Ex. 1 — [3 point] Given a network, let $\mathcal{J}$ denote the set of links and $\mathcal{R}$ the set of routes used by different source-destination pairs. We also denote the flow rate on route $r \in \mathcal{R}$ by x_r and the aggregate traffic rate on link $j \in \mathcal{J}$ by y_j (then $y_j = \sum_{r \in \mathcal{R} | j \in r} x_r$). Assume that the network operator incurs a cost at each link equal to $F_j(y_j) = \frac{y_j}{C_j - y_j}$, where C_j is the capacity of link j. The operator would like at the same time to provide a rate allocation close to proportional fairness and to reduce its cost. It decides then to solve the following maximization problem:

$$\begin{array}{ll} \text{maximize} & \displaystyle\sum_{r \in \mathcal{R}} \ln x_r - \sum_{j \in \mathcal{J}} F_j \left(\sum_{r \in \mathcal{R} | j \in r} x_r \right) \\ \text{over} & x \geq 0. \end{array}$$

1. Can you propose a distributed rate adaptation mechanism that solves the above optimization problem?

Ex. 2 — [2 point] In an ads auction for a given keyword, there are three possible positions with expected click rates per-day 12, 5 and 1. Three companies bid for these positions. They value one click respectively 5$, 4$ and 2$.

1. In the case of a Generalized Second Price (GSP) auction, do the following bids produce a Nash equilibrium $b_1 = 3$, $b_2 = 4$, $b_3 = 2$?
2. In the case of a VCG auction, how are the ads priced? How does the seller's revenue changes in comparison to the GSP auction studied above?