UBINET, Master 2 IFI Algorithms for telecommunications

First Exam, October 2017

2 hours

Course and manuscript notes are allowed. No computers, cellphones, books.

Instruction and comments: the points awarded for your answer will be based on the correctness of your answer as well as the clarity of the main steps in your reasoning. All proposed solutions must be proved. **All the exercises are independent.** *The points are indicated so you may adapt your effort.*

1 Linear Programming

Exercise 1 (Modeling as a linear program (5 points, 25 minutes)) *We have n objects each having a weight a_j and a value c_j ($j = 1, ..., n$). We must choose a subset $J \subseteq \{1, ..., n\}$, whose total weight is less than or equal to a given number b and whose total value (sum of the values of the objects selected) is maximum.*

1. *Formulate the problem as an integer linear program. Explain well all constraints.*
2. *To find the best solution we can enumerate all possible solutions, how many are there in the worst case?*
3. *Finally, what is the best solution with $n = 4$, $b = 30$, $a = (19, 17, 15, 13)$ and $c = (9, 7, 5, 4)$?*

Exercise 2 (Duality and Optimality certificates. (4 points, 20 minutes)) *Consider the following linear program.*

$$\begin{array}{lrcrcrcrcrcl}
\textit{Maximize} & 7x_1 & + & 6x_2 & + & 5x_3 & - & 2x_4 & + & 3x_5 & & \\
\textit{Subject to:} & & & & & & & & & & & \\
& x_1 & + & 3x_2 & + & 5x_3 & - & 2x_4 & + & 2x_5 & \leq & 4 \\
& 4x_1 & + & 2x_2 & - & 2x_3 & + & x_4 & + & x_5 & \leq & 3 \\
& 2x_1 & + & 4x_2 & + & 4x_3 & - & 2x_4 & + & 5x_5 & \leq & 5 \\
& 3x_1 & + & x_2 & + & 2x_3 & - & x_4 & - & 2x_5 & \leq & 1 \\
& \multicolumn{5}{r}{x_1, x_2, x_3, x_4, x_5} & \geq & 0 & & & &
\end{array}$$

1. *Write the dual program of this linear program.*
2. *Is $x_1 = 0, x_2 = \frac{4}{3}, x_3 = \frac{2}{3}, x_4 = \frac{5}{3}, x_5 = 0$ an optimal solution of the linear programme above? To answer, use the method to provide optimality certificates seen during the class. Detail all steps.*