Direct Manipulation of RDMS Free Form Deformation

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Abstract

In this paper, we investigate the direct manipulation problem of free form deformation with rational DMS spline volume (RDMS-FFD). For the weights based direct manipulation method, the solution of the weights can be achieved by solving a linear system; for the control points based method, the explicit solution of displacements of the control points can be obtained, and some properties are also derived from the results. For the constraint points inside the control lattices, we use the weights based methods; for the constraint points outside the control lattices, the control points based method is adopted. Several examples are presented to show the effectiveness of the proposed methods.

1. Introduction

Free-form deformation (FFD) has been one of the most popular modeling tools in Computer Graphics and Computer Aided Design. One limitation of the existing FFD methods is the indirect manipulation of the deformation by modifying control points or weights of the embedding tools. In order to improve the controllability of FFD methods, it is a key issue to make the deformed object pass through some constraint points. That is, given some source points on an object to be deformed and the corresponding target points, the adjusted position of the control points or the changed values of weights can be computed automatically. By using the direct manipulation methods, it is more intuitive to model complex objects with FFD methods, because the users don't need to understand the mathematics background of FFD methods.

In [1], we proposed a new FFD method called RDMS-FFD by using rational DMS spline volumes. RDMS-FFD inherits some good properties of rational DMS-spline volume and combines more deformation techniques than previous FFD methods in a consistent framework, such as local deformation, control lattice of arbitrary topology, smooth deformation, multiresolution deformation and direct manipulation of deformation. For a given objective point, we can only move some vertices of the tetrahedral domain towards the objective point, and increase the weight of the corresponding control point that is closest to the objective point, then all affected points with the rational DMS-volume will move towards the objective point.

However, the exact displacements of the control points and the weights have not been proposed in [1]. In this paper, we will give the exact solution for direct manipulation problem of RDMS-FFD. In general, we adopt the weights based method for the constraint points inside the control lattices,, and for the constraint points outside the control lattices, we use the control points based method.

The rest of the paper is organized as follows. In Section 2, we introduce some related work on volume-based FFD techniques and direct manipulation of FFD. Section 3 briefly reviews RDMS volumes and RDMS FFD. Section 4 presents the weights based direct manipulation method by minimizing the change of control polygon. The control points based direct manipulation method and its properties are presented in Section 5. Conclusions and future work are presented in Section 6.

2. Related work

Free-form deformation technique (FFD) is firstly proposed by Sederberg and Parry [2], which is widely available and almost all subsequent methods are based on it. Griessmair and Purgathofer proposed a FFD method based on B-Spline volumes, and optimized the mesh division after deformation [3]. Kalra et al. proposed a Rational Free-Form Deformation (RFFD) method to use a rational parametric volume to simulate the movement of facial muscles [4]. The method proposed by Lamousin and Waggenspack was based on NURBS volumes and succeeded in raising the flexibility of FFD [5]. However, the control lattice of the above methods should be a regular parallelepiped or a uniformly arranged shape. Hence, the users cannot predict the deformation results correctly from the manipulation of the control lattice.

To overcome this limitation, many FFD techniques that use control lattices with arbitrary topology have been proposed in recent years. Coquillart proposed an extension of free-form deformation (EFFD), which uses several low resolution lattices, called "chunk", for deformation [6]. By using the Catmull-Clark subdivision scheme, MacCracken and Joy further extended the capability of FFD by introducing

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lattices of arbitrary topology [7]. Bechmann et al. proposed continuous FFD (CFFD) based on barycentric coordinates and Bézier tetrahedrons [8] . Moccozet and Magnenat-Thalmann proposed Dirichlet FFD approach based on the Voronoi structure defined within the convex hull of a set of points [9]. By employing weighted T-spline volume as deformation tools, Song and Yang proposed a more flexible and adaptive FFD method [10]. Feng et al. proposed a deformation technique with subdivision surface of arbitrary topology [11].

