

# Lecture 8

# Generative Adversarial Networks

Deep Learning for Computer Vision  
Valeriya Strizhkova  
9 November 2021

# About myself

Valeriya Strizhkova

1st year PhD student @ Inria, STARS team

<https://scholar.google.ru/citations?user=6n5PrUAAAAAJ&hl>

<https://github.com/valerystrizh>



# Lecture Structure

- GANs: Valeriya Strizhkova
- DeepFake Detection: Dr. Antitza Dantcheva

# Outline

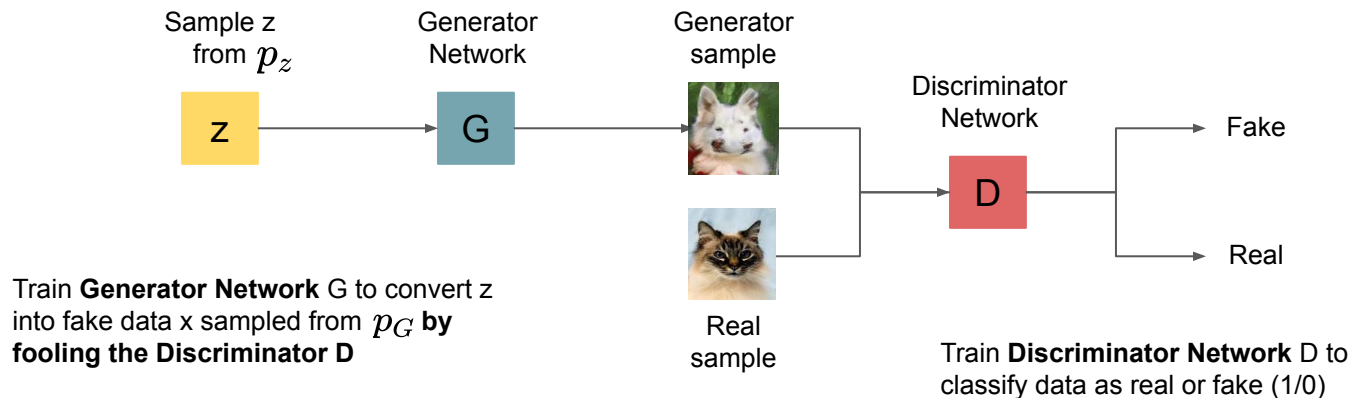
- Basic idea of GAN
- Image generation
  - Conditional GAN
  - Image-to-image translation (Pix2Pix, CycleGAN)
  - StyleGAN
- Video Generation

# Generative Adversarial Networks

- **Setup:** Assume we have data  $x_i$  drawn from distribution  $p_{data}(x)$ . Want to sample from  $p_{data}$ .
- **Idea:** Introduce a latent variable  $z$  with simple prior  $p(z)$ .
- Sample  $z \sim p(z)$  and pass to a Generator Network  $x = G(z)$
- Then  $x$  is a sample from the Generator distribution  $p_G$ . Want  $p_G = p_{data}$

# Generative Adversarial Networks

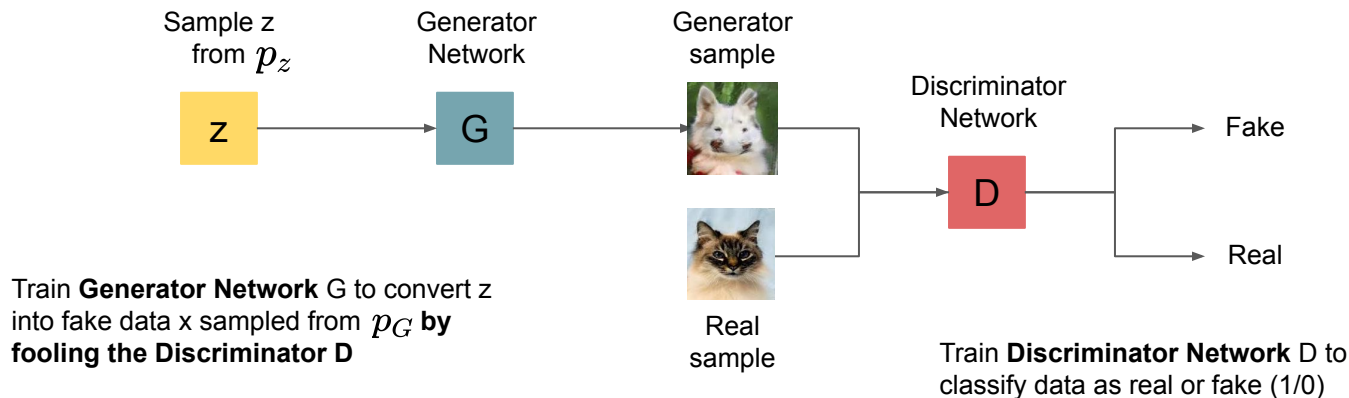
- Setup: Assume we have data  $x_i$  drawn from distribution  $p_{data}(x)$ . Want to sample from  $p_{data}$ .
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# Generative Adversarial Networks: Training Objective

