

# Diverse Routing In Optical Mesh Networks

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## Abstract

In this paper, we study the diverse routing problem in optical mesh networks. We use a general framework based on Shared Risk Link Groups (SRLGs) to model the problem. We prove that the diverse routing problem is indeed NP-complete, a result that has been conjectured by several researchers previously. In fact, we show that even the fiber-span-disjoint paths problem, a special case of the diverse routing problem is also NP-complete. We then develop an integer linear programming formulation and show through numerical results that it is a very viable method to solve the diverse routing problem for most optical networks found in many applications which typically have no more than a few hundreds nodes and fiber spans.

**Index Terms:** optical fiber communication, wavelength division multiplexing, communication system routing, protection, complexity theory

## 1 Introduction

In optical networks, circuits (demands) often require two paths, one working path and one protection path, so that it can withstand certain network failures, such that fiber cuts, hardware and software failures, and power outages. Therefore, in the design of an optical network, not only does one need to build enough working capacity to accommodate working paths and also additional spare capacity so that protection paths can be established. There are two commonly used protection schemes in determining how much the spare capacity is needed for protection paths: shared

path protection and dedicated path protection. In the case of shared path protection, the spare capacity may be shared among different protection paths, while in the case of dedicated path protection, the spare capacity is dedicated to individual protection paths and cannot be shared by others. In general, the requirement for working path and protection path is that they have to be diversely routed so that at least one path can survive a single failure in the network, where a single failure may represent a fiber cut or an individual card failure at a node.

The problem of finding two diversely routed paths in optical networks is much more difficult than the classical edge/node-disjoint paths problem in graph theory. This is because optical networks architecturally have two layers: the physical layer and the optical layer (virtual/logical layer). The physical layer consists of fiber spans and nodes (representing locations where fiber spans terminate) and the optical layer consists of optical links (or light paths) and a subset of nodes contained in the physical layer. An optical link in the optical layer is a path connecting a pair of nodes via a set of fiber spans in the physical layer. The failures, including fiber cuts and optical/electronic component failures at nodes, occur in the physical layer. However, the circuits are routed over optical links in the optical layer. Since an optical link is a path that may traverse several fiber spans and nodes, it can be affected by different failures in the physical layer. On the other hand, several optical links may traverse a single fiber span or node, hence, a single failure in the physical layer can cause multiple optical link failures. The diverse routing problem is to find two paths between a pair of nodes in the optical layer such that no single failure in the physical layer may cause both paths to fail. In [8], an example is provided to illustrate why the traditional edge/node-disjoint algorithms do not work for such a problem.

The problem of finding diversely routed paths in optical networks was first considered in [1] and later in [15, 2, 8, 10]. Heuristic algorithms were developed in [1, 15, 10] for special cases of the diverse routing problem, where only fiber cuts (and node fail-

ures) were considered. An exhaustive search algorithm is proposed in [2] for finding diversely routed paths. In [8], the problem was formulated as a linear programming problem. A problem related to the diverse routing problem is to embed the optical links into the physical layer with the objective of minimizing the effect of physical layer failures on the optical layer (e.g., see [5, 12]).

In this paper, we consider the diverse routing problem within a very general framework in which we use the Shared Risk Link Group (SRLG) to represent a set of optical links that are affected by a single failure in the physical layer. SRLG is generic network protection/restoration terminology that has been proposed in the Internet Engineering Task Force (IETF) and Optical Internetworking Forum (OIF) standard bodies [7, 13]. We first prove that the diverse routing problem is NP-complete, a result that has been suggested by several researchers (e.g., see [2, 8, 10]) and remains to be an open problem until now. A weaker result was given in [10] where it was shown that the problem of finding maximum number of diversely routed paths is NP-complete. We further prove that the fiber-span-disjoint paths problem, a special case of the diverse routing problem considered in [1, 15, 10], is NP-complete as well. We also consider two generalizations of the diverse routing problem: one is the minimum cost diverse routing problem in which our goal is to find two diversely routed paths with minimum total cost, and the other is the least coupled paths problem in case that there do not exist two diverse paths. Obviously, they are both NP-complete since they include the diverse routing problem as a special case. We propose an integer linear programming (ILP) formulation for these two problems, and provide numerical results to show that the ILP formulation offers a very effective way to solve them (and the diverse routing problem itself) for most optical networks found in applications which typically have no more than a few hundreds of nodes and fiber spans. Therefore, in addition to resolving the long-standing complexity issue for the diverse routing problem for optical networks, we also provide a viable method for solving the

problem.

