Localiser une cible dans un graphe

Julien Bensmail\textsuperscript{1}, Dorian Mazauric\textsuperscript{2}, \textbf{Fionn Mc Inerney}\textsuperscript{1}, Nicolas Nisse\textsuperscript{1}, Stéphane Pérennes\textsuperscript{1}

\textsuperscript{1}Université Côte d’Azur, Inria, CNRS, I3S, France
\textsuperscript{2}Université Côte d’Azur, Inria, France

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A facility, city or some other location is modelled by a graph $G$.

A target is hidden at a vertex of $G$ (i.e., a building).

Detector at each vertex; when “probed”, returns its distance to the target.

Metric Dimension of $G$

Min. # of vertices needed to be probed all at once to locate the target in $G$. 
Sequential Locating Game on Graphs [Seager, 2013] & Game of Guess Who?

\[ SL(G) : \text{Sequential location number of } G \]

Min. \# of turns of probing one vertex per turn, to locate a target hidden in \( G \).
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Sequential Metric Dimension of $G$

Given $k, \ell, G$, is it possible to locate the immobile target in $G$ in at most $\ell$ turns by probing at most $k$ vertices each turn.

Related: Locate moving target with $k$? [Bosek et al, 2017]; $k = 1$ [Seager, 2012].

$\frac{MD(G)}{k}$ turns suffice to locate target. But it can be located faster.

$MD(G) = 19$ and for $k = 4$, the target can be located in 2 turns.
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![Diagram of a graph with blue and red vertices, showing sequential probing and locating of a target.]

Answer: \( d = 3 \)
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All vertices in blue probed

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Two vertices in blue NOT probed
$MD(G) > 18$

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Now sequential probing

All vertices in blue probed

Locate in 2 turns (FIRST turn)

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Given $k, \ell, G$, is it possible to locate the immobile target in $G$ in at most $\ell$ turns by probing at most $k$ vertices each turn.

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Now sequential probing

All vertices in blue probed

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$\frac{MD(G)}{k}$ turns suffice to locate target. But it can be located faster.

Now sequential probing

All vertices in blue probed

Locate in 2 turns (SECOND turn)

$MD(G) = 19$ and for $k = 4$, the target can be located in 2 turns.
**Complexity**

*NP-complete* when either number of vertices to be probed or number of turns is fixed.

**Trees**

*NP-complete* in trees.

- Difficulty only comes from first turn.
- Polynomial-time (+1)-approximation algorithm for trees.
**Summary of results in trees**

\[ \lambda_k(T) : \text{min. \# turns to locate target in } T. \]

**(+1)-approximation algorithm**

- Computes strategy that locates target in \( T \) in at most \( \lambda_k(T) + 1 \) steps.
- Time complexity: \( O(n \log n) \).

**Exact algorithm**

- Computes strategy that locates target in \( T \) in at most \( \lambda_k(T) \) steps.
- Time complexity: \( O(n^{k+2} \log n) \).
Trees: why problem is “easy” after 1\textsuperscript{st} turn

Probe any 1 vertex on 1\textsuperscript{st} turn.
Probe any 1 vertex on 1st turn.
Probe any 1 vertex on 1\textsuperscript{st} turn.

Result: Tree $T'$ rooted in $r$ where all leaves are the same distance from $r$ and the target is known to occupy a leaf.
Probe any 1 vertex on 1\textsuperscript{st} turn.

Result: Tree $T'$ rooted in $r$ where all leaves are the same distance from $r$ and the target is known to occupy a leaf.
Second key argument for “easiness”

$T_v$ : subtree rooted in $v$ of $T'$ rooted in $r$, $v$ is a child of $r$.

Probing 1 vertex in $T_v$ allows to know if target is in $T_v$ or in $T' \setminus T_v$. 
Two parameters needed for algorithm

Assume target is known to occupy a leaf of $T_i$. Then,

$\lambda_k(T_i)$: min. # of turns of probings to locate target in $T_i$.

$\pi_k(T_i)$: min. # of vertices to probe in $T_i$ during first turn of probing vertices in $T_i$ to locate target in $\lambda_k(T_i)$ turns.

$k = 3$  \hspace{1cm} $\lambda_3(T) = 3$  \hspace{1cm} $\pi_3(T) = 2$
Need for tradeoff between probing 1 or $\pi(T_{v_i})$ vertices in $T_{v_i}$

$k = 3$

$\lambda(T) = 5$

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Need for tradeoff between probing 1 or $\pi(T_{v_i})$ vertices in $T_{v_i}$

$k = 3$  \hspace{1cm} $\lambda(T) = 5$

\begin{itemize}
  \item $v_1$
  \item $v_2$
  \item $v_3$
  \item $v_4$
  \item $v_5$
  \item $v_6$
\end{itemize}

\begin{itemize}
  \item $\lambda_3(T_{v_i}), \pi_3(T_{v_i})$
\end{itemize}
Need for tradeoff between probing 1 or $\pi(T_{v_i})$ vertices in $T_{v_i}$

$k = 3$  \hspace{1cm} $\lambda(T) = 5$

$\lambda_3(T_{v_i}), \pi_3(T_{v_i})$
Variant with relative distances

Relative distances returned instead of exact distances.

Target

$v_1$  $v_2$  $v_3$  $v_4$  $v_5$  $v_6$  $v_7$  $v_8$  $v_9$

Relative dist. vector from probing blue vertices: $(v_7 < v_4 = v_8 < v_1)$.

Complexity

NP-complete when either number of vertices to be probed or number of turns is fixed.
Future work

- Study the problem with exact distances in other graph classes such as planar and interval graphs.
- Try to get exact values for the problem with relative distances in paths [Foucaud et al, 2014].
- Study the problem with relative distances in trees.
Thanks!