There are many other FFDs based on surfaces or curves. For more information, the reader can refer to the survey on FFD [12].

In order to improve the controllability of FFD techniques, several constraint FFD methods have been proposed. Hsu et al. adopted least-square fitting approach to determine the movement of the control points[13]. Hu et al. solved the direct manipulation problem of NURBS based FFD explicitly using constraint optimization [14]. Zhang and Sun proposed the weights based method for direct manipulation problem of NURBS based FFD [15]. The direct manipulation of a generalized cylinder is presented in [16].

3. Free form deformation with rational DMS-Spline volumes

In this section, we will review the rational DMS-Spline volumes and the corresponding RDMS-FFD method presented in [1].

3.1. Rational DMS spline volumes

DMS-splines, introduced by Dahmen, Micchelli and Seidel in [17], are based on the simplex splines. We will firstly review the formulation of the trivariate simplex splines presented in [18].

A trivariate simplex spline $M(\boldsymbol{x}|\boldsymbol{x}_0,\ldots,\boldsymbol{x}_{n+3})$ of degree n is a function of $\boldsymbol{x}\in\mathbb{R}^3$ over the half open convex hull of a point set $\boldsymbol{V}=[\boldsymbol{x}_0,\ldots,\boldsymbol{x}_{n+3})$, depending on the $n+4$ knots $\boldsymbol{x}_i\in\mathbb{R}^3, i=0,1,\ldots,n+3$. The basis function of trivariate simplex splines can be defined recursively as follows. When $n=0$,

$$M(\boldsymbol{x}|\boldsymbol{x}_0,\ldots,\boldsymbol{x}_{n+3}) = \begin{cases} \frac{1}{6|\mathrm{Vol}_{\mathbb{R}^3}(\boldsymbol{x}_0,\ldots,\boldsymbol{x}_3)|}, & \boldsymbol{x}\in[\boldsymbol{x}_0,\ldots,\boldsymbol{x}_3), \\ 0, & \text{otherwise.} \end{cases}$$

When $n>0$, select four points $\boldsymbol{W}=[\boldsymbol{x}_{k_0},\boldsymbol{x}_{k_1},\boldsymbol{x}_{k_2},\boldsymbol{x}_{k_3}]$ from $\boldsymbol{V}$, such that $\boldsymbol{W}$ is affine independent, then

$$M(\boldsymbol{x}|\boldsymbol{x}_0,\ldots,\boldsymbol{x}_{n+3}) = \sum_{j=0}^{3}\lambda_j(\boldsymbol{x}|\boldsymbol{W})M(\boldsymbol{x}|\mathbf{V}\backslash\{\boldsymbol{x}_{k_j}\}),$$

where $\sum_{j=0}^{3}\lambda_j(\boldsymbol{x}|\boldsymbol{W})=1$ and $\sum_{j=0}^{3}\lambda_j(\boldsymbol{x}|\boldsymbol{W})\boldsymbol{x}_{k_j}=\boldsymbol{x}$. In fact, $\lambda_j(\boldsymbol{x}|\boldsymbol{W})$ are the barycentric coordinates of $\boldsymbol{x}$ with respect to $\boldsymbol{W}$.

In the following, we will review the construction of DMS-spline volume: let Ω be an arbitrary proper tetrahedralization of $\mathbb{R}^3$ or some bounded domain $D\subset\mathbb{R}^3$. The proper tetrahedralization means that every pair of tetrahedral domain are disjoint, or share exactly one vertex, one edge, or one face. Next, with every vertex $\boldsymbol{x}$ of Ω, we associate a cloud of knots $[\boldsymbol{t}_0,\boldsymbol{t}_1,\cdots,\boldsymbol{t}_n]$, where $\boldsymbol{t}_0=\boldsymbol{t}$. For every tetrahedron $\boldsymbol{I}=(\boldsymbol{p},\boldsymbol{q},\boldsymbol{r},\boldsymbol{s})$, we require

• all the tetrahedron $[\boldsymbol{p}_i,\boldsymbol{q}_j,\boldsymbol{r}_k,\boldsymbol{s}_l]$ with $i+j+k+l\le n$ are nondegenerate.

