Int J Software Informatics, Vol.3, No.4, December 2009, pp. 501–511 E-mail: ijsi@iscas.ac.cn
International Journal of Software and Informatics ISSN 1673-7288 http://www.ijsi.org

Construct Control Meshes of Helicoids over Trapezium Domain*

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Abstract In this paper, we present a geometric construction of control meshes of helicoids over trapezium domain. We first introduce the quasi-Bézier basis in the space spanned by $\{1, t, \cos t, \sin t, t\sin t, t\cos t\}$, with $t \in [0, \alpha], \alpha \in [0, 2\pi)$. We denote the curves expressed by the quasi-Bézier basis as algebraic-trigonometric Bézier curves, for short AT-Bézier curves. Then we find out the transform matrices between the quasi-Bézier basis and $\{1, t, \cos t, \sin t, t\sin t, t\cos t\}$. Finally, we present the control mesh representation of the helicoids and the geometric construction of the control mesh. In detail, we construct the control polygon of the planar Archimedean solenoid, which is also expressed with the quasi-Bézier basis, and then generate the mesh vertices by translating points of the control polygon.

Key words: helicoids; Archimedean solenoid; minimal surface; AT-Bézier surfaces; control mesh representation; CAGD/CAM

Chen WY, Xu G, Wang GZ. Construct control meshes of helicoids over trapezium domain. *Int J Software Informatics*, **2009, 3(4): 501–511. http://www.ijsi.org/1673-7288/3/501.htm**

1 Introduction

Modeling of special surfaces is very important for computer aided design and computer graphics because of the beautiful properties of special surfaces[1,3,11]. Recently, minimal surfaces have attracted more attentions in CAGD[2,5−9,12−17]. Helicoid is an important kind of minimal surfaces. Catalan verified that all ruled non-planar minimal surfaces are helicoids[10]. Helicoids have various applications in manufacture and architecture, for example, if a sliding board adopts helicoids, one can acquire constant acceleration when sliding along it. Hence, it is valuable to introduce helicoids into CAGD/CAM systems.

* This effort is sponsored by the National Natural Science Foundation of China under Grant Nos. 60473130, 60773179; the National Basic Research Program (973) of China under Grant No. 2004CB318000; the Natural Science Foundation of Zhejiang Province under Grant No. Y1090718.
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Manuscript received 2009-07-04; revised 2009-11-03; accepted 2009-11-25; published online 2009-12-30.

Most of curves and surfaces in CAD/CAM systems are represented by control polygons/control meshes. However, helicoids cannot be represented by Bézier or NURBS surfaces. Hence, in order to introduce helicoids into CAD/CAM systems, we should first propose the control mesh representation of helicoids. Refs.[7] and [14] proposed two kinds of control mesh representations of helicoids. However, in their presentations, the domain of the parameters is restricted to be a rectangle. It is inconvenient to obtain a trimmed surface of helicoids over a rectangular domain. In particular, rectangular domain is a special case of trapezium domain. Hence, in this paper, we present a new control mesh representation of helicoids, letting the domain of the parameters be a trapezium.

Motivated by this purpose, first, we introduce the quasi-Bézier basis $\{u_{i,5}(t)\}_{i=0}^{5}$ in the space $\Gamma_5 = span\,\{1, t, \cos t, \sin t, t\sin t, t\cos t\}$, with $t \in [0, \alpha], \alpha \in [0, 2\pi)$, which has been discussed in Mainar01. In this paper, we denote the curves expressed by the quasi-Bézier basis $\{u_{i,5}(t)\}_{i=0}^{5}$ as algebraic-trigonometric Bézier curves, for short AT-Bézier curves. Then we find out the transform matrices between $\{u_{i,5}(t)\}_{i=0}^{5}$ and $\{1, t, \cos t, \sin t, t\sin t, t\cos t\}$. Hence, the definition of the basis is explicit, and the control points can be attained expediently. Secondly, the tensor product representation of a patch of helicoids is derived, as well as the control mesh of the patch. Finally, the geometric construction of the control mesh is discussed.

