Available online at www.sciencedirect.com
ScienceDirect
Journal of Banking & Finance 31 (2007) 1351–1373
Journal of BANKING & FINANCE
www.elsevier.com/locate/jbf

Technical analysis compared to mathematical models based methods under parameters mis-specification

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Available online 11 December 2006

Abstract

In this study, we compare the performance of trading strategies based on possibly mis-specified mathematical models with a trading strategy based on a technical trading rule. In both cases, the trader attempts to predict a change in the drift of the stock return occurring at an unknown time. We explicitly compute the trader's expected logarithmic utility of wealth for the various trading strategies. We next rely on Monte Carlo numerical experiments to compare their performance. The simulations show that under parameter mis-specification, the technical analysis technique out-performs the optimal allocation strategy but not the Model and Detect strategies. The latter strategies dominance is confirmed under parameter mis-specification as long as the two stock returns' drifts are high in absolute terms.

MSC: 60G35; 93E20; 91B28; 91B26; 91B70

JEL classification: G11; G14; C15; C65

Keywords: Stochastic models; Model specification; Portfolio allocation; Chartist

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0378-4266/$ - see front matter
doi:10.1016/j.jbankfin.2006.10.017

1. Introduction

The financial services industry typically relies on three main approaches to make investment decisions: the fundamental approach that uses fundamental economic principles to form portfolios, the technical analysis approach that uses price and/or volume histories and the mathematical approach that is based on mathematical models. Technical analysis has been used by professional investors for more than a century. The academic community has looked at its foundations and its performance with a rather skeptical frame of mind. Indeed, technical analysis techniques have limited theoretical justification, and they stand in contradiction to the conclusions of the efficient market hypothesis. More recently, there has been a renewal of academic interest in the performance of technical analysis based methods. Indeed, the pioneering study by Brock et al. (1992) applied 26 trading rules to the Dow Jones Industrial Average and found that they significantly out-perform a benchmark of holding cash. In their impressive study, Sullivan et al. (1999) examine close to 8000 technical trading rules and repeat Brock et al. study while correcting it for data snooping problems. They find that the trading rules examined by Brock et al. do not generate superior performance out-of-sample. Lo et al. (2000) propose to use a non-parametric kernel regressions pattern recognition method in order to automate the evaluation of technical analysis trading techniques. In their comprehensive study they compare the unconditional and the conditional – on technical analysis indicators – distribution of a large number of stocks traded on the NYSE/AMEX and on the NASDAQ. They conclude that "several technical indicators do provide some incremental information and may have some practical value". However, as pointed out by Jegadeesh, 2000 in his comment of the Lo et al. (2000) paper, none of the technical analysis indicators examined by the authors is able to identify profitable investment opportunities. Thus, it seems that the debate about the effectiveness of technical analysis usefulness is still very much alive.

The purpose of our study is to examine chartist and mathematical models based trading strategies by providing a conceptual framework where their performance can be compared. If one considers a non-stationary economy, it is impossible to specify and calibrate mathematical models that can capture all the sources of parameter instability during a long time interval. In such an environment, one can only attempt to divide any long investment period into sub-periods such that, in each of these sub-periods, the financial assets prices can reasonably be supposed to follow some particular distribution (e.g., a stochastic differential system with a fixed volatility function). Due to the investment opportunity set's instability, each sub-period must be short. Therefore, one can only use small amounts of data during each sub-period to calibrate the model, and the calibration errors can be substantial. Yet, any investment strategy's performance depends on the underlying model characterizing the evolution of the investment opportunity set and also on the parameters involved in the model. Thus, in a non-stationary economy, one can use strategies which have been optimally designed under the assumption that the market is well described by a prescribed model, but these strategies can be extremely misleading in practice because the prescribed model does not fit the actual evolution of the investment opportunity set. In such a situation, is one better of using a technical analysis based trading rule which is free of any model dependency? In order to answer that question one should compare the performance obtained by using erroneously calibrated mathematical models with the one associated with technical analysis techniques. To our knowledge, this question has not yet been investigated in the academic literature.

More specifically, we here consider the following test case: the agent in a frictionless continuous-time economy can invest in a riskless asset and in a stock. The instantaneous expected rate of return of the stock changes once at an unknown random time. We compare the performance of traders who respectively use:

- A technical analysis technique, namely the simple moving average technique in order to predict the change in the stock returns' drift.
- A portfolio allocation strategy which is optimal when the mathematical model is perfectly specified and calibrated.
- Two mathematical strategies called "Model and Detect" strategies aimed at detecting the time of the drift change.
- The three previous strategies under mis-specified parameters (due to the error on calibration).

The study is divided into two parts: a mathematical part which, whenever possible, provides analytical formulae for portfolios managed by means of mathematical and technical analysis strategies and a numerical part which provides comparisons between the various strategies' performance. Based on the numerical simulations, we find that the chartist strategy can out-perform optimal portfolio allocation models when there is parameter mis-specification. However, the "Model and Detect mathematical strategies" clearly dominate the chartist trading rule even when they are subject to parameter mis-specification.

The paper is organized as follows: In Section 2, we describe the basic setting underlying our mathematical modeling. In Sections 3 and 4, we examine the performance of a trader whose strategy is based on mathematical models. In Section 3, we examine the optimal portfolio allocation strategy. We give explicit formulas for the optimal wealth and the portfolio strategy of a trader who perfectly knows all the parameters characterizing the investment opportunity set and thus fully describe the best financial performance that one can expect within our model. In Section 4, we consider a trader who uses mathematical models in order to detect the change time τ in the drift of the stock price process as early and reliably as possible: he/she selects a stopping time Θ^* adapted to the filtration generated by (S_t), which serves as an "alarm signal" (this strategy is called "Model and Detect"). In Section 5, we consider the performances of the optimal portfolio allocation strategy and of the Model and Detect strategy when the trader mis-specifies the parameters of the model. In Section 6, we focus on a technical analyst who uses a simple moving average indicator to detect the time at which the drift of the stock return switches. We characterize his/her expected utility of wealth in the logarithmic case. We also numerically illustrate the properties of his/her strategy's performance. Finally, in Section 7, we compare the performances of the mis-specified mathematical strategies to those of the technical analysis technique.[1]

2. Description of the setting

We consider a frictionless continuous-time economy with two assets that are traded continuously. The first one is an asset without systematic risk, typically a

[1] A short version of this paper has been published in Blanchet-Scalliet et al. (2006).

risk-less *bond* (or bank account), whose price at time t evolves according to the following equation

$$\begin{cases} \mathrm{d}S_t^0 = S_t^0 r\,\mathrm{d}t, \\ S_0^0 = 1. \end{cases}$$

The second asset is a stock subject to systematic risk. We model the evolution of its price at time t by the linear stochastic differential equation

$$\begin{cases} \mathrm{d}S_t = S_t\big(\mu_2 + (\mu_1 - \mu_2)\mathbb{1}_{(t\leqslant\tau)}\big)\,\mathrm{d}t + \sigma S_t\,\mathrm{d}B_t, \\ S_0 = S^0, \end{cases} \tag{2.1}$$

where $(B_t)_{0\leqslant t\leqslant T}$ is a one-dimensional Brownian motion on a given probability space $(\Omega, \mathscr{F}, \mathbb{P})$. The random time of the stock return drift change τ is independent of B and has an exponential distribution with parameter λ:

$$\mathbb{P}(\tau > t) = \mathrm{e}^{-\lambda t}, \quad t \geqslant 0. \tag{2.2}$$

At time τ, which is neither known, nor directly observable, the instantaneous expected rate of return changes from μ_1 to μ_2.

We suppose that the parameters μ_1, μ_2, $\sigma > 0$ and $r \geqslant 0$ satisfy

$$\mu_1 - \frac{\sigma^2}{2} < r < \mu_2 - \frac{\sigma^2}{2}.$$

The main purpose of this study will be to use this setting in order to examine if mathematical models used to detect the time change in the drift of the stock price process lead to better predictions and thus to superior performance than a very popular technical analysis method based on a simple moving average signal.

3. The optimal portfolio allocation strategy under a change of drift

3.1. Introduction

We start by characterizing the optimal wealth and portfolio allocation of a trader who perfectly knows all the parameters μ_1, μ_2, λ, r and σ. Of course, this situation is unrealistic. However, it is worth computing the best performance that one can expect within our setting. This performance represents an optimal benchmark for mis-specified allocation strategies relying either on a mathematical model or on technical analysis.