Jointly train generator G and discriminator D with a minimax game

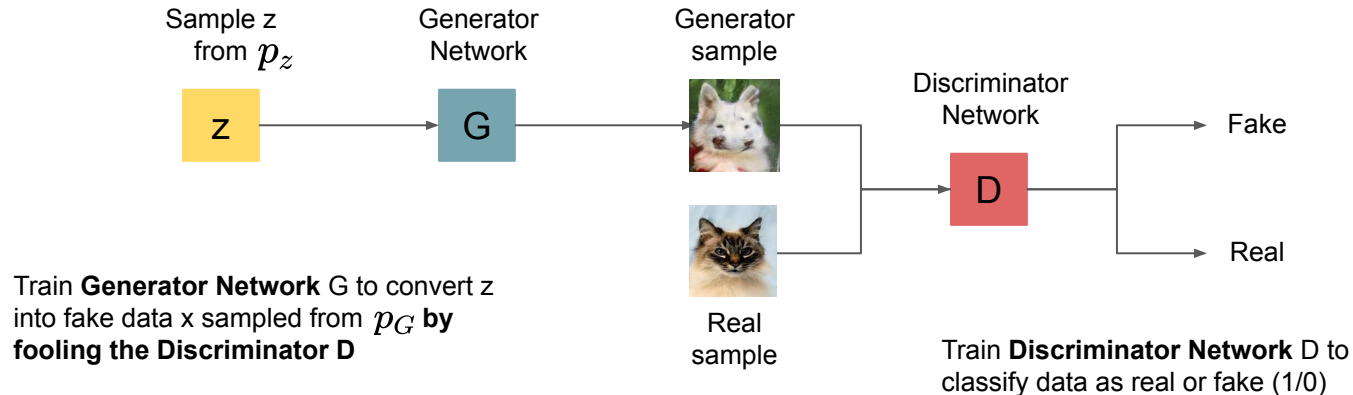
$$\min \max (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$



# Generative Adversarial Networks: Training Objective

Jointly train generator G and discriminator D with a minimax game

$$\min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$





# Generative Adversarial Networks: Training Objective

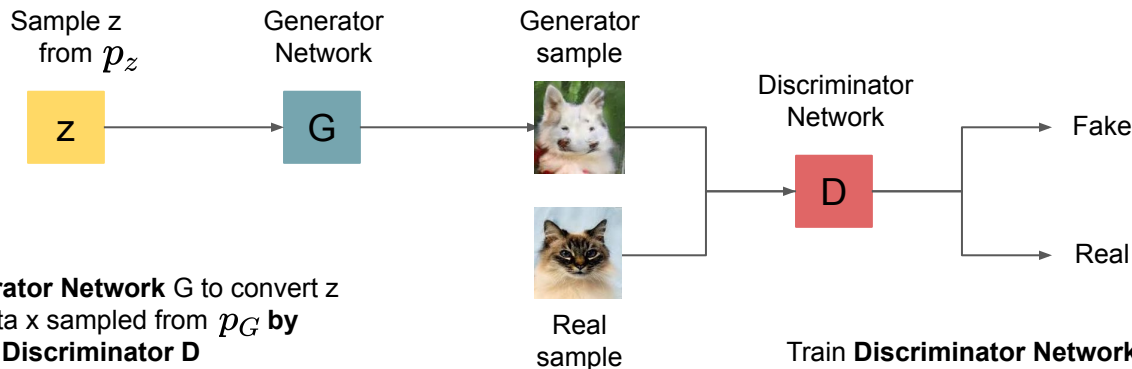
Jointly train generator G and discriminator D with a **minimax game**

$$\min_{\mathbf{G}} \max_{\mathbf{D}} (E_{x \sim p_{data}} [\log \mathbf{D}(x)] + E_{z \sim p(z)} [\log(1 - \mathbf{D}(\mathbf{G}(z)))])$$

Discriminator wants  $D(x)=1$  for real data

Discriminator wants  $D(x)=0$  for fake data

Generator wants  $D(x)=1$  for fake data



Train **Generator Network G** to convert  $z$  into fake data  $x$  sampled from  $p_G$  by **fooling the Discriminator D**

Train **Discriminator Network D** to classify data as real or fake (1/0)

# Generative Adversarial Networks: Training Objective

Jointly train generator  $G$  and discriminator  $D$  with a **minimax game**

$$\min_{\mathbf{G}} \max_{\mathbf{D}} (E_{x \sim p_{data}} [\log \mathbf{D}(x)] + E_{z \sim p(z)} [\log(1 - \mathbf{D}(\mathbf{G}(z)))])$$
$$= \min_{\mathbf{G}} \max_{\mathbf{D}} \mathbf{V}(\mathbf{G}, \mathbf{D})$$

Train  $G$  and  $D$  using alternating gradient updates:

1. Update  $\mathbf{D} = \mathbf{D} + \alpha_{\mathbf{D}} \frac{\delta \mathbf{V}}{\delta \mathbf{D}}$
2. Update  $\mathbf{G} = \mathbf{G} + \alpha_{\mathbf{G}} \frac{\delta \mathbf{V}}{\delta \mathbf{G}}$

# Generative Adversarial Networks: vanishing gradient

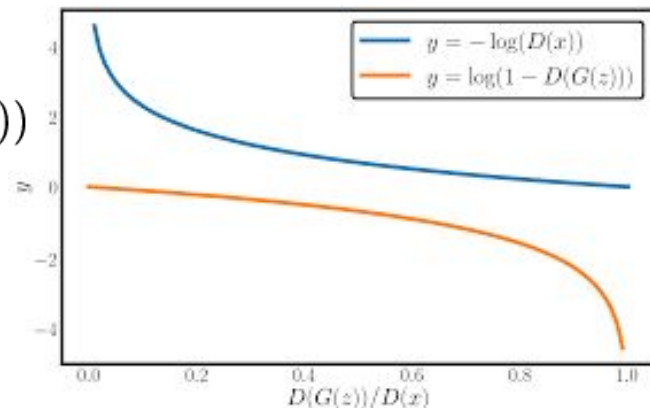
$$\min_G \max_D V(G, D) = \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$

$$\nabla_{\theta_G} V(G, D) = \nabla_{\theta_G} E_{z \sim q(z)} [\log(1 - D(G(z)))]$$

$$\nabla_a \log(1 - \sigma(a)) = \frac{-\nabla_a \sigma(a)}{1 - \sigma(a)} = \frac{-\sigma(a)(1 - \sigma(a))}{1 - \sigma(a)} = -\sigma(a) = -D(G(z))$$

$D(G(z)) \rightarrow 0$

- Gradient goes to 0 if D is confident, i.e.
- Minimize  $-E_{z \sim p(z)} [\log(D(G(z)))]$  for **Generator** instead  
(keep Discriminator as it is)



# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \max_D \int_X (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \\ &= \min_G \int_X \max_D (p_{data}(x) \log D(x) + p_G(x) \log(1 - D(x))) dx \end{aligned}$$

$$f(y) = a \log y + b \log(1 - y)$$

$$f'(y) = \frac{a}{y} - \frac{b}{1-y}$$

$$f'(y) = 0 \Leftrightarrow y = \frac{a}{a+b}$$

**Optimal Discriminator:**

$$D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \int_X (p_{data}(x) \log D_G^*(x) + p_G(x) \log(1 - D_G^*(x))) dx \end{aligned}$$

$$\text{Optimal Discriminator: } D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{x \sim p_G} [\log(1 - D(x))]) \\ &= \min_G \int_X (p_{data}(x) \log D_G^*(x) + p_G(x) \log(1 - D_G^*(x))) dx \\ &= \min_G \int_X (p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} + p_G(x) \log \frac{p_G(x)}{p_{data}(x) + p_G(x)}) dx \end{aligned}$$

$$\text{Optimal Discriminator: } D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \int_X (p_{data}(x) \log D_G^*(x) + p_G(x) \log(1 - D_G^*(x))) dx \\ &= \min_G \int_X (p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} + p_G(x) \log \frac{p_G(x)}{p_{data}(x) + p_G(x)}) dx \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}]) \end{aligned}$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G \int_X (p_{data}(x) \log D_G^*(x) + p_G(x) \log(1 - D_G^*(x))) dx \\ &= \min_G \int_X (p_{data}(x) \log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)} + p_G(x) \log \frac{p_G(x)}{p_{data}(x) + p_G(x)}) dx \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}]) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \end{aligned}$$



# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))] ) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \end{aligned}$$

**Kullback-Leibler Divergence:**  $KL(p, q) = E_{x \sim p} [\log \frac{p(x)}{q(x)}]$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \\ &= \min_G (KL(p_{data}, \frac{p_{data} + p_G}{2}) + KL(p_G, \frac{p_{data} + p_G}{2}) - \log 4) \end{aligned}$$

**Kullback-Leibler Divergence:**  $KL(p, q) = E_{x \sim p} [\log \frac{p(x)}{q(x)}]$

# Generative Adversarial Networks: Optimality

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$$\text{Jensen-Shannon Divergence: } JSD(p, q) = \frac{1}{2} KL(p, \frac{p+q}{2}) + \frac{1}{2} KL(q, \frac{p+q}{2})$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \\ &= \min_G (KL(p_{data}, \frac{p_{data} + p_G}{2}) + KL(p_G, \frac{p_{data} + p_G}{2}) - \log 4) \\ &= \min_G (2 \times JSD(p_{data}, p_G) - \log 4) \end{aligned}$$

$$\text{Jensen-Shannon Divergence: } JSD(p, q) = \frac{1}{2} KL(p, \frac{p+q}{2}) + \frac{1}{2} KL(q, \frac{p+q}{2})$$

# Generative Adversarial Networks: Optimality

$$\begin{aligned} & \min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))] ) \\ &= \min_G (E_{x \sim p_{data}} [\log \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}] + E_{x \sim p_G} [\log \frac{p_G(x)}{p_{data}(x) + p_G(x)}] - \log 4) \\ &= \min_G (KL(p_{data}, \frac{p_{data} + p_G}{2}) + KL(p_G, \frac{p_{data} + p_G}{2}) - \log 4) \\ &= \min_G (2 \times JSD(p_{data}, p_G) - \log 4) \end{aligned}$$

JSD is always nonnegative and zero when the two distributions are equal

=> the global minimum is  $p_{data} = p_G$

$$\text{Jensen-Shannon Divergence: } JSD(p, q) = \frac{1}{2} KL(p, \frac{p+q}{2}) + \frac{1}{2} KL(q, \frac{p+q}{2})$$

# Generative Adversarial Networks: Optimality

$$\min_G \max_D (E_{x \sim p_{data}} [\log D(x)] + E_{z \sim p(z)} [\log(1 - D(G(z)))])$$

$$= \min_G (2 * JSD(p_{data}, p_G) - \log 4)$$

**Summary:** The global minimum of the minimax game happens when:

1.  $D_G^*(x) = \frac{p_{data}(x)}{p_{data}(x) + p_G(x)}$  (Optimal discriminator for any G)
2.  $p_G(x) = p_{data}(x)$  (Optimal generator for optimal D)

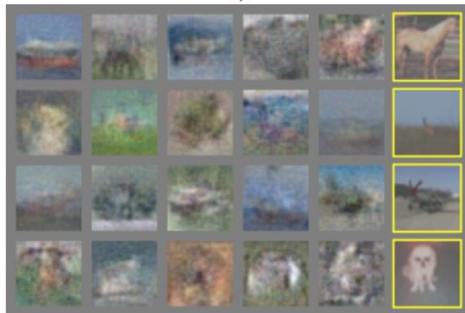
# Generative Adversarial Networks: results



a)



b)