Section 2 introduces the AT-Bézier curves with AT-Bézier basis. Section 3 provides the representation of the helicoid patch defined on a trapezium domain. The geometric construction of the control mesh of each helicoid patch is presented in Section 4. Section 5 concludes the research and discusses the future work.

2 AT-Bézier Curves

2.1 AT-Bézier basis

Mainar01 defines the quasi-Bézier basis in the space $\Gamma_5 = span\{1, t, \cos t, \sin t, t\sin t, t\cos t\}$, $t \in [0, \alpha], \alpha \in [0, 2\pi)$. We denote it as $\{u_{i,5}(t)\}_{i=0}^{5}$. Let

$$\begin{aligned} F(t) &= 3(t - \sin t) - t(1 - \cos t), \\ G(t) &= t - \sin t. \end{aligned}$$

Set the derivatives of $F(t)$ be $f_i = F^{(i)}(\alpha), g_i = G^{(i)}(\alpha)$, and denote

$$\begin{aligned} &e = f_1^2 - f_0 f_2, \quad g = f_0 f_3 - f_1 f_2, \quad h = f_2^2 - f_1 f_3, \quad H = \tfrac{h}{e(f_2 h + f_3 g + f_4 e)}, \\ &c = \cos\alpha, \quad s = \sin\alpha, \quad d = g + \alpha h. \end{aligned}$$

Then the quasi-Bézier basis is

$$\begin{aligned} u_{5,5}(t) &= F(t)/F(\alpha), \\ u_{4,5}(t) &= \tfrac{f_1}{e}(F(t) - f_1 \cdot u_{0,5}(t)), \\ u_{3,5}(t) &= H(h \cdot F(t) + g \cdot F'(t) + e \cdot F''(t)), \\ u_{2,5}(t) &= u_{3,5}(\alpha - t), \\ u_{1,5}(t) &= u_{4,5}(\alpha - t), \\ u_{0,5}(t) &= u_{5,5}(\alpha - t). \end{aligned}$$

From the definition, $\{u_{i,5}(t)\}_{i=0}^{5}$ satisfy:

(1) 0 is i-fold zero of $u_{i,5}(t)$, and α is $(5-i)$-fold zero of $u_{i,5}(t)$.

(2) $\sum\limits_{i=0}^{5} u_{i,5}(t) = 1$.

For expedience, we can rewrite the definition explicitly, i.e.

$$\left(u_{0,5}(t), u_{1,5}(t), u_{2,5}(t), u_{3,5}(t), u_{4,5}(t), u_{5,5}(t)\right)^{\mathrm{T}} = A\left(1, t, \sin t, \cos t, t\sin t, t\cos t\right)^{\mathrm{T}},$$

where the transform matrix A equals

$$\begin{pmatrix} \frac{2\alpha}{f_0} & \frac{-2}{f_0} & \frac{2+c-f_1}{f_0} & \frac{f_0-2\alpha}{f_0} & \frac{-s}{f_0} & \frac{-c}{f_0} \\ \frac{2f_1(f_0-\alpha f_1)}{f_0 e} & \frac{2f_1^2}{f_0 e} & \frac{f_1(e-g_1^2)}{f_0 e} & \frac{2f_1(\alpha f_1-f_0)}{f_0 e} & \frac{f_1(sf_1-cf_0)}{f_0 e} & \frac{f_1(g_0 s-f_1)}{f_0 e} \\ 2dH & -2hH & 2f_1 g_1 H & -2dH & -2f_0 g_1 H & 2(h-f_1 g_1)H \\ 2gH & 2hH & (e-3h)H & -2gH & -gH & (h-e)H \\ \frac{2f_1}{e} & \frac{-2f_1^2}{f_0 e} & \frac{3f_1^2}{f_0 e} & \frac{-2f_1}{e} & \frac{-f_1}{e} & \frac{-f_1^2}{f_0 e} \\ 0 & \frac{2}{f_0} & \frac{-3}{f_0} & 0 & 0 & \frac{1}{f_0} \end{pmatrix}$$

