UBINET, Master 2 IFI Algorithms for telecommunications

First Exam, October 2014

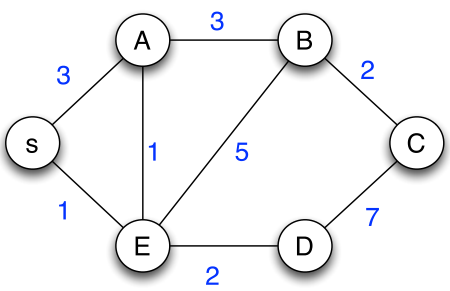
1 hour 1/2

Course and manuscript notes allowed. Not computers, cellphones, books.

Instruction and comments: the points awarded for your answer will be based on the correctness of your answer as well as the clarity of the main steps in your reasoning. All proposed solutions must be proved. **All the exercises are independent.** *The points are indicated so you may adapt your effort.*

1 Algorithmics on graphs

Exercise 1 (4 points) Consider the following graph H.



Apply the Dijkstra algorithm on H starting from node s. All steps of the algorithm must be detailed (**at most three lines per steps**).

In particular, indicate the order in which vertices are considered during the execution of the algorithm. Moreover, give the distance between s and any other node.

Exercise 2 (4 points) Consider the elementary network flow N depicted in Figure 1 (left) and the initial flow f from s to t in Figure 1 (right).

- What must be checked to show that f is a flow? What is the value of the flow f?
- Apply the Ford-Fulkerson Algorithm to N starting from the flow f. All steps of the execution (in particular each auxiliary graph) of the algorithm must be detailed.
- Give the flow and the cut obtained. Conclusion?

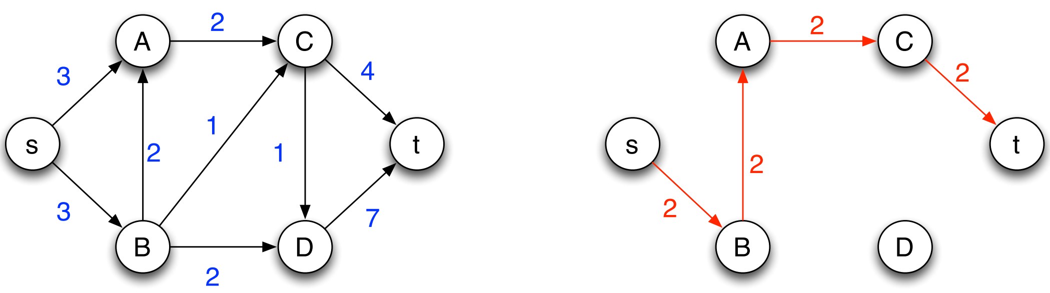


Figure 1: (left) Elementary network flow with arcs' capacity. (right) An initial flow f from s to t: a number close to an arc indicates the amount of flow along it. Arcs that are not represented have no flow.

Exercise 3 (5 points) Recall that a graph $G = (V, E)$ is *bipartite* if its vertex set can be partionned into two sets A and B (that is, $A \cup B = V$ and $A \cap B = \emptyset$) such that all edges of G are between a node in A and a node in B. That is there is no edge between two nodes in A, and there is no edge between two nodes in B.

A cycle in a graph is said *odd* if it has an odd number of vertices.

For any two nodes x and y in G, let $dist(x, y)$ denote the distance between x and y, i.e., the length of a shortest path from x to y.

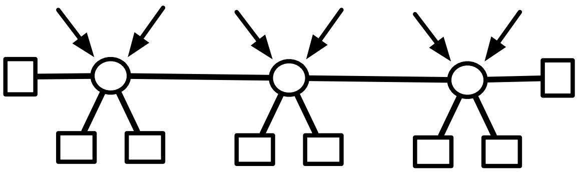
1. Let G be a graph that contains an odd cycle. Show that G is not bipartite.
2. Let $G = (V, E)$ be a graph that does not contain any odd cycle. Show that G is bipartite.

   Hint: Let $s \in V$. Let $x, y \in V$ such that $dist(s, x) = dist(s, y)$, show that x and y cannot be adjacent.
3. Conclusion?

2 Satellite Networks

Exercise 4 (7 points) We study in this exercise a variant of the Alcatel's problem considered in the first lecture. Consider a network with n inputs (signals) $n + k$ outputs (amplifiers) and which is valid k-fault tolerant. We suppose $k \geq 1$. But, here, we suppose that **each switch has 6 ports** (in the lecture, they had only 4 ports). The objective is to design such a network with the minimum number of switches.

1. Give the cut criteria for this variant of the problem. (Proof not needed).
2. Show that the network below with $n = 6$ inputs, 8 outputs (amplifiers) tolerates $k = 2$ failures.



3. Prove that in a valid (n, k)-network, with $k \geq 1$ failures, there is no switch connected to 4 or more inputs (each switch is connected to at most 3 inputs).
4. Prove that in a minimum valid (n, k)-network, with $k \geq 1$ failures, there is no switch connected to 3 or more inputs (each switch is connected to at most 2 inputs).
5. Deduce that the minimum number of switches in a valid (n, k)-network is at least $\frac{n}{2}$.
6. Design for $k = 2$ and n even a valid network with $\frac{n}{2}$ switches. Is it optimal?
7. Design for $k = 4$ and $n = 6$ a valid $(6, 4)$-network with 4 switches. Is it optimal?
8. **(Bonus question. Do only after all the other questions.)** Propose minimum valid $(n, 4)$-networks and show they are optimal, for any n.