Let π_t be the proportion of the trader's wealth invested in the stock at time t; the remaining proportion $1 - \pi_t$ is invested in the bond. For a given, non random, initial capital $x > 0$, let $W_\cdot^{x,\pi}$ denote the wealth process corresponding to the portfolio $(\pi_\cdot)$. We have

$$\mathrm{d}W_t^{x,\pi} = W_t^{x,\pi}(r\,\mathrm{d}t + \pi_t[(\mu_1 - r + (\mu_2 - \mu_1)\mathbb{1}_{(\tau\leqslant t)})\,\mathrm{d}t + \sigma\,\mathrm{d}B_t]). \tag{3.1}$$

Now, let $U(\cdot)$ denote a *utility function*. We suppose that U is strictly increasing, concave, of class $C^2(0,\infty)$ and satisfies

$$U'(0+) = \lim_{x\downarrow 0} U'(x) = \infty, \quad U'(\infty) = \lim_{x\uparrow\infty} U'(x) = 0.$$

Let $\mathscr{A}(x)$ denote the set of admissible strategies, that is, the set of processes π which take values in $[0,1]$ and are progressively measurable with respect to the filtration $\mathbb{F}^S$ generated by the observed prices S_t.

It is easy to see that, for all process π in $\mathscr{A}(x)$, $W_t^{x,\pi} > 0$ for all t.

The investor's objective is to maximize his/her expected utility of wealth at the terminal date T. He/she solves the following optimization problem:

$$V(x) := \sup_{\pi. \in \mathscr{A}(x)} \mathbb{E}[U(W_{\mathrm{T}}^{\pi}) | W_0^{\pi} = x]. \tag{3.2}$$

In order to compare the performance of the optimal strategy defined by (3.2) to the one pursued by a technical analyst, we impose constraints on the portfolio weights. Indeed, we will below assume that the technical analyst invests all of his/her wealth either in the stock or in the bond depending upon the moving average signal. We therefore assume that the portfolio weights of the trader pursuing an optimal strategy are also constrained to lie within the interval $[0,1]$ in the absence of short selling.

3.2. The case of general utility functions

In order to compute the constrained optimal wealth of the trader, we use the martingale approach to stochastic control problems as developed by Karatzas, Shreve, Cvitanić, etc. More precisely, we follow and carefully adapt the martingale approach to the well known optimal consumption-portfolio problem studied by Merton (1971). We emphasize that our trader's situation differs from the Merton problem for the following three reasons:

- The drift coefficient of the dynamics of the risky asset return is not constant over time (since it changes at the random time τ).
- We face some subtle measurability issues since the trader's strategy needs to be adapted with respect to the filtration generated by (S_t). As already mentioned, the drift change at the random time τ makes this filtration different from the filtration generated by the Brownian motion (B_t).
- The portfolio weight π is constrained to lie in a finite interval ($[0,1]$).

These three features of our problem make it hard to construct optimal strategies. Indeed, in the case of a general utility function, we need an additional hypothesis described below to prove the existence of an optimal constrained allocation strategy π^*, and to exhibit an abstract representation for the corresponding optimal portfolio process.

As in Karatzas and Shreve (1998), we introduce an auxiliary unconstrained market $\mathscr{M}_v$ defined as follows: Let $\mathscr{D}$ be the subset of $\{\mathscr{F}_t^S\}$ – progressively measurable processes $v : [0, T] \times \Omega \to \mathbb{R}$ such that

$$\mathbb{E} \int_0^{\mathrm{T}} v(t)^- \, \mathrm{d}t < \infty, \quad \text{where } v(t)^- := -\inf(0, v(t)).$$

The bond price process $S^0(v)$ and the stock price $S(v)$ satisfy

$$\frac{\mathrm{d}S_t^0(v)}{S_t^0(v)} = \frac{\mathrm{d}S_t^0}{S_t^0} + v^-(t)\,\mathrm{d}t,$$

$$\frac{\mathrm{d}S_t(v)}{S_t(v)} = \frac{\mathrm{d}S_t}{S_t} + (v(t) + v^-(t))\,\mathrm{d}t.$$

We compute the optimal allocation strategy for each auxiliary unconstrained market driven by a process v (see Proposition 3.1). We conclude with Proposition 3.2 which links the optimal strategy for the constrained problem with the set of optimal strategies for auxiliary unconstrained markets.

For each auxiliary unconstrained market, let $\mathscr{A}(x,v)$ denote the set of admissible strategies, that is,

$$\mathscr{A}(x,v) := \{\pi_\cdot - \mathscr{F}_t^S\text{–progressively measurable process s.t.}$$
$$W_0^{v,\pi} = x, \quad W_t^{v,\pi} > 0 \quad \text{for all} \quad t > 0\}.$$

We have to solve the following problem $\mathscr{P}_v$:

$$V(v,x) := \sup_{\pi_\cdot \in \mathscr{A}(v,x)} \mathbb{E}[U(W_{\mathrm{T}}^{v,\pi})|W_0^{v,\pi} = x],$$

where

$$\frac{\mathrm{d}W_t^{v,\pi}}{W_t^{v,\pi}} = \pi_t \frac{\mathrm{d}S_t(v)}{S_t(v)} + (1-\pi_t)\frac{\mathrm{d}S_t^0(v)}{S_t^0(v)} = \frac{\mathrm{d}W_t^\pi}{W_t^\pi} + (v^-(t) + \pi_t v(t))\,\mathrm{d}t. \tag{3.3}$$

To characterize optimal allocation strategies and their associated wealth level, we need to introduce four processes which are adapted to the filtration $\mathbb{F}^S$ generated by the observed price process.

- The exponential likelihood-ratio process L_t is defined by

$$L_t = \left(\frac{S_t}{S_0}\right)^{\frac{\mu_2-\mu_1}{\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}\left((\mu_2-\mu_1)^2 + 2(\mu_2-\mu_1)\left(\mu_1 - \frac{\sigma^2}{2}\right)\right)t\right\}. \tag{3.4}$$

- The conditional a posteriori probability F_t that the change point has appeared before time t is $F_t := \mathbb{P}(\tau \leqslant t|\mathscr{F}_t^S)$.

$$F_t = \frac{\lambda e^{\lambda t} L_t \int_0^t e^{-\lambda s} L_s^{-1}\,\mathrm{d}s}{1 + \lambda e^{\lambda t} L_t \int_0^t e^{-\lambda s} L_s^{-1}\,\mathrm{d}s} \tag{3.5}$$

- The innovation process $\overline{B}$ is defined by

$$\overline{B}_t = \frac{1}{\sigma}\left(\log(S_t) - \left(\mu_1 - \frac{\sigma^2}{2}\right)t - (\mu_2-\mu_1)\int_0^t F_s\,\mathrm{d}s\right), \quad t \geqslant 0. \tag{3.6}$$

  The process $\overline{B}$ is a Brownian motion for the filtration $\mathbb{F}^S$.
- The exponential process H_t^v:

$$H_t^v = \exp\left(-\int_0^t \left(\frac{\mu_1 - r + v(s)}{\sigma} + \frac{(\mu_2-\mu_1)F_s}{\sigma}\right)\mathrm{d}\overline{B}_s - \frac{1}{2}\int_0^t \left(\frac{\mu_1 - r + v(s)}{\sigma} + \frac{(\mu_2-\mu_1)F_s}{\sigma}\right)^2 \mathrm{d}s\right).$$

Proposition 3.1. *For each $v \in \mathscr{D}$, the optimal wealth is*

$$W_{\mathrm{T}}^{v,*} = (U')^{-1}\left(yH_{\mathrm{T}}^v e^{-rT - \int_0^{\mathrm{T}} v^-(t)\,\mathrm{d}t}\right)$$

$$W_t^{v,*} = \frac{e^{rt + \int_0^t v^-(s)\,\mathrm{d}s}}{H_t^v}\,\mathbb{E}\left(H_{\mathrm{T}}^v e^{-rT - \int_0^{\mathrm{T}} v^-(s)\,\mathrm{d}s}(U')^{-1}\left(yH_{\mathrm{T}}^v e^{-rT - \int_0^{\mathrm{T}} v^-(s)\,\mathrm{d}s}\right)|\mathscr{F}_t^S\right).$$

Moreover, the optimal strategy satisfies

$$\boxed{\pi_t^{v,*} = \sigma^{-1}\left(\frac{\mu_1 - r + (\mu_2 - \mu_1)F_t + v(t)}{\sigma} + \frac{\phi_t}{H_t^v W_t^{v,*} \mathrm{e}^{-rt - \int_0^t v^-(s)\,\mathrm{d}s}}\right),} \tag{3.7}$$

where F_t is defined as in (3.5), y stands for the Lagrange multiplier, that is, such that

$$\mathbb{E}\left[H_{\mathrm{T}}^v \exp\left(-rT - \int_0^{\mathrm{T}} v^-(t)\,\mathrm{d}t\right)(U')^{-1}\left(yH_{\mathrm{T}}^v \exp\left(-r\mathrm{T} - \int_0^{\mathrm{T}} v^-(t)\,\mathrm{d}t\right)\right)\right] = x.$$

and ϕ is a $\mathscr{F}_t^S$ adapted process which satisfies

$$\mathbb{E}\left(H_{\mathrm{T}}^v \mathrm{e}^{-rT - \int_0^{\mathrm{T}} v^-(t)\,\mathrm{d}t}(U')^{-1}\left(yH_{\mathrm{T}}^v \mathrm{e}^{-rT - \int_0^T v^-(t)\,\mathrm{d}t}\right)|\mathscr{F}_t^S\right) = x + \int_0^t \phi_s \mathrm{d}\overline{B}_s.$$

The proof is postponed to Appendix.