And the inverted matrix A^{-1} equals

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \frac{f_0}{f_1} & -\frac{g}{h} & \frac{d}{h} & -\frac{2(h+f_0 s)e+gd}{2f_1 h_0} & \alpha \\ 0 & \frac{f_0}{f_1} & -\frac{g}{h} & \frac{2f_0 g_1}{h} & -\frac{(gg_1+es)f_0+eh}{f_1 h_0} & s \\ 1 & 1 & \frac{g_0^2-e}{h} & \frac{2(f_1 g_1-h)}{h} & \frac{gh-(gg_1+es)f_1}{f_1 h_0} & c \\ 0 & 0 & \frac{2e}{h} & \frac{2(3h-2f_1 g_1)}{h} & \frac{(2f_1 g_1-3h)g-ed}{f_1 h_0} & \alpha s \\ 0 & \frac{f_0}{f_1} & -\frac{g}{h} & \frac{2(3f_0 g_1-d)}{h} & \frac{(d-3f_0 g_1)g-(h+f_0 s)e}{f_1 h_0} & \alpha c \end{pmatrix} \tag{0.1}$$

2.2 AT-Bézier curves

The AT-Bézier curve can be defined as

$$p(t) = \sum_{i=0}^{5} P_i u_{i,5}(t), t \in [0, \alpha],$$

where $\{u_{i,5}(t)\}_{i=0}^{5}$ are the AT-Bézier basis functions and $\{P_i\}_{i=0}^{5}$ are control points. Several transcendental curves can be expressed as an AT-Bézier curve, for instance, the Archimedean solenoid and the conical solenoids. The followings are some examples.

A piece of the conical solenoids can be expressed as (Fig.1):

$$p(t) = (t\cos t, t\sin t, t) = \sum_{i=0}^{5} P_i^0 u_{i,5}(t), t \in [0, \alpha],$$

where the six controls points $\left\{P_i^0\right\}_{i=0}^{5}$ are

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \frac{f_0}{f_1} \\ 0 \\ \frac{f_0}{f_1} \end{pmatrix} \begin{pmatrix} -\frac{g}{h} \\ \frac{2e}{h} \\ -\frac{g}{h} \end{pmatrix} \begin{pmatrix} \frac{2(3f_0 g_1-2d)}{h} \\ \frac{2(3h-2f_1 g_1)}{h} \\ \frac{d}{h} \end{pmatrix} \begin{pmatrix} \frac{(3f_0 g_1-2d)g+(h-f_0 s)e}{f_1(2f_0 g_1-d)} \\ \frac{(3h-2f_1 g_1)g+ed}{f_1(2f_0 g_1-d)} \\ \frac{2(h-f_0 s)e+gd}{2f_1(2f_0 g_1-d)} \end{pmatrix} \begin{pmatrix} \alpha c \\ \alpha s \\ \alpha \end{pmatrix} \tag{0.2}$$

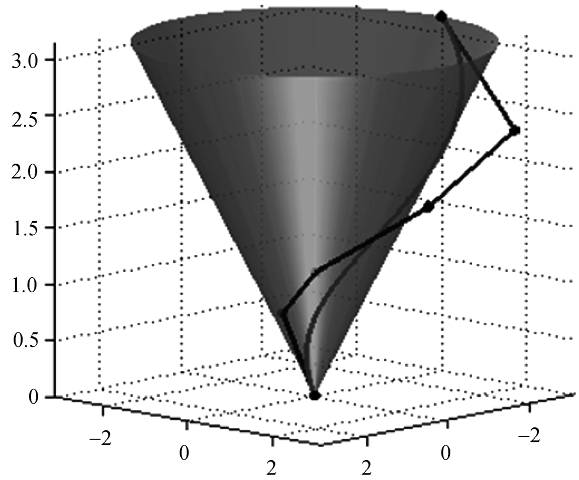


Figure 1. A piece of the conical solenoids and its control polygon under $\alpha = \pi$