The rest of the paper is organized as follows. In Section 2, we provide a basic framework for the diverse routing problem and introduce necessary notation, definitions, and terminologies. In Section 3, we study the complexity issue and prove that the diverse routing problem is NP-complete. Our NP-completeness results include one for the fiber-span-disjoint paths problem, which was considered by several researchers previously. In Section 4, we develop an integer linear programming formulation and also provide some numerical results to demonstrate its effectiveness for the diverse routing problem. Finally, a conclusion is provided in Section 5.

## 2 Problem Formulation

For an optical network, we use graph  $G_f = (V_f, E_f)$  to represent its physical layer, where  $E_f$  is the set of edges representing fiber spans and  $V_f$  is the set of nodes representing locations (where the fiber spans terminate). We use graph  $G_o = (V_o, E_o)$  to represent its optical layer, where  $E_o$  is the set of edges representing optical links (light paths) and  $V_o \subset V_f$  representing end points of the optical links. In general, each edge in  $E_o$  may correspond to a path in graph  $G_f$ . Let  $R$  be the set of risks (failures). Each risk may represent a fiber cut, a card failure at a node, a piece of software failure, an operational error, or any combination of these factors. Let  $E_r \subset E_o$  denote the subset of optical links that can be affected by  $r$  ( $r \in R$ ), and we refer  $E_r$  as a Shared Risk Link Group (SRLG). For a path  $p$  in  $G_o$ , we say it contains  $r \in R$  if any edge on the path belongs to  $E_r$ , and let

$$r_p = \{r \in R : \text{path } p \text{ contains } r\}.$$

Then the SRLG diverse routing problem can be defined as follows:

**Definition 1 (The SRLG Diverse Routing Problem)** *Find two paths  $p_1$  and  $p_2$  between a pair of nodes such that  $r_{p_1} \cap r_{p_2} = \emptyset$ , i.e.,  $p_1$  and  $p_2$  do not contain a common*

element of  $R$ . We also say that  $p_1$  and  $p_2$  are two SRLG diverse paths (with respect to  $R$ ).

In case that there do not exist two SRLG diverse paths, then a related problem is the following least coupled SRLG paths problem:

**Definition 2 (The Least Coupled SRLG Paths Problem)** *Find two paths  $p_1$  and  $p_2$  between a pair of nodes such that  $|r_{p_1} \cap r_{p_2}|$  is minimized, i.e., the number of common elements in  $R$  shared by both paths is minimized.*

The least coupled SRLG paths problem was also considered in [8], where it was set up as an ILP problem based on the cut set formulation. Clearly, the SRLG diverse routing problem can be viewed as a special case of the least coupled SRLG paths problem in which  $\min_{p_1, p_2} |r_{p_1} \cap r_{p_2}| = 0$ . If there is a cost associated with each edge in  $G_o$ , then the SRLG diverse routing problem can be extended to the minimum cost SRLG diverse routing problem.

**Definition 3 (The Minimum Cost SRLG Diverse Routing Problem)** *Find two SRLG diverse paths between a pair of nodes such that their total edge cost is minimized (the total edge cost is defined as the summation of costs of all edges on the two paths).*

A special case of the SRLG diverse routing problem is the fiber-span-disjoint paths problem in which  $R = E_f$  (only failures considered in the physical layer are fiber cuts) and every edge in  $E_o$  represents a path in  $G_f$ . Other special cases of the SRLG diverse routing problem include the edge-disjoint paths problem ( $R = E_o$  and  $E_r = \{r\}$ ) and the node-disjoint paths problem ( $R = V_o$  and  $E_r$  contains all edges in  $E_o$  which are connected to node  $r$ ).

