



## Part C

Applications to image analysis problems

# Outlines

## Polygonal superpixels

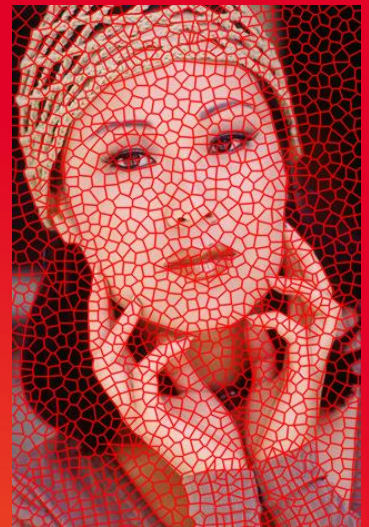
- with Voronoi diagrams
- with Kinetic data-structures
- application to object contouring

## Delaunay point processes

- principle
- application to object contouring
- application to line-network extraction
- application to image compression

# 1

## Polygonal superpixels



# Superpixels



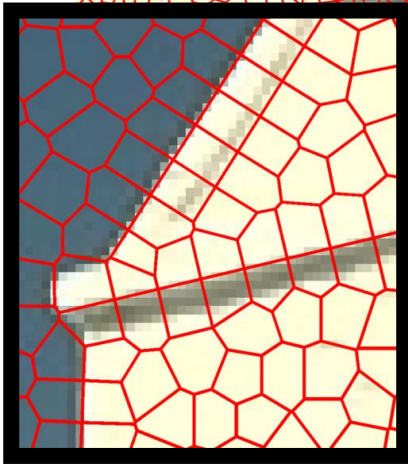
## Over-segmentation of images

- ✓ contour preservation
- ✓ algorithmic complexity
- ✗ storage
- ✗ control on region shapes
- ✗ region adjacency

Can we improve this with geometric data-structures ?



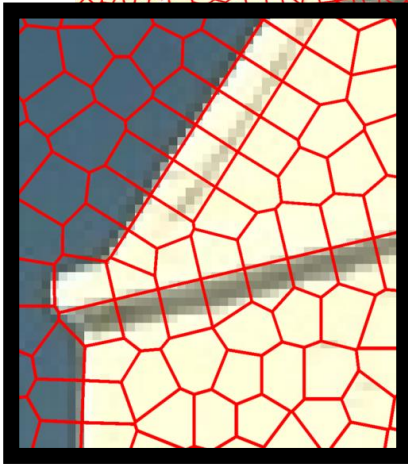
# Superpixels as Voronoi cells



How to do it?

- ✓ storage  
(2D Delaunay triangulation)
- ✓ control on region shapes  
(convex polygons)
- ✓ region adjacency  
(uniqueness)

# Superpixels as Voronoi cells



- ✓ storage  
(2D Delaunay triangulation)
- ✓ control on region shapes  
(convex polygons)
- ✓ region adjacency  
(uniqueness)

How to do it?

Guide the partition by geometric shapes

# Voronoi-based Image partitioning



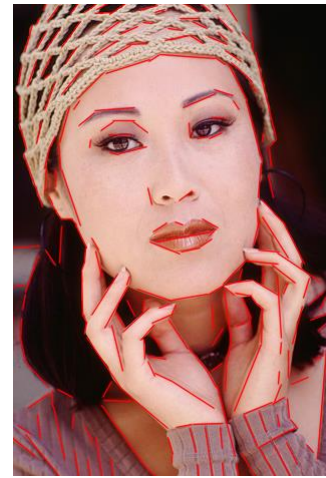
[Duan and Lafarge, Partitioning images into convex polygons,  
CVPR 2015]

# Step 1: extraction of geometric shapes

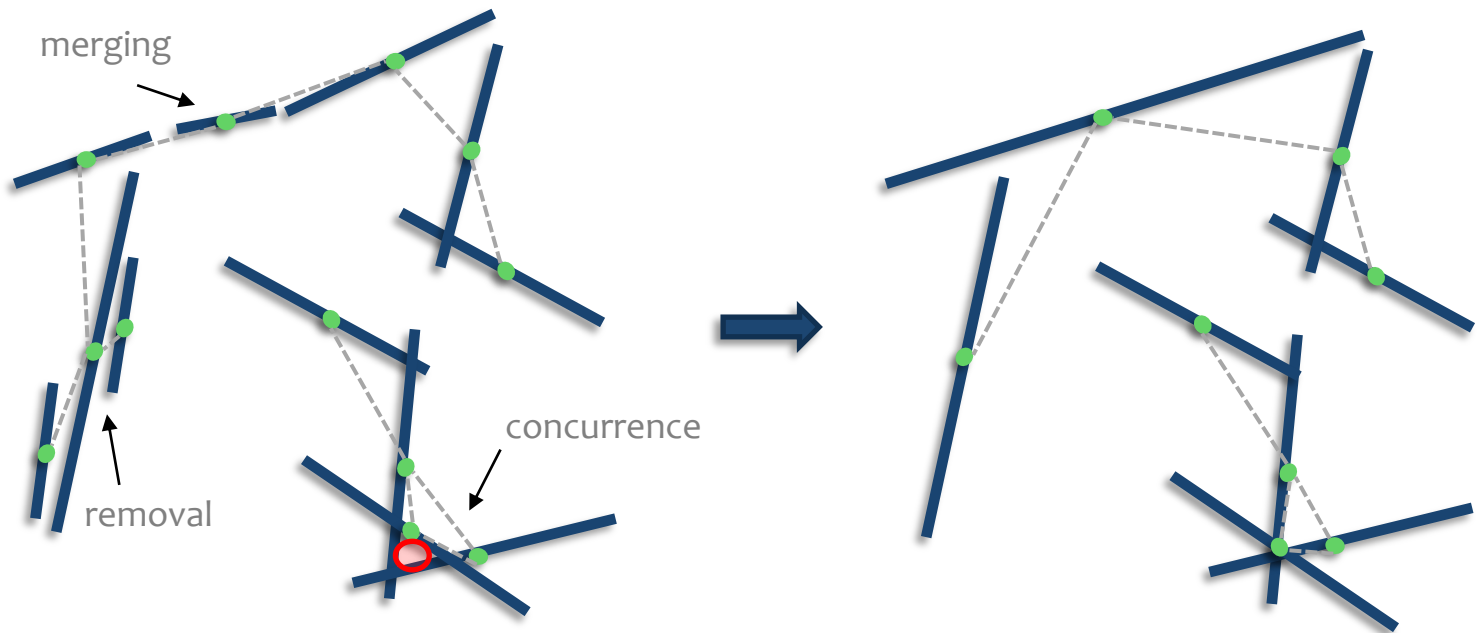
## Detection of line-segments



[Von Gioi et al., Lsd: A fast line segment detector with a false detection control, PAMI 2010]

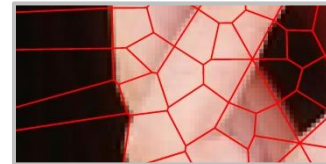
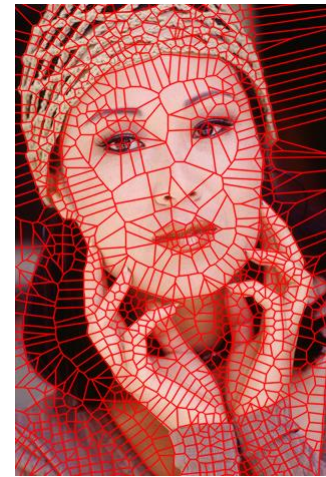
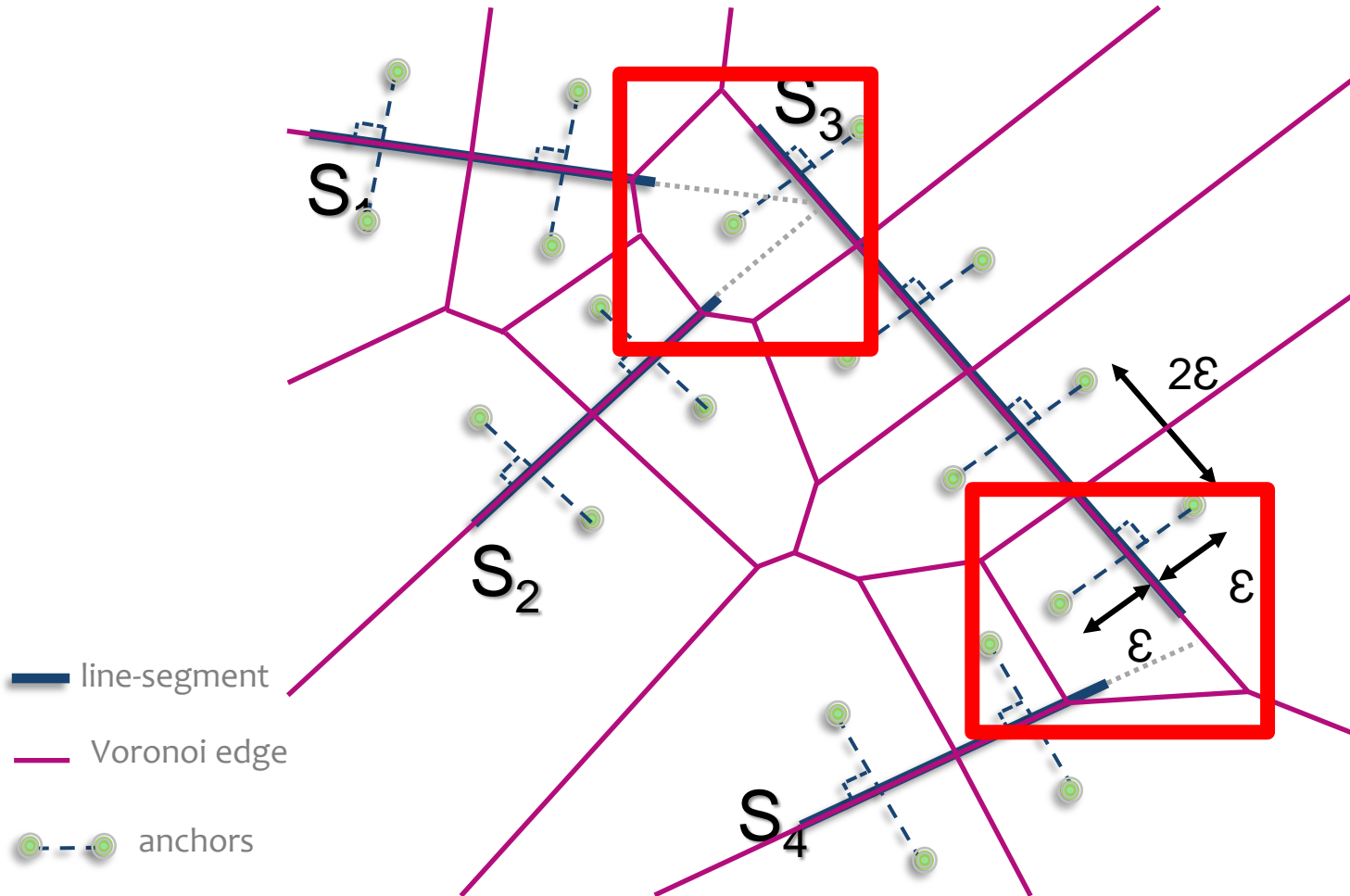


## Consolidation of line-segments

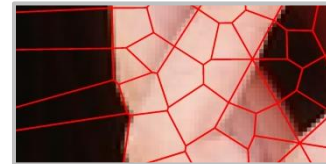
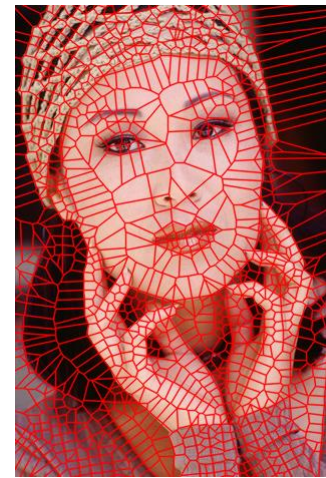
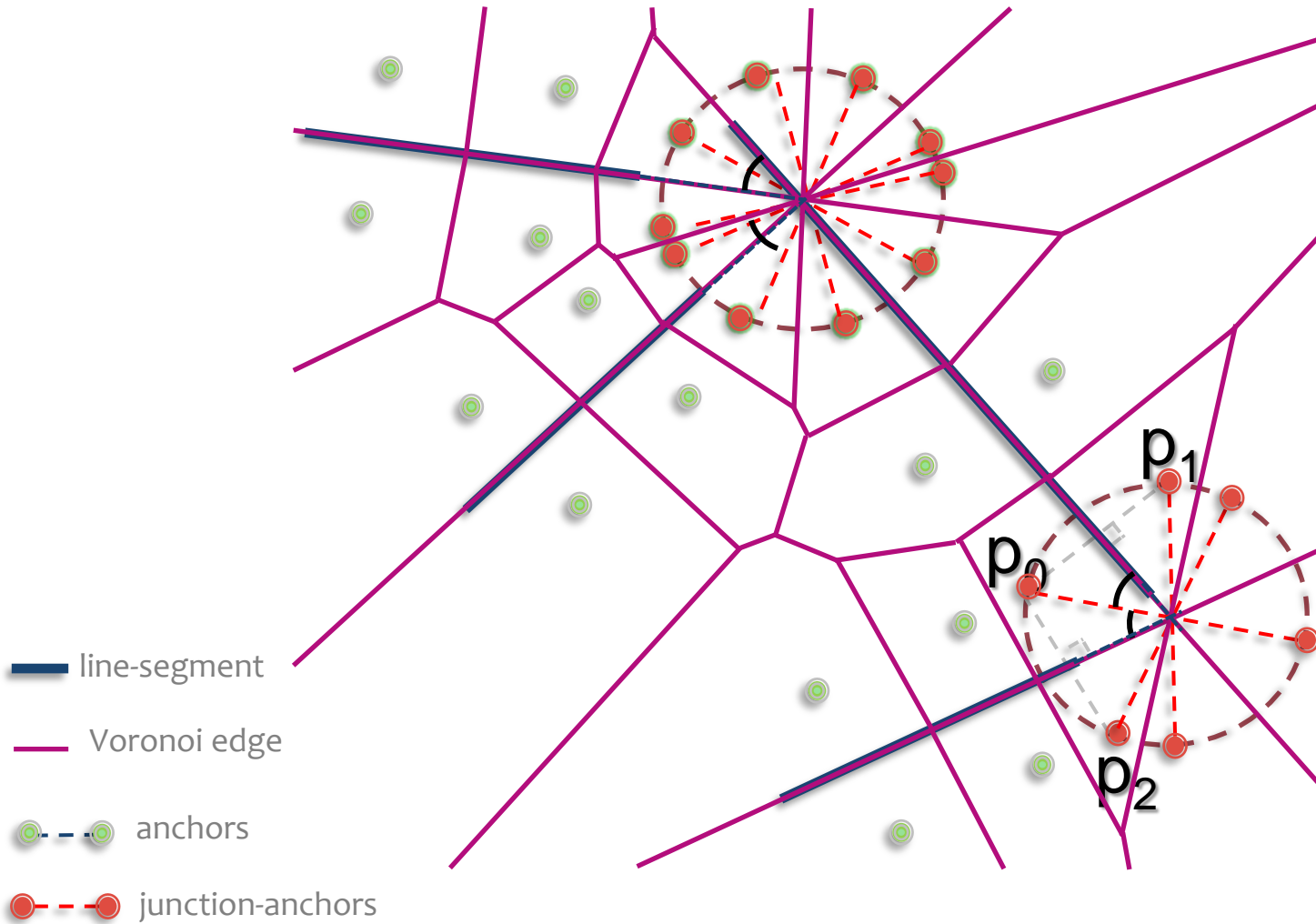




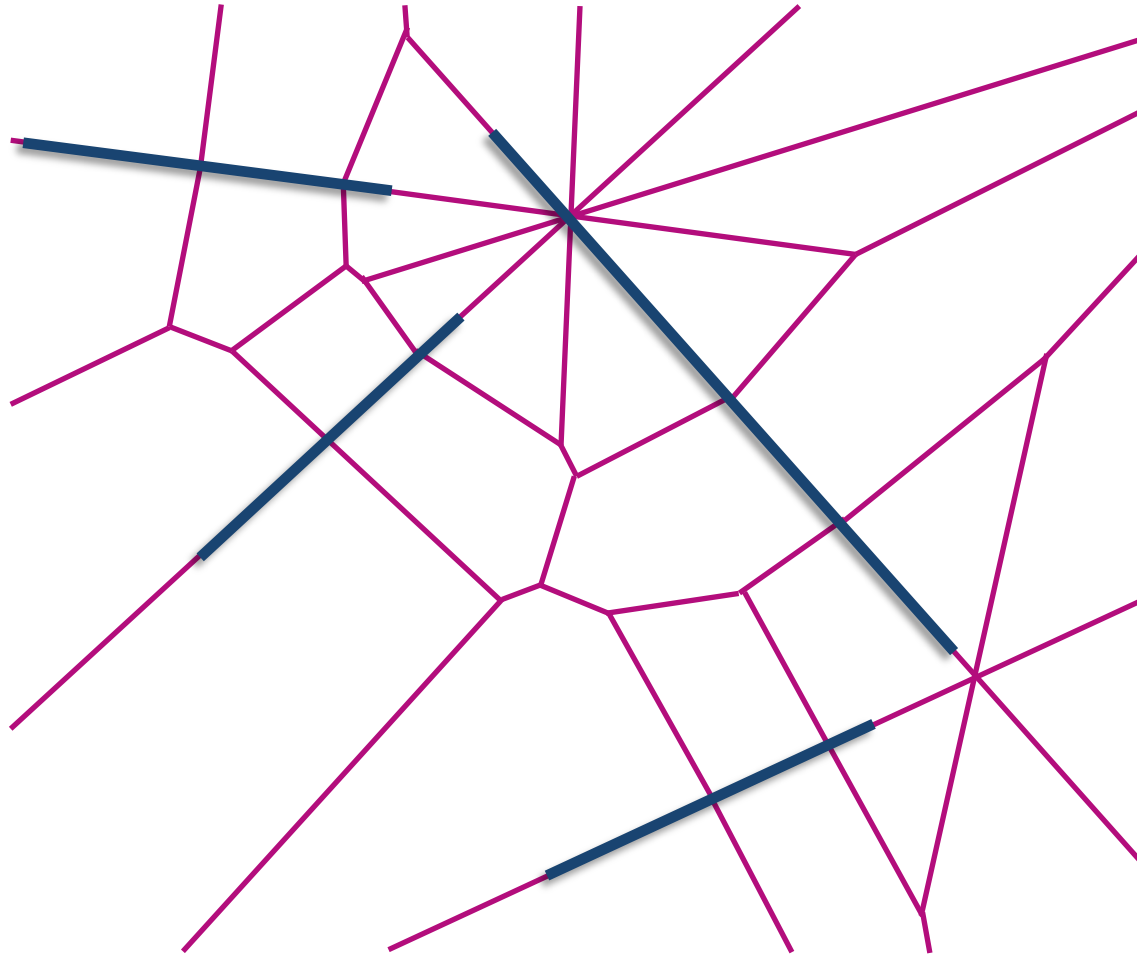
## Step 2: anchoring



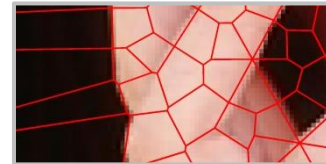
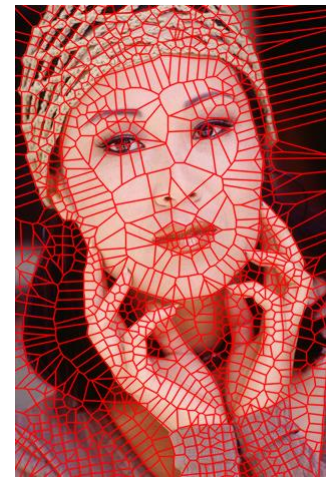
## Step 2: anchoring



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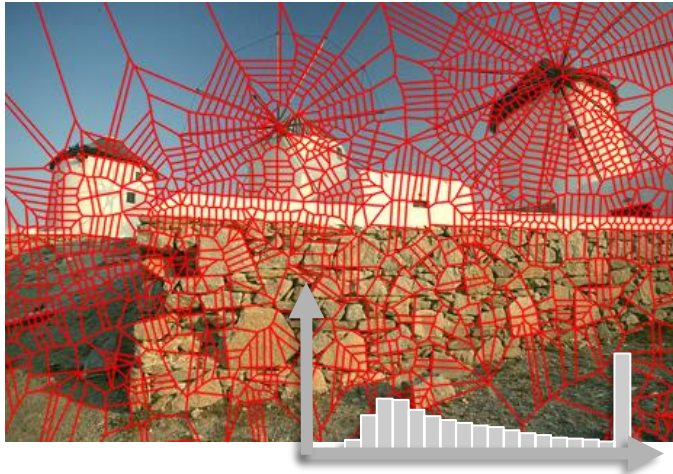