Similarly, a piece of circular arc can be represented as

$$q(t) = (\cos t, \sin t, 0) = \sum_{i=0}^{5} P_i^1 u_{i,5}(t), t \in [0, \alpha],$$

where the six controls points $\left\{P_i^1\right\}_{i=0}^5$ are

$$\begin{pmatrix}1\\0\\0\end{pmatrix} \begin{pmatrix}1\\ \frac{f_0}{f_1}\\0\end{pmatrix} \begin{pmatrix}\frac{g_0^2-e}{h}\\ -\frac{g}{h}\\0\end{pmatrix} \begin{pmatrix}\frac{2(f_1g_1-h)}{h}\\ \frac{2f_0g_1}{h}\\0\end{pmatrix} \begin{pmatrix}\frac{(gg_1+es)f_1-gh}{f_1(2f_0g_1-d)}\\ \frac{(gg_1+es)f_0+eh}{f_1(2f_0g_1-d)}\\0\end{pmatrix} \begin{pmatrix}c\\s\\0\end{pmatrix} \tag{0.3}$$

3 Representation of the Helicoid Patch

Refs.[7, 16] discuss two representations of helicoids, which is defined over a rectangular domain, that is,

$$r(w, v) = (w\cos v, w\sin v, v), 0 \le v \le \alpha, \gamma_0 \le w \le \gamma_1.$$

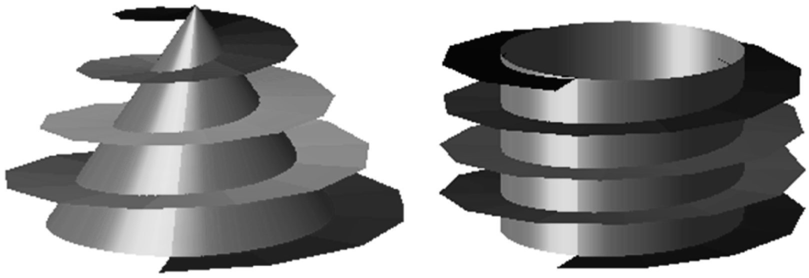
Here we will generalize the domain of the parameters to be a trapezium. Consider a helicoid patch defined as

$$r(w, v) = (w\cos v, w\sin v, v), 0 \le v \le \alpha, \gamma_0 + \beta_0 v \le w \le \gamma_1 + \beta_1 v.$$

Fig.2 shows the shape of the patches defined on trapezium domain and rectangular domain. In Fig.2(a), $\beta_0 = \beta_1 = 1$; in Fig.2(b), $\beta_0 = \beta_1 = 0$.

It is obvious that the w-parameter curves are all straight lines and the two v-boundaries are AT-Bézier curves, that is

$$\begin{aligned} r(\gamma_i + \beta_i v, v) &= ((\gamma_i + \beta_i v)\cos v, (\gamma_i + \beta_i v)\sin v, v),\\ &= \gamma_i(\cos v, \sin v, 0) + \beta_i(v\cos v, v\sin v, v), \quad 0 \le v \le \alpha, \ \ i = 0, 1. \end{aligned}$$



(a) Trapezium domain (b) Rectangular domain

Figure 2. Helicoids over trapezium domain and rectangular domain

Suppose $P_j^1, P_j^0, j = 0, \cdots, 5$ be given in Equation 2.2 and Equation 2.3, then, the control points of the corresponding boundaries are

$$\gamma_i P_j^1 + \beta_i P_j^0, \quad j = 0, \cdots, 5, \quad i = 0, 1,$$

Hence, we can represent the helicoids patch as a tensor product representation of AT-Bézier basis and Bézier basis of degree one. Suppose

$$u = \frac{w - (\gamma_0 + \beta_0 v)}{(\gamma_1 + \beta_1 v) - (\gamma_0 + \beta_0 v)}.$$