• the set

$$Z = interior(\bigcap_{i+j+k+l\le n}[\boldsymbol{p}_i,\boldsymbol{q}_j,\boldsymbol{r}_k,\boldsymbol{s}_l])$$

satisfies

$$Z\neq\emptyset$$

• if $\boldsymbol{I}$ has a boundary triangle, the knots associated to the boundary triangle must lie outside of Ω.

Then the trivariate DMS-spline basis function $N_\beta^{\boldsymbol{I}}(\boldsymbol{u})$ is defined by means of trivariate simplex spline $M(\boldsymbol{u}|\boldsymbol{V}_\beta^{\boldsymbol{I}})$ as

$$N_\beta^{\boldsymbol{I}}(\boldsymbol{u}) = |d(\boldsymbol{p}_i,\boldsymbol{q}_j,\boldsymbol{r}_k,\boldsymbol{s}_l)|M(\boldsymbol{u}|\boldsymbol{V}_\beta^{\boldsymbol{I}}).$$

where β is the 4-tuple(i,j,k,l),

$$\boldsymbol{V}_\beta^{\boldsymbol{I}} = [\boldsymbol{p}_0,\cdots,\boldsymbol{p}_i,\boldsymbol{q}_0,\cdots,\boldsymbol{q}_j,\boldsymbol{r}_0,\cdots,\boldsymbol{r}_k,\boldsymbol{s}_0,\cdots,\boldsymbol{s}_l].$$

$d(\mathbf{p}_i,\mathbf{q}_j,\mathbf{r}_k,\mathbf{s}_l)$ is six times of the volume of $(\boldsymbol{p}_i,\boldsymbol{q}_j,\boldsymbol{r}_k,\boldsymbol{s}_l)$.

A degree n DMS-spline volume $\boldsymbol{S}(\mathbf{u})$ over Ω is then defined as

$$\boldsymbol{S}(\boldsymbol{u}) = \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n}\boldsymbol{c}_\beta^I N_\beta^I(\boldsymbol{u}). \quad (1)$$

where $\boldsymbol{c}_\beta^{\boldsymbol{I}}\in\mathbb{R}^3$ are the control points.

Generalizing (1) by associating a weight ω_β^I with each control point, we define rational DMS-spline volume as the combination of a set of piecewise rational functions:

$$\boldsymbol{F}(\boldsymbol{u}) = \frac{\boldsymbol{P}(\boldsymbol{u})}{\boldsymbol{Q}(\boldsymbol{u})} = \frac{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n}\omega_\beta^{\boldsymbol{I}}\boldsymbol{c}_\beta^{\boldsymbol{I}}N_\beta^{\boldsymbol{I}}(\boldsymbol{u})}{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n}\omega_\beta^{\boldsymbol{I}}N_\beta^{\boldsymbol{I}}(\boldsymbol{u})}.$$

The rational DMS-spline volumes can be also considered as the trivariate generalization of the triangular NURBS presented in [19]. They have many properties of the non-rational schemes, such as convex hull property, local support and affine invariance. Moreover, they have some additional properties:

- Like the DMS-spline volumes, rational DMS-spline volumes and their rational basis functions are also

C^{n-1} continuous if the knots are in general position, where n is the degree of rational DMS-spline.
- The weights of rational DMS-spline volumes are extra degrees of freedom which influence local shape. If a weight is increased, the volume will move towards the corresponding control point.