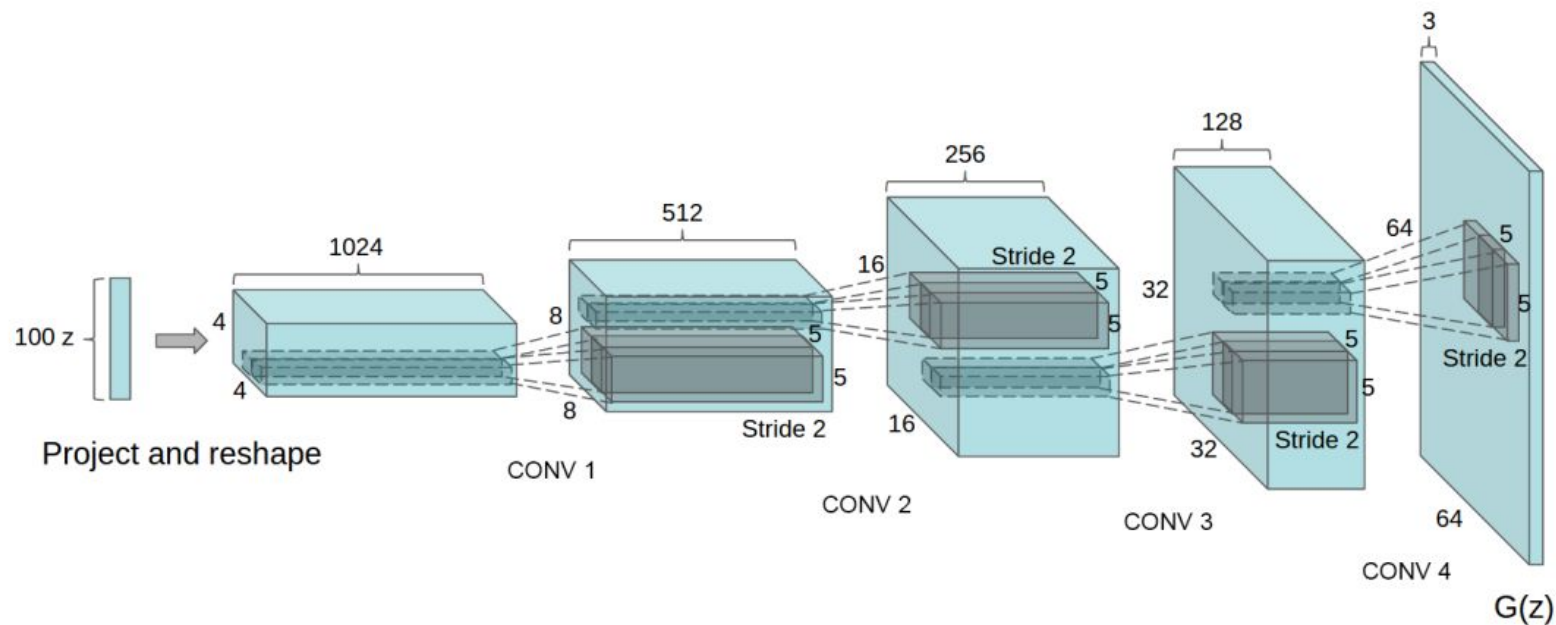


c)



d)