Before closing this section, we should point out that another two generalizations of the SRLG diverse routing problem are: i) Find  $k > 2$  SRLG diverse paths between

a pair of nodes, and ii) Find maximum number of SRLG diverse paths between a pair of nodes.

### 3 NP-Completeness

In this section, we first prove that the SRLG diverse routing problem is NP-complete, which immediately implies that both the least coupled SRLG paths problem and the minimum cost SRLG diverse routing problem are NP-complete. We then prove that the fiber-span-disjoint paths problem is also NP-complete. To proceed, we first need to introduce a special type of optical layer graph  $G_o^l = (V_o^l, E_o^l)$ , which is depicted in Figure 1 (where solid circles, both large and small, represent nodes). The graph  $G_o^l$  can be divided into  $S$  subgraphs, which are denoted as  $G_1, G_2, \dots, G_S$ . In subgraph  $G_i$  ( $i = 1, 2, \dots, S$ ), there are two end-nodes  $i - 1$  and  $i$  that are connected by a set of parallel paths. If multiple edges are allowed between a pair of nodes, then each parallel path between nodes  $i - 1$  and  $i$  can simply be a single edge; otherwise, we can assume that each parallel path is made of two edges. We further assume that the (two) edges on each of these parallel paths belong to the same SRLG and the edges on different parallel paths belong to different SRLGs. Let

$$R_i = \{r \in R : r \text{ is contained by any edge in } G_i\},$$

i.e.,  $R_i$  is the subset of  $R$  for subgraph  $G_i$ . We are now ready to present our first NP-complete result.

**Theorem 1** *The SRLG diverse routing problem is NP-complete for nodes 0 and  $S$  in  $G_o^l$ , hence, it is NP-complete in general.*

*Proof.* We first note that finding a path between nodes 0 and  $S$  is equivalent to finding a subset of  $R$  which contains at least one element in each  $R_i$  ( $i = 1, 2, \dots, S$ ). Hence, finding two SRLG diverse paths between nodes 0 and  $S$  is equivalent to finding two

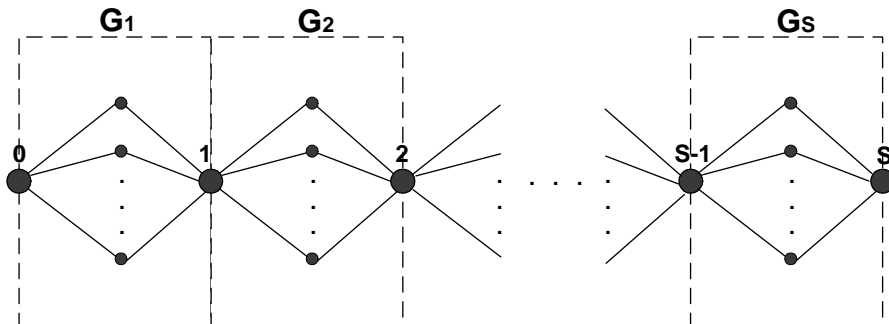


Figure 1: A Special Graph  $G_o^l$  with  $S$  Subgraphs

disjoint subsets of  $R$  such that each of them contains at least one element from each subset  $R_i$  ( $i = 1, 2, \dots, S$ ). We now show that the well-known set splitting problem can be reduced to the SRLG diverse routing problem. The set splitting problem can be stated as follows: Given a collection of subsets,  $\{C_i : i = 1, \dots, K\}$ , of a finite set  $C$ , are there two disjoint subsets of  $C$  such that each subset contains at least one element in  $C_i$  ( $i = 1, 2, \dots, K$ )? So, if we set  $R = C$ ,  $S = K$ , and  $R_i = C_i$  ( $i = 1, 2, \dots, S$ ), then the set splitting problem is reduced to the SRLG diverse routing problem. Since the set splitting problem is NP-complete (e.g., see [4, p.221, SP4]), the SRLG diverse routing problem is NP-complete.

