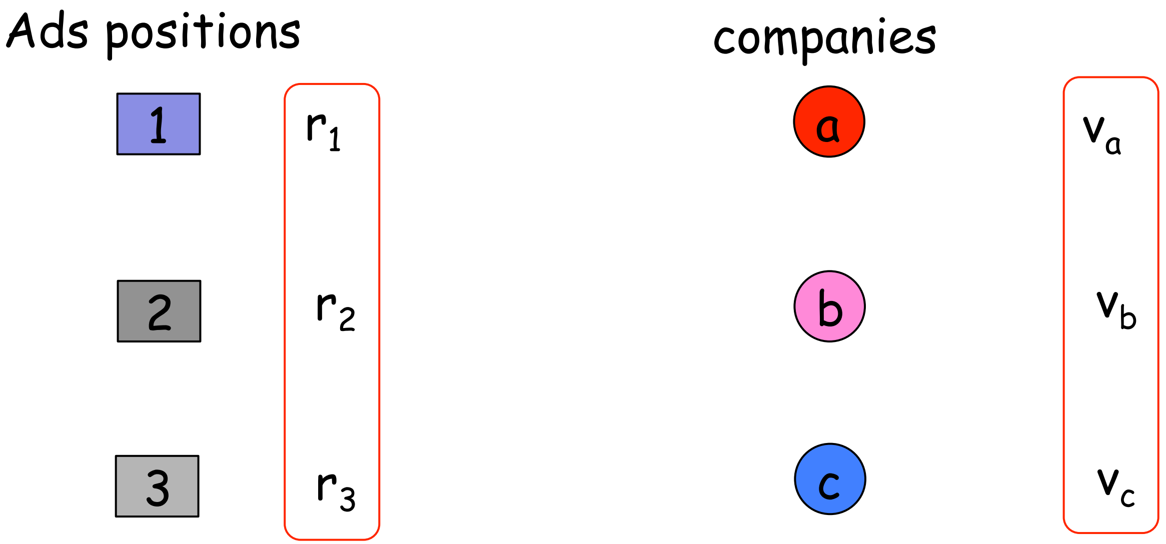
Ads pricing



r_i: click rate for an ad in position i
(assumed to be independent
from the ad and known a priori)

v_i: value that company i
gives to a click

How to rank ads from different companies

Ads pricing as a matching market

Ads positions		companies	
1	r_1	a	$v_a r_1, v_a r_2, v_a r_3$
2	r_2	b	$v_b r_1, v_b r_2, v_b r_3$
3	r_3	c	$v_c r_1, v_c r_2, v_c r_3$

r_i: click rate for an ad in position i (assumed to be independent from the ad and known a priori)

v_i: value that company i gives to a click

- Problem: Valuations are not known!
- … but we could look for something as 2nd price auctions

The VCG mechanism

- The correct way to generalize 2nd price auctions to multiple goods
- Vickrey-Clarke-Groves
- Every buyers should pay a price equal to the social value loss for the others buyers
  - Example: consider a 2nd price auction with $v_a > v_b > \ldots v_N$
    - With *a* present the others buyers get 0
    - Without *a*, *b* would have got the good with a value v_b
    - then the social value loss for the others is v_b

The VCG mechanism

- The correct way to generalize 2nd price auctions to multiple goods
- Vickrey-Clarke-Groves
- Every buyers should pay a price equal to the social value loss for the others buyers
  - If V_B^S is the maximum total valuation over all the possible perfect matchings of the set of sellers S and the set of buyers B,
  - If buyer β gets good i, he/she should be charged $V_{B-\beta}^{S} - V_{B-\beta}^{S-i}$