# Generative Adversarial Networks: DC-GAN

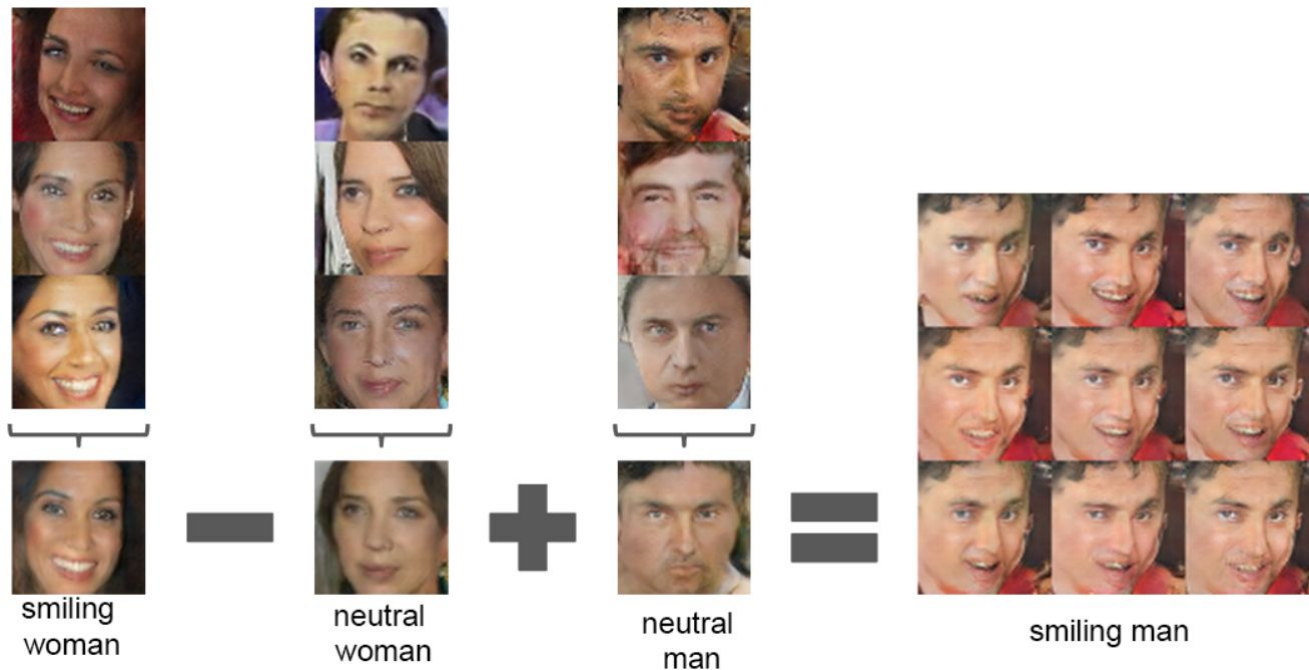




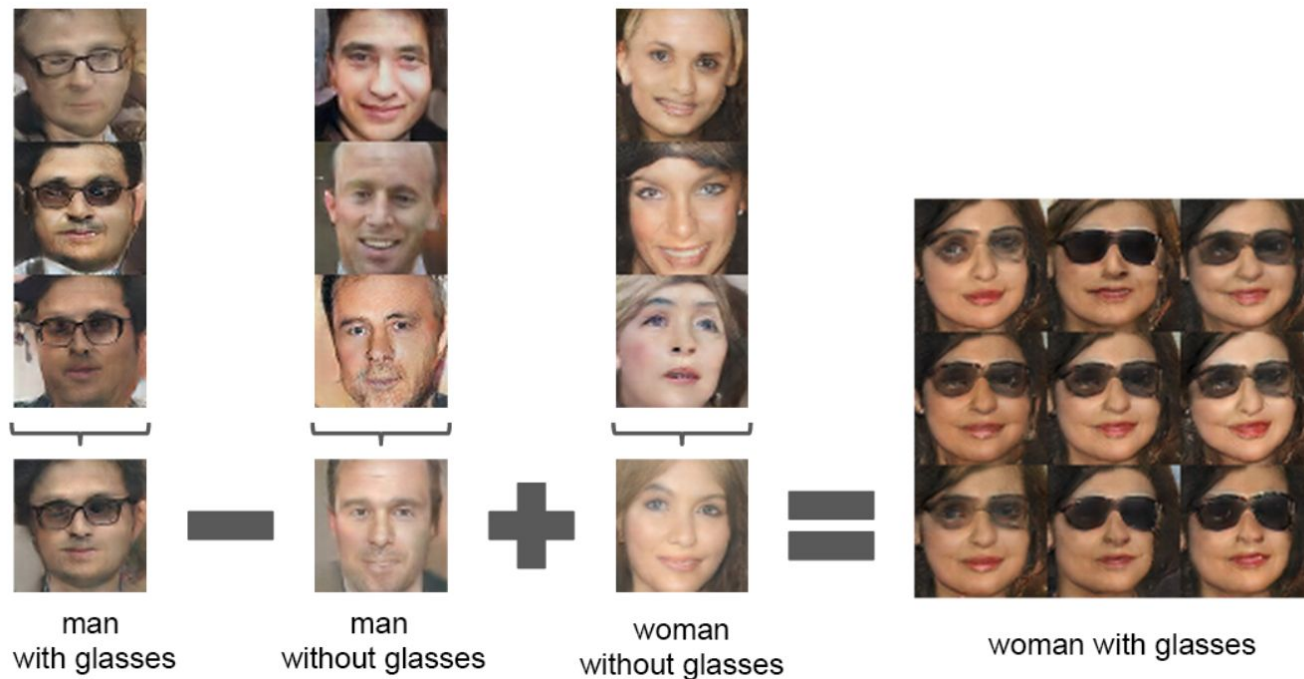
# Generative Adversarial Networks: Interpolation



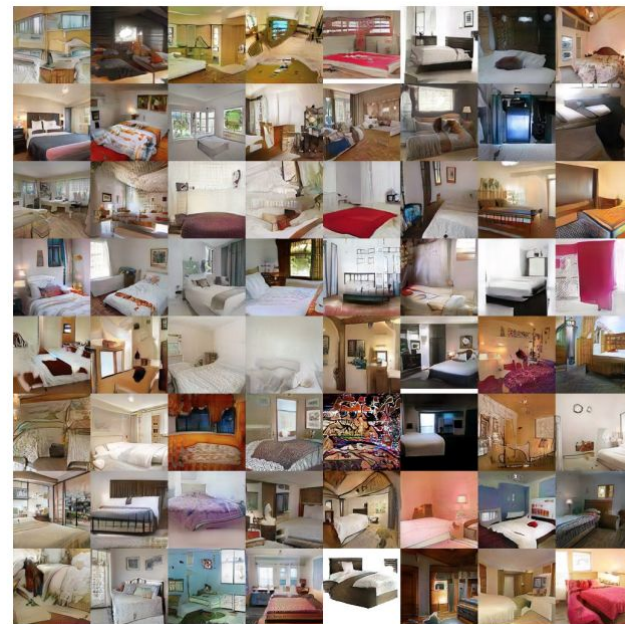
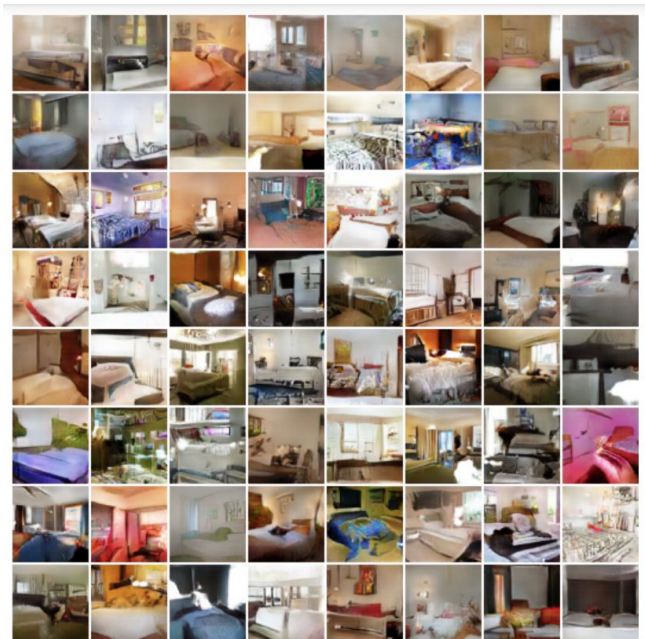
# Generative Adversarial Networks: Vector Math



# Generative Adversarial Networks: Vector Math



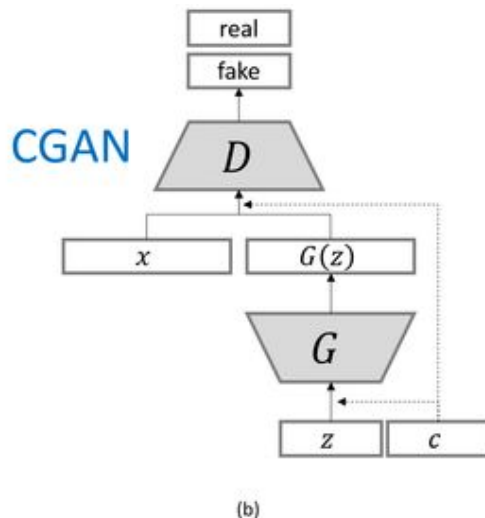
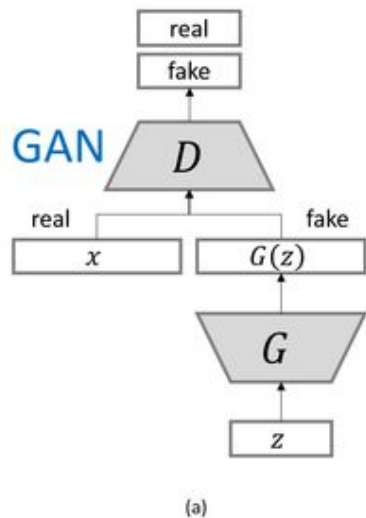
# GAN: Improved Loss Functions