- line-segment
- Voronoi edge
- anchors
- junction-anchors

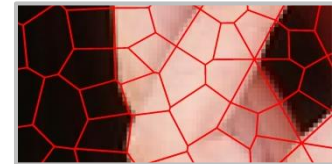
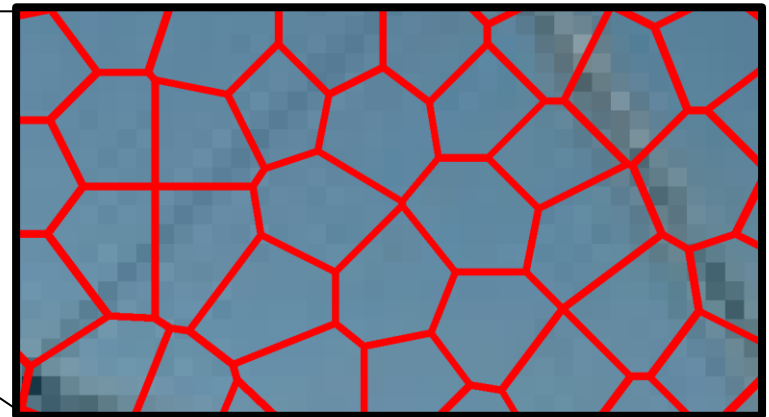
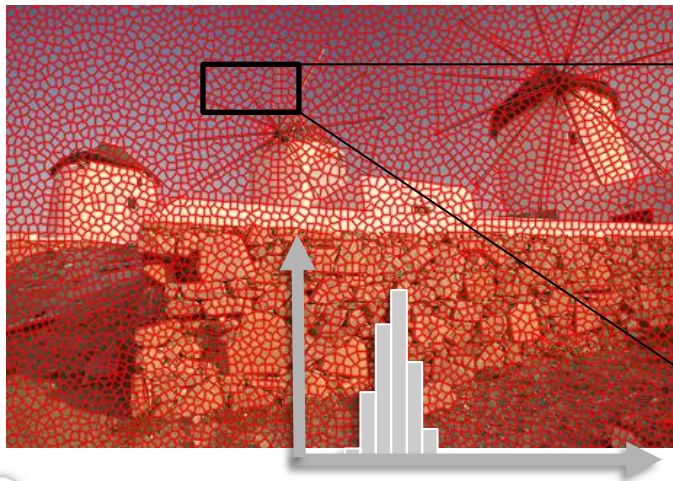




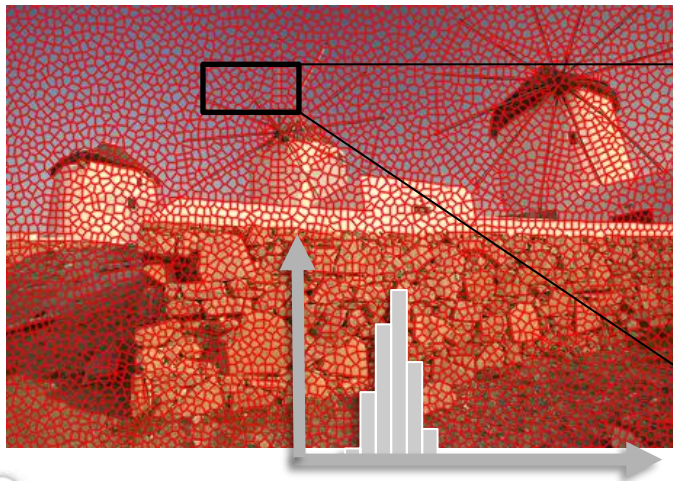
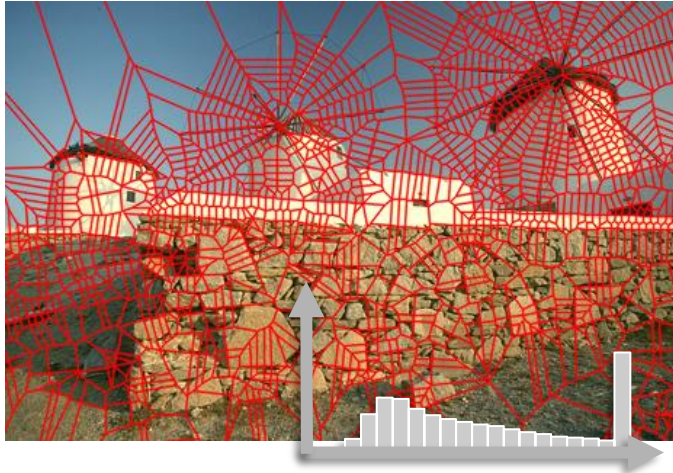
## Step 3: homogeneization



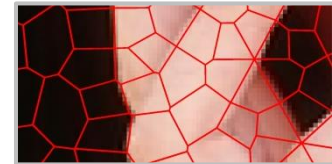
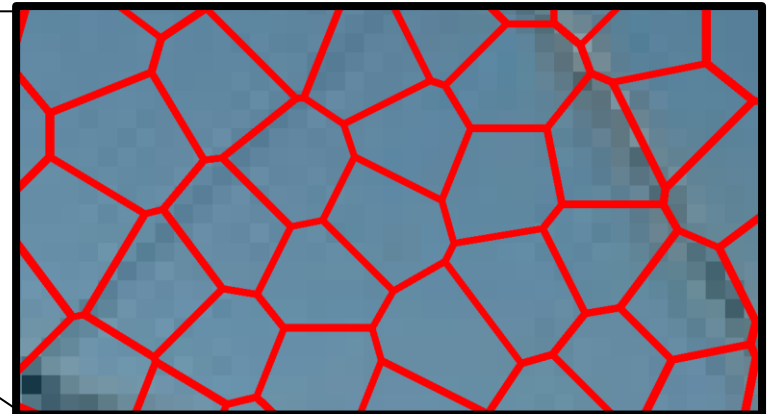
Poisson disk sampling



## Step 3: homogeneization

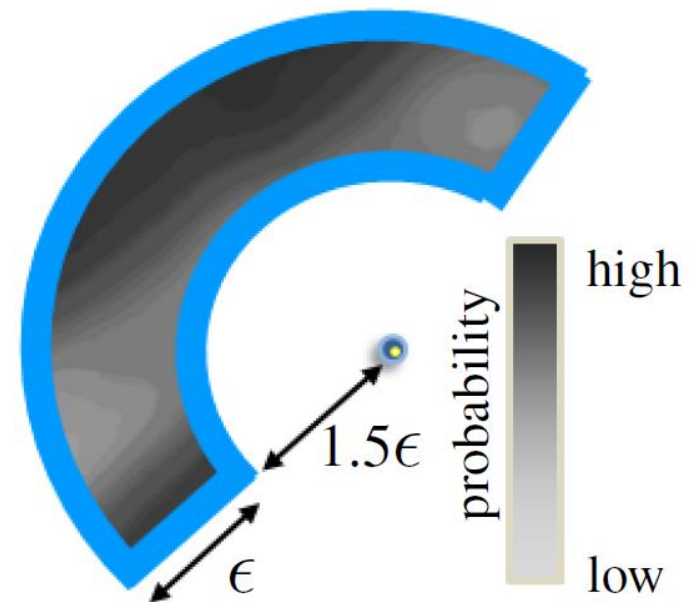
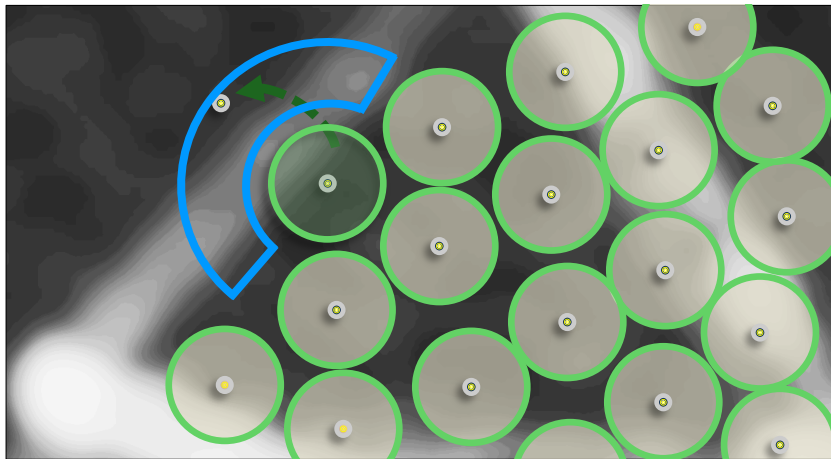
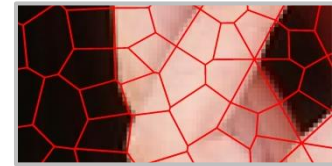
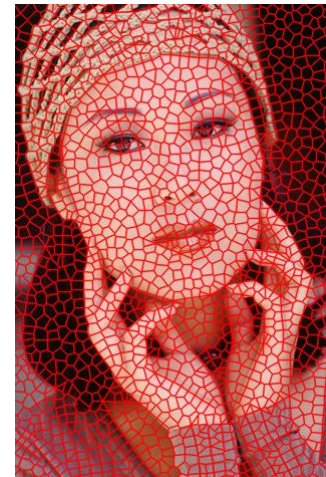