Then, the patch can be rewritten as

$$r(u,v) = \begin{pmatrix} (((\gamma_1 + \beta_1 v) - (\gamma_0 + \beta_0 v))u + (\gamma_0 + \beta_0 v)) \cos v \\ (((\gamma_1 + \beta_1 v) - (\gamma_0 + \beta_0 v))u + (\gamma_0 + \beta_0 v)) \sin v \\ v \end{pmatrix}$$

Let the control points be

$$P_{ij} = \gamma_i P_j^1 + \beta_i P_j^0, i = 0, 1, j = 0, \cdots, 5, \tag{0.4}$$

Then,

$$r(u,v) = \sum_{i=0}^{5} \sum_{j=0}^{1} P_{ij} u_{i,5}(v) B_{j,1}(u), v \in [0, \alpha], u \in [0, 1],$$

where $u_{i,5}(v)$ is AT-Bézier basis function and $B_{j,1}(u)$ is Bézier basis function of degree one. Fig.3 shows two examples with different α, with $\gamma_1 = 2\pi, \gamma_0 = 0, \beta_0 = \beta_1 = 1$.

4 Geometric Construction of the Control Meshes

4.1 Geometric construction of the control polygon of the Archimedean solenoid

Before discussing the geometric construction of the control mesh of the helicoids, we exploit the geometric construction of the planar Archimedean solenoid. In the next section, we will use it to construct the control meshes of the helicoids patches.

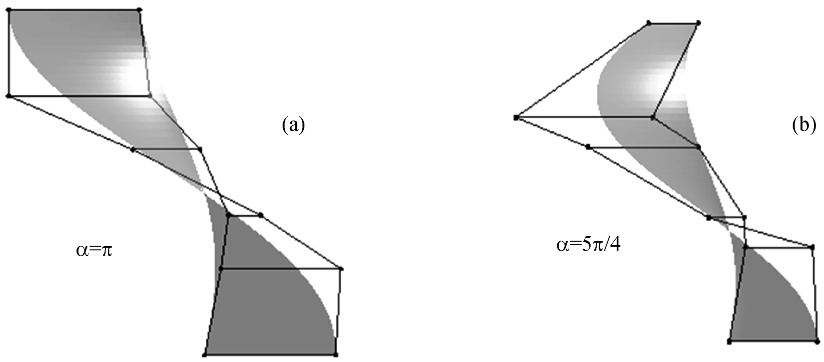


Figure 3. The helicoid patches and their control meshes

The planar Archimedean solenoid can be expressed as

$$p(t) = (t\cos t, t\sin t).$$

For $t_0 \geq 0$, we exploit the geometric construction of $p'(t_0), p''(t_0)$ (see Fig. 4):

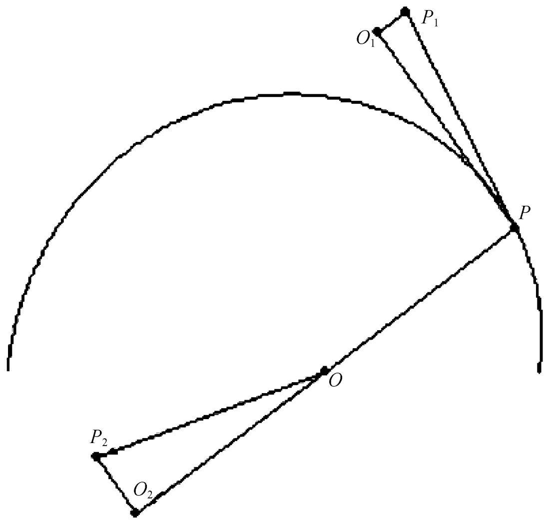


Figure 4. Geometric construction of $p'(t_0), p''(t_0)$

Theorem 1. Let $P = p(t_0)$ and O be the origin. Clockwise rotate PO along P to O_1 with a right angle. Set the point P_1 satisfy

$$|O_1P_1| = 1, \quad O_1P_1//OP.$$

Then $PP_1 = p'(t_0)$. Let point O_2 be the symmetry point of P along O. Set the point P_2 satisfy

$$|O_2P_2| = 2, \ O_2P_2//PO_1.$$

Then $OP_2 = p''(t_0)$.