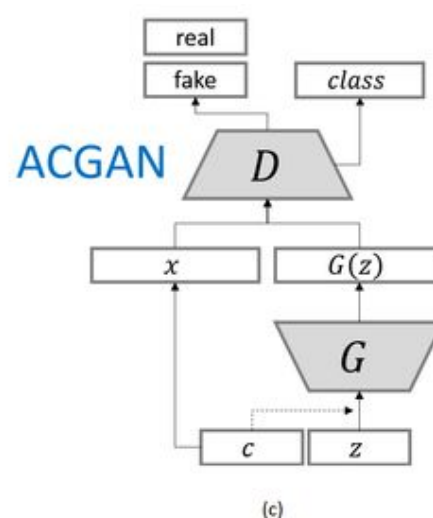
[1] Martin Arjovsky, Soumith Chintala, Léon Bottou. Wasserstein GAN. 2017

[2] Ishaan Gulrajani, Faruk Ahmed, Martin Arjovsky, Vincent Dumoulin, Aaron Courville. Improved Training of Wasserstein GANs. NeurIPS, 2017.

# Conditional GANs



[1]



[2]

[1] Mehdi Mirza, Simon Osindero. Conditional Generative Adversarial Nets. 2014

[2] Augustus Odena, Christopher Olah, Jonathon Shlens. Conditional Image Synthesis With Auxiliary Classifier GANs. ICML 2016

# Conditional GANs



monarch butterfly



goldfinch



daisy



redshank



grey whale

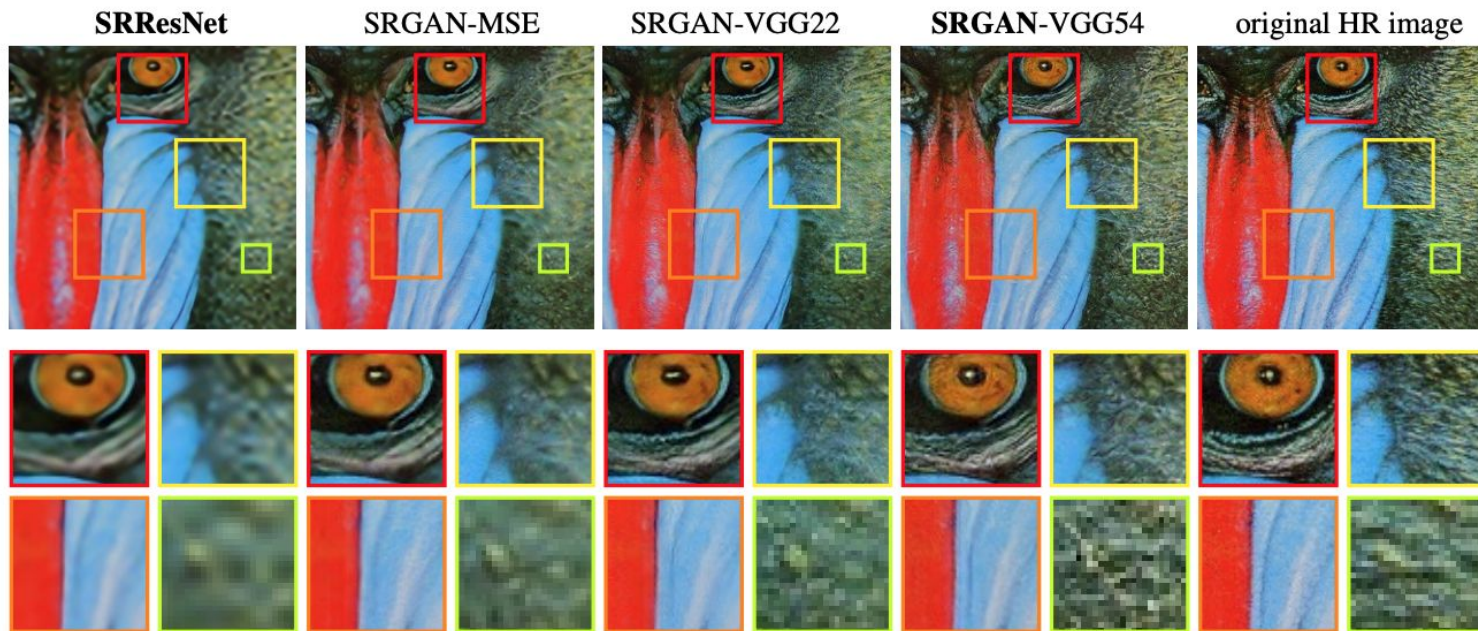
# Conditional GANs: BigGAN



Figure 6: Samples generated by our BigGAN model at  $512 \times 512$  resolution.