Poisson disk sampling guided by image gradient





## Step 3: homogeneization

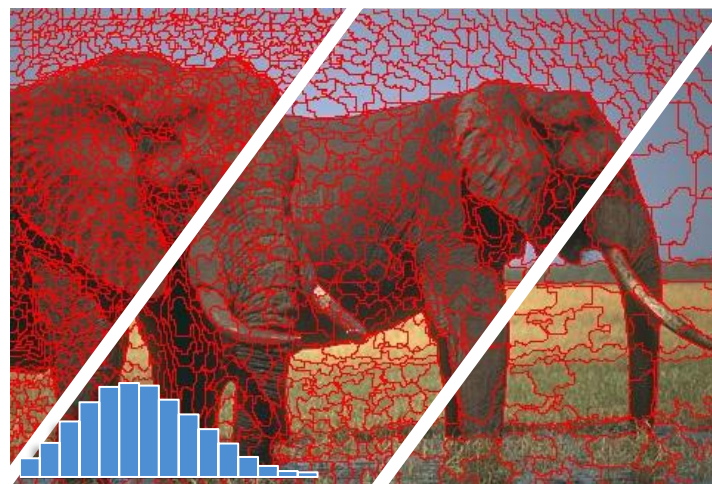


# Comparisons with superpixel methods

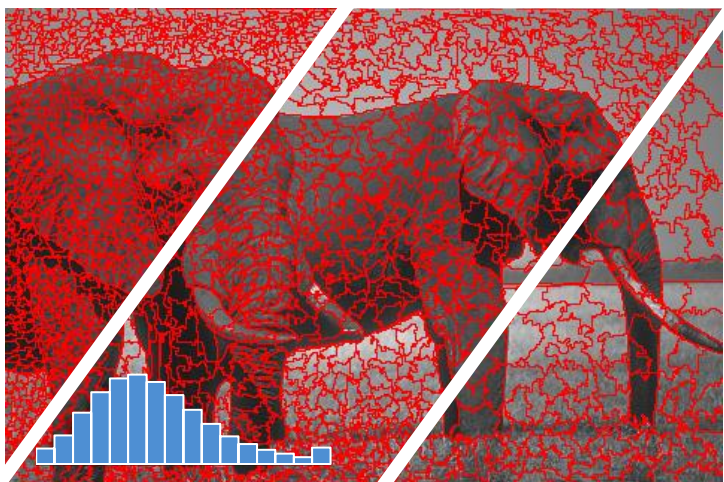
SLIC



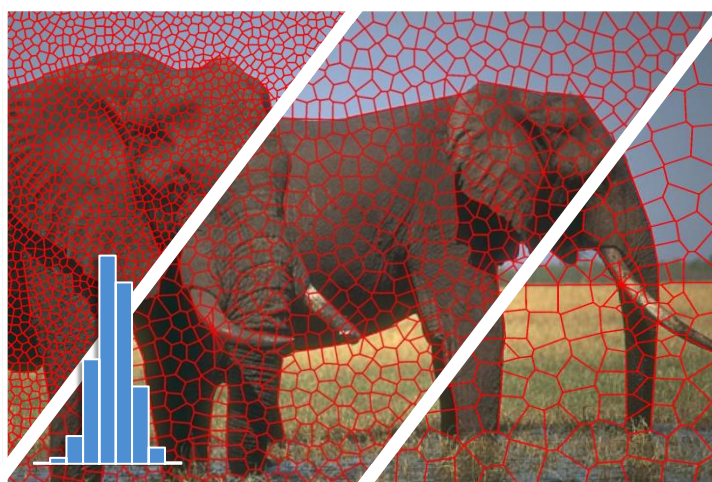
SEEDS



ERS



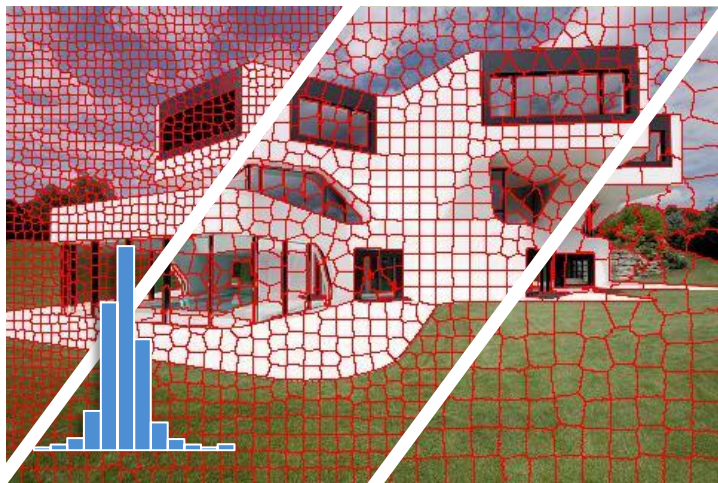
Voronoi



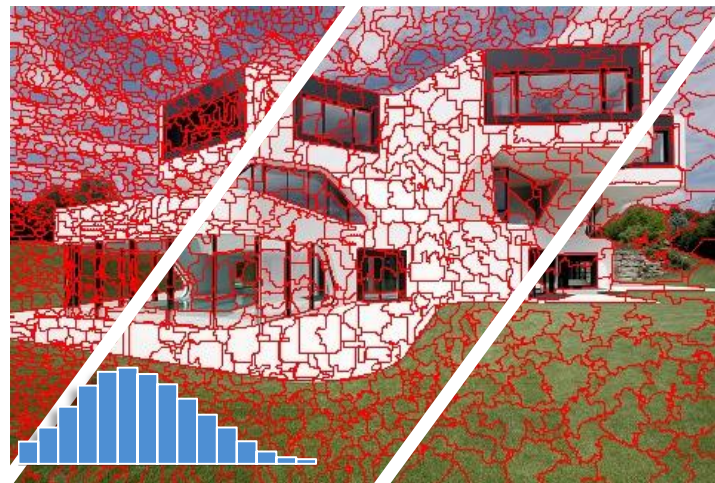


# Comparisons with superpixel methods

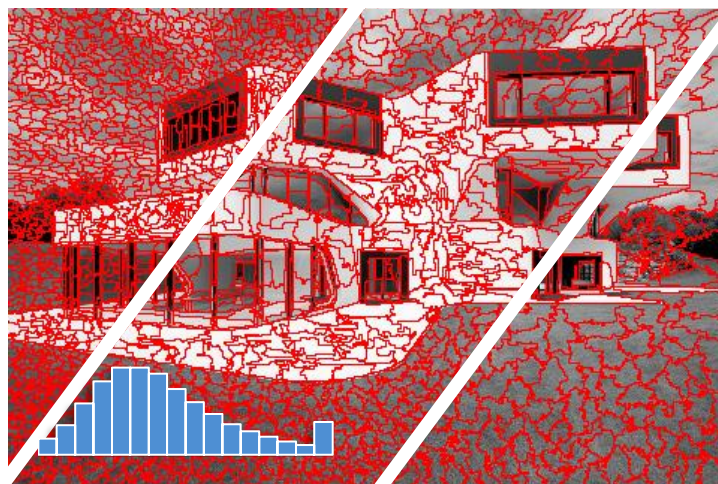
SLIC



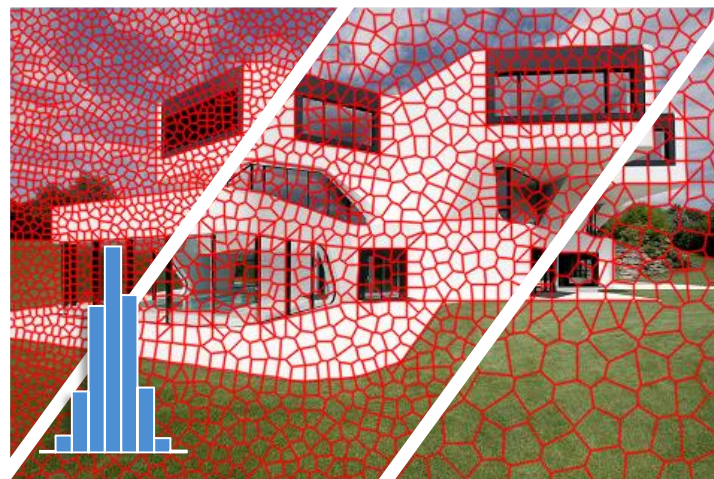
SEEDS



ERS



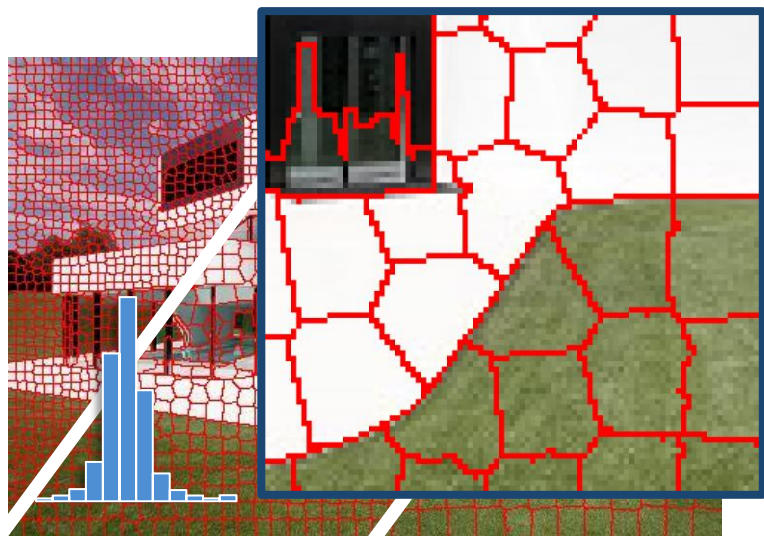
Voronoi



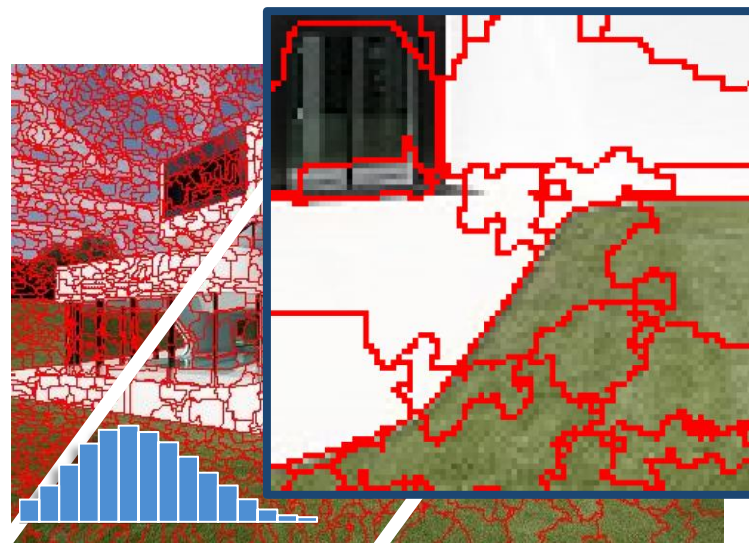


# Comparisons with superpixel methods

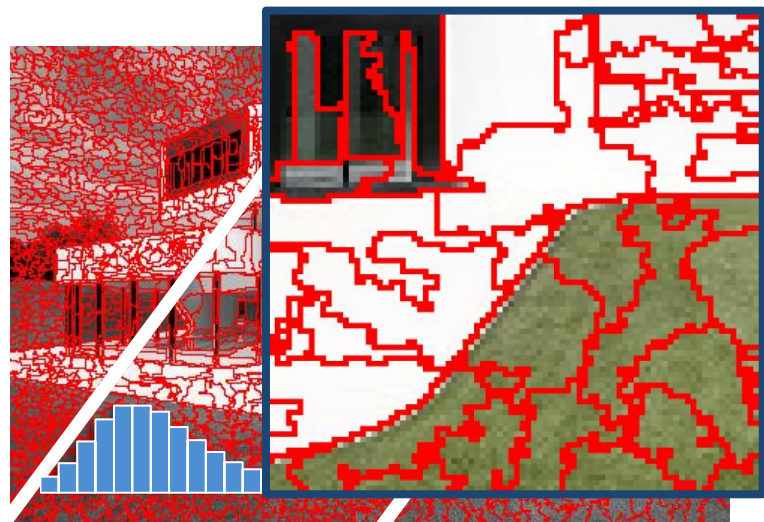
SLIC



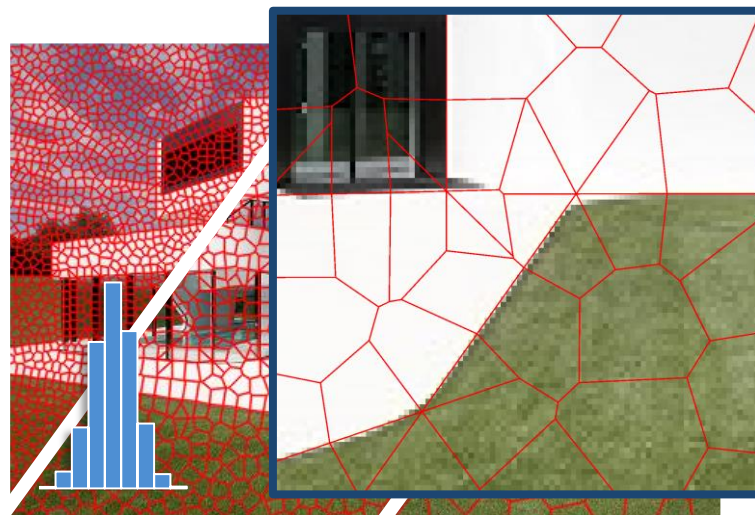
SEEDS



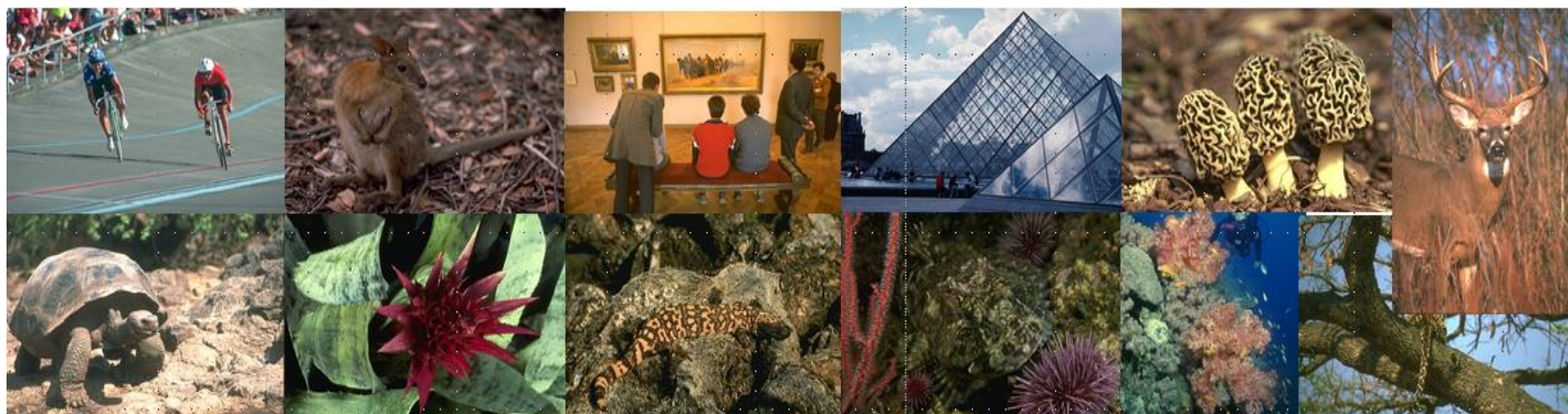
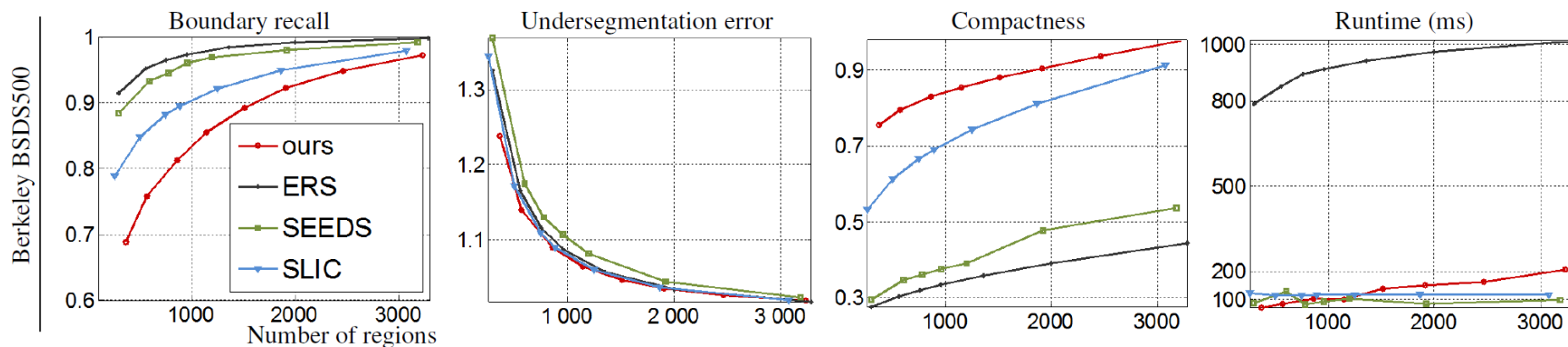
ERS



Voronoi



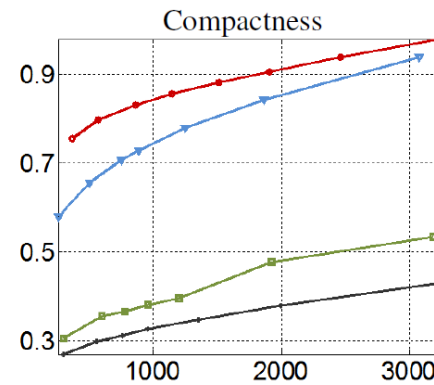
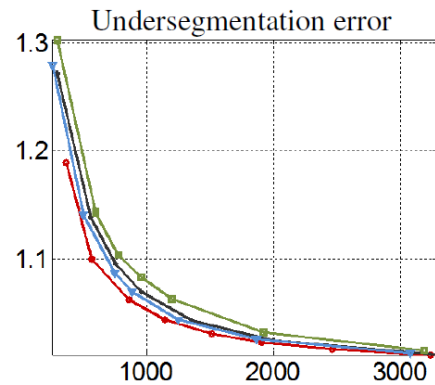
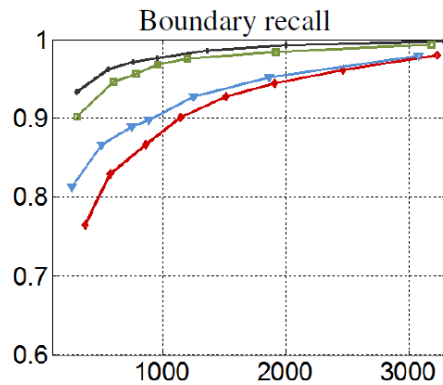
# Comparisons with superpixel methods





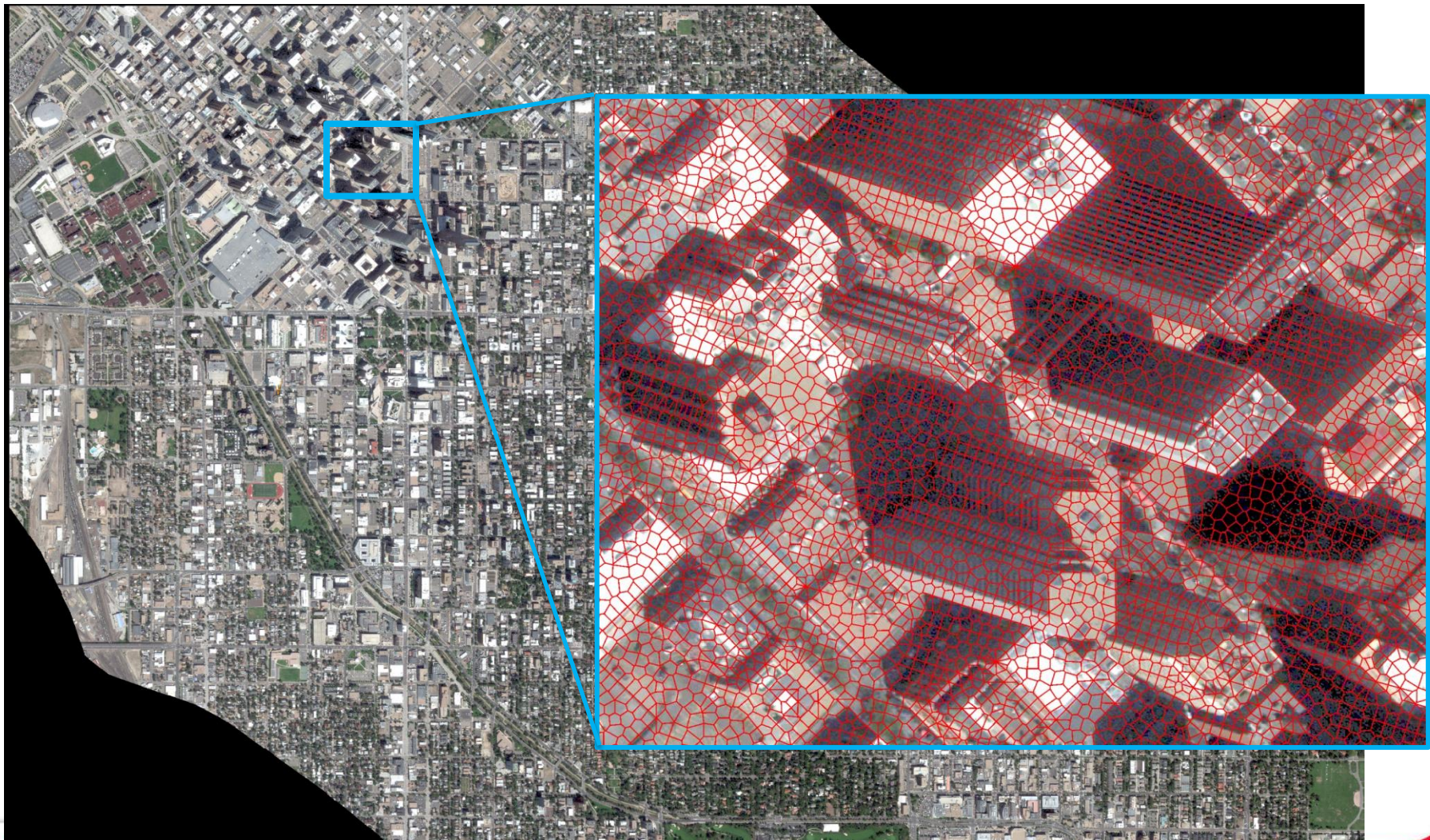
# Comparisons with superpixel methods

subset with man-made objects



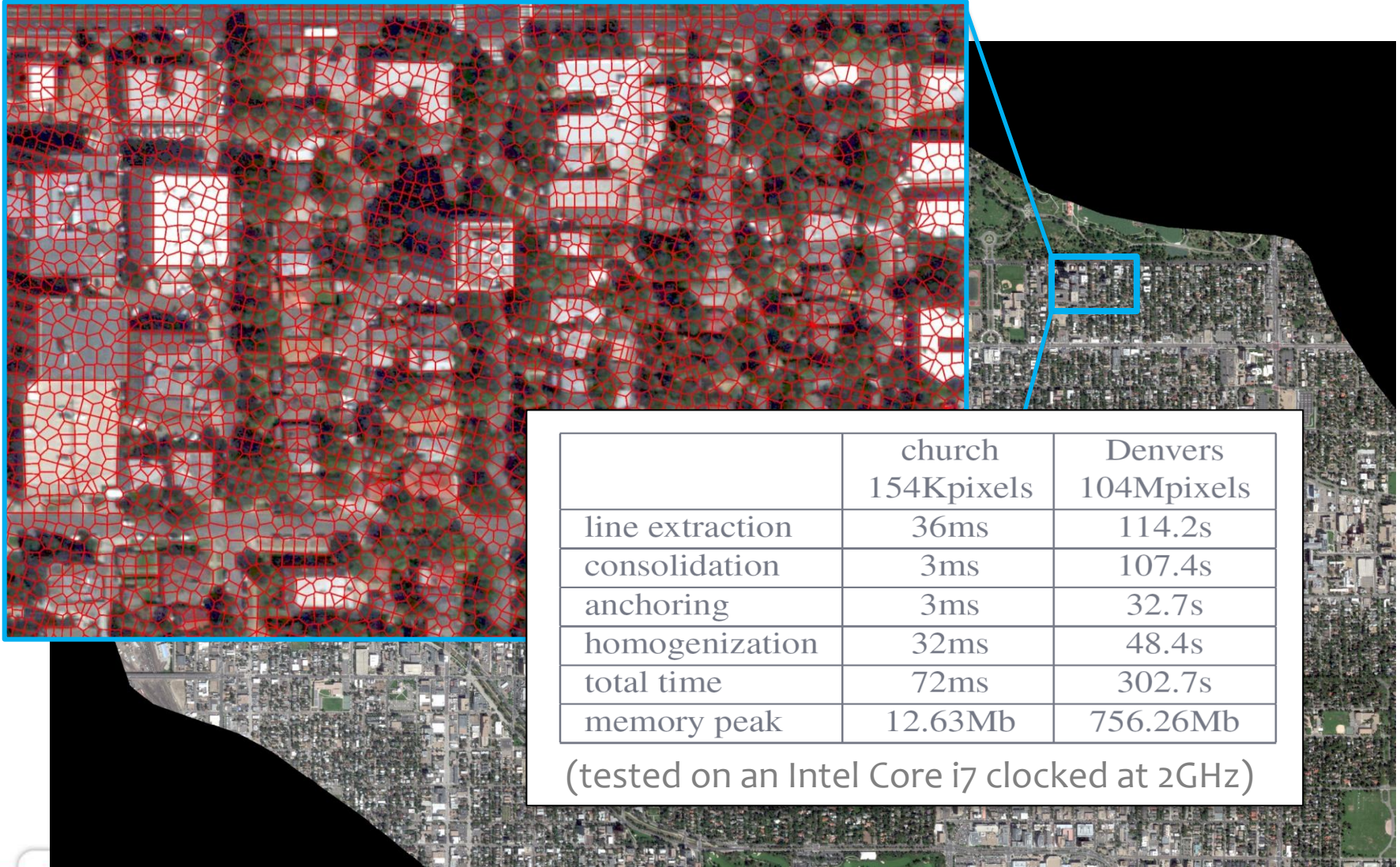


# Results on very big images





# Results on very big images



	church 154Kpixels	Denvers 104Mpixels
line extraction	36ms	114.2s
consolidation	3ms	107.4s
anchoring	3ms	32.7s
homogenization	32ms	48.4s
total time	72ms	302.7s
memory peak	12.63Mb	756.26Mb

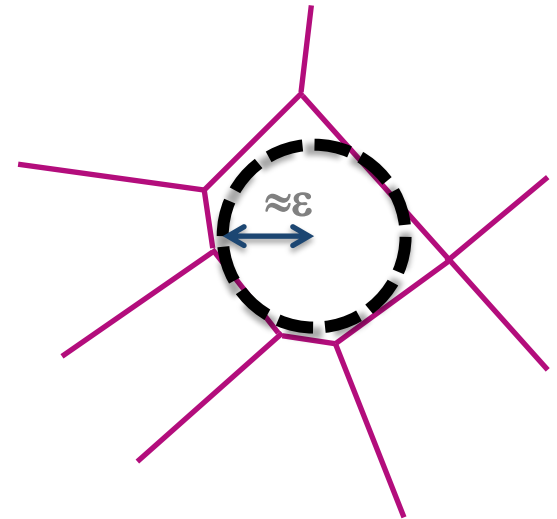
(tested on an Intel Core i7 clocked at 2GHz)

# Demo



One parameter

$\epsilon$  : expected width of the Voronoi cells



# Image partitioning with a kinetic data-structure



[Bauchet and Lafarge, KIPPI: Kinetic Polygonal Partitioning of Images, CVPR 2018]



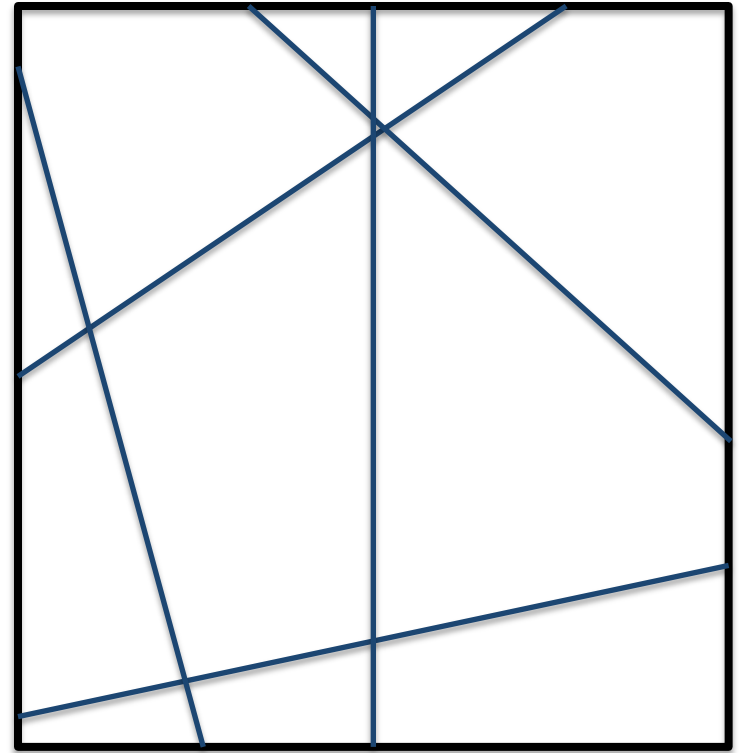
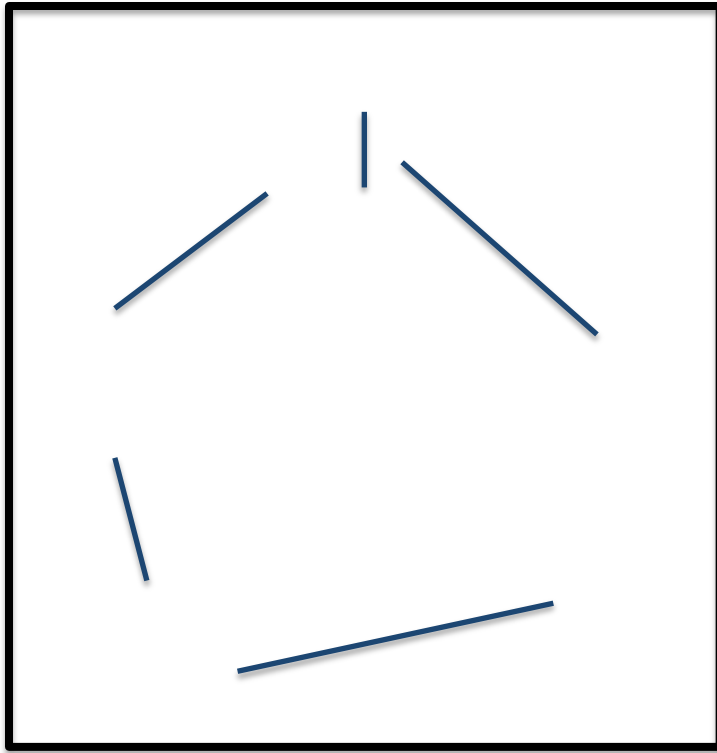
# Cells with heterogeneous size



Less cells but  
more meaningful



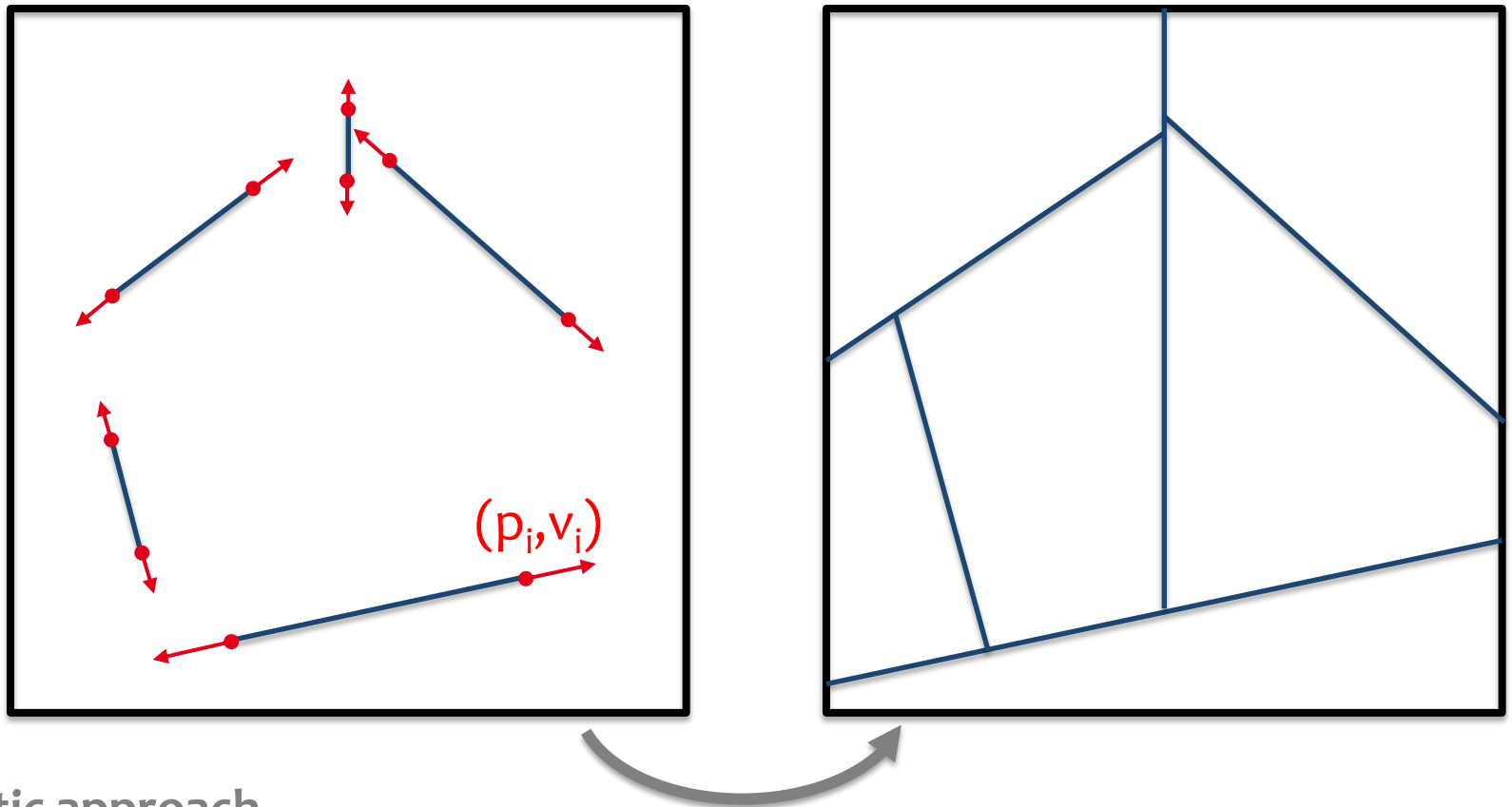
# Polygonal partitioning as space cutting