It is worth pointing out that if  $|R_i| = 2$ , then the SRLG diverse routing problem for nodes 0 and  $S$  in  $G_o^l$  can be solved in polynomial time (see [4, p.221, SP4]) (note that if any  $R_i$  has only one element, then there do not exist two SRLG diverse paths). As we have mentioned in Section 2 that the SRLG diverse routing problem can be viewed as a special case of the least coupled SRLG paths problem, hence the latter problem is also NP-complete. However, in what follows we provide a different proof for the NP-completeness of the least coupled SRLG paths problem, which in fact leads to a slightly stronger result. First, we need some preliminary results.

**Lemma 2** *For a given graph  $G = (V, E)$ , its maximum bipartite subgraph problem, i.e., finding a bipartite subgraph with maximum number of nodes, is NP-complete.*

A proof of Lemma 2 can be found in [16, 9] (also see [4, p.195, GT21]).

**Lemma 3** *For a given graph  $G = (V, E)$ , the problem of finding two node-covers which share minimum number of common nodes is NP-complete.*

*Proof.* We prove that this problem is equivalent to the maximum bipartite subgraph problem, hence it is NP-complete. Consider two node-covers:  $V_1$  and  $V_2$ . Let  $V_c = V_1 \cap V_2$ ,  $\bar{V}_c = V \setminus V_c$ ,  $\bar{V}_1 = V \setminus V_1$ , and  $\bar{V}_2 = V \setminus V_2$ . Since  $V_1$  is a node-cover, there is no edge between nodes in  $\bar{V}_1$ . Similarly, there is no edge between nodes in  $\bar{V}_2$ . Since  $V_1 \setminus V_c \subset \bar{V}_2$ , there is no edge between nodes in  $V_1 \setminus V_c$ . Note that  $\bar{V}_c = \bar{V}_1 + V_1 \setminus V_c$ , therefore the subgraph generated by  $\bar{V}_c$  is a bipartite. On the other hand, suppose the subgraph generated by  $V_b = (V_3, V_4) \subset V$  is a bipartite. Let  $V_1 = V_3 \cup (V \setminus V_b)$  and  $V_2 = V_4 \cup (V \setminus V_b)$ . It is not difficult to verify that both  $V_1$  and  $V_2$  are node-covers. Also note that  $V_1 \cap V_2 = V \setminus V_b$ . To summarize, we have shown that for every two node-covers we can generate one bipartite subgraph and vice versa, and furthermore, the set of the common nodes shared by the two node-covers is the complementary node set of the corresponding bipartite subgraph. Therefore, the problem of finding two node-covers with minimum common nodes is equivalent to the maximum bipartite subgraph problem.

We now prove that the least coupled SRLG paths problem for nodes 0 and  $S$  in  $G_o^l$  is NP-complete, even if  $|R_i| = 2$  for  $i = 1, 2, \dots, S$ . As mentioned earlier, the SRLG diverse routing problem for nodes 0 and  $S$  in  $G_o^l$  can be solved in polynomial time if  $|R_i| = 2$ . Hence, the following NP-completeness result for the least coupled SRLG paths problem is stronger.

**Theorem 4** *The least coupled SRLG paths problem for nodes 0 and  $S$  in  $G_o^l$  is NP-complete, even if  $|R_i| = 2$  for  $i = 1, 2, \dots, S$ .*

*Proof.* As we have pointed out in the proof of Theorem 1, finding a path between nodes 0 and  $S$  is equivalent to finding a subset of  $R$  which contains at least one



element from each subset  $R_i$  ( $i = 1, 2, \dots, S$ ). On the other hand, the problem of finding a node-cover for a graph can polynomially transform to the latter problem with  $|R_i| = 2$  (e.g., see [6]). Therefore, the problem of finding two node-covers which share minimum number of common nodes can polynomially transform to the least coupled SRLG paths for nodes 0 and  $S$  in  $G_o^l$  with  $|R_i| = 2$ . Based on Lemma 3 we can then conclude that the latter problem is NP-complete.

