UBINET, Master 2 IFI Algorithms for telecommunications

First Exam, October 2013

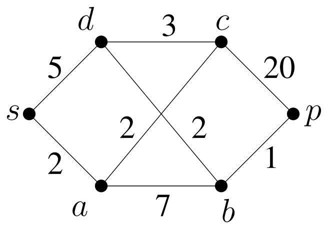
1 hour 1/2

Course and manuscript notes allowed. Not computers, cellphones, books.

Instruction and comments: the points awarded for your answer will be based on the correctness of your answer as well as the clarity of the main steps in your reasoning. All proposed solutions must be proved. **All the exercises are independent.** *The points are indicated so you may adapt your effort.*

1 Algorithmics on graphs

Exercise 1 (8 points) Consider the following graph H.



1. Apply the Dijkstra algorithm on H to compute a shortest-path tree rooted in s and the distance between any vertex and vertex s. The first two steps of the algorithm must be detailed (**at most three or four lines per steps**). Moreover, indicate the order in which vertices are considered during the execution of the algorithm.
2. Apply the Kruskal algorithm on H (detail in few sentences the first two steps).

Exercise 2 (4 points) *Questions:*
Let $G = (V, E)$ be a graph. Its complement is the graph $\bar{G} = (V, (V \times V) \setminus E)$.

1. *Is the complement of a non-connected graph always connected?*

   Hint: *let G be a non-connected graph, $u \in V(G)$ and C_u be the connected component of G containing u. For any $x, y \in V(G)$, show that there is a path from x to y in $\bar{G}$ by considering the three cases depending on whether x and y are in C_u or not.*

2. *Is the complement of a connected graph always non-connected?*

Exercise 3 (6 points) A graph is *regular* if all its nodes have same degree.
Let G be a graph on at least 4 vertices such that for every vertex v, $G - v$ is regular.

Question: Show that G is either a complete graph or an empty graph.

Hint: *Assume G is neither complete nor empty. Show that there are $u, v, w \in V(G)$ such that $\{u, v\} \in E(G)$ and $\{u, w\} \notin E(G)$. Let d_x be the degree of node x in G. Show that $d_w = d_v - 1$. Then, show that $2|E(G)| - 2d_w = (|V(G)| - 1)d_u$. Give a similar equality linking d_v and d_u. Conclude.*

2 Linear Programming

Exercise 4 (1 points) We consider the following linear programme.

$$
\begin{array}{lrcrcrcr}
\text{Minimize} & -x_1 & + & 2x_2 & - & 5x_3 & & \\
\text{Subject to:} & & & & & & & \\
& 3x_1 & - & x_2 & - & 2x_3 & \geq & -4 \\
& 4x_1 & + & 5x_2 & - & 4x_3 & \leq & -2 \\
& -2x_1 & - & 8x_2 & + & 3x_3 & \geq & 7 \\
& & & & & x_1, x_2, x_3 & \geq & 0
\end{array}
$$

Question: Write this linear programme under the standard form.

Exercise 5 (8 points) A farmer has 80 hectares of his farm available for planting corn and cabbages. He must grow at least 10 hectares of corn and 20 hectares of cabbages to meet demands. He prefers to plant more corn than cabbages but his work force and equipment will only allow him to cultivate a maximum of three times more corn than cabbage. If the profit on corn is 800 euros per ha and on cabbages 500 euros per ha, how should the farmer plant the two crops to make a maximum profit and what is this profit?

a) Formulate the problem of maximizing the farmer profit as a linear program.

b) Solve the problem using the graphical method.

Exercise 6 (6 points) We consider a scenario in which n cellular phones want to connect to m antennas. Each phone should be connected to exactly one antenna. An antenna can serve a maximum of 10 cellular phones. The throughput that the phone p ($p \in 1, ..., n$) can obtain from the antenna a ($a \in 1, ..., m$) depends on their respective position and is given by the constant $t_{p,a}$.

Question: Write a linear program maximizing the sum of the throughput of the cellular phones.