High algorithmic complexity  
Generate many cells



# Polygonal partitioning as space cutting



## Kinetic approach

Shape intersection not greedy anymore, but based on shape proximity

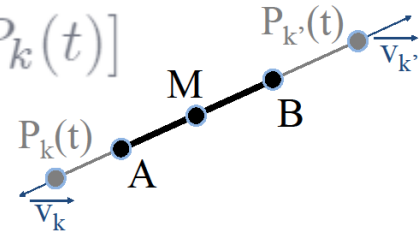
# kinetic formulation

Kinetic data-structure: a dynamic planar graph  $G_t = (V_t, E_t)$

# kinetic formulation

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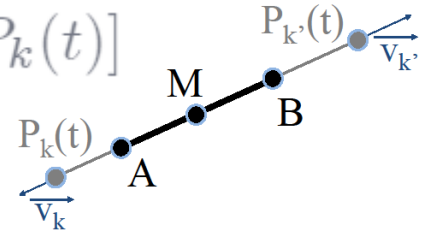
Primitive: a dynamic segment  $s_k(t) = [MP_k(t)]$   
with  $P_k(t) = A + \vec{v}_k \times t$



# kinetic formulation

Kinetic data-structure: a dynamic planar graph  $G_t = (V_t, E_t)$

Primitive: a dynamic segment  $s_k(t) = [MP_k(t)]$   
with  $P_k(t) = A + \vec{v}_k \times t$



## Certificate

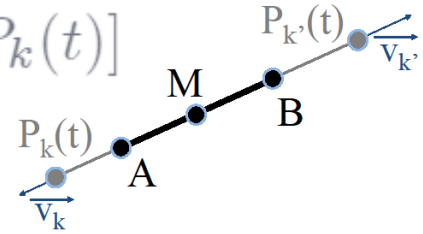
Function testing the intersection of primitive  $i$  with other primitives at time  $t$

$$C_i(t) = \prod_{\substack{j=1 \\ j \neq i}}^N Pr_{i,j}(t) \quad \text{with} \quad Pr_{i,j}(t) = \begin{cases} 1 & \text{if } d(P_i(t), s_j(t)) > 0 \\ 0 & \text{otherwise} \end{cases}$$

# kinetic formulation

Kinetic data-structure: a dynamic planar graph  $G_t = (V_t, E_t)$

Primitive: a dynamic segment  $s_k(t) = [MP_k(t)]$   
with  $P_k(t) = A + \vec{v}_k \times t$



## Certificate

Function testing the intersection of primitive  $i$  with other primitives at time  $t$

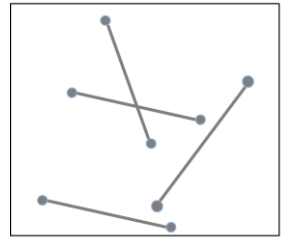
$$C_i(t) = \prod_{\substack{j=1 \\ j \neq i}}^N Pr_{i,j}(t) \quad \text{with} \quad Pr_{i,j}(t) = \begin{cases} 1 & \text{if } d(P_i(t), s_j(t)) > 0 \\ 0 & \text{otherwise} \end{cases}$$

## Queue of events

List of times  $t$  indicating when a certificate is equal to 0  
(ranked by ascending order)

# kinetic formulation

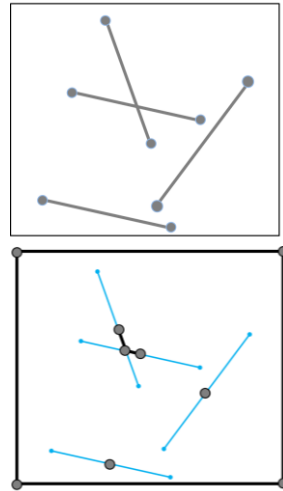
Algorithm



# kinetic formulation

## Algorithm

- Initialize the data-structure by inserting points where two line-segments intersect

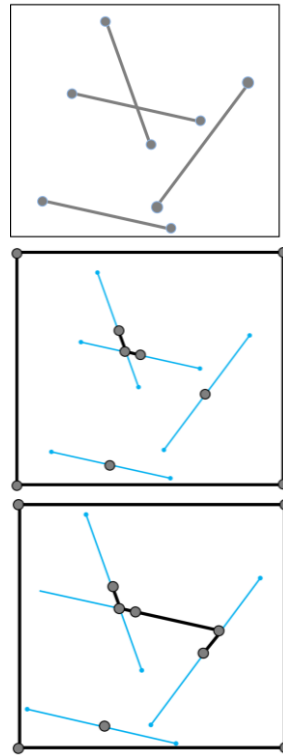




# kinetic formulation

## Algorithm

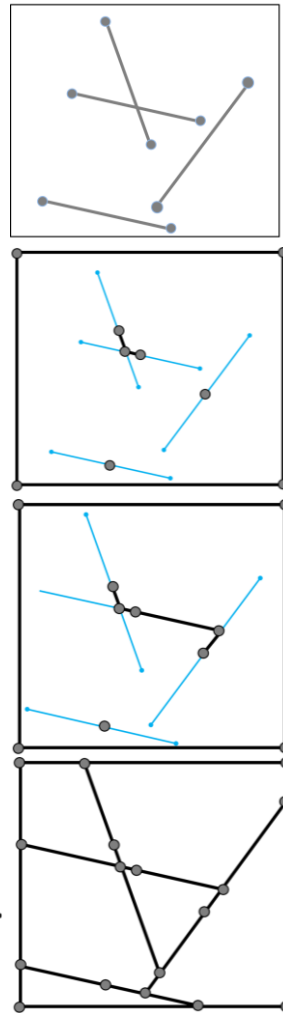
- Initialize the data-structure by inserting points where two line-segments intersect
- For each event of the queue,
  - update the data-structure
  - test the deactivation of the primitive
  - update the queue of events



# kinetic formulation

## Algorithm

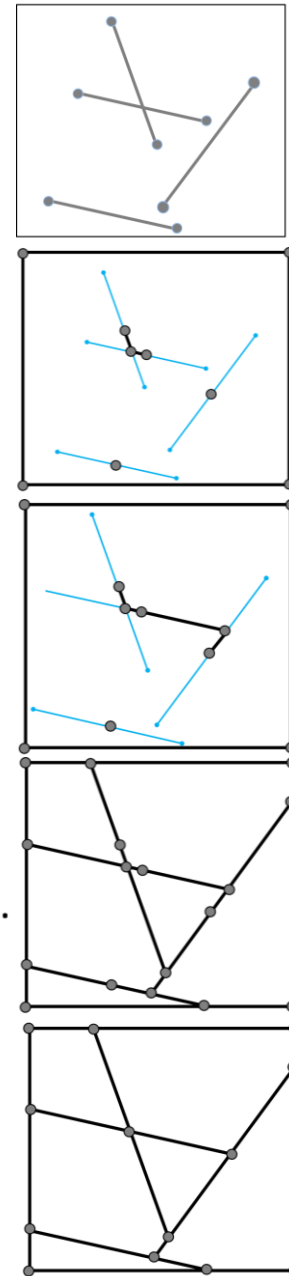
- Initialize the data-structure by inserting points where two line-segments intersect
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  - update the data-structure
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  - update the queue of events



# kinetic formulation

## Algorithm

- Initialize the data-structure by inserting points where two line-segments intersect
- For each event of the queue,
  - update the data-structure
  - test the deactivation of the primitive
  - update the queue of events
- Finalization



# kinetic formulation

## Flexibility

- Use the confidence for each detected line-segments to better adapt the partition (increase speed of good line-segments)

# kinetic formulation

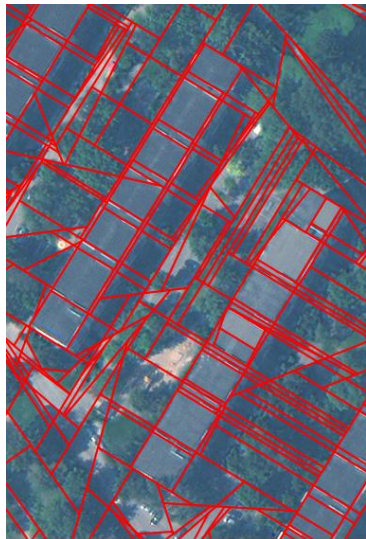
## Flexibility

- Use the confidence for each detected line-segments to better adapt the partition (increase speed of good line-segments)
- Policy for deactivating a primitive
  - Impose a maximal number of intersection  $K$  per primitive
  - Check the alignment of a potential prolongation with image gradients

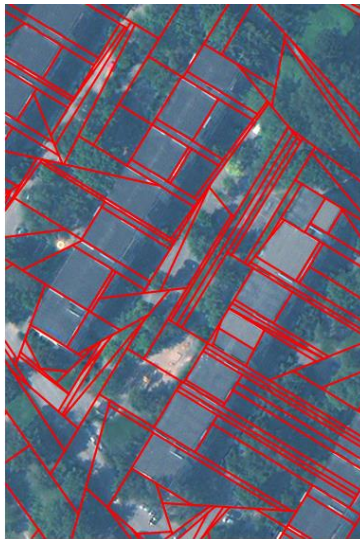
# kinetic formulation

## Flexibility

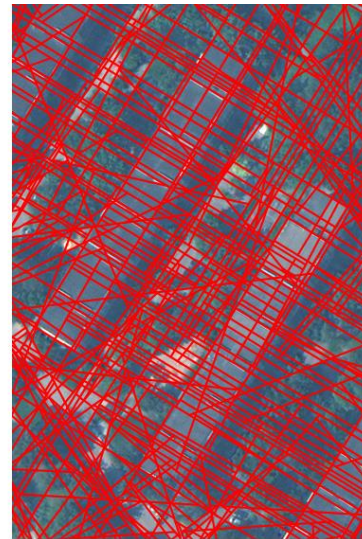
- Use the confidence for each detected line-segments to better adapt the partition (increase speed of good line-segments)
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  - Impose a maximal number of intersection  $K$  per primitive
  - Check the alignment of a potential prolongation with image gradients



$K=1$  + gradients



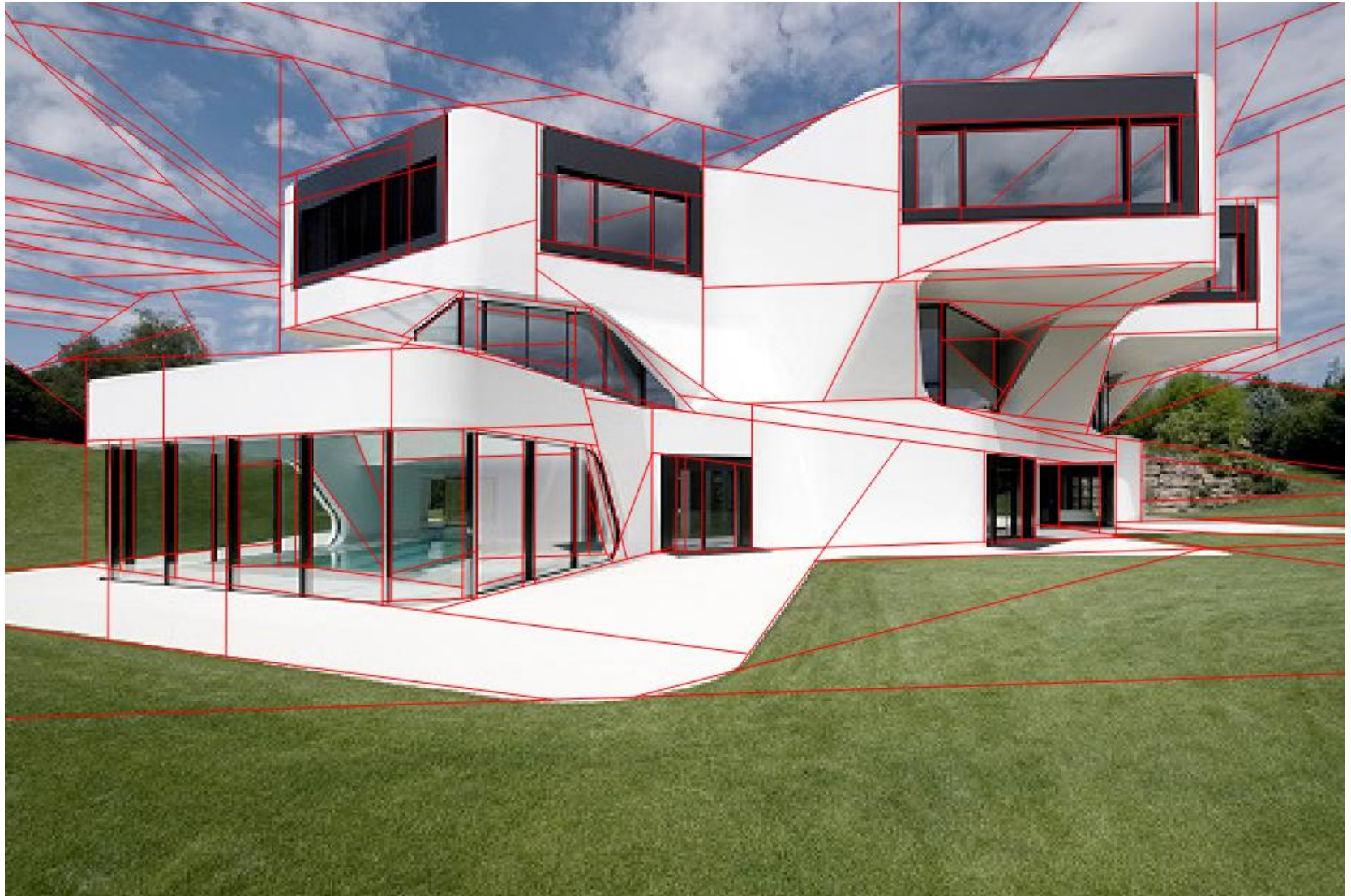
$K=1$   
(motorcycle graph)