Finally, we consider the fiber-span-disjoint paths problem. To prove it is NP-complete, we only need to show that the SRLG diverse routing problem polynomially transforms to the fiber-span-disjoint paths problem. To achieve this, we need to construct the physical layer graph  $G_f = (V_f, E_f)$  for  $G_o^l$  with the following properties: each risk in  $R$  represents a fiber cut in  $G_f$  (i.e.,  $R = E_f$ ) and each edge  $e \in E_o^l$  corresponds to a path in  $G_f$ . Without loss of generality, we assume that multiple edges between a pair of nodes are allowed in  $G_o^l$  and each parallel path is simply an edge (otherwise, each edge can be replaced by two edges and the corresponding risk be replaced by a pair of new risks). We now construct the physical layer graph  $G_f = (V_f, E_f)$  as follows:

1. Add  $4|E_o^l|$  new elements to  $R$  and let  $R^n$  denote the set of these newly added elements. We then assign four (4) elements in  $R^n$  to each edge  $e \in E_o^l$ . So, now every edge  $e \in E_o^l$  belongs to five (5) SRLGs (one original SRLG and four new SRLGs). We note that each new SRLG only contains one edge.
2. Replace each edge in  $G_o^l$  with a path consisting of 5 new edges, one for each risk, and placing the edge corresponding the original risk in the middle.
3. Merge the edges in the same SRLG into a single edge.

To illustrate how the above procedure works, we provide a simple example in Figure 2. The network at the top of Figure 2 is  $G_o^l$  with four nodes and seven edges:

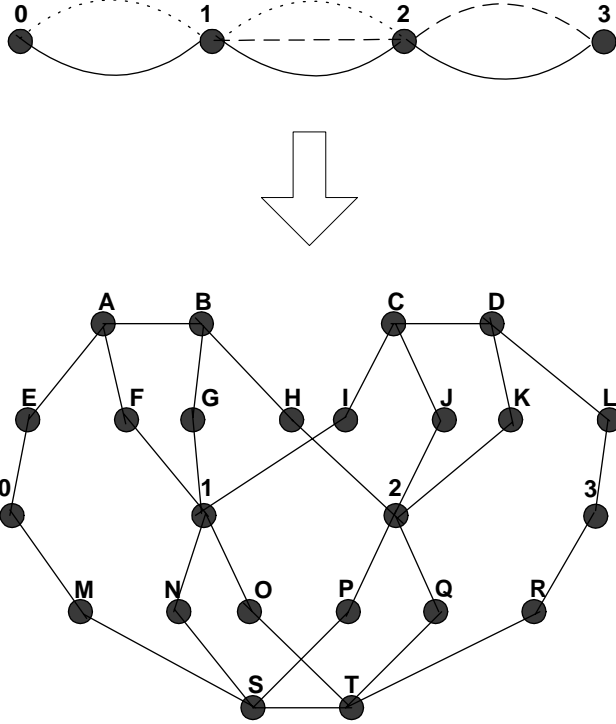


Figure 2: An Illustrative Example on How to Construct  $G_f$  for  $G_o^l$

the edges represented by the same line pattern belong to the same SRLG. So, the seven edges belong to three SRLGs: one is represented by solid lines (three), one by short dotted lines (two), and one by long dotted lines (two). The physical layer is depicted at the bottom. Specifically, the seven paths in  $G_f$  that represent the seven edges in  $G_o^l$  are given in the following table:

Paths in $G_f$	Edges in $G_o^l$
0-E-A-B-G-1	short dotted line between nodes 0 and 1
1-F-A-B-H-2	short dotted line between nodes 1 and 2
1-I-C-D-K-2	long dotted line between nodes 1 and 2
2-J-C-D-L-3	long dotted line between nodes 2 and 3
0-M-S-T-O-1	solid line between nodes 0 and 1
1-N-S-T-Q-2	solid line between nodes 0 and 1
2-P-S-T-R-3	solid line between nodes 0 and 1

It is not difficult to see that graph  $G_f$  constructed based on our proposed procedure has a total of  $4|E_o^l| + |R|$  edges and  $2|E_o^l| + 2|R| + S + 1$  nodes, and also the construction

can be carried out in polynomial time. We note that the reason we add 4 rather than fewer, say 2, risks for each edge  $e \in E_o^l$  is to avoid multiple edges between a pair of nodes in  $G_f$ . Also, it is not difficult to see that two paths between nodes 0 and  $S$  in  $G_o^l$  are SRLG diverse with respect to  $R$  if and only if they are SRLG diverse with respect to  $R \cup R^n$ , i.e., the SRLG diverse routing problem remains the same with the expanded set of risks. To summarize, we present