3.2. RDMS FFD

In [1], the RDMS-FFD method is proposed by using rational DMS-spline volumes. The main steps for the RDMS-FFD algorithm are as follows

- Construct the tetrahedral domain and DMS-lattice. Set initial weights for rational DMS-spline volume. For details, see [1].
- Calculate the parametric coordinates $\boldsymbol{u} = (u, v, w)$ for each point on the object to be deformed. Nonlinear conjugate gradient method is adopted to implement the parametrization process. For details, see [1].
- Manipulate the vertices of the tetrahedral domain or weights of the control points and update the DMS-lattice, evaluate the new locations of the points according to the new tetrahedral domain and DMS-lattice. We will obtain a raw global deformation.
- Use multiresolution deformation technique to obtain satisfied results.

4. Weights based direct manipulation of RDMS FFD

4.1. Case of single point constraint

In this section, we will investigate the following problems: given one source point $\boldsymbol{S}$ on the object with parameter value $\boldsymbol{u}_0$ and the corresponding target point $\boldsymbol{T}$, we want to modify the weights of the rational DMS volume such that $\boldsymbol{S}$ is moved to $\boldsymbol{T}$ while making the changes of weights as small as possible.

In [15], Zhang et al proposed the solution of this problem for NURBS based FFD. Instead of solving a linear system for the Lagrange multiplier in NURBS based FFD, we directly solve a linear system for the displacements of the weights for RDMS-FFD.

If we denote the displacement of each weight $\omega_{\beta,\boldsymbol{I}}$ by $\varepsilon_{\beta,\boldsymbol{I}}$, then we have

$$\boldsymbol{S} = \frac{\sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} \omega_{\beta,\boldsymbol{I}}\boldsymbol{c}_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}{\sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} \omega_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}$$

$$\boldsymbol{T} = \frac{\sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} (\omega_{\beta,\boldsymbol{I}}+\varepsilon_{\beta,\boldsymbol{I}})\boldsymbol{c}_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}{\sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} (\omega_{\beta,\boldsymbol{I}}+\varepsilon_{\beta,\boldsymbol{I}})N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}$$

If we denote $\boldsymbol{T}-\boldsymbol{S}$ by $\boldsymbol{D}$, then the constraint condition can be written as

$$\boldsymbol{T}-\boldsymbol{S}-\boldsymbol{D} = 0,$$

and the objective function is

$$Min \sum_{\boldsymbol{I}\in\Omega}\sum_{\beta|=n} \varepsilon^2_{\beta,\boldsymbol{I}}.$$

Following the Lagrange multiple method, we can construct the Lagrange function as following,

$$L = \sum_{\boldsymbol{I}\in\Omega}\sum_{\beta|=n} \varepsilon^2_{\beta,\boldsymbol{I}} + \lambda(\boldsymbol{T}-\boldsymbol{S}-\boldsymbol{D}) \tag{2}$$

where $\lambda = (\lambda^1, \lambda^2, \lambda^3)$ is the Lagrange multiplier. By setting $\frac{\partial L}{\partial\lambda^1} = \frac{\partial L}{\partial\lambda^2} = \frac{\partial L}{\partial\lambda^3} = 0$, $\frac{\partial L}{\partial\varepsilon^1_{\beta,I}} = \frac{\partial L}{\partial\varepsilon^2_{\beta,I}} = \frac{\partial L}{\partial\varepsilon^3_{\beta,I}} = 0$, we have

$$\begin{cases} \boldsymbol{T} = \boldsymbol{S}+\boldsymbol{D} \\ 2\varepsilon_{\beta,\boldsymbol{I}} = \lambda\dfrac{\partial\boldsymbol{D}}{\partial\varepsilon_{\beta,\boldsymbol{I}}} \end{cases} \tag{3}$$

After some computation, we have

$$\boldsymbol{D} = \frac{\sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}\boldsymbol{c}_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)S^\top - S^\perp \sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}{(S^\top + \sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0))S^\top}$$

$$\frac{\partial\boldsymbol{D}}{\partial\varepsilon_{\beta,\boldsymbol{I}}} = \frac{(\boldsymbol{c}_{\beta,\boldsymbol{I}}-\boldsymbol{T})N^2_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}{S^\top + \sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)} \tag{4}$$

where

$$S^\top = \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)$$

$$S^\perp = \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}}\boldsymbol{c}_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)$$