$K=20$



# kinetic formulation

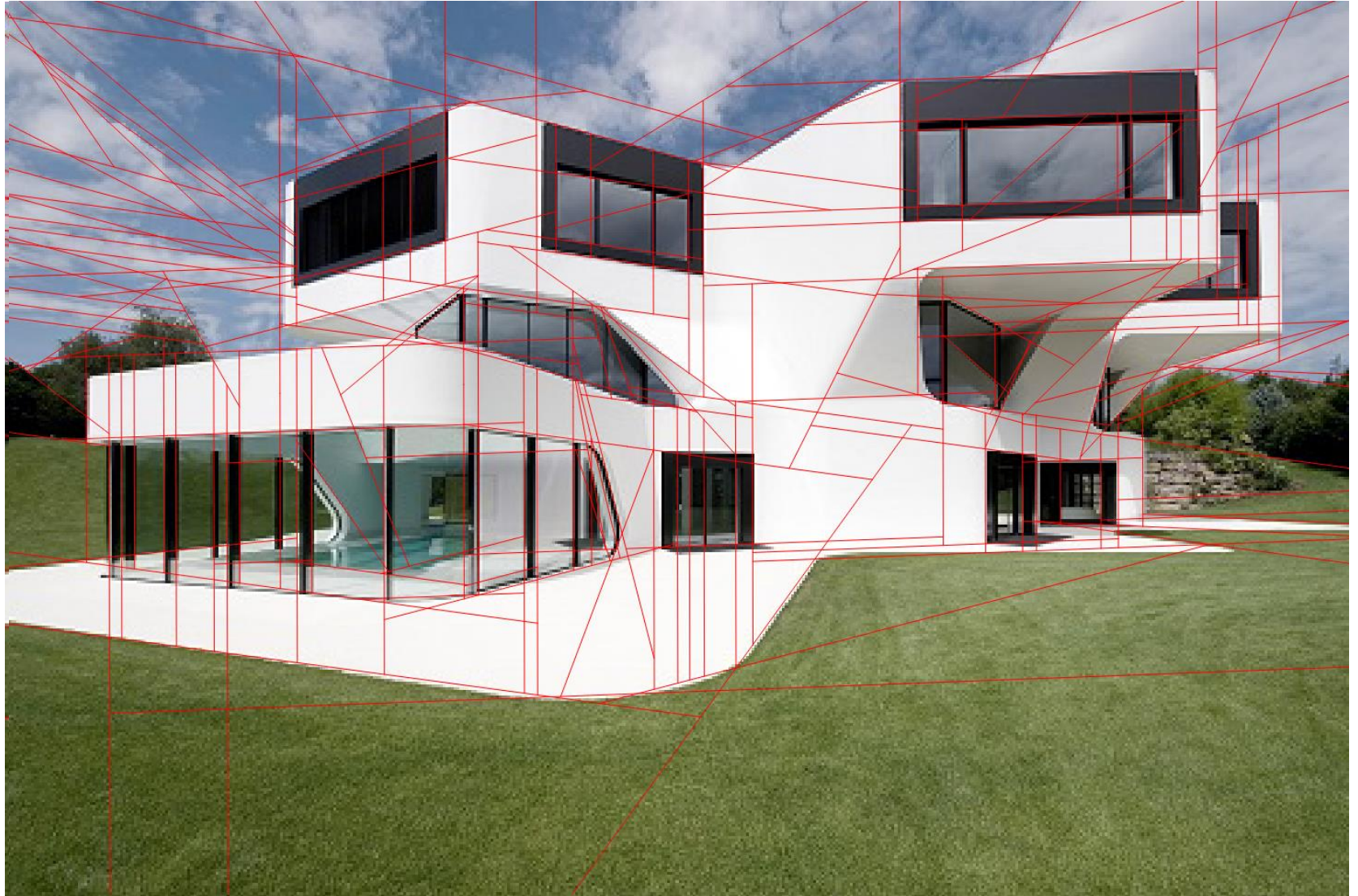


$K=1$

226 cells



# kinetic formulation

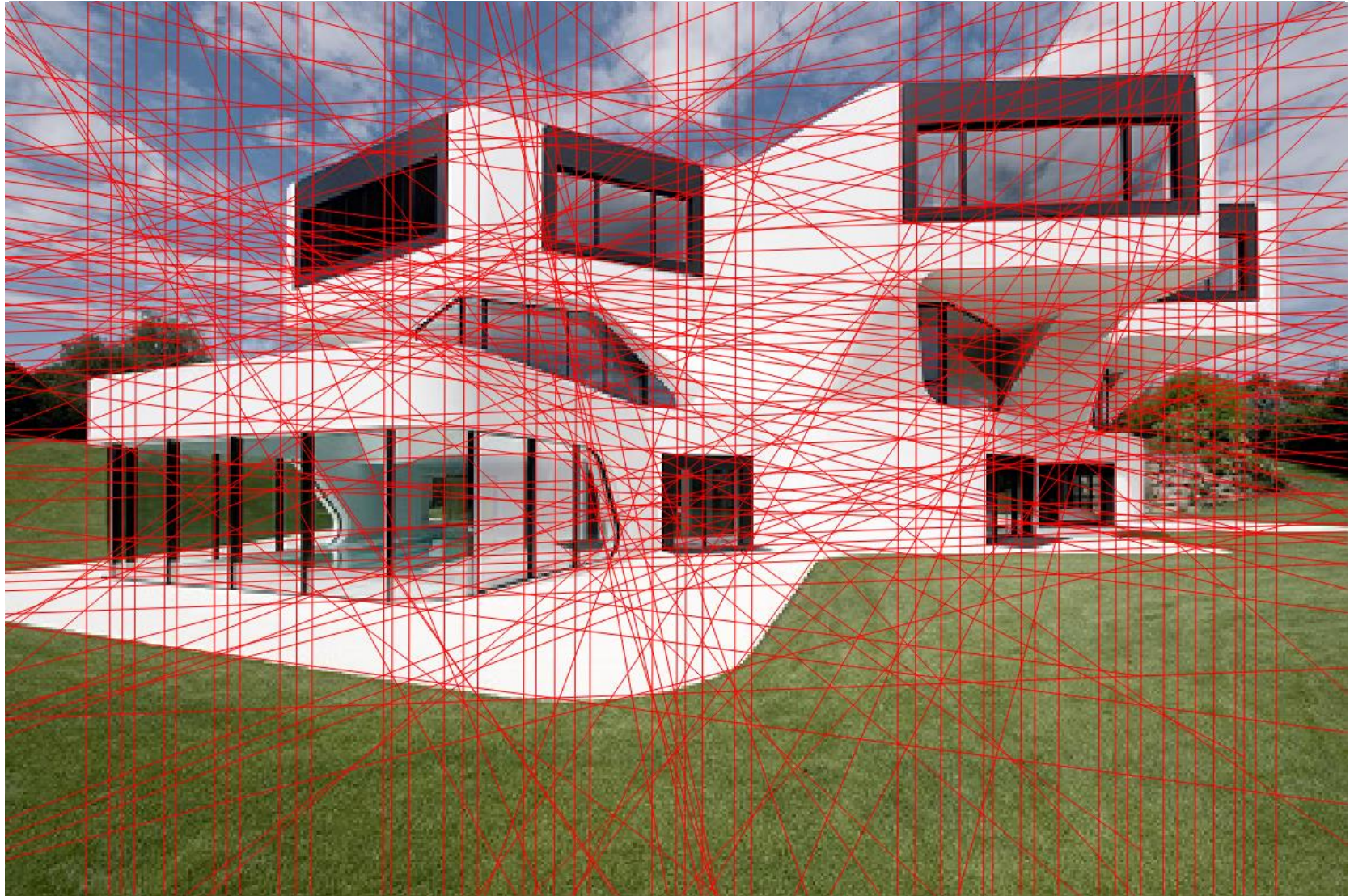


$K=2$

593 cells



# kinetic formulation

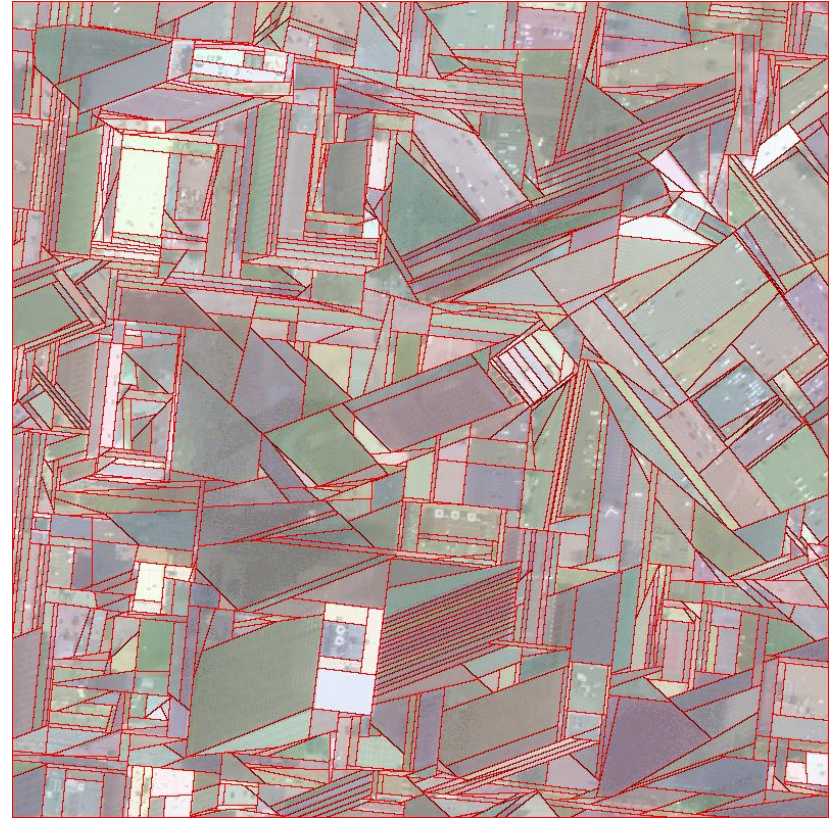


$K = \infty$

9.8k cells



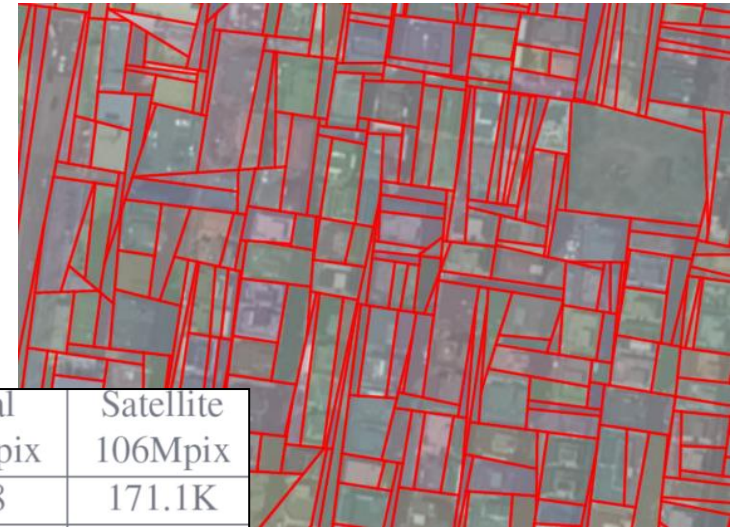
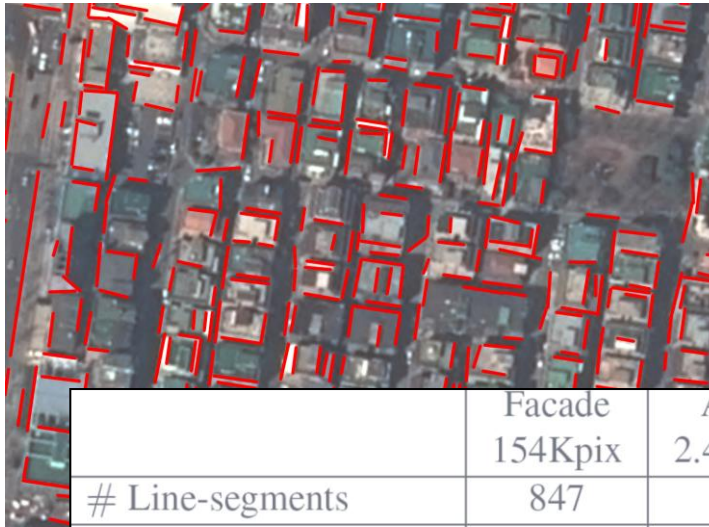
# Results on satellite images





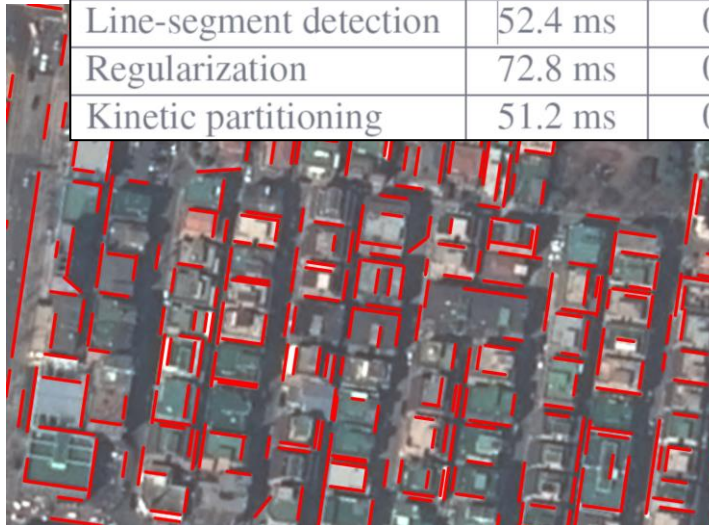
# Results on satellite images

Without line-segment  
regularization

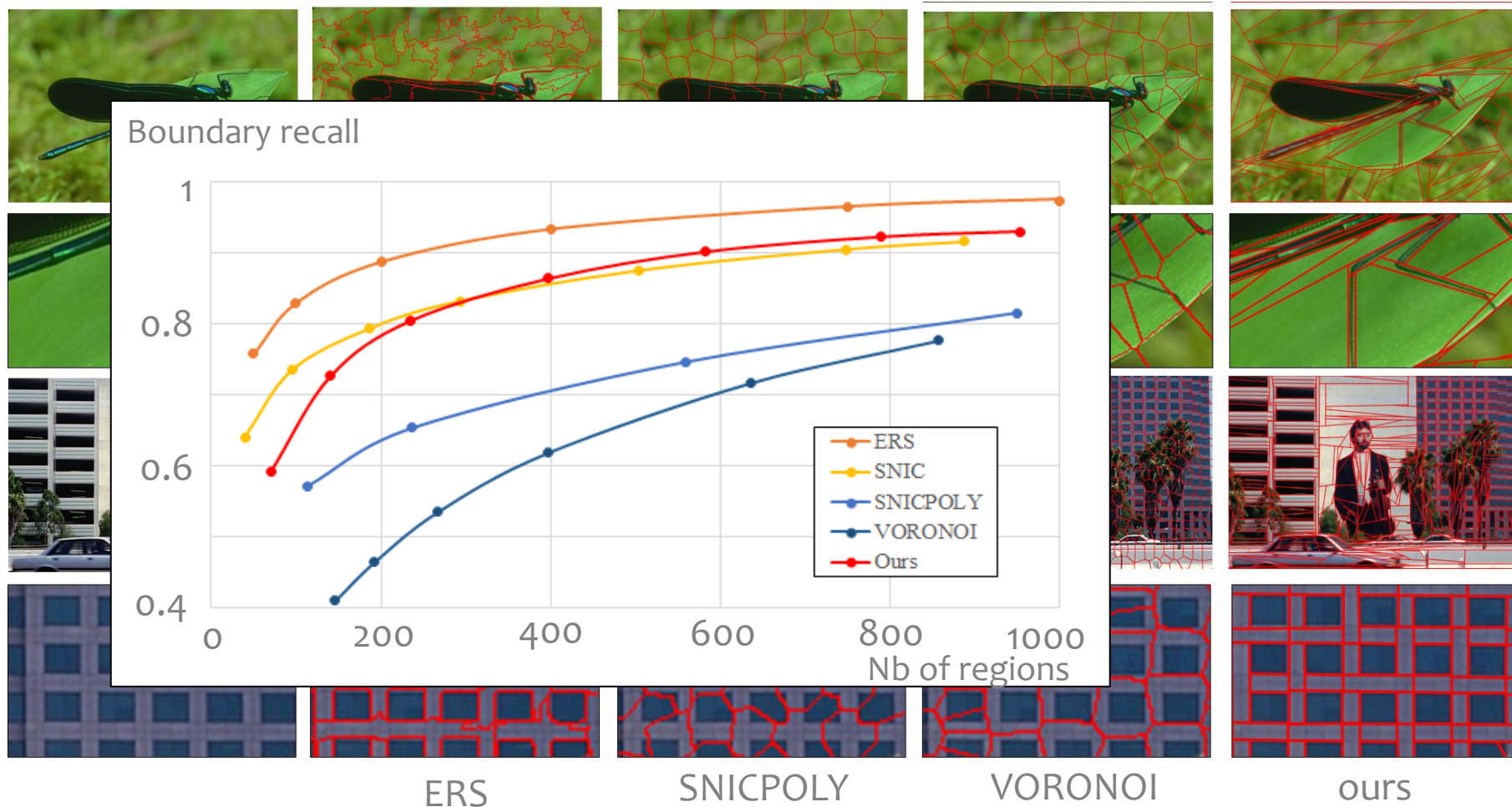


	Facade 154Kpix	Aerial 2.46Mpix	Satellite 106Mpix
# Line-segments	847	3178	171.1K
# Output polygons	530	2488	124.5K
Line-segment detection	52.4 ms	0.59 s	70.7 s
Regularization	72.8 ms	0.35 s	654.5 s
Kinetic partitioning	51.2 ms	0.23 s	45.1 s

With line-segment  
regularization

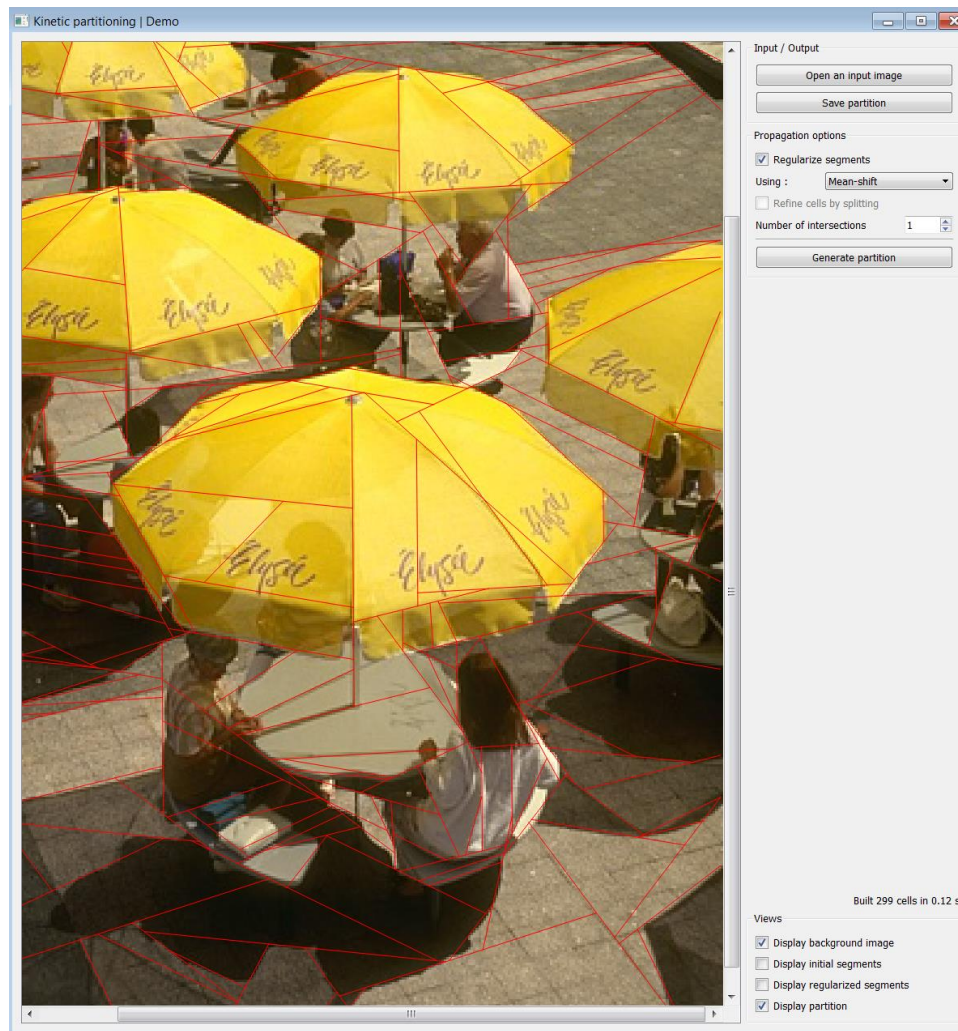


# Comparisons with over-segmentation methods





# Demo





**Application to object contouring**

# Object contouring

Label each cell as inside or outside the objects of interest

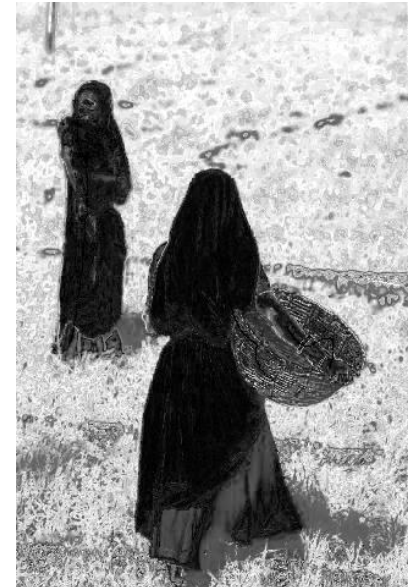
# Object contouring

Label each cell as inside or outside the objects of interest

Graph-cut

Data term: distance to a saliency map

$$H(i|m_f) = \frac{\min_{j \in S_{m_f}} \|I(i) - \hat{I}(j)\|_2^2}{\min_{j \in S_0} \|I(i) - \hat{I}(j)\|_2^2 + \min_{j \in S_1} \|I(i) - \hat{I}(j)\|_2^2}$$





# Object contouring

Label each cell as inside or outside the objects of interest

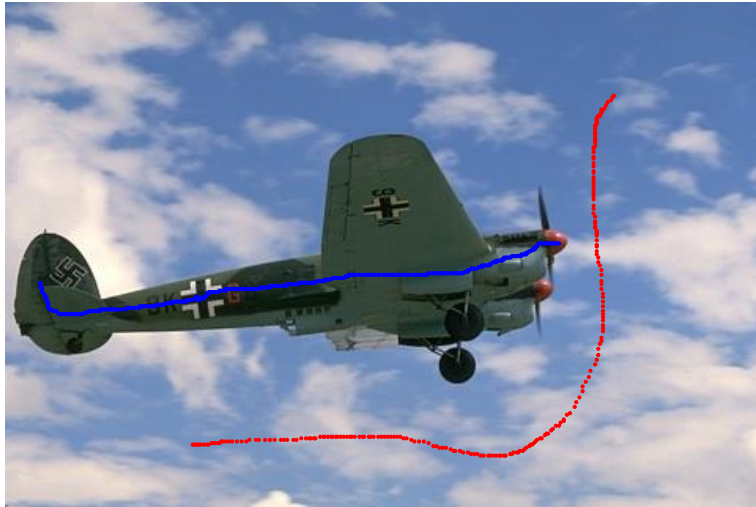
Graph-cut

Data term: distance to a saliency map

Potential: Potts model



# Object contouring



with Voronoi

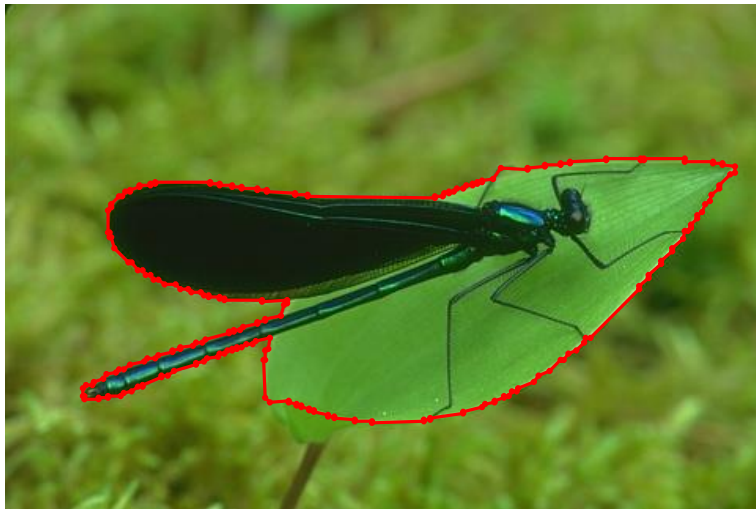


ours

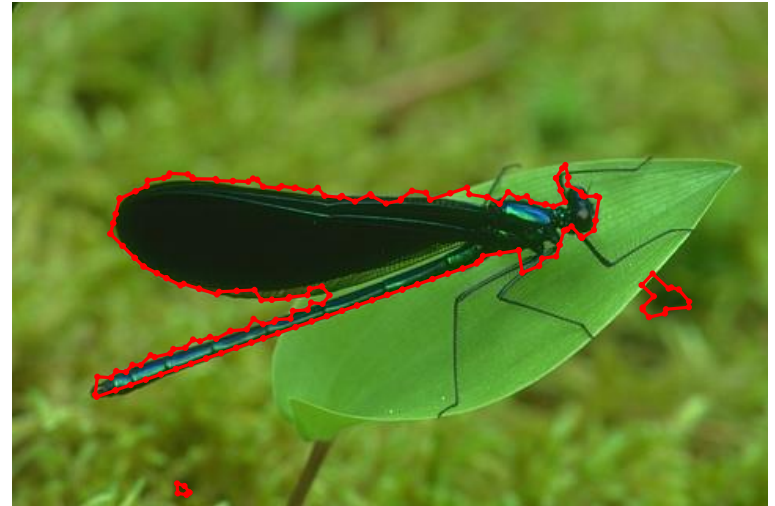
Grabcut + vectorization

# Object contouring

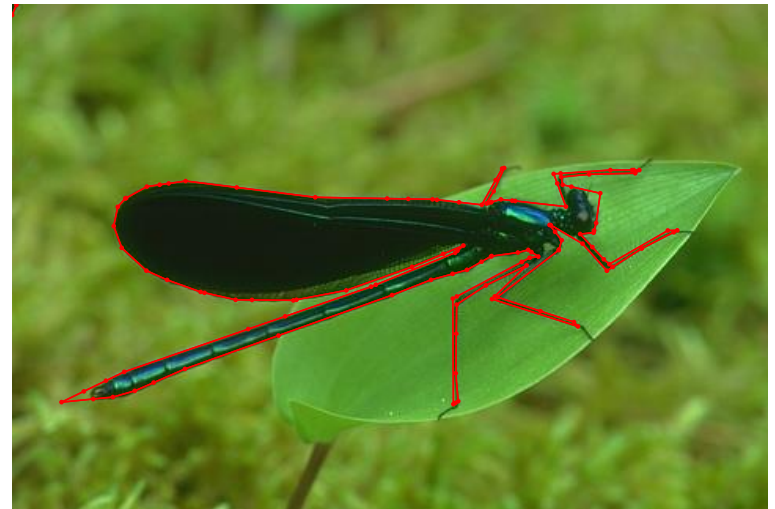
Grabcut + vectorization



with Voronoi



ours





# Object contouring



with Voronoi



Grabcut + vectorization



ours



# Object contouring



Grabcut + vectorization



with Voronoi



ours





# Object contouring



Nb of edges: 130



Nb of edges: 308



Nb of edges: 476

# Application to city modeling from satellite images



[Duan and Lafarge, Towards large-scale city reconstruction from satellites, ECCV 2016]