**Theorem 5** *The SRLG diverse routing problem can polynomially transform to the fiber-span-disjoint paths problem for nodes 0 and  $S$  in  $G_o^l$ , which implies that the fiber-span-disjoint paths problem is also NP-complete.*

## 4 Integer Linear Programming Formulation

In this section, we formulate the problems discussed in the previous two sections as the integer linear programming (ILP) problems. Though we have proved that these problems are NP-complete, as we will demonstrate later that the ILP formulation provides a very viable tool for solving them, in particular for most optical networks in real-world which typically have less than a few hundreds nodes and fiber spans. Throughout this section, we will focus exclusively on the optical layer graph  $G_o = (V_o, E_o)$  and use  $s$  to denote the source node and  $t$  the destination node and the paths that we are interested are between  $s$  and  $t$ . Again, for ease of exposition, we assume that  $G_o$  is a directed graph.

We first introduce some necessary notation:

$\mathbf{0}$ : =  $[0, 0, \dots, 0]^T$ , the zero column vector;

$\mathbf{1}$ : =  $[1, 1, \dots, 1]^T$ , the unit column vector;

$c$ : =  $[c_e]_{e \in E_o}$ , the cost row vector for the edges, where  $c_e$  is the cost of edge  $e \in E_o$ ;

$u$ : =  $[u_v]_{v \in V_o}^T$ , the source-destination column vector, where

$$u_v = \begin{cases} 1 & \text{if } v = s, \\ -1 & \text{if } v = t, \\ 0 & \text{otherwise;} \end{cases}$$

$A$ : =  $[a_{v,e}]_{|V_o| \times |E_o|}$ , the node-edge incidence matrix, where

$$a_{v,e} = \begin{cases} 1 & \text{if edge } e \text{ originates from node } v, \\ -1 & \text{if edge } e \text{ terminates at node } v, \\ 0 & \text{otherwise;} \end{cases}$$

$H$ : =  $[h_{r,e}]_{|R| \times |E_o|}$ , the risk-edge incidence matrix, where

$$h_{r,e} = \begin{cases} 1 & \text{if } e \in E_r, \\ 0 & \text{otherwise;} \end{cases}$$

$x_i$ : =  $[x_{e,i}]_{e \in E_o}^T$ , the column vector containing edge decision variables for path  $p_i$

( $i = 1, 2$ ), where

$$x_{e,i} = \begin{cases} 1 & \text{if edge } e \text{ is on path } p_i \\ 0 & \text{otherwise;} \end{cases}$$

$z_i$ : =  $[z_{r,i}]_{r \in R}^T$ , the column vector containing failure decision variables for path  $p_i$

( $i = 1, 2$ ), where

$$z_{r,i} = \begin{cases} 1 & \text{if } r \text{ is contained in path } p_i \\ 0 & \text{otherwise.} \end{cases}$$

## 4.1 The Minimum Cost SRLG Diverse Routing Problem

The problem of finding two SRLG diverse paths from  $s$  to  $t$  with minimum total edge cost can be formulated as the following ILP problem:

$$\begin{aligned} \min \quad & c(x_1 + x_2) \\ \text{s.t.} \quad & Ax_i = u \quad i = 1, 2 \end{aligned} \tag{1}$$

$$Hx_i \leq |E_o|z_i \quad i = 1, 2 \tag{2}$$

$$z_1 + z_2 \leq \mathbf{1} \tag{3}$$

$x$  and  $z$  are binary decision variables.