From (3) and (4), we obtain

$$\lambda = \frac{2(S^\top + \sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0))^2(\boldsymbol{T}-\boldsymbol{S})}{\sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\beta|=n} (\boldsymbol{c}_{\beta,\boldsymbol{I}}-\boldsymbol{T})(\boldsymbol{c}_{\beta,\boldsymbol{I}}-\boldsymbol{S})N^3_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)} \tag{5}$$

Submitting (5) into (3), we have

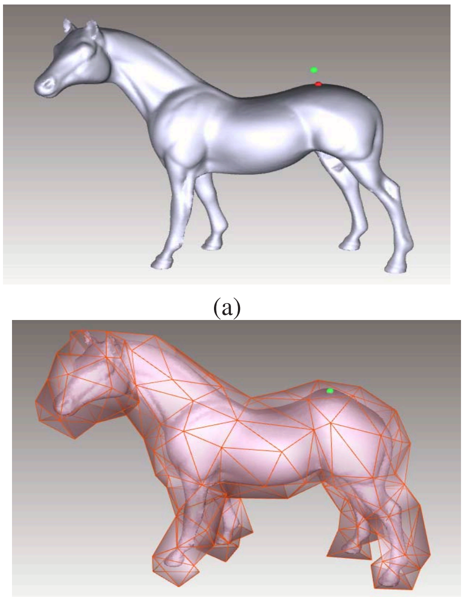
$$\varepsilon_{\beta,\boldsymbol{I}} = \frac{(S^\top + \sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\alpha|=n} \varepsilon_{\alpha,\boldsymbol{I}}N_{\alpha,\boldsymbol{I}}(\boldsymbol{u}_0))(\boldsymbol{T}-\boldsymbol{S})(\boldsymbol{c}_{\beta,\boldsymbol{I}}-\boldsymbol{T})N^2_{\beta,\boldsymbol{I}}}{\sum\limits_{\boldsymbol{I}\in\Omega}\sum\limits_{|\alpha|=n} (\boldsymbol{c}_{\alpha,\boldsymbol{I}}-\boldsymbol{T})(\boldsymbol{c}_{\alpha,\boldsymbol{I}}-\boldsymbol{S})N^3_{\alpha,\boldsymbol{I}}(\boldsymbol{u}_0)} \tag{6}$$

Hence, we can obtain m equations as (6), where m is the number of control points of RDMS volume. These equations can be rewritten as matrix form as follows,

$$\boldsymbol{M}\varepsilon = \boldsymbol{b}, \tag{7}$$

where ε is the vector of displacement of each weight. Hence, we obtain

$$\varepsilon = \boldsymbol{M}^{-1}\boldsymbol{b}. \tag{8}$$



(a)

(b)

Figure 1. Weight-based direct manipulation of RDMS-FFD:(a)original horse model and the constraint points, where the red point is the source point, and the green one is the target point; (b) deformation result.

Figure 1 shows an example of weight-based direct manipulation. In order to show the results more clearly, we only present the boundary control points of the control lattices in this paper.

4.2. Case of multiple constraint points

Suppose that $\boldsymbol{S}_i$ are the source points on the object with parameter value $\boldsymbol{u}_i$, and $\boldsymbol{T}_i$, are the corresponding target points, $i = 1, \cdots, l$. We denote $\boldsymbol{D}_i = \boldsymbol{T}_i - \boldsymbol{S}_i$, then the corresponding Lagrange function can be constructed as

$$L = \sum_{\boldsymbol{I}\in\Omega}\sum_{\beta|=n} \varepsilon^2_{\beta,\boldsymbol{I}} + \sum_{i=0}^{l} \lambda_i(\boldsymbol{T}_i - \boldsymbol{S}_i - \boldsymbol{D}_i) \quad (9)$$