In view of (3.3), we observe that for each constrained strategy π, $v^-(t) + \pi_t v(t) \geqslant 0$, and therefore $W_t^{v,\pi} \geqslant W_t^\pi$ and $V(v, x) \geqslant V(x)$. So, $V(x) \leqslant \inf_{v\in\mathscr{D}} V(v,x)$. The following proposition tells us that, if the minimum is attained, then the minimizing auxiliary strategy $\widetilde{v}$ provides the optimal constrained strategy π^*.

Proposition 3.2. *If there exists $\widetilde{v}$ in $\mathscr{D}$ such that*

$$V(\widetilde{v},x) = \inf_{v\in\mathscr{D}} V(v,x), \tag{3.8}$$

then an optimal portfolio $\pi^{*,\tilde{v}}$ for the unconstrained problem $\mathscr{P}_{\tilde{v}}$ is also an optimal portfolio for the constrained original problem $\mathscr{P}$, such that

$$W_t^* = W_t^{\pi^{*,\tilde{v}},\widetilde{v}} \quad \text{and} \quad V(x) = V(\widetilde{v},x). \tag{3.9}$$

An optimal portfolio allocation strategy is

$$\pi_t^* := \sigma^{-1}\left(\frac{\mu_1 - r + (\mu_2 - \mu_1)F_t + \widetilde{v}(t)}{\sigma} + \frac{\phi_t}{H_t^{\widetilde{v}} W_t^* \mathrm{e}^{-rt - \int_0^t \widetilde{v}^-(s)\,\mathrm{d}s}}\right), \tag{3.10}$$

where F_t is defined as in (3.5).

Proof. See the proof in Karatzas and Shreve (1998, p. 275). □

3.3. The case of the logarithmic utility function

When the agent has a logarithmic utility function, we can verify the existence of a $\widetilde{v}$ satisfying (3.8) and explicit the optimal allocation strategy π^* and its associated wealth $W^{*,x}$.

Proposition 3.3. *If $U(\cdot) = log(\cdot)$ and the initial endowment is x, then the optimal wealth process and strategy are*

$$W_t^{*,x} = \frac{x \exp(rt + \int_0^t \widetilde{v}^-(s)\,\mathrm{d}s)}{H_t^{\widetilde{v}}}, \tag{3.11}$$

where

$$\widetilde{v}(t)=\begin{cases}-(\mu_1-r+(\mu_2-\mu_1)F_t) & \textit{if}\ \dfrac{\mu_1-r+(\mu_2-\mu_1)F_t}{\sigma^2}<0,\\ 0 & \textit{if}\ \dfrac{\mu_1-r+(\mu_2-\mu_1)F_t}{\sigma^2}\in[0,\,1],\\ \sigma^2-(\mu_1-r+(\mu_2-\mu_1)F_t) & \textit{otherwise},\end{cases}\tag{3.12}$$

and, as above,

$$\widetilde{v}^-(t)=-\inf(0,\widetilde{v}(t)).$$

In addition,

$$\pi_t^*=\mathrm{proj}_{[0,\,1]}\left(\frac{\mu_1-r+(\mu_2-\mu_1)F_t}{\sigma^2}\right).\tag{3.13}$$

Proof. If $U(x)=\log(x)$ for each $v\in\mathscr{D}$, the solution of the unconstrained problem is

$$\begin{aligned}\pi_t^{v,*}&=\frac{\mu_1-r+(\mu_2-\mu_1)F_t+v(t)}{\sigma^2},\\ W_t^{v,*}&=\frac{x\exp\left(rt+\int_0^t v^-(s)\,\mathrm{d}s\right)}{H_t^v},\\ V(v,x)&=\log x+rT+\mathbb{E}\left[\int_0^T v(t)^-\,\mathrm{d}t\right]+\mathbb{E}\left[\int_0^T\frac{1}{2}\left(\frac{\mu_1-r+(\mu_2-\mu_1)F_t+v(t)}{\sigma}\right)^2\mathrm{d}t\right].\end{aligned}\tag{3.14}$$

Then the process $\widetilde{v}$ defined by (3.12) satisfies

$$\widetilde{v}(t)^-+\frac{1}{2}\left(\frac{\mu_1-r+(\mu_2-\mu_1)F_t+\widetilde{v}(t)}{\sigma}\right)^2=\inf_{v\in\mathscr{D}}\left(v(t)^-+\frac{1}{2}\left(\frac{\mu_1-r+(\mu_2-\mu_1)F_t+v(t)}{\sigma}\right)^2\right).\quad\square$$

Remark 3.4. Optimal strategies for the constrained problem are projections on $[0,1]$ of optimal strategies for the unconstrained problem.

Remark 3.5. In the case of the logarithmic utility function, when t is small and thus smaller than the change time τ with high probability, F_t is close to 0; since, by hypothesis, one also has $\frac{\mu_1-r}{\sigma^2}\leqslant 0$, the optimal risky asset portfolio weight is close to 0 ; after the change time τ, F_t is close to 1, and the optimal risky asset portfolio weight is close to $\min(1,\frac{\mu_2-r}{\sigma^2})$. In both cases, we approximately recover the optimal strategies of the constrained Merton problem with drift parameters equal to μ_1 or μ_2 respectively.

Remark 3.6. Notice that, in view of (3.4), (3.5) and (3.13), π_t^* can be expressed in terms of the prices $(S_u,\ 0\leqslant u\leqslant t)$.

Using the explicit value (3.12) of $\widetilde{v}(t)$, one can obtain an explicit formula for the value function $V^{\widetilde{v}}(x)$ corresponding to the optimal strategy.

The derivations of the optimal portfolio weights and of the expected utility of wealth for the logarithmic investor will serve as benchmarks to compare the performances obtained by the Model and Detect strategy and the strategy pursued by a technical analyst. Indeed, we can write (see Appendix A.3) a lengthy formula for the value function $V(x)$ by using a result due to Yor (see Yor, 1992 or Borodin and Salminen, 2002, formula 1.20.8 p.618). See Blanchet-Scalliet et al., 2005 for further details.

4. Two Model and Detect strategies

The optimal portfolio allocation strategy in the previous section supposes that the trader is allowed to continuously change his/her portfolio allocation. In this section, the trader is allowed to change his/her allocation only once. So, the trader uses an optimal detection procedure to decide when to rebalance his/her portfolio. In order to facilitate the comparison with the performance of the technical analyst's strategy, we here assume that the trader arbitrarily sets the stock weight to $\pi = 0$ before the drift change and subsequently to 1. We continue to suppose that the trader perfectly knows all the parameters of the model.

We consider two detection methods: the first one has been proposed by Karatzas (2003), and the other one has been proposed by Shiryaev (2002). The goal of these two methods is to find a stopping rule Θ which detects the instant τ at which the drift of the stock return changes. We compute the wealth of the trader who uses one of the two Model and Detect strategies. In both cases, the trader puts all of his/her money in the bond until Θ, and in the stock after Θ, thus his wealth satisfies:

$$W_{\mathrm{T}} = \frac{xS^0_{\Theta}}{S_{\Theta}} S_{\mathrm{T}} \mathbb{1}_{(\Theta \leqslant T)} + xS^0_{\mathrm{T}} \mathbb{1}_{(\Theta > \mathrm{T})}.$$

The time Θ is interpreted as the "alarm" time, it can occur before τ (in this case, it corresponds to a "false alarm"), or after τ. So, the amount of time by which the stopping rule Θ misses the true change of drift point τ is given by $|\Theta - \tau|$.