# City modeling from satellite images

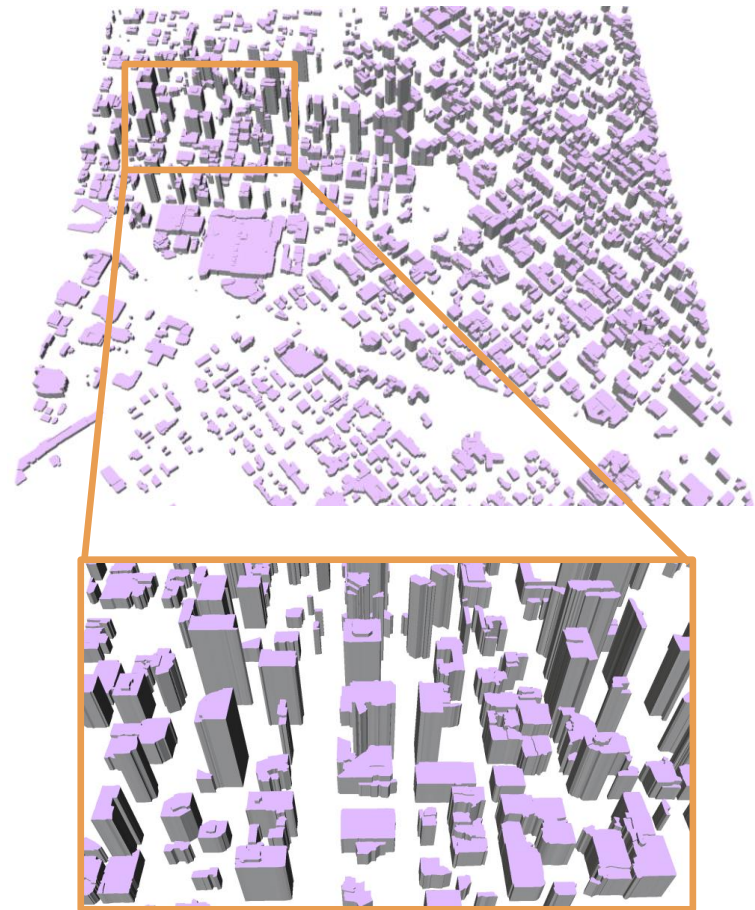
Input

Stereo pair of satellite images



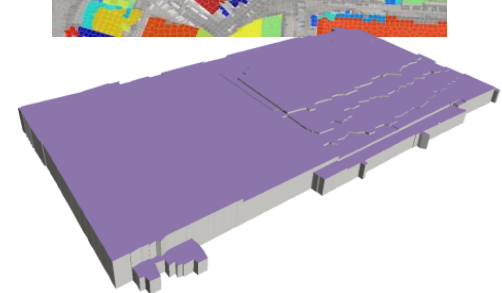
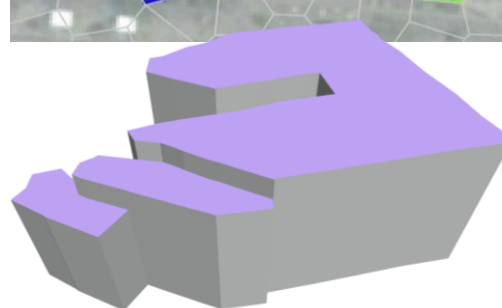
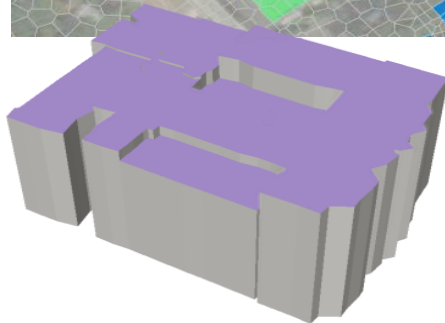
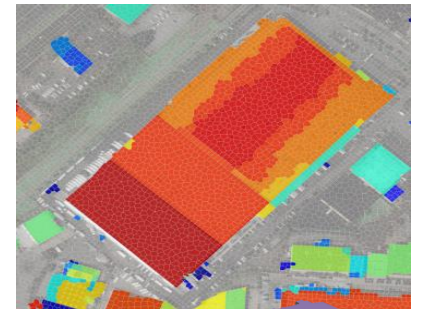
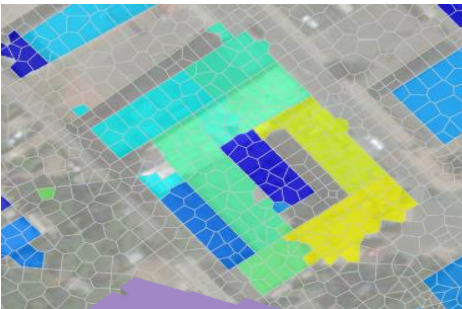
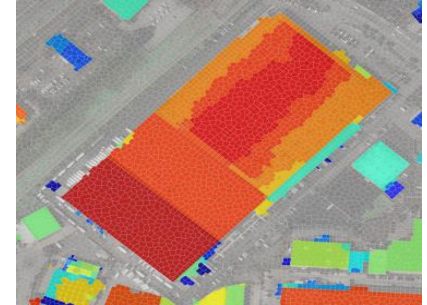
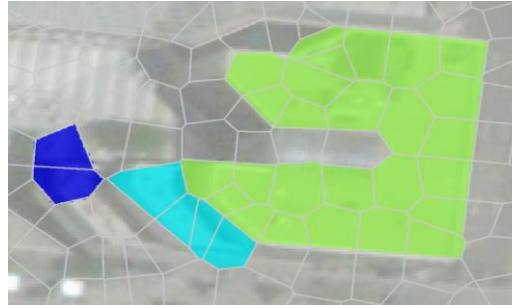
Output

3D model at LOD1





# City modeling from satellite images

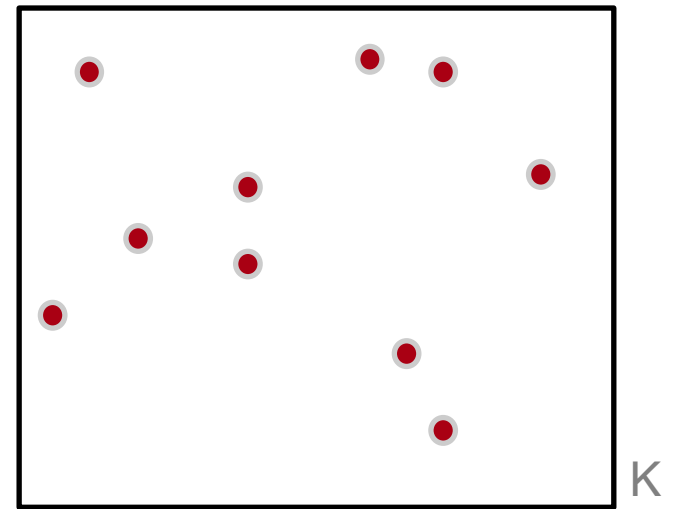


# 2

## Delaunay point processes

# Background on point process

Random configurations of points distributed in a bounded domain  $K$



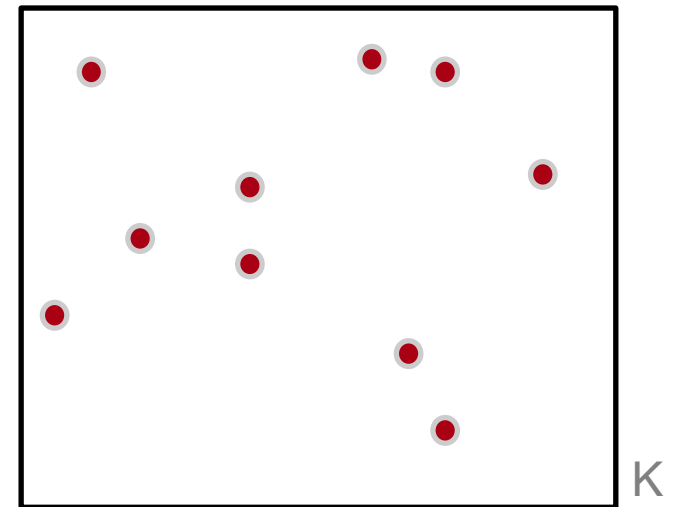


# Background on point process

Random configurations of points distributed in a bounded domain  $K$

Interesting characteristics

- #points is a random variable

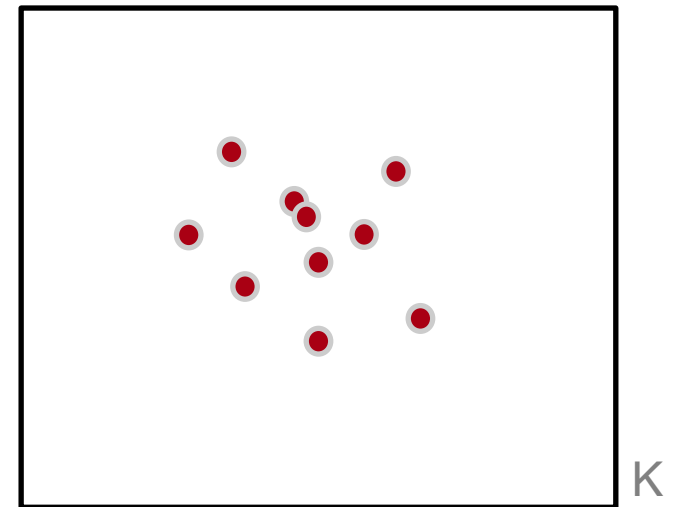
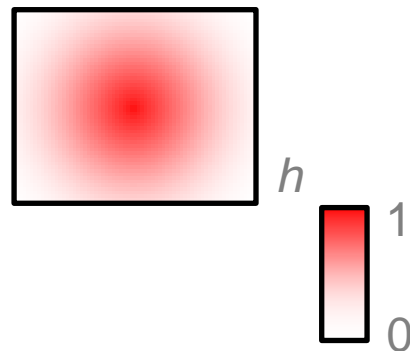


# Background on point process

Random configurations of points distributed in a bounded domain  $K$

Interesting characteristics

- #points is a random variable
- can be guided by a density  $h$

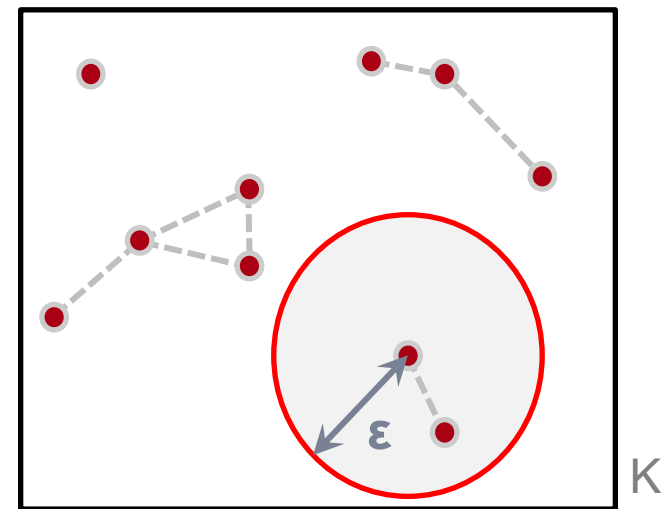


# Background on point process

Random configurations of points distributed in a bounded domain  $K$

Interesting characteristics

- #points is a random variable
- can be guided by a density  $h$
- **with spatial interactions**



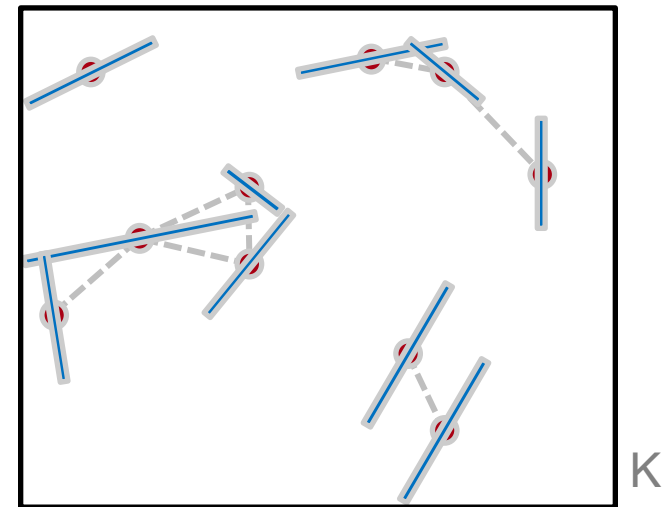


# Background on point process

Random configurations of points distributed in a bounded domain  $K$

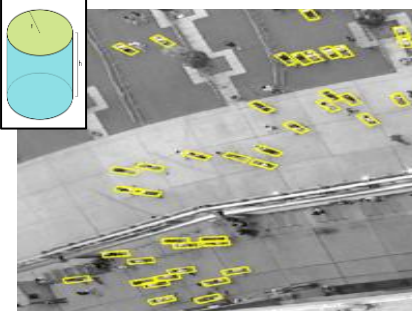
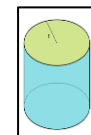
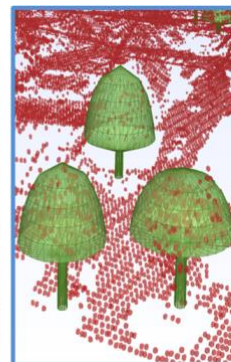
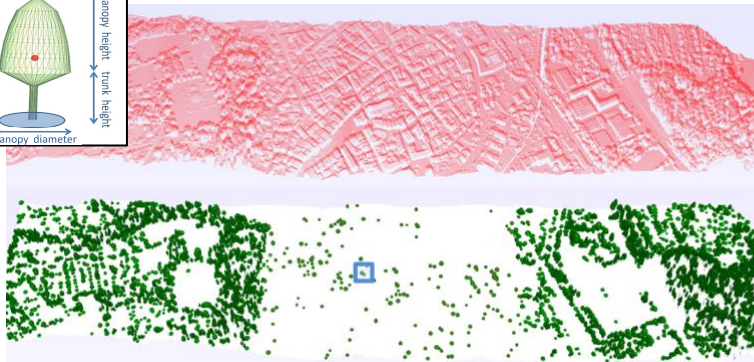
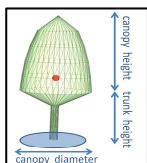
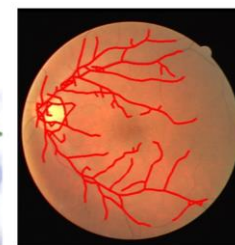
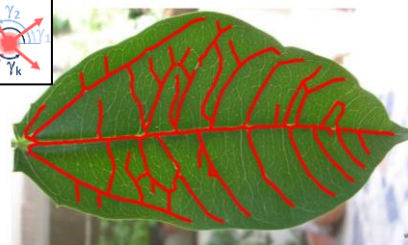
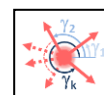
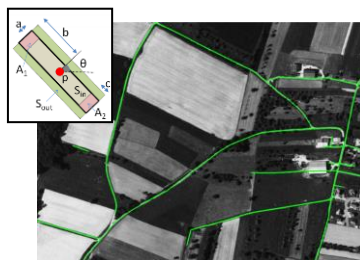
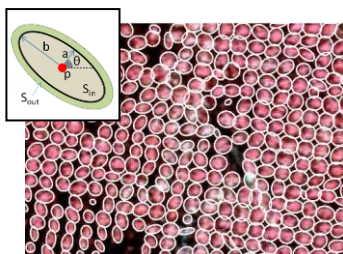
Interesting characteristics

- #points is a random variable
- can be guided by a density  $h$
- with spatial interactions
- **each point can be associated with a parametric object**



# Applications

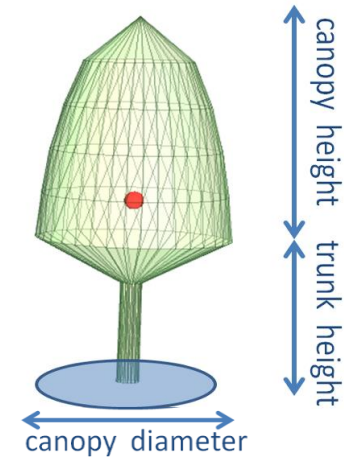
## Parametric object detection



# Three important ingredients

A parametric object

- points = object centroids
- some additional parameters



# Three important ingredients

A parametric object

An energy  $U$

- measures the quality of an object configuration
- specifies the density  $h$  of the process  $h(.) \propto \exp -U(.)$



# Three important ingredients

A parametric object

An energy  $U$

- measures the quality of an object configuration
- specifies the density  $h$  of the process  $h(.) \propto \exp -U(.)$
- typical form:

$$\forall x \in \mathcal{C}, \quad U(x) = \underbrace{\sum_{x_i \in x} D(x_i)}_{\text{Data term}} + \underbrace{\sum_{x_i \sim x_j} V(x_i, x_j)}_{\text{Pairwise interactions}}$$

# Three important ingredients

A parametric object

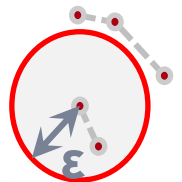
An energy  $U$

- measures the quality of an object configuration
- specifies the density  $h$  of the process  $h(.) \propto \exp -U(.)$
- typical form:

$$\forall \mathbf{x} \in \mathcal{C}, \quad U(\mathbf{x}) = \underbrace{\sum_{x_i \in \mathbf{x}} D(x_i)}_{\text{Data term}} + \underbrace{\sum_{x_i \sim x_j} V(x_i, x_j)}_{\text{Pairwise interactions}}$$

Markovian property: interactions restricted to a local neighborhood

$$x_i \sim x_j = \{(x_i, x_j) \in \mathbf{x}^2 : i > j, \|x_i - x_j\|_2 < \epsilon\}$$



# Three important ingredients

A parametric object

An energy  $U$

A sampler

- Find an approximate solution of the global minimum of  $U$   
typically RJMCMC sampler [Green95]



# Three important ingredients

A parametric object

An energy  $U$

A sampler

- Find an approximate solution of the global minimum of  $U$   
typically RJMCMC sampler [Green95]
- **Principle:** iterative mechanism that simulates a discrete Markov chain on the configuration space

# Three important ingredients

A parametric object

An energy  $U$

A sampler

- Find an approximate solution of the global minimum of  $U$   
typically RJMCMC sampler [Green95]
- **Principle:** iterative mechanism that simulates a discrete Markov chain on the configuration space
- **At each iteration,**
  - (i) proposition of a local modification
  - (ii) acceptation/rejection of the modification depending on energy variation, proposal densities and a relaxation parameter

# Three important ingredients

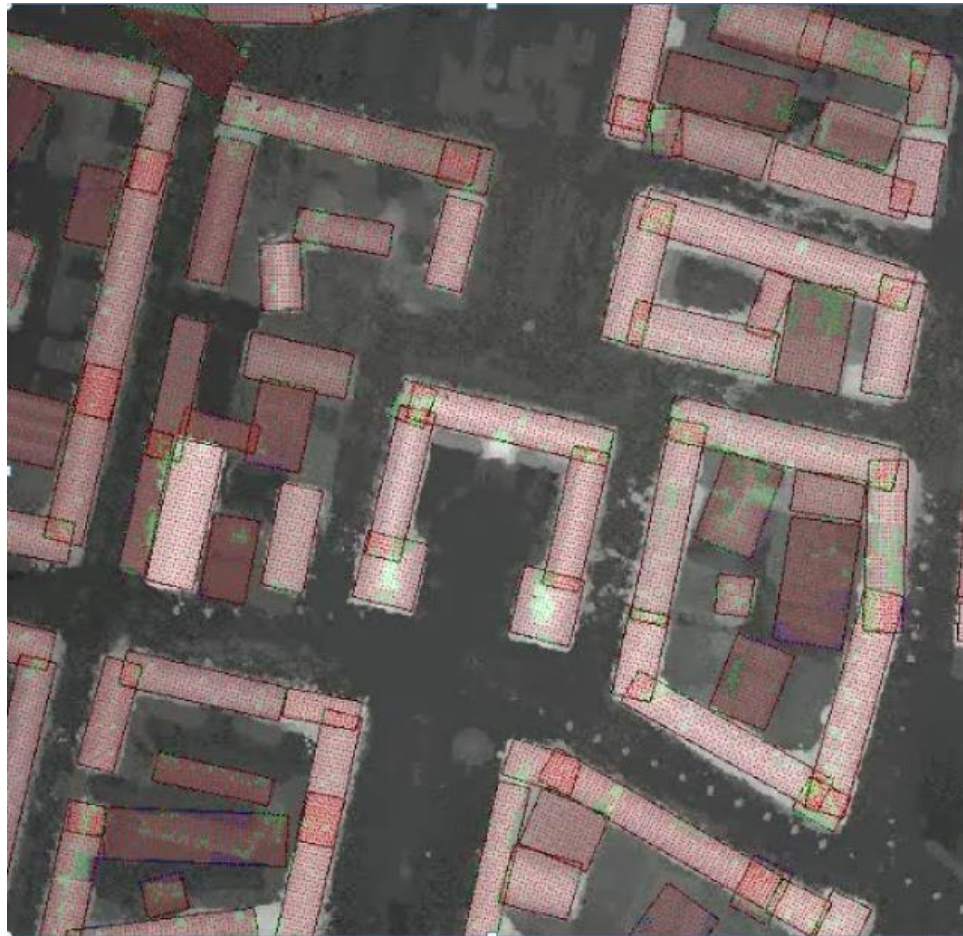
An example: extracting buildings from Elevation maps



(time  $\times 150$ )

# Three important ingredients

An example: extracting buildings from Elevation maps



**Geometric structures  
not guaranteed by  
construction!**

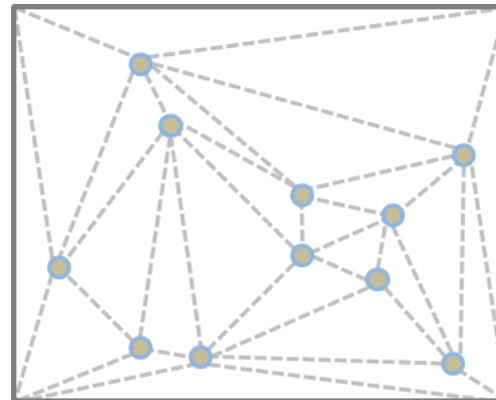
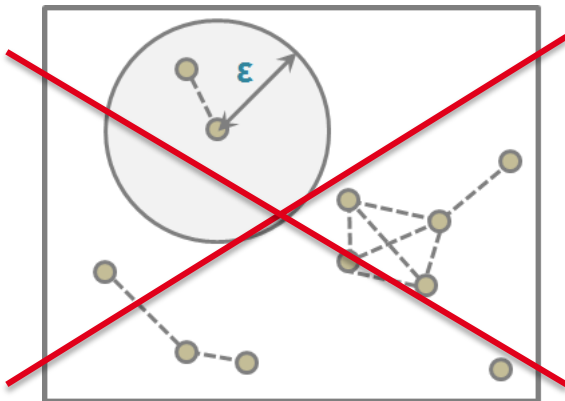
(time  $\times 150$ )



# Delaunay neighborhood

Idea: use Delaunay triangulation to define neighboring relationship (instead a traditional Euclidean distance)

$$p_i \sim_D p_j = \{(p_i, p_j) \in \mathcal{P}^2 : (p_i, p_j) \in C_2(\mathcal{P})\}$$