where $\lambda_i = (\lambda_i^1, \lambda_i^2, \lambda_i^3)$ is the Lagrange multiplier. By setting $\frac{\partial L}{\partial \lambda_i^1} = \frac{\partial L}{\partial \lambda_i^2} = \frac{\partial L}{\partial \lambda_i^3} = 0$, $\frac{\partial L}{\partial \varepsilon^1_{\beta,\boldsymbol{I}}} = \frac{\partial L}{\partial \varepsilon^2_{\beta,\boldsymbol{I}}} = \frac{\partial L}{\partial \varepsilon^3_{\beta,\boldsymbol{I}}} = 0$, we have

$$\begin{cases} \boldsymbol{T}_i = \boldsymbol{S}_i + \boldsymbol{D}_i, \\ 2\varepsilon_{\beta,\boldsymbol{I}} = \sum_{i=0}^{l} \lambda_i \dfrac{\partial \boldsymbol{D}_i}{\partial \varepsilon_{\beta,\boldsymbol{I}}} \end{cases} \quad (10)$$

where

$$\boldsymbol{D}_i = \frac{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}\boldsymbol{c}_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i) S_i^\top - S_i^\perp \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)}{(S_i^\top + \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)) S_i^\top},$$

$$\frac{\partial \boldsymbol{D}_i}{\partial \varepsilon_{\beta,\boldsymbol{I}}} = \frac{(\boldsymbol{c}_{\beta,\boldsymbol{I}} - \boldsymbol{T}) N^2_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)}{S_i^\top + \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)},$$

$$S_i^\top = \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i),$$

$$S_i^\perp = \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}}\boldsymbol{c}_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i).$$

Then the displacement of each control point can be obtained by solving the linear system (10).

5. Control points based direct manipulation of RDMS FFD

5.1. Case of single point constraint

In this section, we will investigate the following problems: given one source point $\boldsymbol{S}$ on the object and the corresponding target point $\boldsymbol{T}$, we want to modify the control points of the rational DMS volume such that $\boldsymbol{S}$ is moved to $\boldsymbol{T}$ while making the change of control polygon as small as possible.

In [14], Hu et al proposed the solution of this problem for NURBS based FFD. For RDMS-FFD, the similar result can be obtained.

If we denote the displacement of each control point $\boldsymbol{c}_{\beta,\boldsymbol{I}}$ by $\varepsilon_{\beta,\boldsymbol{I}}$, then the constraint condition is

$$\boldsymbol{T} - \boldsymbol{S} - \frac{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}\omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)} = 0, \quad (11)$$

and the objective function is

$$Min \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \|\varepsilon_{\beta,\boldsymbol{I}}\|^2. \quad (12)$$

Following the Lagrange multiple method, we can construct the Lagrange function as following,

$$L = \sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \|\varepsilon_{\beta,\boldsymbol{I}}\|^2 + \lambda(\boldsymbol{T} - \boldsymbol{S} - \frac{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}\omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}) \quad (13)$$

where $\lambda = (\lambda^1, \lambda^2, \lambda^3)$ is the Lagrange multiplier. By setting $\frac{\partial L}{\partial \lambda^1} = \frac{\partial L}{\partial \lambda^2} = \frac{\partial L}{\partial \lambda^3} = 0$, $\frac{\partial L}{\partial \varepsilon^1_{\beta,\boldsymbol{I}}} = \frac{\partial L}{\partial \varepsilon^2_{\beta,\boldsymbol{I}}} = \frac{\partial L}{\partial \varepsilon^3_{\beta,\boldsymbol{I}}} = 0$, we have

$$\begin{cases} \boldsymbol{T} = \boldsymbol{S} + \dfrac{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}}\omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}, \\ 2\varepsilon_{\beta,\boldsymbol{I}} = \lambda \dfrac{\omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)}{\sum_{\boldsymbol{I}\in\Omega}\sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0)} \end{cases} \quad (14)$$