- Constraint (1) guarantees that the edges selected based on  $x_i$  is a path from  $s$  to  $t$ .
- Constraint (2) implies that if  $r$  is contained in path  $p_i$  then any edge in  $E_r$  may be selected for the path, otherwise, no edge in  $E_r$  can be selected. Clearly, if an edge belongs to several SRLGs, then it can be selected for path  $p_i$  if and only if the corresponding elements in  $R$  are contained in  $p_i$ . The reason we need to have coefficient  $|E_o|$  in (2) is because  $p_i$  may contain several edges that belong to the same SRLG ( $|E_o|$  can be replaced by any large positive number).
- Constraint (3) guarantees that no one element in  $R$  is contained in both paths.

We should also point out that the binary constraint on  $x$  can be replaced by  $0 \leq x \leq 1$ . This is because that for a given pair of  $z_1$  and  $z_2$  (assuming that (3) is satisfied) the following mixed ILP (MILP) problem has an integer optimal solution:

$$\begin{aligned}
\min \quad & c(x_1 + x_2) \\
\text{s.t.} \quad & Ax_i = u \quad i = 1, 2 \\
& Hx_i \leq |E_o|z_i \quad i = 1, 2 \\
& \mathbf{0} \leq x_i \leq \mathbf{1} \quad i = 1, 2 \\
& z \text{ is binary decision variable.}
\end{aligned}$$

In fact, the above MILP problem can be treated as two independent shortest path problems between  $s$  and  $t$ : the first one only uses edges that belong to  $\cup_{\{r:z_{r,1}=1\}}E_r$  and the second one only uses edges that belong to  $\cup_{\{r:z_{r,2}=1\}}E_r$ . Obviously, the computation time can be significantly reduced if the binary constraint on  $x$  is removed (see the numerical results presented in Section 5.3).

## 4.2 The Least Coupled SRLG Paths Problem

Let  $z = [z_r]_{r \in R}^T$  denote a column vector with  $|R|$  elements, where  $z_r = \min\{z_{r,1}, z_{r,2}\}$  indicating whether  $r$  is contained in both paths  $p_1$  and  $p_2$  or not. Then, the least coupled SRLG paths problem can be formulated as the following ILP:

$$\begin{aligned} \min \quad & \sum_{r \in R} z_r \\ \text{s.t.} \quad & Ax_i = u \quad i = 1, 2 \end{aligned} \quad (4)$$

$$Hx_i \leq |E_o|z_i \quad i = 1, 2 \quad (5)$$

$$z_r \geq z_{r,1} + z_{r,2} - 1 \quad r \in R \quad (6)$$

$x$  and  $z$  are binary decision variables.

Again, the binary constraint on  $x$  and  $z_r$  can be replaced by  $0 \leq x \leq 1$  and  $z_r \geq 0$ , respectively. Finally, we note that a different ILP formulation for the least coupled SRLG problem was given in [8]. It is based on the cut set formulation.

## 4.3 Some Numerical Results

We tested the ILP formulation for the minimum cost SRLG diverse routing problem on four networks with different sizes by using CPLEX 7.0 on a Pentium III 800MHz PC. The results are provided in the following table.

Network	Nodes	Fiber Spans	Optical Links	Run Time (seconds)	
				ILP	MILP
I	47	65	47	0.28	0.20
II	49	72	185	0.54	0.29
III	144	198	298	2.27	1.26
IV	226	303	353	4.42	1.78

The run times given in the last two columns are average times (in seconds) to obtain two SRLG diverse paths with minimum cost based on the ILP formulation and the MILP formulation (where the binary constraint on the edge decision variables  $x$  is removed), respectively. It is clear that the run time is reduced quite significantly when the binary constraint on the edge decision variables  $x$  is removed.

## 5 Conclusion

In this paper, we considered the SRLG diverse routing problem in optical networks. We proved that the problem is NP-complete. Furthermore, we showed that the fiber-span-disjoint paths problem, a special case of the SRLG diverse routing problem that was considered by several researchers before, is also NP-complete. We then provided an ILP formulation for the minimum cost SRLG diverse routing problem, which we demonstrated is quite effective for networks with a few hundreds nodes and fiber spans. One possible future research direction is to develop more efficient algorithms based on some heuristics and approximation methods, or their combination with our ILP formulation. In addition, it also needs to be investigated how the SRLG diverse routing problem should be considered in combination of other important issues in optical networks, such as capacity planning and overall equipment cost.

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