The main mathematical tool used to obtain these two stopping rules is the process F_t, the (conditional) probability that the (unknown) change point appeared before the running time t. For each procedure, the trader decides to invest his/her wealth in the stock when F_t is bigger than a given quantity respectively equal to p^* for the Karatzas' method (see (4.2)) and to A^* for the Shiryaev method (see (4.3)). These two quantities will be defined in Sections 4.1 and 4.2.

With the Karatzas' method, the trader minimizes the expected miss $\mathbb{E}|\Theta - \tau|$. With Shiryayev's one, the trader minimizes $\{\mathbb{P}(\Theta < \tau) + c\mathbb{E}(\Theta - \tau)^+\}$, i.e., he/she does not give the same weight to errors generated by false alarms ($\Theta < \tau$) and errors generated by late alarms ($\Theta \geqslant \tau$).

4.1. Karatzas' method

We first adapt Karatzas's detection method to compute the optimal stopping rule Θ^{K} that minimizes the *expected miss*

$$\mathscr{R}(\Theta) := \mathbb{E}|\Theta - \tau| \tag{4.1}$$

over all stopping rules Θ, when τ is assumed to have the a priori exponential distribution (2.2).

Proposition 4.1. *The stopping rule Θ^K which minimizes the expected miss $\mathbb{E}|\Theta-\tau|$ over all the stopping rules Θ with $\mathbb{E}(\Theta)<\infty$ is*

$$\Theta^{\mathrm{K}} = \inf\{t \geqslant 0 | F_t \geqslant p^*\}, \tag{4.2}$$

where F_t is defined as in (3.5) and p^* is the unique solution in $(\frac{1}{2},1)$ of the equation

$$\int_0^{1/2} \frac{(1-2s)\mathrm{e}^{-\beta/s}}{(1-s)^{2+\beta}} s^{2-\beta}\,\mathrm{d}s = \int_{1/2}^{p^*} \frac{(2s-1)\mathrm{e}^{-\beta/s}}{(1-s)^{2+\beta}} s^{2-\beta}\,\mathrm{d}s,$$

where $\beta = 2\lambda\sigma^2/(\mu_2-\mu_1)^2$.

Proof. We adapt Karatzas's method in Karatzas (2003) to our specific case. Denote by $\mathscr{S}$ the collection of stopping rules $\Theta : \Omega \to [0,\infty)$ such that $\mathbb{E}(\Theta)<\infty$. Rewrite (4.1) as follows:

$$\mathscr{R}(\Theta) = \mathbb{E}[(\Theta-\tau)^+ + (\tau-\Theta)^+] = \mathbb{E}(\Theta-\tau)^+ + \mathbb{E}(\tau) - \mathbb{E}(\Theta\wedge\tau).$$

Then, using (2.2) and the notation (3.5), we get

$$\begin{aligned}\mathscr{R}(\Theta) &= \frac{1}{\lambda} + \mathbb{E}\left(\int_0^{\Theta} \mathbb{1}_{(\tau\leqslant s)}\,\mathrm{d}s - \int_0^{\Theta} \mathbb{1}_{(\tau>s)}\,\mathrm{d}s\right)\\ &= \frac{1}{\lambda} + \mathbb{E}\left(\int_0^{\infty} (2\mathbb{1}_{(\tau\leqslant s)}-1)\mathbb{1}_{(s\leqslant\Theta)}\,\mathrm{d}s\right)\\ &= \frac{1}{\lambda} + 2\mathbb{E}\int_0^{\Theta}\left(F_s-\frac{1}{2}\right)\mathrm{d}s.\end{aligned}$$

We thus obtain

$$\inf_{\Theta\in\mathscr{S}} \mathscr{R}(\Theta) = \frac{1}{\lambda} + 2\inf_{\Theta\in\mathscr{S}} \mathbb{E}\int_0^{\Theta}\left(F_s-\frac{1}{2}\right)\mathrm{d}s.$$

It now remains to follow Karatzas' arguments (see Karatzas, 2003). □

The terminal wealth of a trader who uses Karatzas detection procedure satisfies

$$\begin{aligned}W_{\mathrm{T}} = x\mathrm{e}^{r\Theta^{\mathrm{K}}} \exp\Bigg(&\sigma(B_{\mathrm{T}}-B_{\Theta^{\mathrm{K}}}) + \left(\mu_1-\frac{\sigma^2}{2}\right)(T-\Theta^{\mathrm{K}})\\ &+(\mu_2-\mu_1)[(T-\tau)^+ - (\Theta^{\mathrm{K}}-\tau)^+]\Bigg)\mathbb{1}_{(\Theta^{\mathrm{K}}\leqslant T)} + x\exp(rT)\mathbb{1}_{(\Theta^{\mathrm{K}}>T)}.\end{aligned}$$

4.2. Shiryaev method

The detection method proposed by Shiryaev (namely the **Variant** *B* in Shiryaev (2002)) consists in computing

$$B(c) := \inf_{\Theta}\{\mathbb{P}(\Theta<\tau) + c\mathbb{E}(\Theta-\tau)^+\}$$

and the corresponding optimal stopping time for a given $c>0$.

For all $c > 0$, this time is given by

$$\Theta^S(A^*) := \inf\{t \geqslant 0; F_t \geqslant A^*\} \tag{4.3}$$

where F is the conditional a posteriori probability solution of (A.6) and the parameter A^* is defined as the root in $(0, 1)$ of the equation

$$\int_0^{A^*} \exp\left(-\frac{2\lambda\sigma^2}{(\mu_2-\mu_1)^2}\frac{1}{y}\right)\frac{1}{y(1-y)^2}\left(\frac{y}{1-y}\right)^{\frac{2\lambda\sigma^2}{(\mu_2-\mu_1)^2}} \mathrm{d}y$$
$$= \frac{(\mu_2-\mu_1)^2}{2\sigma^2 c}\exp\left(-\frac{2\lambda\sigma^2}{(\mu_2-\mu_1)^2}\frac{1}{A^*}\right)\left(\frac{A^*}{1-A^*}\right)^{\frac{2\lambda\sigma^2}{(\mu_2-\mu_1)^2}}.$$

For both Karatzas and Shiryaev's detection procedures, we are able to write explicit formulae for the expected utility of terminal wealth, $\mathbb{E}(\log(W_\mathrm{T}))$ (similar to the formula in Proposition 6.1 below). Due to their complexity, these formulae do not allow us to compare the expected utility of terminal wealth associated with the Model and Detect strategies to the one obtained when pursuing the optimal strategy or the technical analysis based strategy. In Section 7, we therefore rely on numerical simulations to make comparisons between the various strategies' performances.

Remark 4.2. Beibel and Lerche (1997) have considered the model (2.1) with $\mu_1 - \frac{\sigma^2}{2} \geqslant r \geqslant \mu_2 - \frac{\sigma^2}{2}$. They have studied the problem of maximizing $\mathbb{E}(\mathrm{e}^{-r\Theta}S_\Theta \mathbb{1}_{(\Theta<\infty)})$ over all stopping times Θ adapted to the filtration generated by (S_t). We do not examine their detection procedure in our study.

5. Mis-specified trading models

In reality, it is extremely difficult to know the parameters characterizing the investment opportunity set exactly. It may be possible to calibrate the first drift coefficient μ_1 and the volatility coefficient σ relatively well owing to historical data, but the value of μ_2 cannot be determined a priori (i.e. before the occurrence of the drift change), and the law of τ cannot be calibrated accurately because of the lack of data associated with τ. It is thus reasonable to assess the impact of estimation risk on the performance of the various model-based detection strategies.

5.1. Mis-specification of the parameters

We consider the case where each parameter is estimated with error. In other words, the trader believes that the stock price satisfies the following stochastic differential equation:

$$\mathrm{d}S_t = S_t(\bar{\mu}_2 + (\bar{\mu}_1 - \bar{\mu}_2)\mathbb{1}_{(t\leqslant\bar{\tau})})\,\mathrm{d}t + \bar{\sigma}S_t\,\mathrm{d}B_t, \tag{5.1}$$

where the law of $\bar{\tau}$ is exponential with parameter $\bar{\lambda}$, while the true stock price is still given by (2.1). Our aim is to study the mis-specified optimal allocation strategy and the mis-specified Model and Detect strategy.