## Why is it interesting?

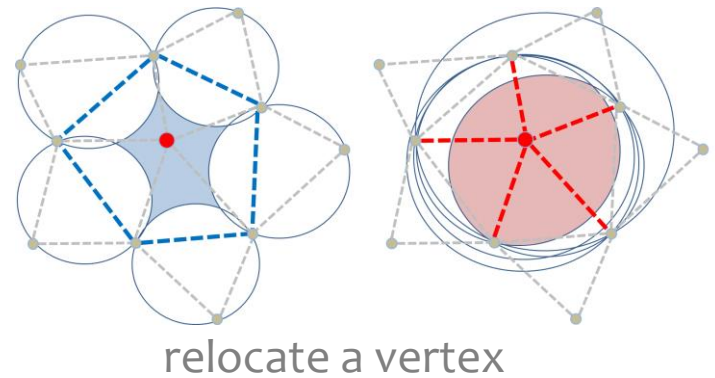
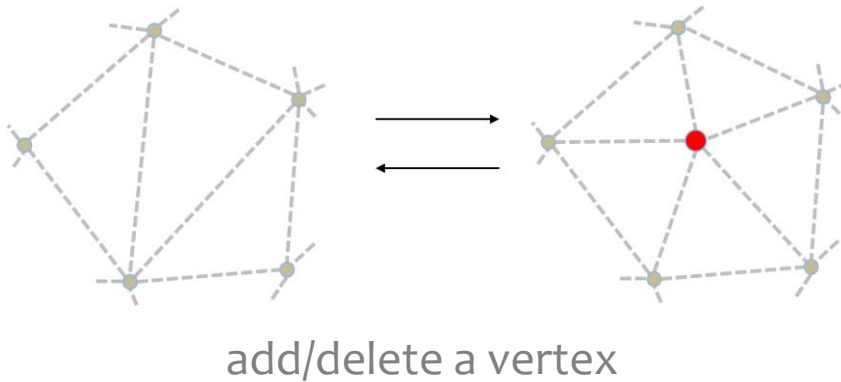
Each configuration relies on space decomposition that can be used as a mean to sample points but also as goal to segment data

- Parameter-free neighborhood

# Why is it interesting?

Each configuration relies on space decomposition that can be used as a mean to sample points but also as goal to segment data

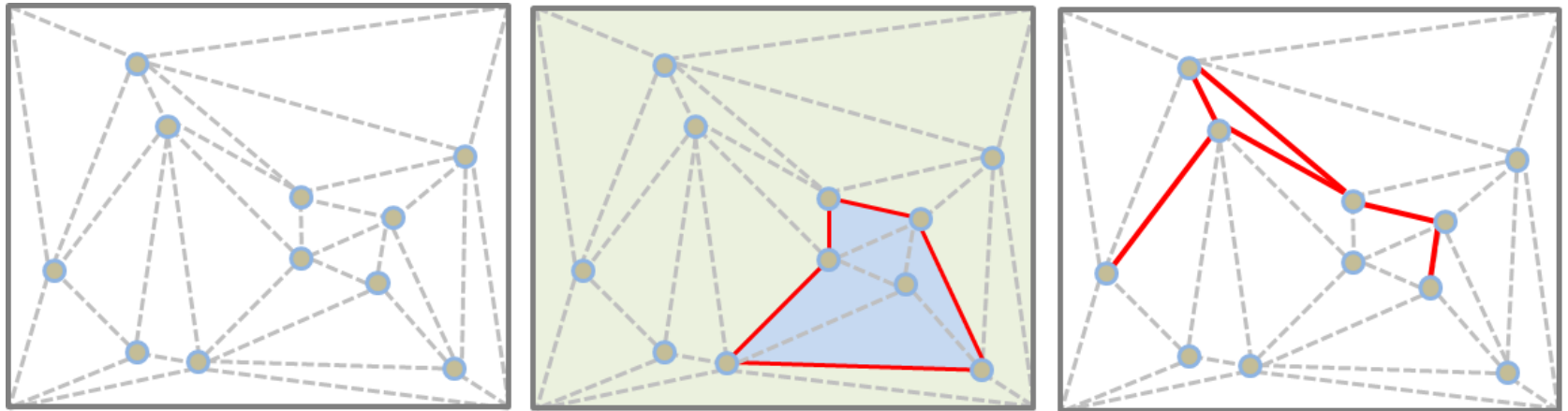
- Parameter-free neighborhood
- Efficient sampling



# Why is it interesting?

Each configuration relies on space decomposition that can be used as a mean to sample points but also as goal to segment data

- Parameter-free neighborhood
- Efficient sampling
- flexibility for a large range of applications



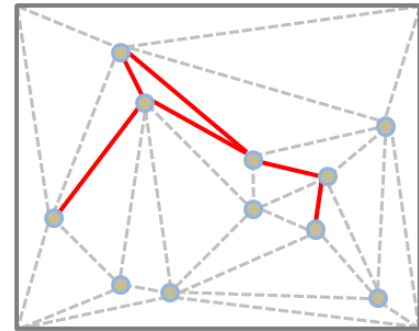


**Application to line-network extraction**

# Energy formulation

A configuration  $\mathbf{x} = (\mathbf{p}, \mathbf{m})$  a set of points and some additional parameter on points, edges or facets of the triangulation

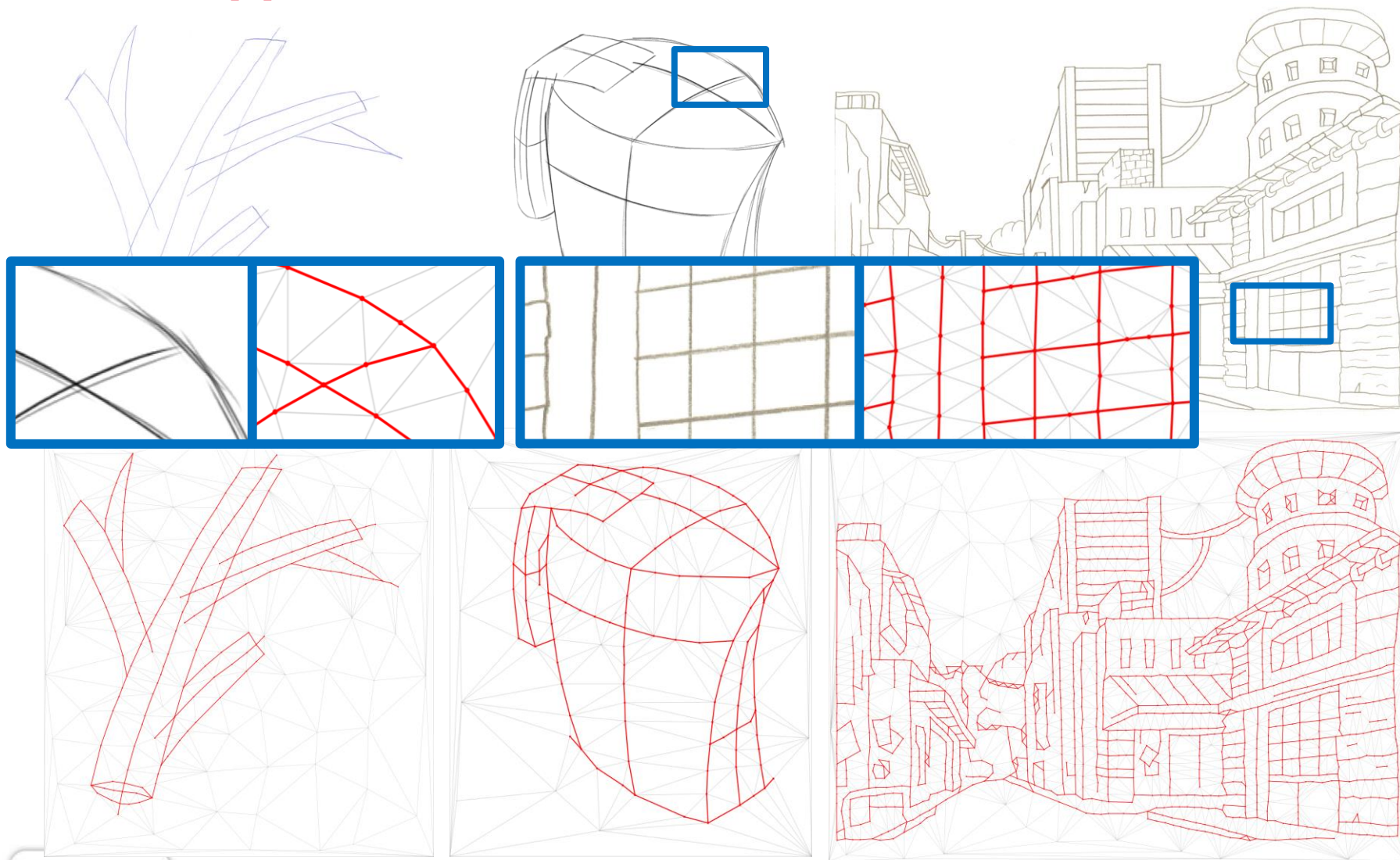
Energy of the form  $U(\mathbf{x}) = U_{fidelity}(\mathbf{x}) + U_{prior}(\mathbf{x})$



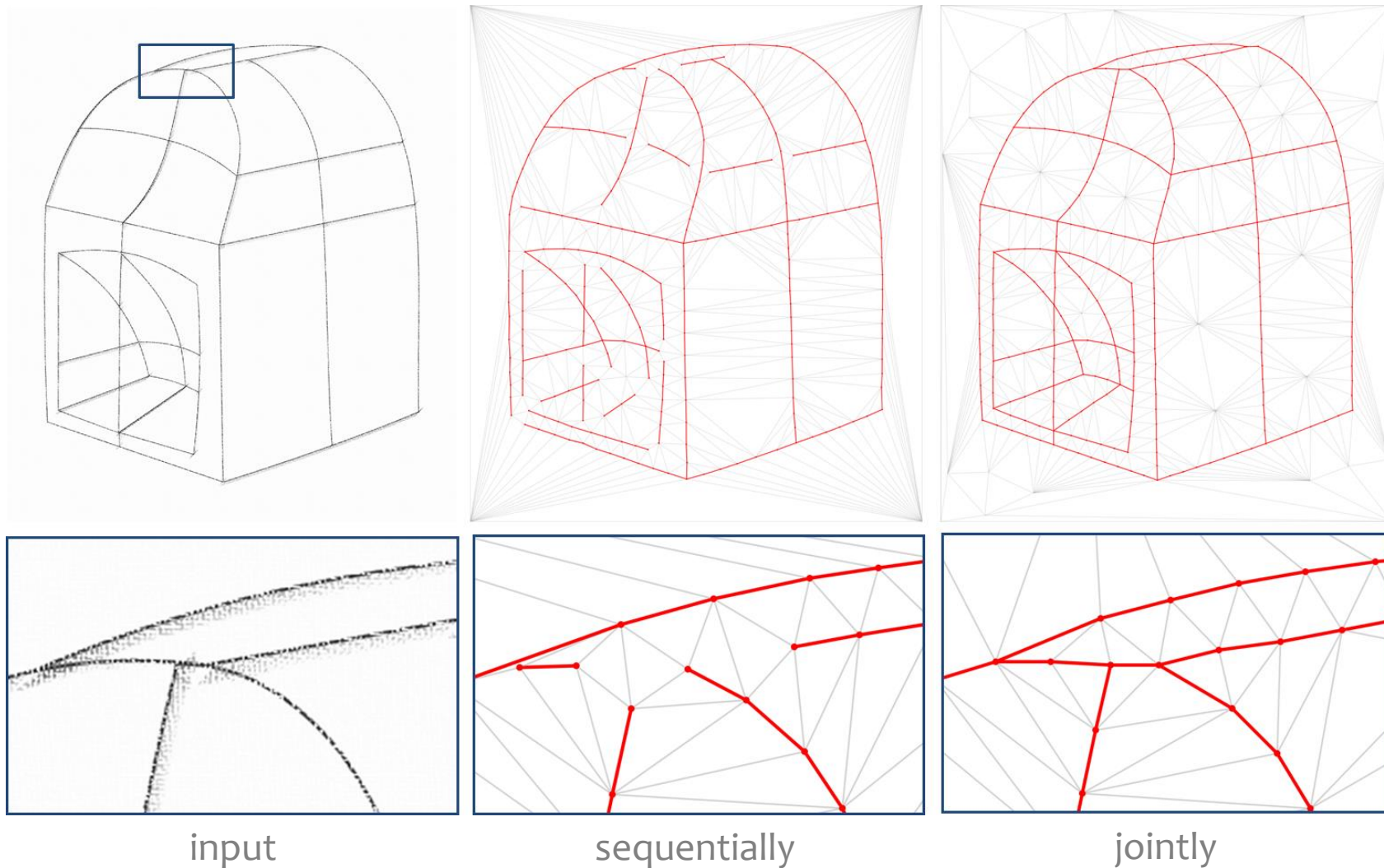
## Example with line-network extraction

- $\mathbf{m} = (m_e)_{e \in C_2(\mathbf{p})}$  with  $m_e \in \{0, 1\}$
- $U_{fidelity}$  : coherence with data (active edges should align with strong gradients)
- $U_{prior}$  : penalty for short edges + penalty for badly connected edges
- Sampling: RJMCMC with points distributed with density following an image gradient

# Application to line-network extraction



# Application to line-network extraction

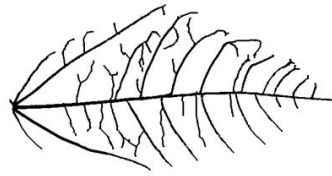




# Application to line-network extraction



Input



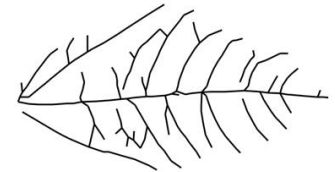
GT



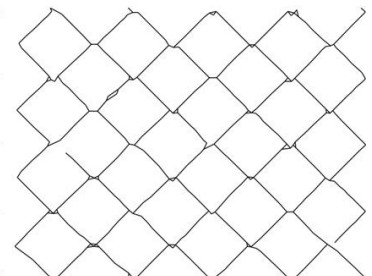
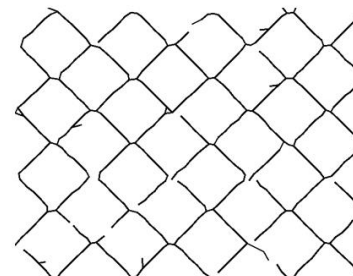
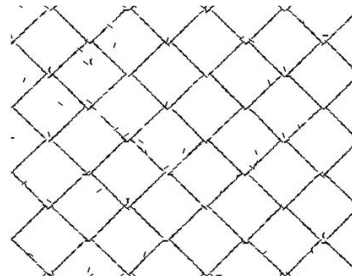
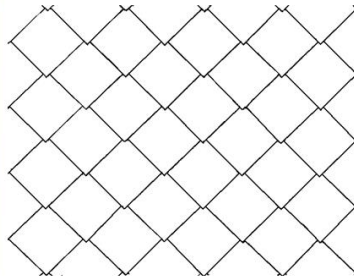
Marked point process  
[Verdie et al, IJCV14]



Junction point process  
[Chai et al, CVPR13]

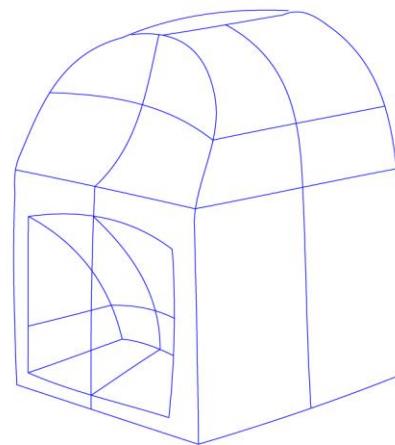
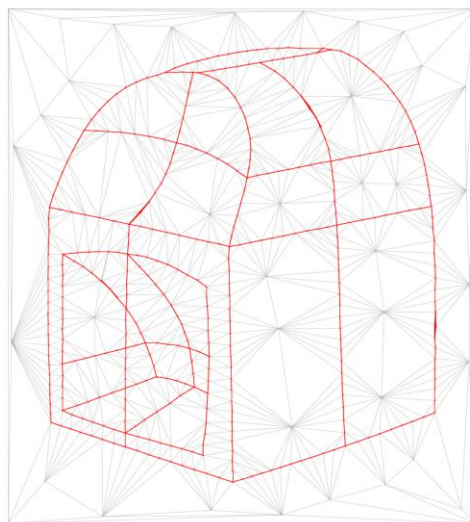
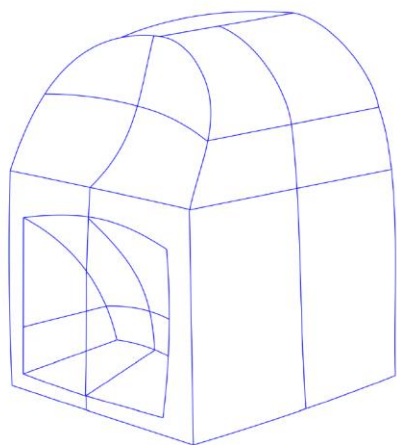
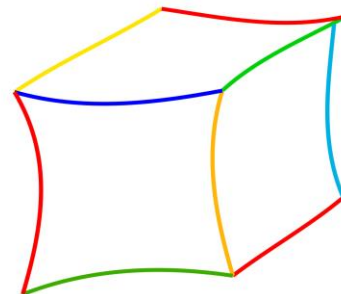
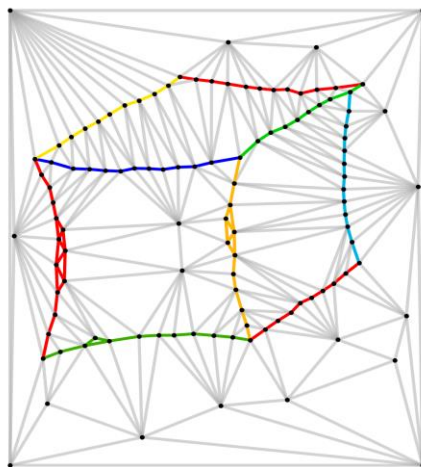
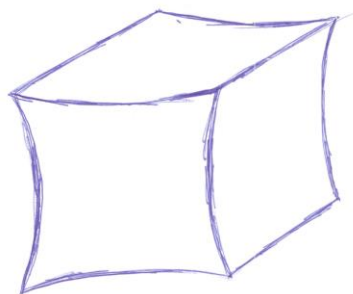


Ours



		Precision	F-measure	Time
Leaf	Junction-point process	0.59	0.64	73s
	Marked point process	0.76	0.70	33s
	ours	<b>0.79</b>	<b>0.73</b>	20s
Tiles	Junction-point process	0.46	0.54	227s
	Marked point process	0.67	0.72	103s
	ours	<b>0.70</b>	<b>0.74</b>	70s

# Application to line-network extraction



input

Output with Bezier curves

**Application to object contouring**

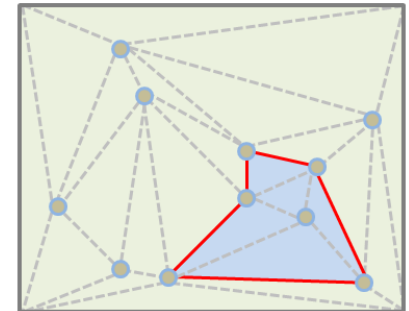
# Application to object contouring

A configuration  $\mathbf{x} = (\mathbf{p}, \mathbf{m})$  a set of points and some additional parameter on points, edges or facets of the triangulation

Energy of the form  $U(\mathbf{x}) = U_{fidelity}(\mathbf{x}) + U_{prior}(\mathbf{x})$

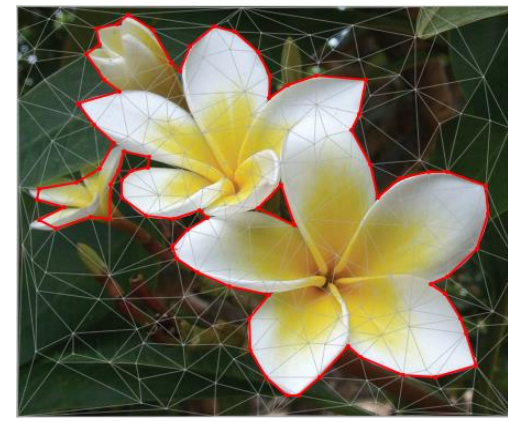
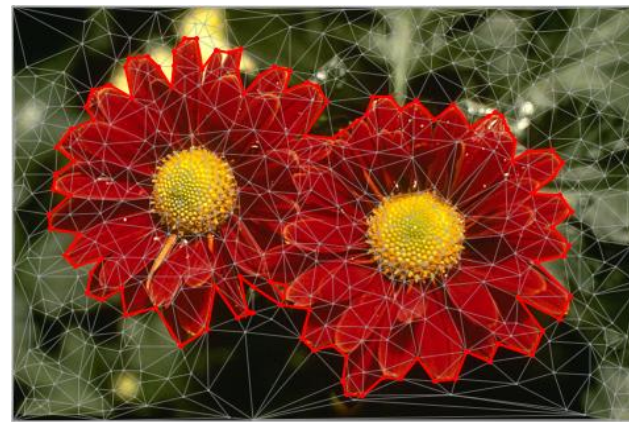
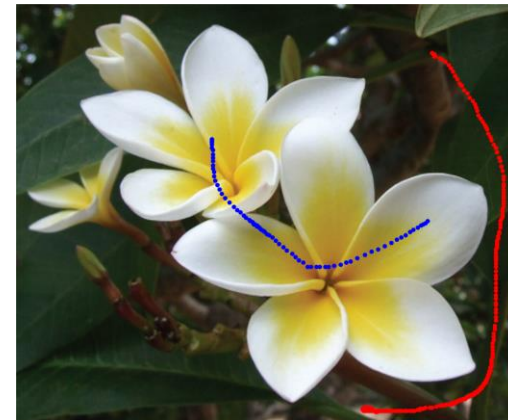
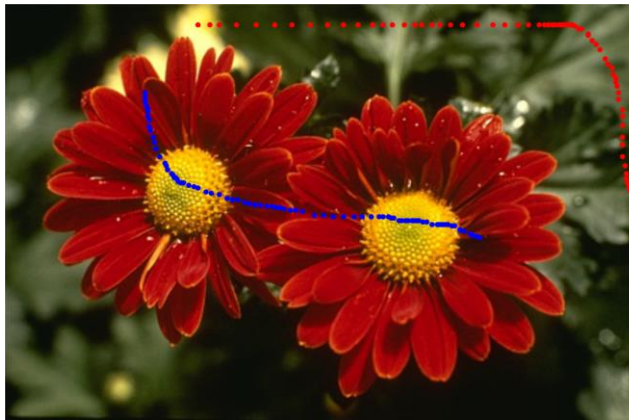
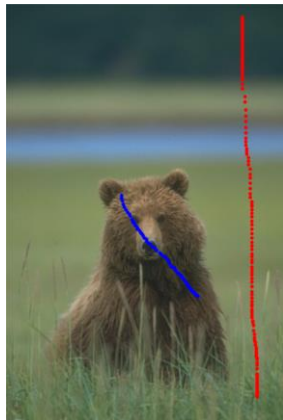
## Example with object countouring