VCG example

Ads positions

1 $r_1=10$

2 $r_2=5$

3 $r_3=2$

companies

a $v_a=3$

b $v_b=2$

c $v_c=1$

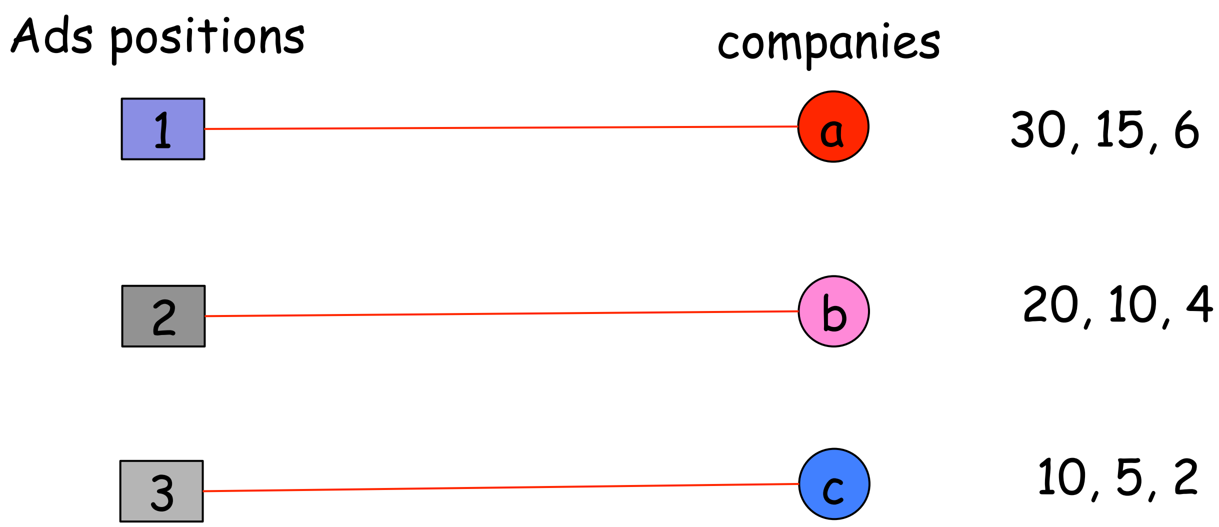
r_i: click rate for an ad in position i
(assumed to be independent
from the ad and known a priori)

v_i: value that company i
gives to a click

VCG example

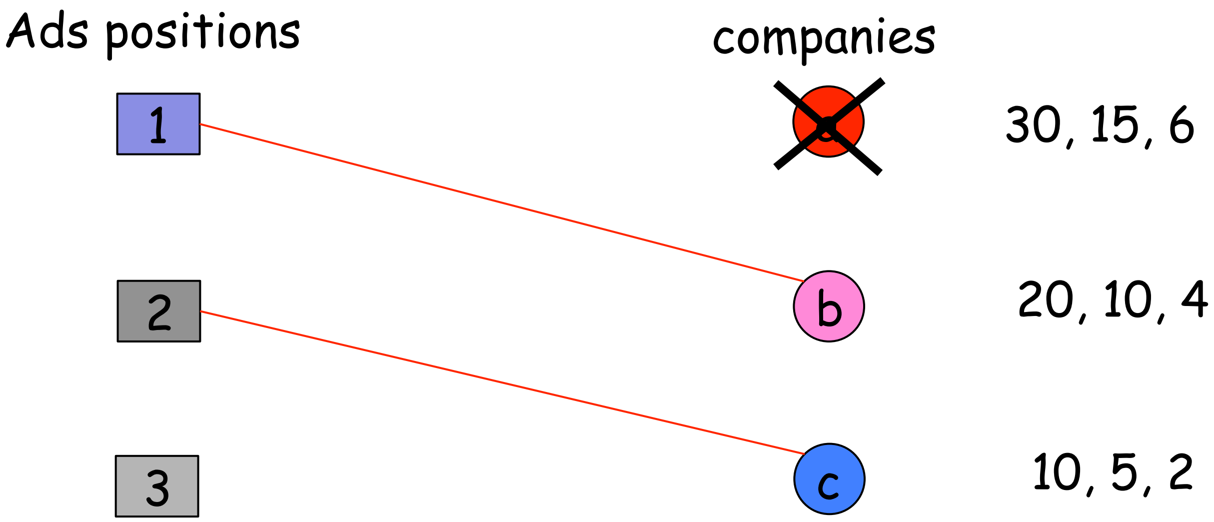
Ads positions	companies	
1	a	30, 15, 6
2	b	20, 10, 4
3	c	10, 5, 2