Proof From the definition of P_1, P_2 and $OP = (t_0 \cos t_0, t_0 \sin t_0)$, we gain

$$p'(t_0) = (\cos t_0, \sin t_0) + (-t_0 \sin t_0, t_0 \cos t_0) = O_1P_1 + PO_1 = PP_1,$$
$$p''(t_0) = 2(-\sin t_0, \cos t_0) + (-t_0 \cos t_0, -t_0 \sin t_0) = O_2P_2 + OO_2 = OP_2.$$

Then, Theorem 1 holds true.

Now, from Theorem 1, we discuss the geometric construction of the control polygon of the Archimedean solenoids. A piece of the Archimedean solenoids can be represented as a AT-Bézier curve, i.e.

$$p(t) = ((t+t_0)\cos(t+t_0), (t+t_0)\sin(t+t_0)) = \sum_{i=0}^{5} Q_{i,5} u_{i,5}(t), \quad t \in [0, \alpha].$$

where $Q_{i,5} \in R^2$ are controls points. Then we can construct the points as follows.

Theorem 2. Suppose $Q_{0,5} = p(0), Q_{5,5} = p(\alpha)$. Following Theorem 1, we set

$$Q_{0,5}T_0^1 = p'(0), \quad OT_0^2 = p''(0),$$
$$Q_{5,5}T_1^1 = p'(\alpha), \quad OT_1^2 = p''(\alpha).$$

Let $Q_{1,5}, Q_{4,5}$ satisfy

$$Q_{0,5}Q_{1,5} = \frac{f_0}{f_1} Q_{0,5}T_0^1, \quad Q_{5,5}Q_{4,5} = -\frac{f_0}{f_1} Q_{5,5}T_1^1.$$

Let T_0^3, T_1^3 satisfy

$$Q_{1,5}T_0^3 = \frac{(2g_0 - f_0)e}{hf_1} Q_{0,5}T_0^1, \quad Q_{4,5}T_1^3 = -\frac{(2g_0 - f_0)e}{hf_1} Q_{5,5}T_1^1.$$

Let $Q_{2,5}, Q_{3,5}$ satisfy

$$T_0^3 Q_{2,5} = \frac{e}{h} OT_0^2, T_1^3 Q_{3,5} = \frac{e}{h} OT_1^2.$$

Then $\{Q_{i,5}\}_{i=0}^5$ are the control points (see Fig.5).

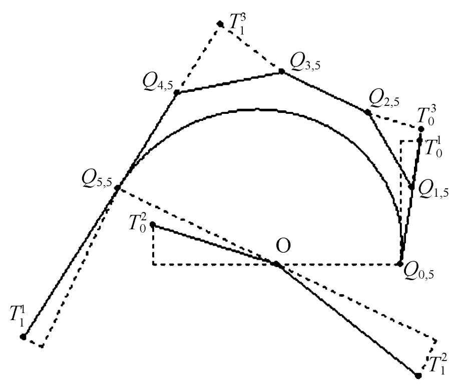


Figure 5. Geometric construction of the control polygon of the Archimedean solenoids

Proof Differentiating $p(t) = \sum_{i=0}^{5} Q_{i,5} u_{i,5}(t)$ at the point $t = 0$ and $t = \alpha$ to the first and the second order, we obtain

$$\begin{cases} \Delta Q_{0,5} = \frac{f_0}{f_1} p'(0), \\ \Delta Q_{1,5} = \frac{(2g_0 - f_0)e}{hf_1} p'(0) + \frac{e}{h} p''(0). \end{cases} \qquad \begin{cases} \Delta Q_{4,5} = \frac{f_0}{f_1} p'(\alpha), \\ \Delta Q_{3,5} = -\frac{(2g_0 - f_0)e}{hf_1} p'(\alpha) + \frac{e}{h} p''(\alpha). \end{cases}$$

