XXIVth International Biometric Conference
University College Dublin — July 13th to 18th 2008

Stochastic spatio-temporal modeling of forest dynamics

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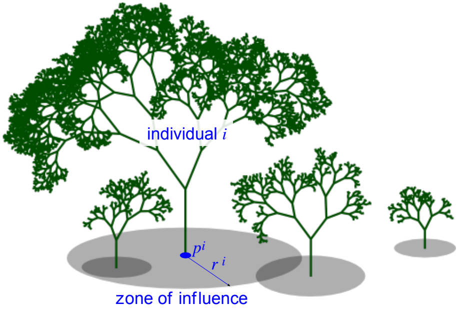
aim

- a population of N_t individuals

$$i \longleftrightarrow x_t^i \quad \text{(state of the individual } i \text{ at time } t\text{)}$$

  that evolves in continuous time and space
- basic ingredients :
  - birth
  - death
    - natural
    - competition
  - growth

zone of influence



- $\mathcal{D}_x$ disk with center p and radius r
- represents the amount of nutrient needed for growth
- used to model the competition for access to the resources

stand state

- at time t : $\nu_t \stackrel{\text{def}}{=} \{x_t^i\,;\, i = 1, \dots, N_t\}$ where

$$x_t^i \stackrel{\text{def}}{=} (p_t^i, r_t^i)$$

and

$$\begin{cases} p_t^i & \text{position} \\ r_t^i & \text{radius of the zone of influence} \end{cases}$$

ingredients

- each individual x in ν is submitted to 3 punctual and 1 continuous phenomena :
  - recruitment/dispersion
  - natural mortality
  - competition death
  - growth

ingredients (I/III)

- recruitment/dispersion : birth rate

$$\lambda^{\mathrm{b}}(x) = \lambda^{\mathrm{b}}_{\max}\,\frac{r}{r^{\max}}\,1_{\{r\geq r^{\mathrm{b}}\}}$$

  new individual $x' \sim D(x, \mathrm{d}y)$ (dispersion kernel)

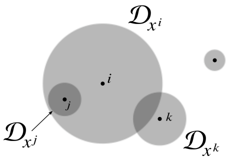
- natural mortality : individual located at x dies at rate λ^{d}

ingredients (II/III)

- competition death : x in ν die at rate

$$\lambda^{\mathrm{c}}(x,\nu) = \sum_{y\in\nu} u(x,y)$$

$$u(x,y) = \begin{cases} c^{\max}\,\frac{\mathrm{Area}(\mathcal{D}_x \cap \mathcal{D}_y)}{\mathrm{Area}(\mathcal{D}_x)} & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$$

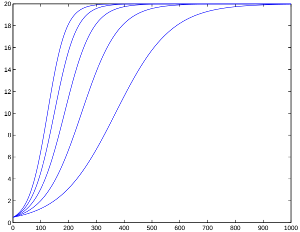


ingredients (III/III)

- growth : between two events (death/birth)

$$\dot{r}_t^i = \alpha^{\mathrm{g}}(x_t^i,\nu_t)\,\frac{1}{1-\beta^{\mathrm{g}}}\,r_t^i\,\Big[1-\Big(\frac{r_t^i}{r^{\max}}\Big)^{\beta^{\mathrm{g}}-1}\Big]\,, \quad r_0 = r^{\min}$$

with $\alpha^{\mathrm{g}}(x_t^i,\nu_t) = \alpha^{\mathrm{g}}_{\max} \times \Big(1 - \frac{\lambda^{\mathrm{c}}(x_t^i,\nu_t)}{c^{\max}} \wedge 1\Big)$



- it could be stochastic

maximum events' rate

- at population level :

$$\bar{\lambda} \stackrel{\text{def}}{=} \bar{\lambda}^{\text{d}} + \bar{\lambda}^{\text{b}} + \bar{\lambda}^{\text{c}}$$

with :

$$\bar{\lambda}^{\text{b}} \stackrel{\text{def}}{=} \lambda^{\text{b}}_{\text{max}}\, N \qquad \bar{\lambda}^{\text{d}} \stackrel{\text{def}}{=} \lambda^{\text{d}}_{\text{max}}\, N \qquad \bar{\lambda}^{\text{c}} \stackrel{\text{def}}{=} u_{\text{max}}\, N^2$$

and hypotheses :

$$\lambda^{\text{b}}(x) \leq \lambda^{\text{b}}_{\text{max}} \qquad\qquad u(x,y) \leq u_{\text{max}}$$

$\nu_{T_{k-1}} \to \nu_{T_k}$ (I/II)