VCG example



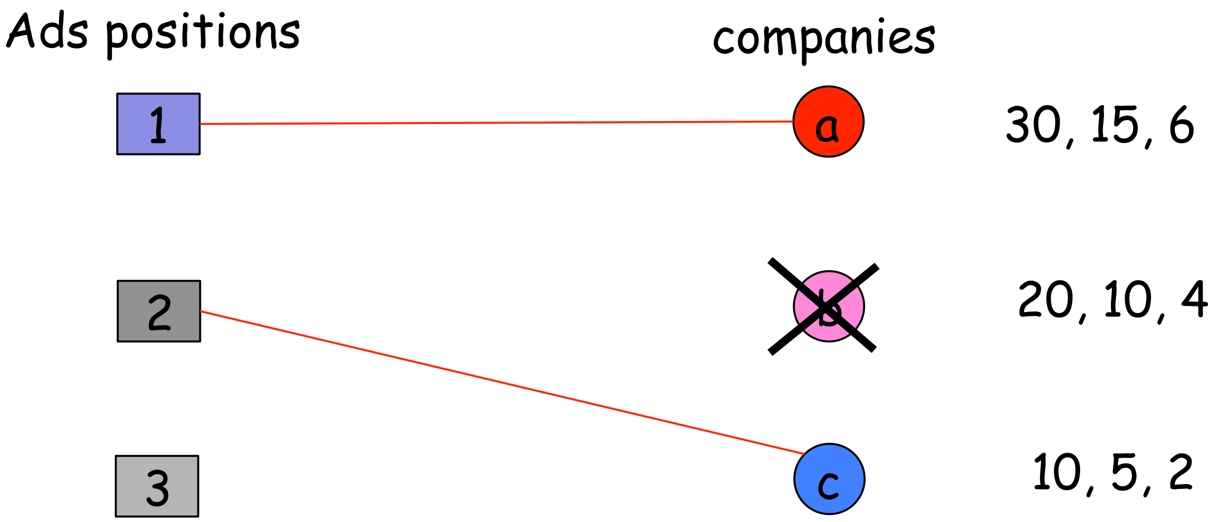
- This is the maximum weight matching
- *a* gets 30, *b* gets 10 and *c* gets 2

VCG example



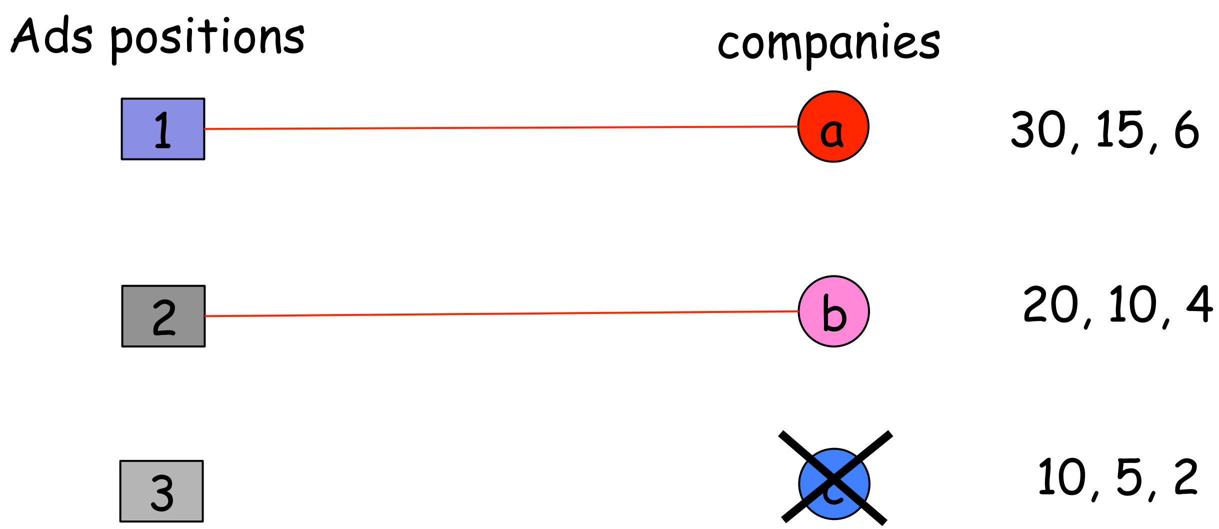
- If *a* weren't there, *b* and *c* would get 25 instead of 12,
- Then *a* should pay 13