Then

$$\begin{aligned} Q_{1,5} &= Q_{0,5} + \tfrac{f_0}{f_1} p'(0) = Q_{0,5} + \tfrac{f_0}{f_1} Q_{0,5} T_0^1, \\ Q_{4,5} &= Q_{5,5} - \tfrac{f_0}{f_1} p'(\alpha) = Q_{5,5} - \tfrac{f_0}{f_1} Q_{5,5} T_1^1, \\ Q_{2,5} &= Q_{1,5} + \tfrac{(2g_0 - f_0)e}{hf_1} p'(0) + \tfrac{e}{h} p''(0) = Q_{1,5} + Q_{1,5} T_0^3 + \tfrac{e}{h} O T_0^2, \\ Q_{3,5} &= Q_{4,5} - \tfrac{(2g_0 - f_0)e}{hf_1} p'(\alpha) + \tfrac{e}{h} p''(\alpha) = Q_{4,5} + Q_{4,5} T_1^3 + \tfrac{e}{h} O T_1^2. \end{aligned}$$

Hence, Theorem 2 holds true.

4.2 Geometric construction of the control mesh of the helicoids patch

As method mentioned above, we can obtain the control polygon for a segment of the Archimedean solenoid. In the following, we will present the geometric construction of the control mesh of the helicoids patch. For convenience, we only consider a special case of Equation 3.4. Suppose the helicoid patch is defined on the domain

$$0 \le v \le \alpha, 2k\pi + v \le w \le 2(k+1)\pi + v.$$

That is, let $\beta_0 = \beta_1 = 1, \gamma_0 = 2k\pi, \gamma_1 = 2(k+1)\pi$. After projecting the helicoids patch to the xy-plane, we get

$$((2\pi u + (\gamma_0 + v)) \cos v, (2\pi u + (\gamma_0 + v)) \sin v), v \in [0, \alpha], u \in [0, 1].$$

The four boundary curves are

$$(\gamma_0 + 2\pi u, 0), \quad u \in [0, 1]. \tag{0.5}$$

$$(((\gamma_0 + \alpha) + 2\pi u) \cos \alpha, ((\gamma_0 + \alpha) + 2\pi u) \sin \alpha), \quad u \in [0, 1]. \tag{0.6}$$

$$((\gamma_i + v) \cos(\gamma_i + v), (\gamma_i + v) \sin(\gamma_i + v)), \quad v \in [0, \alpha], \quad i = 0, 1. \tag{0.7}$$

So, the projected area is surrounded by two segments of the Archimedean solenoid (Equation 4.7) and two line segments (Equation 4.5 and Equation 4.6). Fig.6 shows the projected area with $\gamma_0 = 2\pi, \gamma_1 = 4\pi, \alpha = \frac{5}{4}\pi, \beta_0 = \beta_1 = 1$.

So, our goal is first to construct the control points Q_{ij} of the projected area, and secondly translate Q_{ij} to obtain the control points P_{ij}, where $i = 0, 1, j = 0, \cdots, 5$. From the above section, the control polygon $\{Q_{ij}\}_{j=0}^{5}$ is easy to derive for $i = 0, 1$, which correspond to the boundaries Equation 4.7. So the work turns to found how to translate Q_{ij} along the z-axis to get P_{ij}. Setting the 6 control points $\{l_i\}_{i=0}^{5}$ be

$$0, \quad \frac{f_0}{f_1}, \quad -\frac{g}{h}, \quad \frac{d}{h}, \quad \frac{2(h - f_0 s)e + gd}{2f_1(2f_0 g_1 - d)}, \quad \alpha,$$

we get $v = \sum_{i=0}^{5} l_i u_{i,5}(v)$, which is the z-coordinate of the helicoids patch. So, moving Q_{ij} along the z-axis with the above lengths, we can get P_{ij}.