- $\mathbf{m} = (l_f)_{f \in C_3(\mathbf{p})}$  with  $l_f = 0, 1$
- $U_{fidelity}$  : radiometric coherence inside each facet
- $U_{prior}$  : penalty for short edges + label smoothness for adjacent facets
- Sampling: RJMCMC with points distributed with density following an image gradient





# Application to object contouring





# Application to object contouring

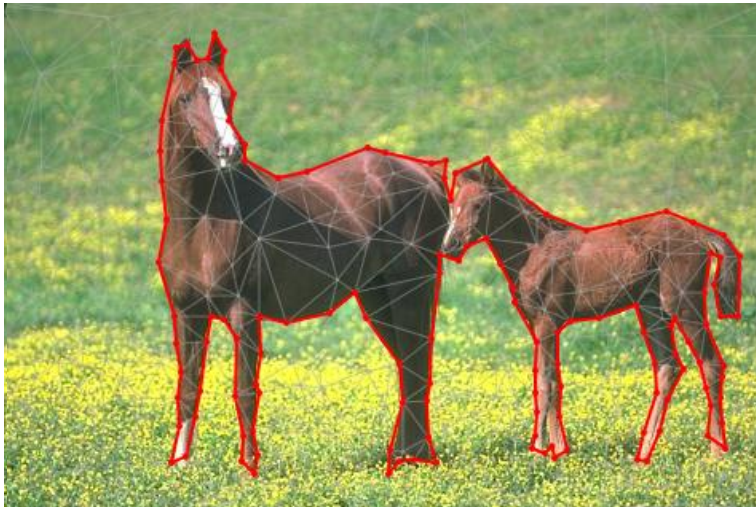




# Application to object contouring

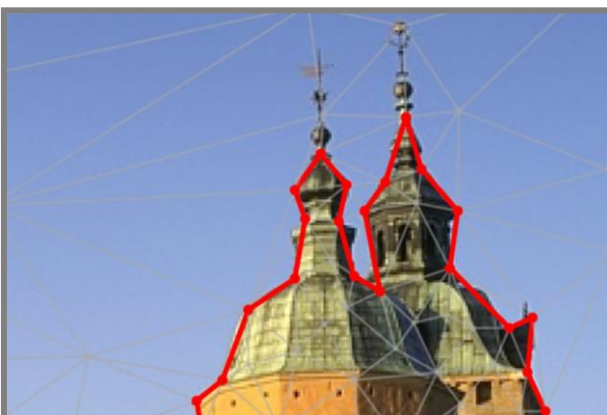
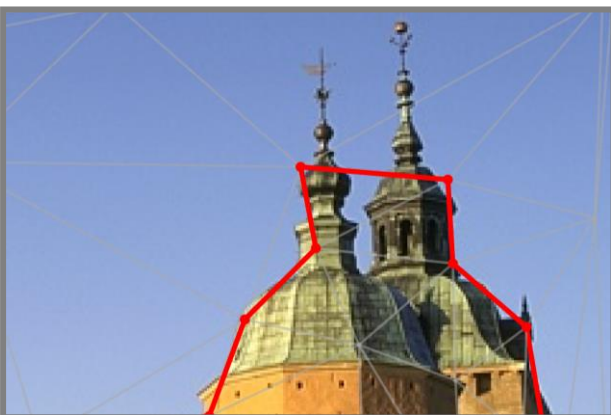
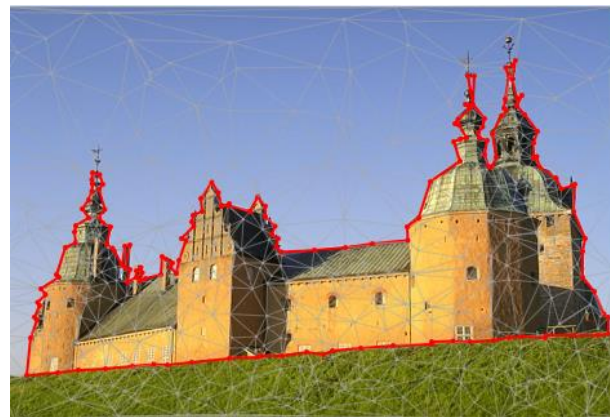


# Applications to object contouring





# Application to object contouring



low point density

high point density

**Application to image compression**

# Application to image compression

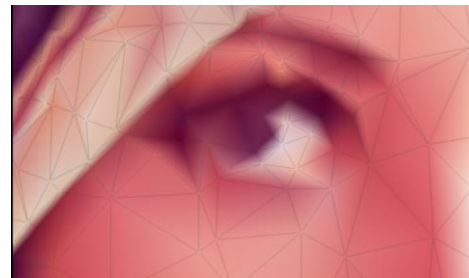
A configuration  $\mathbf{x} = (\mathbf{p}, \mathbf{m})$  a set of points and some additional parameter on points, edges or facets of the triangulation

Energy of the form  $U(\mathbf{x}) = U_{fidelity}(\mathbf{x}) + U_{prior}(\mathbf{x})$

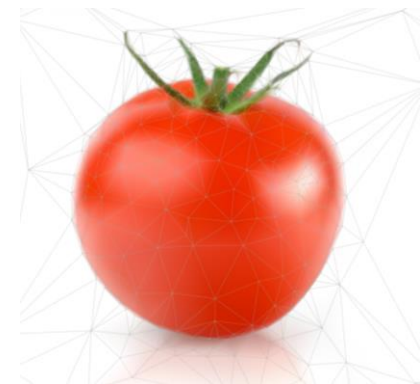
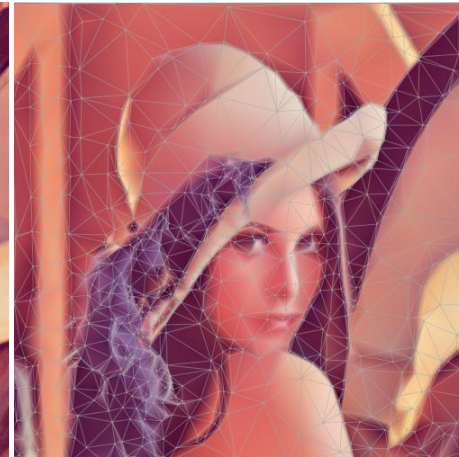


## Example with image compression

- $\mathbf{m} = (c_p)_{p \in \mathbf{p}}$  where  $c_p$  is a RGB color
- $U_{fidelity}$  : per-pixel error between input/output
- $U_{prior}$  : penalty for high number of points
- Sampling: RJMCMC with points distributed with density following an image gradient

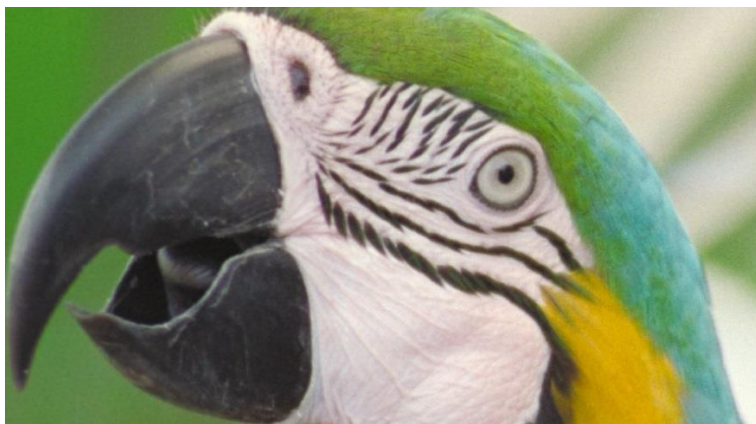
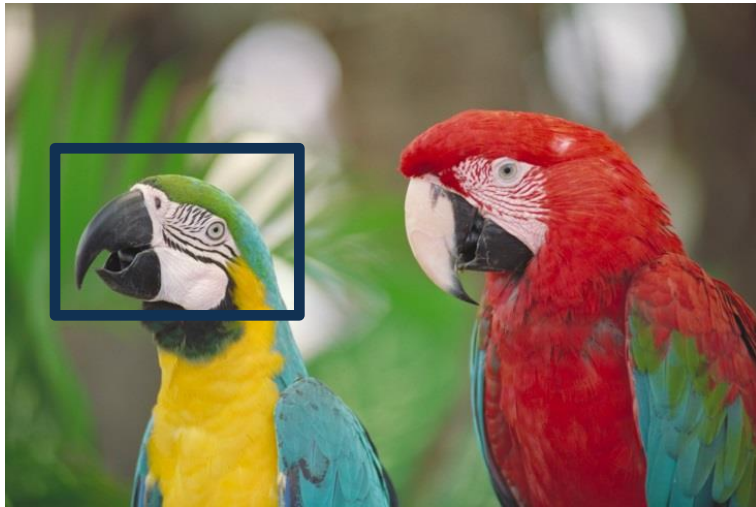


# Application to image compression

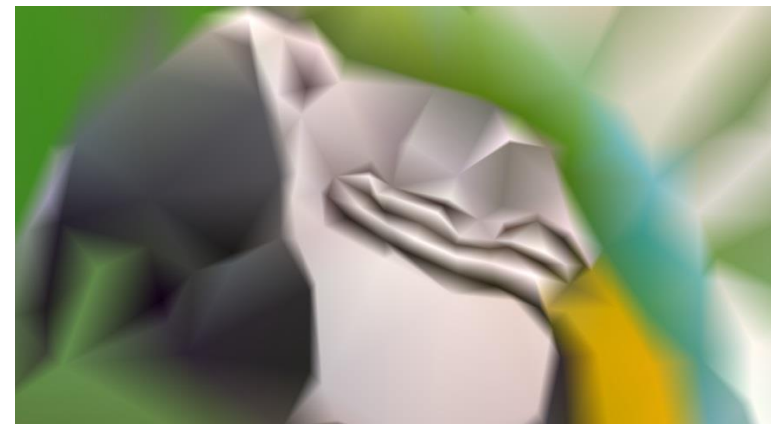




# Application to image compression



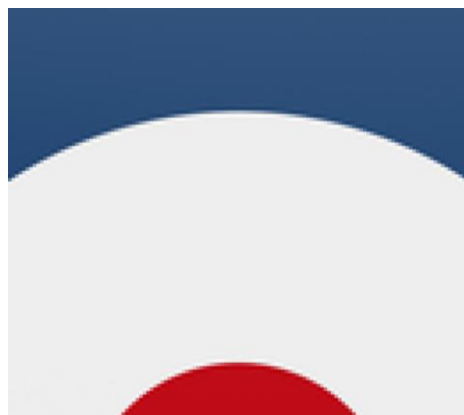
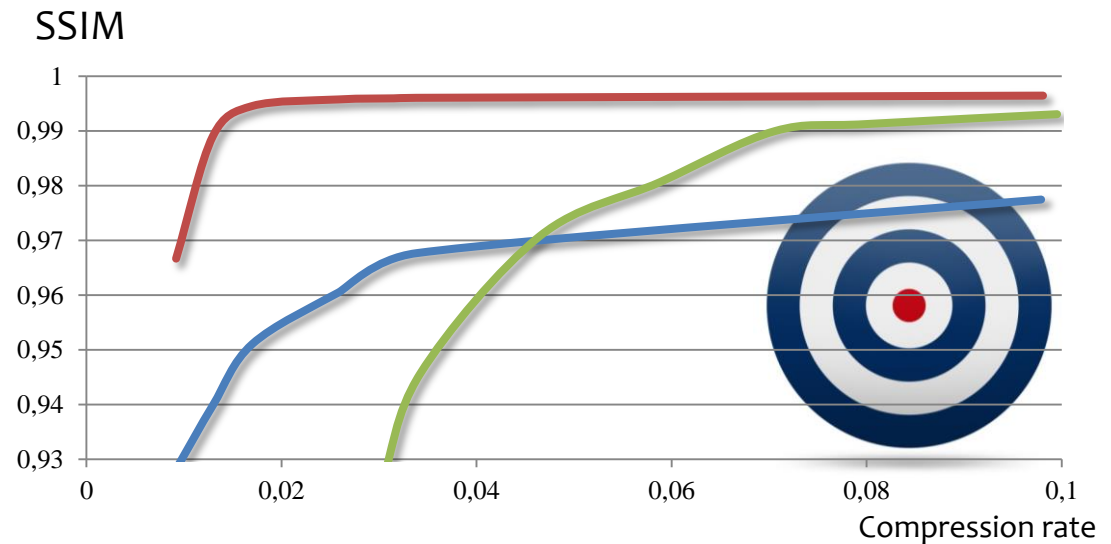
Input (6,3Mpix)



Output (700 vertices)

# Application to image compression

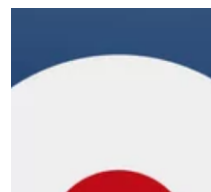
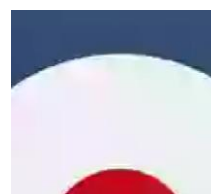
on synthetic  
image



original



JPEG2000



WEBP



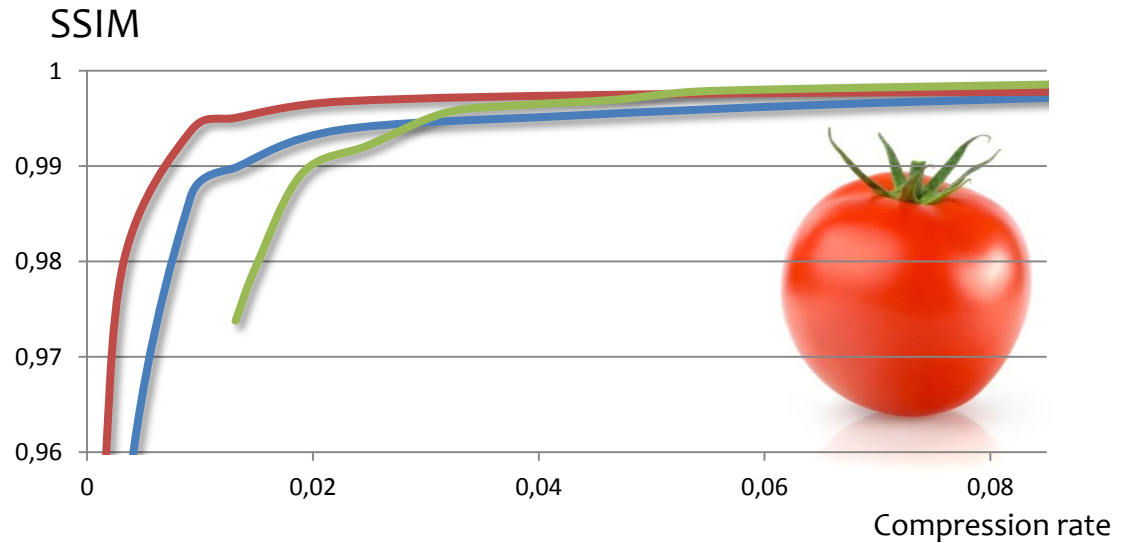
Our

3% compression

10% compression

# Application to image compression

on studio picture



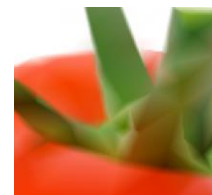
original



JPEG2000



WEBP



Our

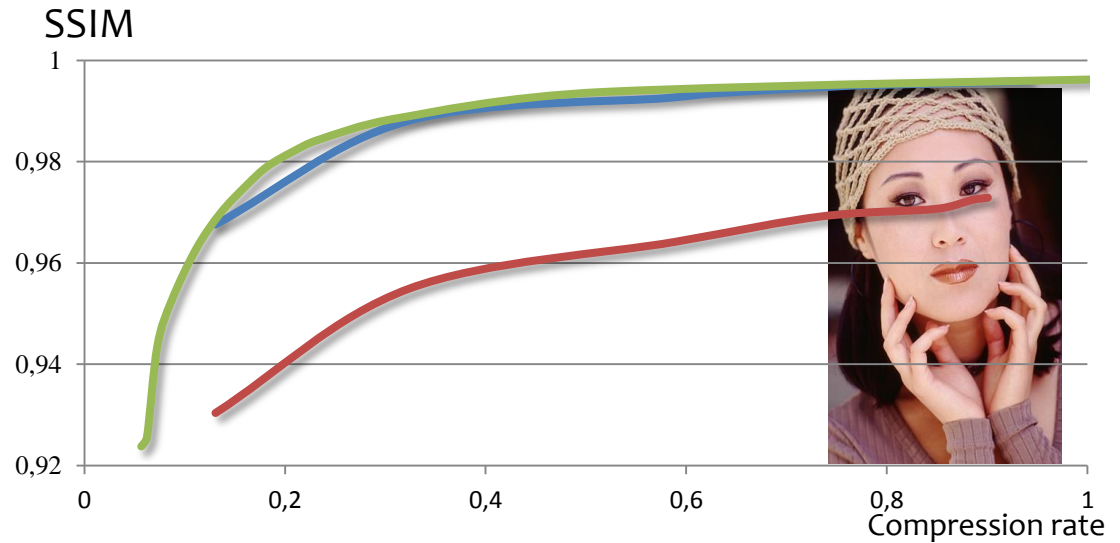
0.1% compression



5% compression

# Application to image compression

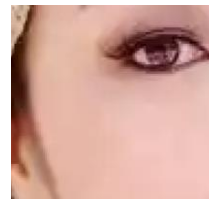
on studio picture



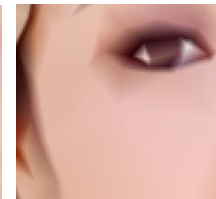
original



JPEG2000



WEBP



Our

13% compression



70% compression



# Conclusion

Partitioning images into geometric data structures:

- fast preprocessing
- man-made objects and scenes
- Scalability
- easy-to-use
- comes with geometric guarantees (cell adjacency, convexity..)

## Extensions

- Integrate image partitioning into the application process
- more types of shape
- 3D

# Code online

The screenshot shows the 'Repository' page of the 'Titane' website, which is dedicated to 'Geometric Modeling of 3D Environments'. The page features a navigation bar with links to Home, Research, Team, Publications, Teaching, Projects, Repository (active), Jobs, Seminars, Contact, and Internal. The main content area is titled 'Repository' and is divided into two sections: 'Software' and 'Data'. The 'Software' section lists several projects with their respective conference proceedings: 'The Computational Geometry Algorithms Library', 'Image Partitioning into Convex Polygons (CVPR 2015)', 'Line drawing vectorization (SIGGRAPH 2016)', 'Structure-Aware Mesh Decimation (CGF 2015)', 'Surface Reconstruction through Structuring (Eurographics 2013)', 'Noise Adaptive Shape Reconstruction (SGP 2013)', and 'Monte Carlo Sampler in Parallel (IJCV 2014)'. The 'Data' section lists 'Visionair repository'. On the right side, there are two sidebars: 'Recent Posts' and 'Categories'. The 'Recent Posts' sidebar lists four posts: 'Software Engineer - 3D Geometry Compression', 'David Bommes: Applied Optimization (not only) for Geometry Processing', 'Pierre Alliez: Low Distortion Inter-surface Mapping via Optimal Mass Transport', and 'Stefanie Wuhler: Alignment and Analysis of 3D Human Motion Sequences'. The 'Categories' sidebar lists four categories: 'Jobs', 'Project', 'Seminars', and 'Uncategorized'. A 'Go back to the front page' link is located in the top right corner of the page.

Go back to the front page

## Titane

Geometric Modeling of 3D Environments

Home Research Team Publications Teaching Projects **Repository** Jobs Seminars Contact Internal

### Repository

#### Software

- **"**  
The Computational Geometry Algorithms Library  
Image Partitioning into Convex Polygons (CVPR 2015)  
Line drawing vectorization (SIGGRAPH 2016)  
Structure-Aware Mesh Decimation (CGF 2015)  
Surface Reconstruction through Structuring (Eurographics 2013)  
Noise Adaptive Shape Reconstruction (SGP 2013)  
Monte Carlo Sampler in Parallel (IJCV 2014)

#### Data

- **"**  
Visionair repository

#### Recent Posts

- Software Engineer - 3D Geometry Compression
- David Bommes: Applied Optimization (not only) for Geometry Processing
- Pierre Alliez: Low Distortion Inter-surface Mapping via Optimal Mass Transport
- Stefanie Wuhler: Alignment and Analysis of 3D Human Motion Sequences
- Jean-Dominique Favreau: Fidelity vs. Simplicity: a Global Approach to Line Drawing Vectorization

#### Categories

- Jobs
- Project
- Seminars
- Uncategorized

<https://team.inria.fr/titane/software/>