Distributed Optimization and Games

Auctions
Giovanni Neglia
INRIA – EPI Maestro
7 February 2018
Outline

- Preliminaries
  - Auctions
  - Matching markets
- Possible approaches to ads pricing
- Google mechanism

References

- Easley, Kleinberg, "Networks, Crowds and Markets", ch.9,10,15
Google’s GSP auction

- Generalized Second Price
- Once all the bids are collected $b_1 > b_2 > \ldots > b_N$
- Company $i$ pays $b_{i+1}$
- In the case of a single good (position), GSP is equivalent to a 2nd price auction, and also to VCG
- But why Google wanted to implement something different???
GSP properties

- Truth-telling may not be an equilibrium
GSP example

Ads positions

1
$\text{r}_1=10$

2
$\text{r}_2=4$

3
$\text{r}_3=0$

companies

• $a$
  $\text{v}_a=7$

• $b$
  $\text{v}_b=6$

• $c$
  $\text{v}_c=1$

$\text{r}_i$: click rate for an ad in position $i$ (assumed to be independent from the ad and known a priori)

$\text{v}_i$: value that company $i$ gives to a click

- If each player bids its true evaluation, $a$ gets a payoff equal to 10
- If $a$ bids 5, $a$ gets a payoff equal to 24
GSP properties

- Truth-telling may not be an equilibrium
- There is always at least 1 socially optimal NE
GSP example

Ads positions

1  \( r_1 = 10 \)

2  \( r_2 = 4 \)

3  \( r_3 = 0 \)

companies

a  \( v_a = 7 \)

b  \( v_b = 6 \)

c  \( v_c = 1 \)

\( r_i \): click rate for an ad in position \( i \) (assumed to be independent from the ad and known a priori)
\( v_i \): value that company \( i \) gives to a click

- Multiple NE
  - a bids 5, b bids 4 and c bids 2
  - a bids 3, b bids 5 and c bids 1
GSP properties

- Truth-telling may not be an equilibrium
- There is always at least 1 socially optimal NE
- Revenues can be higher or lower than VCG
  - Attention: the revenue equivalence principle does not hold for auctions with multiple goods!
  - Google was targeting higher revenues...
  - ... not clear if they did the right choice.
## GSP example

<table>
<thead>
<tr>
<th>Ads positions</th>
<th>companies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>r_1=10</td>
</tr>
<tr>
<td>2</td>
<td>r_2=4</td>
</tr>
<tr>
<td>3</td>
<td>r_3=0</td>
</tr>
</tbody>
</table>

- **Multiple NE**
  - a bids 5, b bids 4, c bids 2  ➔ google’s revenue=48
  - a bids 3, b bids 5, c bids 1  ➔ google’s revenue=34

- **With VCG, google’s revenue=44**
Other issues

- Click rates are unknown and depend on the ad!
  - Concrete risk: low-quality advertiser bidding high may reduce the search engine’s revenue
  - Google’s solution: introduce and ad-quality factor taking into account actual click rate, relevance of the page and its ranking
    - Google is very secretive about how to calculate it => the market is more opaque

- Complex queries, nobody paid for
  - Usually engines extrapolate from simpler bids