Notation. As in the previous developments without estimation error, S_t denotes the current stock price. We need to compute L_t and F_t (see (3.4) and (3.5)) to apply the optimal allocation strategy and the Model and Detect strategies. We define the approximated quantities computed when the model is mis-specified $\overline{L}_t$ and $\overline{F}_t$ as follows:

$$\overline{L}_t = \exp\left\{\frac{1}{\bar{\sigma}^2}(\bar{\mu}_2 - \bar{\mu}_1)\log(S_t) - \frac{1}{2\bar{\sigma}^2}\left((\bar{\mu}_2 - \bar{\mu}_1)^2 + 2(\bar{\mu}_2 - \bar{\mu}_1)\left(\bar{\mu}_1 - \frac{\bar{\sigma}^2}{2}\right)\right)t\right\},$$

$$\overline{F}_t = \frac{\bar{\lambda}e^{\bar{\lambda}t}\overline{L}_t\int_0^t e^{-\bar{\lambda}s}\overline{L}_s^{-1}\,ds}{1 + \bar{\lambda}e^{\bar{\lambda}t}\overline{L}_t\int_0^t e^{-\bar{\lambda}s}\overline{L}_s^{-1}\,ds}.$$

5.2. On the mis-specified optimal allocation strategy

Observing the stock price S_t, the trader computes a "pseudo optimal" portfolio allocation by using the erroneous parameters $\bar{\mu}_1$, $\bar{\mu}_2$, $\bar{\sigma}$ and $\bar{\lambda}$. Thus, the stock proportion of his/her mis-specified optimal allocation strategy satisfies

$$\bar{\pi}_t^* = \mathrm{proj}_{[0,1]}\frac{\bar{\mu}_1 - r + (\bar{\mu}_2 - \bar{\mu}_1)\overline{F}_t}{\bar{\sigma}^2},$$

and the corresponding wealth satisfies

$$\overline{W}_t^* = e^{rt}\exp\left(\int_0^t \bar{\pi}_u^* d(e^{-ru}S_u)\right).$$

5.3. On mis-specified Model and Detect strategies

The erroneous stopping rule for the Karatzas detection time rule satisfies

$$\overline{\Theta}^K = \inf\{t \geqslant 0, \overline{F}_t \geqslant \bar{p}^*\}$$

where $\bar{p}^*$ is the unique solution in $(\frac{1}{2}, 1)$ of the equation

$$\int_0^{1/2}\frac{(1-2s)e^{-\bar{\beta}/s}}{(1-s)^{2+\bar{\beta}}}s^{2-\bar{\beta}}\,ds = \int_{1/2}^{\bar{p}^*}\frac{(2s-1)e^{-\bar{\beta}/s}}{(1-s)^{2+\bar{\beta}}}s^{2-\bar{\beta}}\,ds$$

with $\bar{\beta} = 2\bar{\lambda}\bar{\sigma}^2/(\bar{\mu}_2 - \bar{\mu}_1)^2$.

The wealth of the Model and Detect trading strategy satisfies

$$\overline{W}_T = xS^0_{\overline{\Theta}^K}\frac{S_T}{S_{\overline{\Theta}^K}}\mathbb{1}_{(\overline{\Theta}^K \leqslant T)} + xS^0_T\mathbb{1}_{(\overline{\Theta}^K > T)}.$$

A similar approach based on the results obtained in Section 4.2 can be followed to compute the erroneous stopping rule and its associated wealth for the Shiryaev detection rule.

6. The chartist investment strategy

6.1. Introduction

Technical analysis makes predictions about the evolution of an asset's price and defines trading rules using only the asset's price (or/and volume) history. Thus, technical analysts compute indicators based on the asset's past transaction prices and volumes.

These indicators are used as trading signals assuming that (see, e.g., the book by Achelis, 2000):

- The price of a stock is governed by the law of supply and demand.
- The stock price evolves according to trends during discernible periods.
- These discernible tendencies repeat themselves in a regular fashion.

A very large number of technical analysis indicators are used by practitioners. In their impressive study, Sullivan et al. (1999) provide a parameterization for 7846 distinct trading rules. Here, we limit ourselves to the simple moving average indicator because it is easy to compute and widely used to detect trend patterns in stock prices. In order to compute its value, one averages the closing prices of the stock during the δ most recent time periods. When prices are trending, this indicator reacts quickly to recent price changes.

6.2. Moving average indicator for the stock prices

Consider a chartist trader who takes decisions at discrete times during the interval $[0, T]$ with time increments $\Delta t = \frac{T}{N}$:

$$0 = t_0 < t_1 < \ldots < t_N = T; \quad t_k = k\Delta t.$$

We denote by $\pi_t \in \{0,1\}$ the proportion of the agent's wealth invested in the risky asset at time t, and by M_t^δ the simple moving average indicator of the prices defined as

$$M_t^\delta = \frac{1}{\delta}\int_{t-\delta}^{t} S_u \,\mathrm{d}u. \tag{6.1}$$

The parameter δ denotes the size of the time window used to compute the moving average.

At time 0, the agent knows the past prices of the stock and has enough data to compute M_0^δ. At each t_n, $n \in [1 \cdots N]$, the chartist follows a very simple trading strategy: all the wealth is invested into the risky asset if the price S_{t_n} is larger than the moving average $M_{t_n}^\delta$. Otherwise, all the wealth is invested into the riskless asset. This portfolio investment strategy is thus analogous to the one followed by the Model and Detect trader.

Consequently,

$$\pi_{t_n} = \mathbb{1}_{(S_{t_n} \geqslant M_{t_n}^\delta)}. \tag{6.2}$$

Denote by x the initial wealth of the trader. The wealth at time t_{n+1} is

$$W_{t_{n+1}} = W_{t_n}\left(\frac{S_{t_{n+1}}}{S_{t_n}}\pi_{t_n} + \frac{S_{t_{n+1}}^0}{S_{t_n}^0}(1-\pi_{t_n})\right),$$

and therefore, since $S_{t_{n+1}}^0 / S_{t_n}^0 = \exp(r\Delta t)$,

$$W_{\mathrm{T}} = x\prod_{n=0}^{N-1}\left[\pi_{t_n}\left(\frac{S_{t_{n+1}}}{S_{t_n}} - \exp(r\Delta t)\right) + \exp(r\Delta t)\right]. \tag{6.3}$$

6.3. The particular case of the logarithmic utility function

We now assume that the chartist trader displays a logarithmic utility function. Then, his expected utility of wealth can be explicitly characterized.

Proposition 6.1. *Consider a technical analyst whose strategy is defined as in (6.2). Then his expected logarithmic utility of wealth satisfies*

$$\begin{aligned}\mathbb{E}\left[\log\left(\frac{W_{\mathrm{T}}\mathrm{e}^{-rT}}{x}\right)\right] &= \left(\mu_2-\frac{\sigma^2}{2}-r\right)Tp_\delta^{(1)}\\ &\quad+\Delta t\left(\mu_2-\frac{\sigma^2}{2}-r\right)\frac{1-\mathrm{e}^{-\lambda T}}{1-\mathrm{e}^{-\lambda\Delta t}}\left((p_\delta^{(2)}-p_\delta^{(1)})\mathrm{e}^{\lambda\delta}+p_\delta^{(3)}\right)\\ &\quad-\Delta t(\mu_2-\mu_1)(\mathrm{e}^{-\lambda\Delta t}-\lambda\Delta t)\frac{1-\mathrm{e}^{-\lambda T}}{1-\mathrm{e}^{-\lambda\Delta t}}p_\delta^{(3)},\end{aligned}$$