VCG example



- If *b* weren't there, *a* and *c* would get 35 instead of 32,
- Then *b* should pay 3

VCG example



- If *c* weren't there, nothing would change for *a* and *b*,
- Then *c* should pay 0

The VCG mechanism

- Every buyers should pay a price equal to the social value loss for the others buyers
  - If V_B^S is the maximum total valuation over all the possible perfect matchings of the set of sellers S and the set of buyers B,
  - If buyer j gets good i, he/she should be charged $V_{B-\beta}^{S} - V_{B-\beta}^{S-i}$
- Under this price mechanism, truth-telling is a dominant strategy

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Google's GSP auction

- Generalized Second Price
- Once all the bids are collected $b_1 > b_2 > \ldots b_N$
- Company i pays b_{i+1}
- In the case of a single good (position), GSP is equivalent to a 2^{nd} price auction, and also to VCG
- But why Google wanted to implement something different???

GSP properties

- Truth-telling may not be an equilibrium

GSP example

Ads positions		companies	
1	$r_1=10$	a	$v_a=7$
2	$r_2=4$	b	$v_b=6$
3	$r_3=0$	c	$v_c=1$

r_i: click rate for an ad in position i (assumed to be independent from the ad and known a priori)

v_i: value that company i gives to a click

- If each player bids its true evaluation, a gets a payoff equal to 10
- If *a* bids 5, *a* gets a payoff equal to 24

GSP properties

- Truth-telling may not be an equilibrium
- There is always at least 1 socially optimal NE

GSP example

Ads positions		companies	
1	$r_1=10$	a	$v_a=7$
2	$r_2=4$	b	$v_b=6$
3	$r_3=0$	c	$v_c=1$

r_i: click rate for an ad in position i (assumed to be independent from the ad and known a priori)

v_i: value that company i gives to a click

- Multiple NE
  - *a* bids 5, *b* bids 4 and *c* bids 2
  - *a* bids 3, *b* bids 5 and *c* bids 1

GSP properties

- Truth-telling may not be an equilibrium
- There is always at least 1 socially optimal NE
- Revenues can be higher or lower than VCG
  - Attention: the revenue equivalence principle does not hold for auctions with multiple goods!
  - Google was targeting higher revenues…
  - … not clear if they did the right choice.

GSP example

Ads positions		companies	
1	$r_1=10$	a	$v_a=7$
2	$r_2=4$	b	$v_b=6$
3	$r_3=0$	c	$v_c=1$

- Multiple NE
  - *a* bids 5, *b* bids 4, *c* bids 2 ➔ google's revenue=48
  - *a* bids 3, *b* bids 5, *c* bids 1 ➔ google's revenue=34
- With VCG, google's revenue=44

Other issues

- Click rates are unknown and depend on the ad!
  - Concrete risk: low-quality advertiser bidding high may reduce the search engine's revenue
  - Google's solution: introduce and ad-quality factor taking into account actual click rate, relevance of the page and its ranking
    - Google is very secretive about how to calculate it => the market is more opaque
- Complex queries, nobody paid for
  - Usually engines extrapolate from simpler bids