- $N \leftarrow |\nu_{T_{k-1}}|$
- $\bar\lambda \leftarrow \bar\lambda^{\mathrm{d}} + \bar\lambda^{\mathrm{b}} + \bar\lambda^{\mathrm{c}}$

$$\bar\lambda^{\mathrm{b}} \stackrel{\mathrm{def}}{=} \lambda^{\mathrm{b}}_{\max}\, N$$
$$\bar\lambda^{\mathrm{d}} \stackrel{\mathrm{def}}{=} \lambda^{\mathrm{d}}_{\max}\, N$$
$$\bar\lambda^{\mathrm{c}} \stackrel{\mathrm{def}}{=} u_{\max}\, N^2$$

- next event instant : $T_k = T_{k-1} + S$ with $S \sim \mathrm{Exp}(\bar\lambda)$
- growth : $T_{k-1} \to T_k$, i.e. compute $\nu_{T_k^-}$ (Euler)
- pick x at random in $\nu_{T_k^-}$

$\nu_{T_{k-1}} \to \nu_{T_k}$ (II/II)

- with probabilities $(\bar\lambda^{\mathrm{b}}/\bar\lambda, \bar\lambda^{\mathrm{d}}/\bar\lambda, \bar\lambda^{\mathrm{c}}/\bar\lambda)$:
  - birth :

$$\nu_{T_k} = \begin{cases} \nu_{T_k^-} \cup \{x'\} & \text{with proba } \frac{\lambda^{\mathrm{b}}(x)}{\lambda^{\mathrm{b}}_{\max}} \\ \nu_{T_k^-} & \text{with proba } 1 - \frac{\lambda^{\mathrm{b}}(x)}{\lambda^{\mathrm{b}}_{\max}} \end{cases} \qquad x' \sim D(x, \mathrm{d}z)$$

  - natural death :

$$\nu_{T_k} = \begin{cases} \nu_{T_k^-} \setminus \{x\} & \text{with proba } \frac{\lambda^{\mathrm{d}}(x)}{\lambda^{\mathrm{d}}_{\max}} \\ \nu_{T_k^-} & \text{with proba } 1 - \frac{\lambda^{\mathrm{d}}(x)}{\lambda^{\mathrm{d}}_{\max}} \end{cases}$$

  - death by competition : pick y at random in $\nu_{T_k^-}$ and let

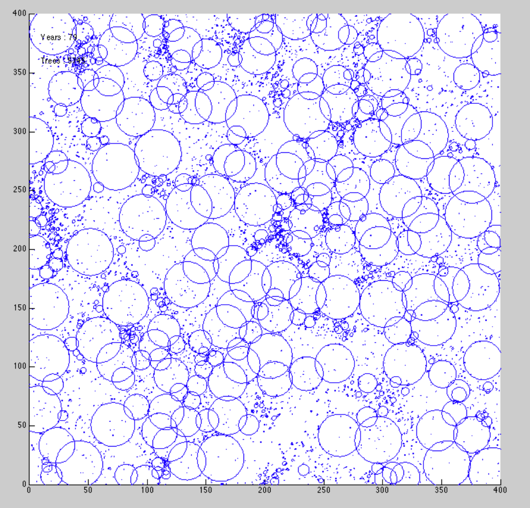
$$\nu_{T_k} = \begin{cases} \nu_{T_k^-} \setminus \{x\} & \text{with proba } \frac{u(x,y)}{u_{\max}} \\ \nu_{T_k^-} & \text{with proba } 1 - \frac{u(x,y)}{u_{\max}} \end{cases}$$

process $(\nu_t)_{t\geq 0}$

- Markov
  - branching
  - diffusion
  - interacting particles system
- law characterized by the infinitesimal generator (explicit)

$$\begin{aligned}
\mathcal{L}\Phi(\nu) = &\int \lambda^{\mathrm{b}}(x) \int [\Phi(\nu + \{x'\}) - \Phi(\nu)]\, D(x, \mathrm{d}x')\, \nu(\mathrm{d}x) \\
&+ \int \Big(\lambda^{\mathrm{d}} + \int u(x,y)\,\nu(\mathrm{d}y)\Big)\,[\Phi(\nu \setminus \{x\}) - \Phi(\nu)]\, \nu(\mathrm{d}x) \\
&+ \int L^{\mathrm{g}}\Phi(\nu)\, \nu(\mathrm{d}x)
\end{aligned}$$

simulation



perspective

- improve, improve...
- coupling the model to data
- few species
- mathematical analysis