where we have set

$$\begin{aligned}p_\delta^{(1)} &= \int_0^\infty\int_y^\infty \frac{z^{\mu_2-3/2}}{2y}\mathrm{e}^{-\frac{(\mu_2/\sigma-\sigma/2)^2\delta}{2}-\frac{(1+z^2)}{2\sigma^2y}}i_{\sigma^2\delta/2}\left(\frac{z}{\sigma^2y}\right)\mathrm{d}z\,\mathrm{d}y,\\ p_\delta^{(2)} &= \int_0^\delta\int_{\mathbb{R}^4}\mathbb{1}_{\left\{\delta y_2\geqslant\frac{z_1}{y_1}+z_2\right\}}\frac{z_2^{\mu_2-3/2}}{2y_2}\mathrm{e}^{-\frac{(\mu_2/\sigma-\sigma/2)^2(\delta-v)}{2}-\frac{(1+z_2^2)}{2\sigma^2y_2}}i_{\sigma^2(\delta-v)/2}\left(\frac{z_2}{\sigma^2y_2}\right)\\ &\quad\times\frac{z_1^{\mu_1-3/2}}{2y_1}\mathrm{e}^{-\frac{(\mu_1/\sigma-\sigma/2)^2v}{2}-\frac{(1+z_1^2)}{2\sigma^2y_1}}i_{\sigma^2v/2}\left(\frac{z_1}{\sigma^2y_1}\right)\mathrm{e}^{-\lambda v}\,\mathrm{d}y_1\,\mathrm{d}z_1\,\mathrm{d}y_2\,\mathrm{d}z_2\,\mathrm{d}v,\\ p_\delta^{(3)} &= \int_0^\infty\int_y^\infty \frac{z^{\mu_1-3/2}}{2y}\mathrm{e}^{-\frac{(\mu_1/\sigma-\sigma/2)^2\delta}{2}-\frac{(1+z^2)}{2\sigma^2y}}i_{\sigma^2\delta/2}\left(\frac{z}{\sigma^2y}\right)\mathrm{d}z\,\mathrm{d}y,\end{aligned}$$

the function i being defined as in (A.11).

Proof. In view of

$$\frac{S_{t_{j+1}}}{S_{t_j}} = \exp\left(\left(\mu_1-\frac{\sigma^2}{2}\right)\Delta t+(\mu_2-\mu_1)\big(t_{j+1}-\max(\tau,t_j)\big)^+ +\sigma(B_{t_{j+1}}-B_{t_j})\right), \tag{6.4}$$

and (6.2), we have

$$\begin{aligned}\mathbb{E}\log\left(\frac{W_{\mathrm{T}}}{x}\right) &= \left(\mu_1-\frac{\sigma^2}{2}-r\right)\Delta t\sum_{j=0}^{N-1}\mathbb{P}\left(S_{t_j}\geqslant M_{t_j}^\delta\right)\\ &\quad+(\mu_2-\mu_1)\sum_{j=0}^{N-1}\mathbb{E}\left\{\mathbb{1}_{(S_{t_j}\geqslant M_{t_j}^\delta)}\big(t_{j+1}-\max(\tau,t_j)\big)^+\right\}\\ &\quad+rT+\sigma\sum_{j=0}^{N-1}\mathbb{E}\left\{\mathbb{1}_{(S_{t_j}\geqslant M_{t_j}^\delta)}(B_{t_{j+1}}-B_{t_j})\right\}.\end{aligned}$$

From here, one obtains the result by lengthy calculations using independence and identity in law arguments, and Lemma A.3 in the Appendix. □

Remark 6.2. Relying on Proposition 6.1, one can use numerical optimization procedures to optimize the choice of δ, the moving average window size. As we will see in the next section, inadequate choices of δ may negatively affect the performance of the technical analyst strategy.

7. A numerical comparison of the various strategies

7.1. Empirical determination of a good windowing

Before turning to a comparison between the various trading strategies, we first show how to optimize the choice of the moving average window size δ by using Proposition 6.1 and deterministic numerical optimization procedures, or by means of Monte Carlo simulations. In this subsection, we present results obtained from Monte Carlo simulations to show that inadequate choices of δ may indeed alter the performance of the technical analyst strategy. For each value of δ we have simulated 500,000 trajectories of the asset price and computed the expected utility of terminal wealth, $\mathbb{E}\log(W_T)$ by a Monte Carlo method. In all our simulations the empirical variance of $\log(W_T)$ is set at 0.04. Thus, the Monte Carlo error on $\mathbb{E}\log(W_T)$ is of the order 5.10^{-4} with probability 0.99. At first, the number of price trajectories used for these simulations may seem too large; however, considered as a function of δ, the quantity $\mathbb{E}\log(W_T)$ varies very slowly, so that we really need a large number of simulations to obtain smooth curves (cf. Fig. 1).

Fig. 1a and b illustrates the relationship between $\mathbb{E}\log(W_T)$ and δ for two different values of the stock returns' volatility. It is clear from these figures that the optimal choice of δ varies with the volatility of the stock returns. When the volatility reaches 0.05, the optimal choice of δ is around 0.3 whereas, when the volatility increases to 0.15, the optimal choice of δ is around 0.8.

On the basis of a comprehensive numerical study performed in Blanchet-Scalliet et al. (2005), we can make the following statements:

- When the volatility decreases, the choice of δ becomes more important: the curve is flat for large volatility levels only.
  - When the volatility is high, the losses decrease when choosing a large window size.
  - In all cases, a too small window length is sub-optimal,

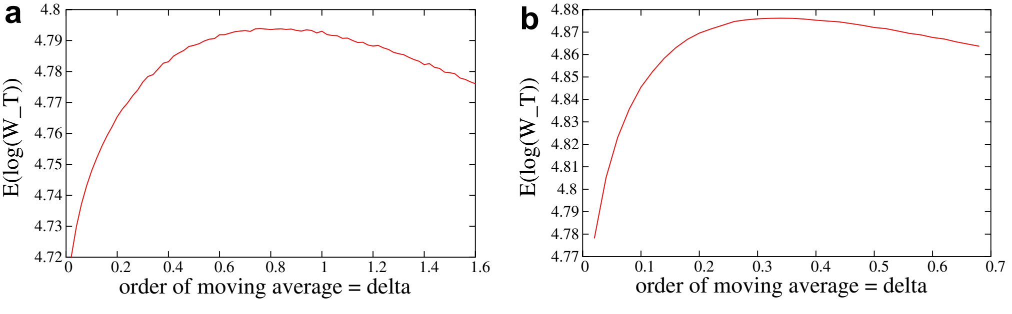

Fig. 1. Expected values of the logarithm of the terminal wealth ($\mathbb{E}(\log(W_T))$) for a trader using the moving average indicator of order δ as a function of δ. (a) $\mu_1 = -0.2$; $\lambda = 2$; $r = 0.0$; $\mu_2 = 0.2$; $\sigma = 0.15$; $T = 2.0$. (b) $\mu_1 = -0.2$; $\lambda = 2$; $r = 0.0$; $\mu_2 = 0.2$; $\sigma = 0.05$; $T = 2.0$.

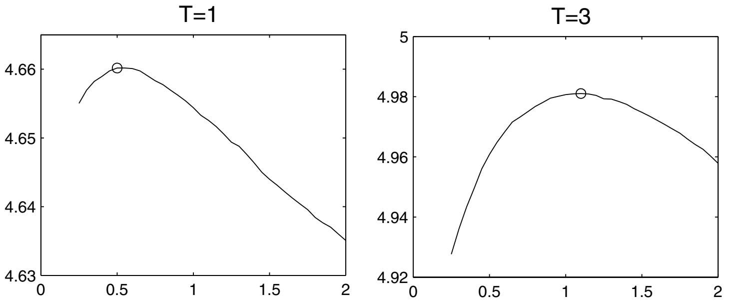

Fig. 2. Expected values of the logarithm of the terminal wealth ($\mathbb{E}(\log(W_T))$) for a trader using the moving average indicator of order δ as a function of δ: the dependence of the optimal δ on the time horizon T.

  – As $|\mu_1|$ or μ_2 decreases (respectively, increases), the choice of δ becomes less (respectively, more) important.
- The choice of δ depends on the arrival rate λ.
- The parameter μ_2 has a strong effect on the importance of the window length. In Blanchet-Scalliet et al. (2005), it is shown that the curves become flatter when μ_2 increases. This observation confirms the intuition; if the future drift is not large enough, the detection of τ will be more difficult.

Fig. 2 shows that, when $\lambda = 2.0$, the time horizon has a significant effect on the optimal choice of δ. When the time horizon is small, for example when $T = 1$, one has better to underestimate δ than to overestimate it. When T is large, one has better to overestimate δ. Of course, the specific values for T highly depend on the chosen level of λ.