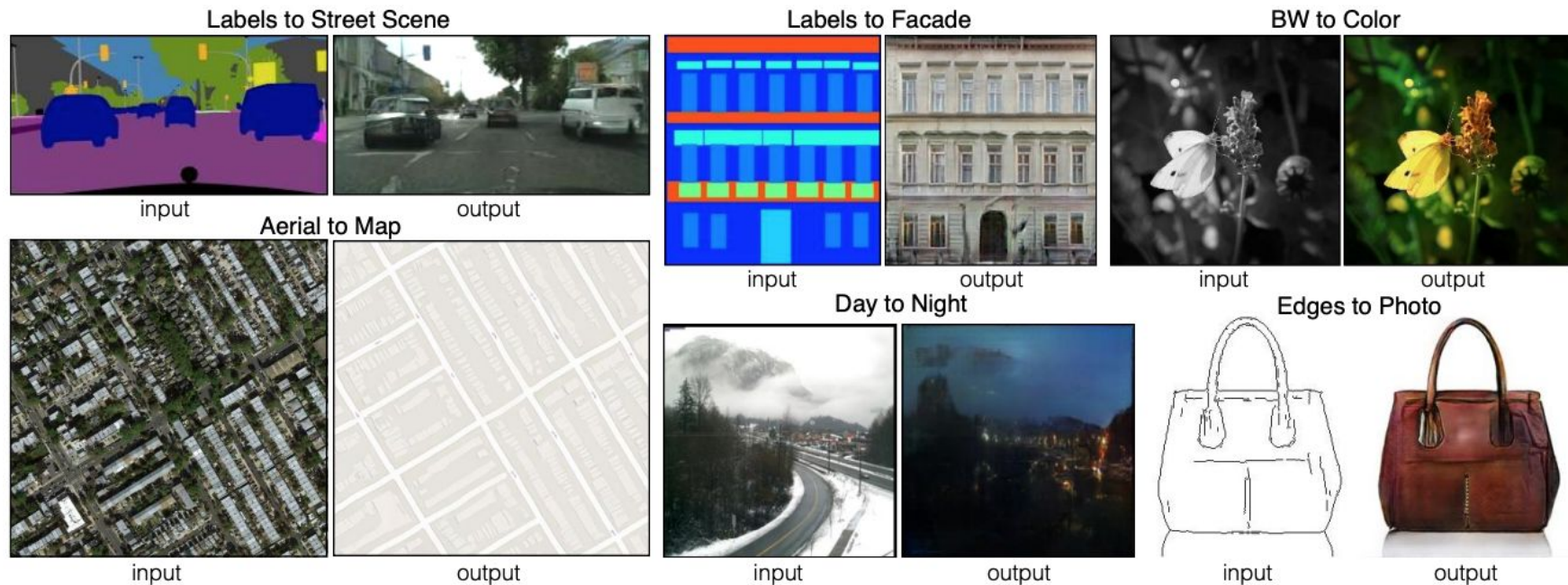


# Image Super-Resolution

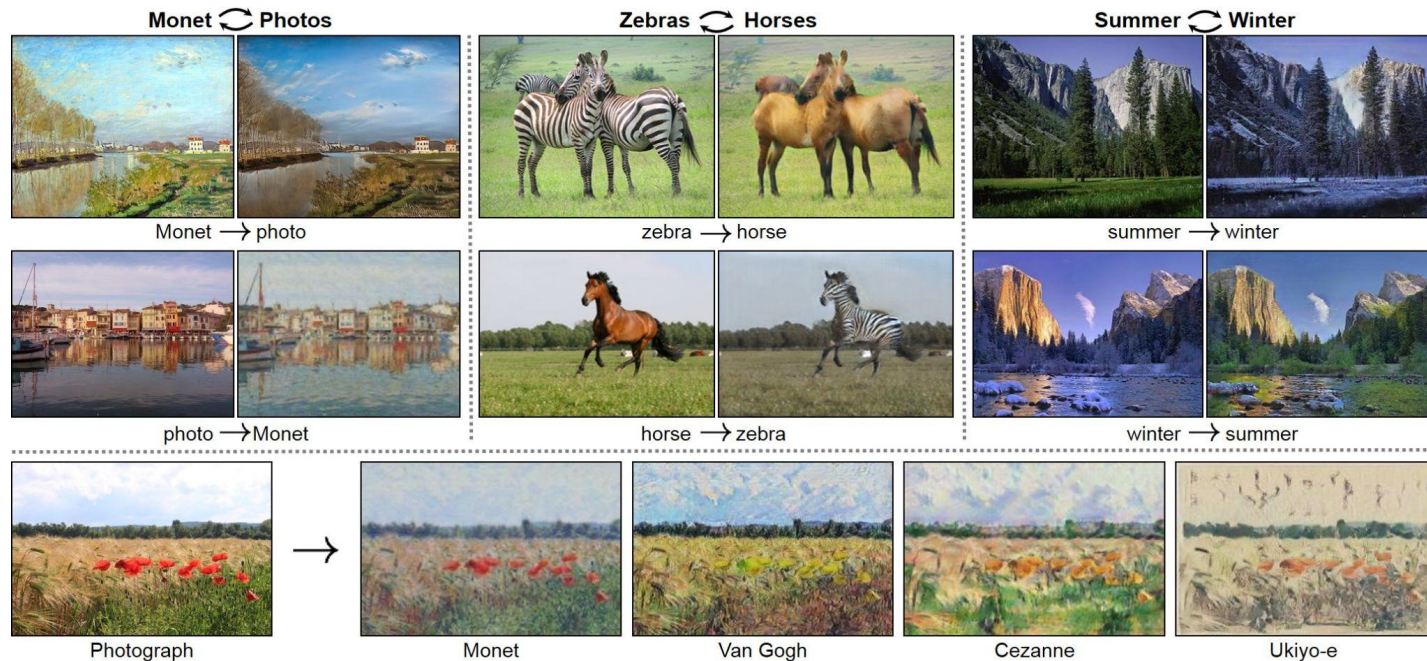




# Image-to-Image Translation: Pix2Pix



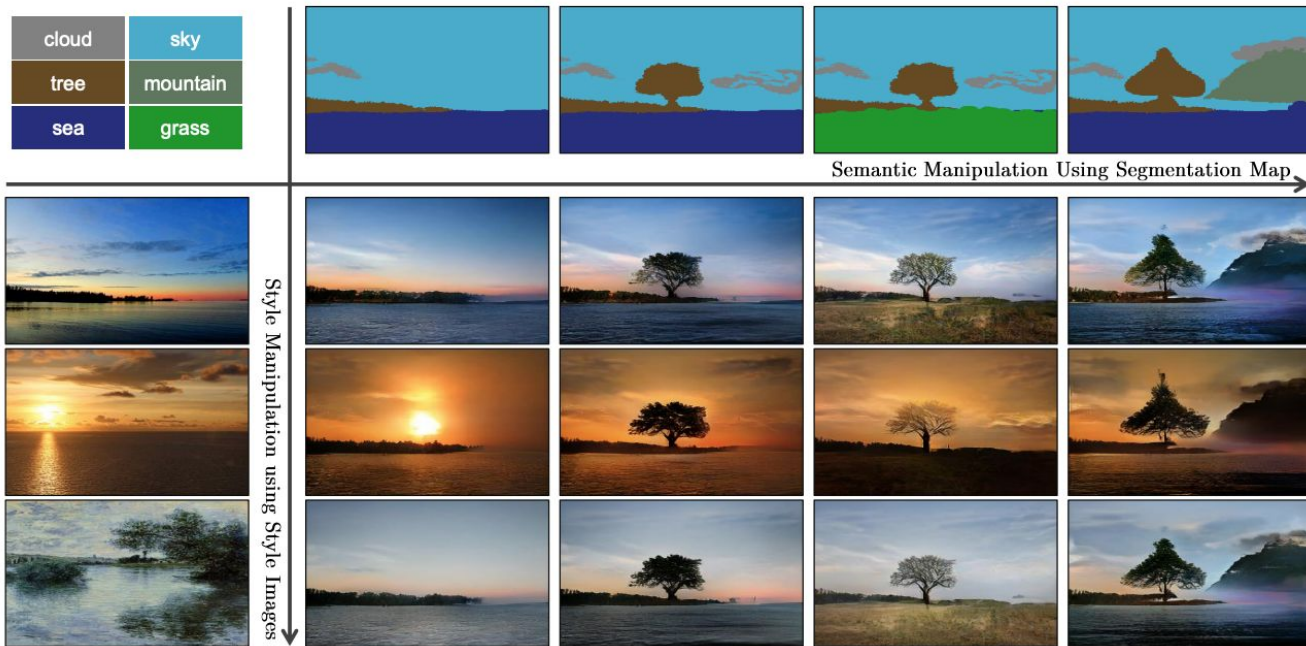
# Unpaired Image-to-Image Translation: CycleGAN



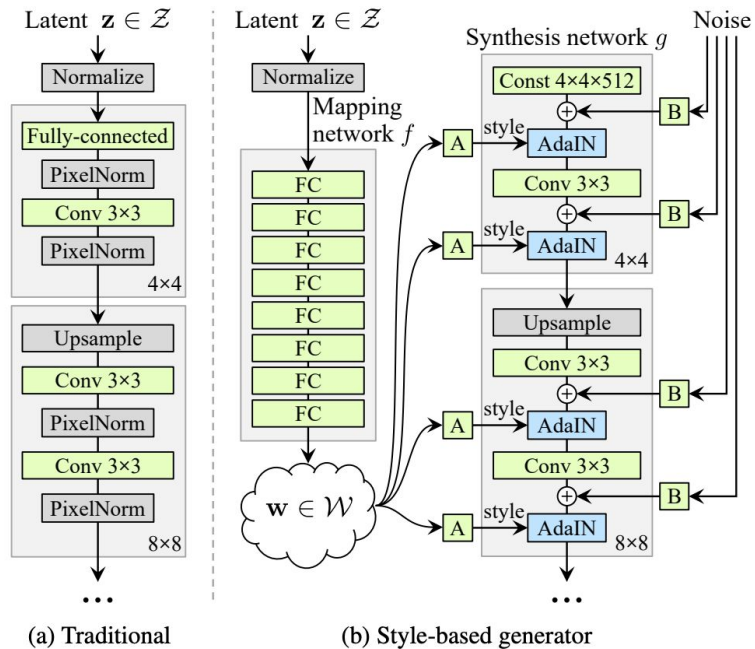