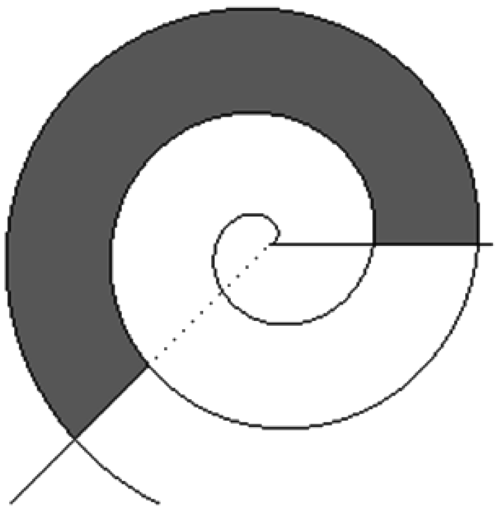


Figure 6. Projection of the helicoid patch to the xy-plane

Theorem 3 Geometric construction of the control mesh:
Step 1. (Fig.6) Choose the value k , and set

$$\gamma_0 = 2k\pi, \gamma_1 = 2(k+1)\pi.$$

Then, on the plane Q, from the planar Archimedean solenoid, we get two boundary curves

$$((\gamma_i + v)\cos(\gamma_i + v), (\gamma_i + v)\sin(\gamma_i + v)) \quad (i = 0, 1).$$

Step 2. (Fig.7(a)) Construct the control polygons of the boundary curves following the theorem 2. Denote the control points as

$$Q_{i0}, Q_{i1}, Q_{i2}, Q_{i3}, Q_{i4}, Q_{i5}, (i = 0, 1).$$

Step 3. (Fig.7(b)) Let $P_{i0} = Q_{i0}$. Corresponding to Q_{ij}, set P_{ij} satisfy

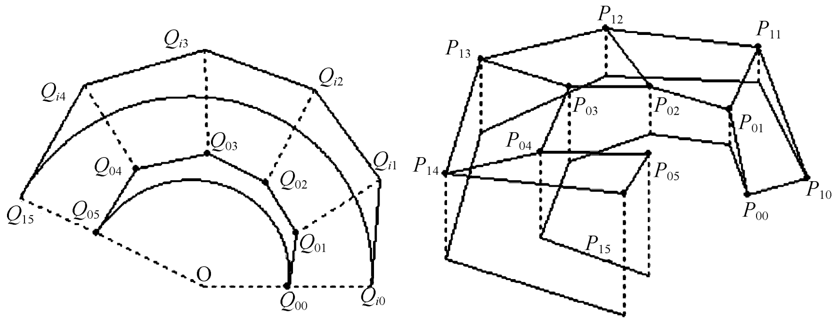
$$P_{ij}Q_{ij} \perp Q, |P_{ij}Q_{ij}| = l_j, i = 0, 1, \quad j = 1, \cdots, 5.$$

Then the tensor product surface

$$r(u, v) = \sum_{i=0}^{5}\sum_{j=0}^{1} P_{ij} u_{i,5}(v) B_{j,2}(u), v \in [0, \alpha], u \in [0, 1],$$

is a helicoids patch defined as

$$r(u, v) = (u\cos v, u\sin v, v), 0 \le v \le \alpha, \gamma_0 + v \le u \le \gamma_1 + v.$$



(a) Control points Q_{ij} of the boundary curves (b) Translate Q_{ij} to get P_{ij}

Figure 7. Geometric construction of the control polygon of the helicoids patch

5 Conclusions and Future Work

In this paper, we propose a geometric construction of control meshes of helicoids over trapezium domain. The result enables us to obtain trimmed minimal surface patches over trapezium domain from helicoids. It is very meaningful for membrane structure design in modern architecture. In the future, we will consider a more general case, i.e.

$$r(w,v) = (w\cos v, w\sin v, v), 0 \le v \le \alpha, p_0(v) \le w \le p_1(v),$$

where $p_0(v), p_1(v)$ are all planar polynomial curves. For this purpose, we should first construct the quasi-Bézier basis in the space

$$\Gamma_{2n+3} = span\left\{1, t, \cos t, \sin t, \cdots, t^n \sin t, t^n \cos t\right\}.$$

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