7.2. Mis-specified mathematical strategies vs technical analysis

We can now address our main question: Is it better to invest according to a mathematical Model and Detect strategy based on a mis-specified model, or according to a strategy which is model – free? Due to the analytical complexity of all the explicit formulae that we have obtained for the various expected utilities of terminal wealth, we have not yet succeeded to find a mathematical answer to this question (even in asymptotic cases, when $\frac{\mu_2-\mu_1}{\sigma^2}$ is large, e.g.). We therefore present numerical results obtained from Monte Carlo simulations to illustrate our comparisons.[2]

Fig. 3 shows that, despite a large degree of parameter mis-specification, the two model and detect strategies represented in panels (a) and (b) yield a good performance and clearly out-perform the technical analyst. However, it can be seen from Fig. 3c that the optimal trading strategy with mis-specified parameters is out-performed by the technical analysis strategy. This result suggests that statistical detection techniques or technical analysis approaches could be more attractive when the parameters are mis-specified.

We have run, a number of other simulations which all confirm that the technical analyst may out-perform the mis-specified optimal allocation strategy but not the mis-specified

[2] A more complete set of simulations on the performance of the mathematical and the chartist strategies can be found in Blanchet-Scalliet et al. (2005).

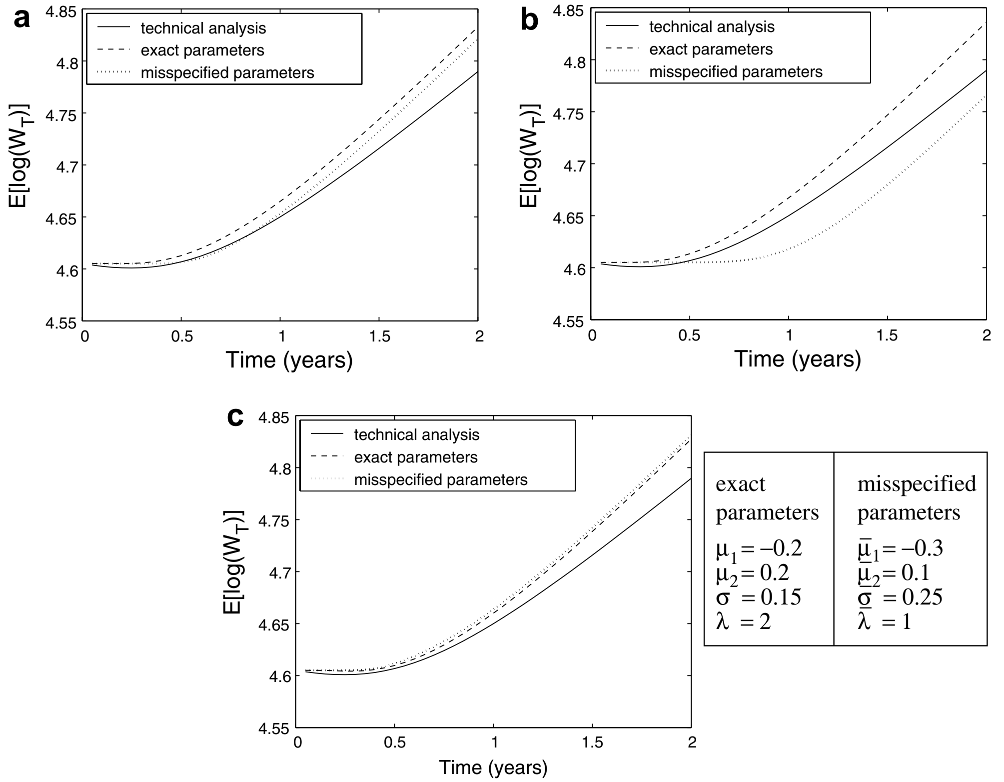

Fig. 3. Comparison of the expected values of the logarithm of wealth when the traders use strategies with exact and mis-specified parameters. (a) Model and Detect strategy based on Karatzas' estimation, (b) Model and Detect strategy based on Shiryaev's estimation and (c) optimal strategy.

Model and Detect strategies. These simulations also show that when μ_2/μ_1 decreases, the performances of well specified and mis-specified Model and Detect strategies decrease.[3]

In conclusion, our numerical study suggests that there is no universal solution to the problem of parameter mis-specification. It seems that when the drifts are high in absolute terms and, in particular, when the upward drift is high, the performance of the Model and Detect strategies can be quite robust and superior to the one of the chartist trading strategy. However, their performance deteriorates rapidly when λ is strongly mis-specified and/or when the upward drift is not very high. Since the second drift is in fact the hardest to estimate due to the fact that we lack a priori information, we recommend caution before asserting that Model and Detect strategies are superior to the technical trading rule.[4] Indeed, the Model and Detect strategies only offer a clear comparative advantage over the chartist trading rule in the presence of strong expected future trends.

[3] The results are available from the authors upon request.

[4] In reality, the technical analyst does not know the length of the optimal window, thus his/her strategy is not free of mis-specification either. However, as we show in Fig. 2, the performance of the technical analyst has only weak sensitivity with respect to the window length.

8. Conclusion and perspectives

In this study, we have compared the performance of trading strategies based on possibly mis-specified mathematical models used to detect the time of the change in the drift of the stock return with a trading strategy based on the simple moving average rule. We have explicitly computed the trader's expected logarithmic utility of wealth for the various trading strategies. Unfortunately, these explicit formulae were not propitious to mathematical comparisons. We have therefore relied on Monte Carlo numerical experiments, and observed from these experiments that under parameter mis-specification, the technical analysis technique out-performs the optimal allocation strategy but not the model and detect strategies. The latter strategies dominance is confirmed under parameter mis-specification as long as the two stock returns' drifts are high in absolute terms.

This study provides a first step towards building a rigorous mathematical framework in which chartist and mathematical model based trading strategies can be compared. We are extending this research along several dimensions. First, we examine and model the performance of other chartist based trading rules (such as filter rules, point and figure charts, etc.). Second, we consider modeling the more realistic case where there are multiple changes in the drift of the stock returns: we examine the case where the instantaneous expected rate of return of the stock changes at the jump times of a Poisson's process, and the value of this rate after each time change is unknown. We follow two new directions to tackle these questions: jointly with B. de Saporta (INRIA), we use stochastic control techniques for switching models and, jointly with M. Martinez (INRIA) and S. Rubenthaler (University of Nice Sophia Antipolis), we use filtering techniques (Martinez et al., 2006). Finally, it would be worth extending our conceptual framework to the more realistic case where the mathematical and the chartist strategies' performances account for market frictions.

Acknowledgement

Financial support by the National Centre of Competence in Research "Financial valuation and Risk Management" (NCCR FINRISK) is gratefully acknowledged. NCCR-FINRISK is a research program supported by the Swiss National Science Foundation. Awa Diop gratefully acknowledges financial support from the National Centre of Competence in Research "Conceptual Issues in Financial Risk Management" (NCCR–FINRISK).

Appendix A

A.1. An explicit expression for the process F_t

Lemma A.1. *The conditional a posteriori probability F_t that the change point has appeared before time t, that is, $F_t := \mathbb{P}(\tau \leqslant t|\mathscr{F}_t^S)$, is*

$$F_t = \frac{\lambda e^{\lambda t} L_t \int_0^t e^{-\lambda s} L_s^{-1}\, ds}{1 + \lambda e^{\lambda t} L_t \int_0^t e^{-\lambda s} L_s^{-1}\, ds} \tag{A.1}$$

where

$$L_t = \left(\frac{S_t}{S_0}\right)^{\frac{\mu_2-\mu_1}{\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2}\left((\mu_2-\mu_1)^2 + 2(\mu_2-\mu_1)\left(\mu_1 - \frac{\sigma^2}{2}\right)\right)t\right\}. \tag{A.2}$$