# Unpaired Image-to-Image Translation: CycleGAN



# Label Map to Image



# StyleGAN



# Video Generation



# Video Generation

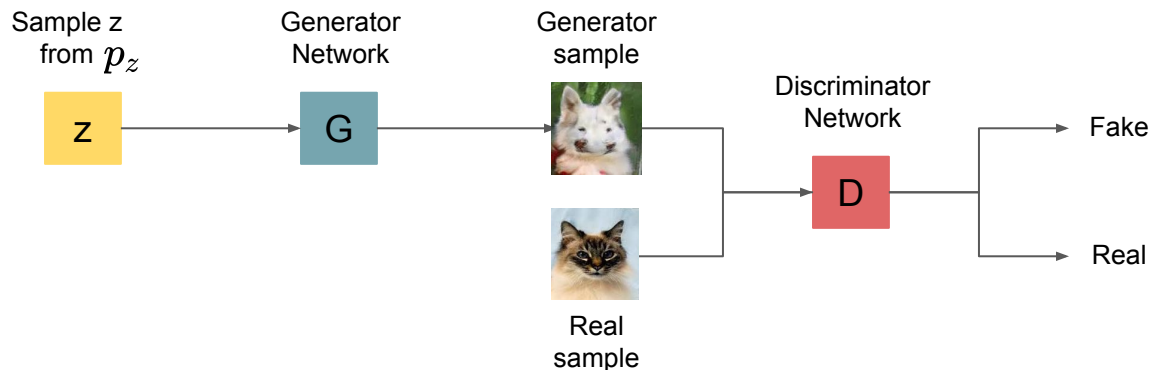


# GAN Summary

Jointly train two networks:

**Discriminator** classifies data as real or fake

**Generator** generates data that fools the discriminator





Q&A