From (14), we can obtain the explicit solution of the displacement,

$$\varepsilon_{\beta,\boldsymbol{I}} = \frac{\omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_0) \sum_{\boldsymbol{I}\in\Omega} \sum_{|\alpha|=n} \omega_{\alpha,\boldsymbol{I}} N_{\alpha,\boldsymbol{I}}(\boldsymbol{u}_0)}{\sum_{\boldsymbol{I}\in\Omega} \sum_{|\alpha|=n} \omega^2_{\alpha,\boldsymbol{I}} N^2_{\alpha,\boldsymbol{I}}(\boldsymbol{u}_0)} (\boldsymbol{T} - \boldsymbol{S}) \quad (15)$$

Because RDMS volume has the local properties, not all the control points are modified during the manipulation process. From(15), some properties of control points based direct manipulation can be derived.

Corollary 1. The displacement of each control point is parallel with the displacement vector of the constraint points $\boldsymbol{T} - \boldsymbol{S}$, and the ratio of the displacement lengths equals the ratio of the coefficient of the control points.

Corollary 2. The sum of the displacement of each control point equals

$$\frac{[\sum_{\boldsymbol{I}\in\Omega} \sum_{|\alpha|=n} \omega_{\alpha,\boldsymbol{I}} N_{\alpha,\boldsymbol{I}}(\boldsymbol{u}_0)]^2}{\sum_{\boldsymbol{I}\in\Omega} \sum_{|\alpha|=n} \omega^2_{\alpha,\boldsymbol{I}} N^2_{\alpha,\boldsymbol{I}}(\boldsymbol{u}_0)} (\boldsymbol{T} - \boldsymbol{S}) \quad (16)$$

That is, the sum of the displacement of each control point is larger than the displacement of the constraints points $\boldsymbol{T} - \boldsymbol{S}$.

The formula (16) can be derived from that the sum of RDMS basis functions equals 1.

As the NURBS based FFD, the control points based direct manipulation of RDMS-FFD also has the commutative, associative and invertible properties.

Figure 2(a) and Figure 2 (b) show an example of control point based direct manipulation with single constraint point.

5.2. Case of multiple constraint points

Suppose that $\boldsymbol{S}_i$ are the source points on the object with parameter value $\boldsymbol{u}_i$, and $\boldsymbol{T}_i$, are the corresponding target points, $i = 1, \cdots, l$. We denote $\boldsymbol{D}_i = \boldsymbol{T}_i - \boldsymbol{S}_i$, then the corresponding Lagrange function can be constructed as

$$L = \sum_{\boldsymbol{I}\in\Omega} \sum_{\beta|=n} \varepsilon^2_{\beta,\boldsymbol{I}} + \sum_{i=0}^{l} \lambda_i (\boldsymbol{T}_i - \boldsymbol{S}_i - \frac{\sum_{\boldsymbol{I}\in\Omega} \sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)}{\sum_{\boldsymbol{I}\in\Omega} \sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)}) \quad (17)$$

where $\lambda_i = (\lambda_i^1, \lambda_i^2, \lambda_i^3)$ is the Lagrange multiplier. By setting $\frac{\partial L}{\partial \lambda_i^1} = \frac{\partial L}{\partial \lambda_i^2} = \frac{\partial L}{\partial \lambda_i^3} = 0$, $\frac{\partial L}{\partial \varepsilon^1_{\beta,\boldsymbol{I}}} = \frac{\partial L}{\partial \varepsilon^2_{\beta,\boldsymbol{I}}} = \frac{\partial L}{\partial \varepsilon^3_{\beta,\boldsymbol{I}}} = 0$, we have

$$\begin{cases} \boldsymbol{T}_i = \boldsymbol{S}_i + \dfrac{\sum_{\boldsymbol{I}\in\Omega} \sum_{|\beta|=n} \varepsilon_{\beta,\boldsymbol{I}} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)}{\sum_{\boldsymbol{I}\in\Omega} \sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)}, \\ 2\varepsilon_{\beta,\boldsymbol{I}} = \sum_{i=0}^{l} \lambda_i \dfrac{\omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)}{\sum_{\boldsymbol{I}\in\Omega} \sum_{|\beta|=n} \omega_{\beta,\boldsymbol{I}} N_{\beta,\boldsymbol{I}}(\boldsymbol{u}_i)} \end{cases} \quad (18)$$

Then the displacement of each control point can be obtained by solving the linear system (18).

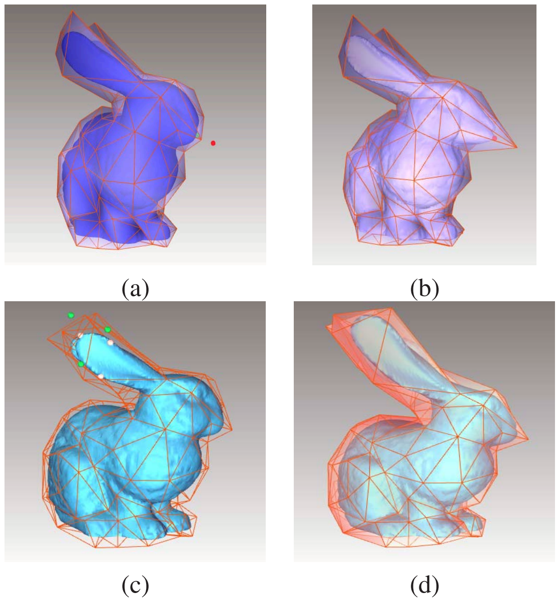


Figure 2. Control points based direct manipulation of RDMS-FFD: (a) original bunny model with one constraint point; (b)deformation result; (c) original model with multiple constraint points; (d) deformation results.

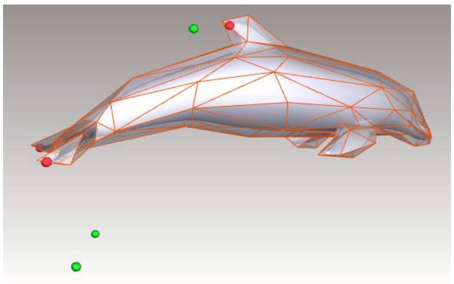
As the NURBS based FFD, the multiple point constraints can also be decomposed into separate manipulations of single point constraints.

Figure 2(c)(d) and Figure 3 present examples of control points based direct manipulation with multiple constraint points.

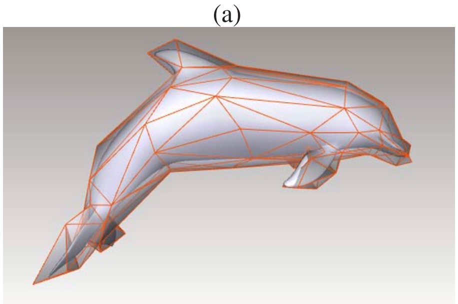
In general, for the constraint points inside the control lattices, we use the weights based methods; for the constraint points outside the control lattices, the control points based method is adopted. In case of the constraint points are both inside and outside of the control lattice, from the decomposition property of the proposed methods, we can firstly use the control points based method for the outside constraint points, and then use the weights based method for the inside constraint points. In particular, because the investigated problems in this paper are linear problems, the manipulation can be implemented in realtime after the parametrization process is finished.

6. Conclusion

In this paper, the direct manipulation problem of RDMS-FFD is addressed. For the weights based direct manipulation method, the solution of the weights can be achieved by solving a linear system; for the control points based method, the explicit solution of displacements of the control points can be obtained. We also present several examples to show the effectiveness of the proposed methods.



(a)



(b)

Figure 3. Direct manipulation of dolphin model: (a) original model with multiple constraint points; (b) deformation results.

In the future, we will add some other object functions into the framework, such as volume-preserving constraints and area-preserving constraints.

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