Proof. In order to compute F_t, we start with a probability space $(\Omega, \mathscr{F}, \mathbb{Q})$ that can support both a standard Brownian motion $\widetilde{B}$ and independent random variable τ: $\Omega \to [0, \infty)$ with distribution $\mathbb{Q}[\tau > t] = \mathrm{e}^{-\lambda t}$ for $t \geqslant 0$; we denote by $(\mathscr{G}_t)_{t\geqslant 0}$ the filtration generated by τ and $\tilde{B}$, which means that $\mathscr{G}_t := \sigma(\tau, \widetilde{B}_s; 0 \leqslant s \leqslant t)$. In view of Girsanov's theorem, the process

$$B_t = \widetilde{B}_t - \frac{(\mu_2-\mu_1)}{\sigma}\int_0^t \mathbb{1}_{(\tau\leqslant s)}\,\mathrm{d}s$$

is a $(\mathscr{G}_t)_{t\geqslant 0}$-Brownian motion under the measure of probability $\mathbb{P}$ such that

$$\begin{aligned}
\left.\frac{\mathrm{d}\mathbb{P}}{\mathrm{d}\mathbb{Q}}\right|_{\mathscr{G}_t} &= \exp\left\{\int_0^t \frac{1}{\sigma}(\mu_2-\mu_1)\mathbb{1}_{(\tau\leqslant s)}\,\mathrm{d}\widetilde{B}_s - \frac{1}{2\sigma^2}\int_0^t (\mu_2-\mu_1)^2\mathbb{1}_{(\tau\leqslant s)}\,\mathrm{d}s\right\}\\
&= \exp\left\{\frac{1}{\sigma}(\mu_2-\mu_1)(\widetilde{B}_t - \widetilde{B}_\tau)\mathbb{1}_{(\tau\leqslant t)} - \frac{1}{2\sigma^2}(\mu_2-\mu_1)^2(t-\tau)^+\right\}\\
&= \exp\left\{\frac{1}{\sigma}(\mu_2-\mu_1)(\widetilde{B}_t - \widetilde{B}_\tau) - \frac{1}{2\sigma^2}(\mu_2-\mu_1)^2(t-\tau)\right\}\mathbb{1}_{(\tau\leqslant t)} + \mathbb{1}_{(\tau>t)}.
\end{aligned}$$

We observe that

$$\widetilde{B}_t = \frac{1}{\sigma}\left(\log\left(\frac{S_t}{S_0}\right) - \left(\mu_1 - \frac{\sigma^2}{2}\right)t\right). \tag{A.3}$$

Observe that under $\mathbb{Q}$, the random variable τ is independent of the $(\mathscr{G}_t; t \geqslant 0)$-Brownian motion $\widetilde{B}$ and thus of R. Thus, the above expression of $\mathrm{d}\mathbb{P}/\mathrm{d}\mathbb{Q}$ can be written as follows:

$$\left.\frac{\mathrm{d}\mathbb{P}}{\mathrm{d}\mathbb{Q}}\right|_{\mathscr{G}_t} = \frac{L_t}{L_\tau}\mathbb{1}_{(\tau\leqslant t)} + \mathbb{1}_{(\tau>t)} := Z_t, \tag{A.4}$$

where L_t is defined as in (3.4). We now check that, on the probability space $(\Omega, \mathscr{F}, \mathbb{P})$, we have the same model as the one presented in the introduction. Indeed, the random variable τ is $\mathscr{G}_0$ -measurable; then, under $\mathbb{P}$, τ is independent of the $(\mathscr{G}_t)_{t\geqslant 0}$-Brownian motion B and we have $\mathbb{P}[\tau > t] = \mathbb{E}_{\mathbb{Q}}[Z_0\mathbb{1}_{\tau>t}] = \mathbb{Q}[\tau > t]$. Using the Bayes rule, one gets

$$F_t = \mathbb{P}[\tau \leqslant t|\mathscr{F}_t^S] = \frac{\mathbb{E}_{\mathbb{Q}}\left[Z_t\mathbb{1}_{(\tau\leqslant t)}|\mathscr{F}_t^S\right]}{\mathbb{E}_{\mathbb{Q}}\left[Z_t|\mathscr{F}_t^S\right]}. \tag{A.5}$$

Using now the independence of $\widetilde{B}$ and τ under $\mathbb{Q}$, one gets

$$\mathbb{E}_{\mathbb{Q}}\left[Z_t|\mathscr{F}_t^S\right] = \mathbb{E}_{\mathbb{Q}}\left[\frac{L_t}{L_\tau}\mathbb{1}_{(\tau\leqslant t)} + \mathbb{1}_{(\tau>t)}|\mathscr{F}_t^S\right] = \int_0^t \frac{L_t}{L_s}\lambda\mathrm{e}^{-\lambda s}\,\mathrm{d}s + \mathrm{e}^{-\lambda t}.$$

On the other hand, we get

$$\mathbb{E}_{\mathbb{Q}}\left[Z_t\mathbb{1}_{(\tau\leqslant t)}|\mathscr{F}_t^S\right] = \int_0^t \frac{L_t}{L_s}\lambda\mathrm{e}^{-\lambda s}\,\mathrm{d}s.$$

Then, going back to (A.5), we get (3.5).

Moreover, we can show that the process $(F_t)_{t\geqslant 0}$ satisfies the following stochastic differential equation

$$\boxed{\mathrm{d}F_t = \lambda(1-F_t)\,\mathrm{d}t + \frac{(\mu_2-\mu_1)}{\sigma}F_t(1-F_t)\,\mathrm{d}\overline{B}_t,} \tag{A.6}$$

where

$$\overline{B}_t = \frac{1}{\sigma}\left(\log(S_t) - \left(\mu_1 - \frac{\sigma^2}{2}\right)t - (\mu_2-\mu_1)\int_0^t F_s\,\mathrm{d}s\right), \quad t \geqslant 0, \tag{A.7}$$

is the innovation process. Indeed, set $V_t := \frac{F_t}{1-F_t}$; an easy computation leads to

$$V_t = \int_0^t \lambda \frac{L_t}{L_s} \mathrm{e}^{\lambda(t-s)}\,\mathrm{d}s.$$

From

$$\mathrm{d}L_t = L_t\left(\frac{1}{\sigma^2}(\mu_2-\mu_1)\,\mathrm{d}(\log S_t) - \frac{1}{\sigma^2}(\mu_2-\mu_1)\left(\mu_1 - \frac{\sigma^2}{2}\right)\mathrm{d}t\right)$$

we deduce

$$\mathrm{d}V_t = \left\{\lambda + V_t\left(\lambda - \frac{1}{\sigma^2}(\mu_2-\mu_1)\left(\mu_1 - \frac{\sigma^2}{2}\right)\right)\right\}\mathrm{d}t + \frac{1}{\sigma^2}(\mu_2-\mu_1)V_t\,\mathrm{d}(\log S_t),$$
$$V_0 = 0.$$

We finally apply Itô's formula to the process $F_t = \frac{V_t}{1+V_t}$, and get the stochastic differential Eq. (A.6). Notice that $\overline{B}_t$ defined as in (3.6) is a $(\mathscr{F}_t^S)_{t\geqslant 0}$-Brownian motion. □

We conclude with a result useful to apply the martingale representation theorem in the next subsection.

Lemma A.2. *The filtration generated by the observations $(\mathscr{F}_\cdot^S)$ is equal to the filtration generated by the innovation process $(\overline{B}_t; t \geqslant 0)$. In particular, each $(\mathscr{F}_\cdot^S)$ martingale M admits a representation as*

$$M_t = M_0 + \int_0^t \phi_s\,\mathrm{d}\overline{B}_s,$$

where ϕ is an $(\mathscr{F}_\cdot^S)$ adapted process.

Proof. Thanks to (3.6), $\overline{B}_t$ is $(\mathscr{F}_\cdot^S)$ adapted. Conversely, we write (3.6) as:

$$\mathrm{d}(\log S_t) = \left(\left(\mu_1 - \frac{\sigma^2}{2}\right) + (\mu_2-\mu_1)F_t\right)\mathrm{d}t + \sigma\,\mathrm{d}\overline{B}_t, \tag{A.8}$$

Thanks to (A.6), F is $(\mathscr{F}_\cdot^{\overline{B}})$ adapted. and we conclude that the process $S_\cdot(=\exp(R_\cdot))$ is also $(\mathscr{F}_\cdot^{\overline{B}})$ adapted. And so $(\mathscr{F}_\cdot^{\overline{B}}) = (\mathscr{F}_\cdot^S)$. □

A.2. Proof of proposition 3.1

We follow Karatzas's method (see for example Karatzas, 1997). For $\pi \in \mathscr{A}(x, v)$, remember that

$$\frac{\mathrm{d}W_t^{\nu,\pi}}{W_t^{\nu,\pi}} = (r + \nu^-(t))\,\mathrm{d}t + \pi_t((\mu_1 - r) + (\mu_2 - \mu_1)F_t + \nu(t))\,\mathrm{d}t + \pi_t\sigma\mathrm{d}\overline{B}_t.$$

Define

$$\gamma_t := (\mu_1 - r) + (u_2 - \mu_1)F_t + \nu(t)$$

$$\overline{W}_t^{\nu,\pi} := W_t^{\nu,\pi} \exp\left(-rt - \int_0^t \nu^-(s)\,\mathrm{d}s\right).$$

We thus have

$$\frac{\mathrm{d}\overline{W}_t^{\nu,\pi}}{\overline{W}_t^{\nu,\pi}} = \pi_t\gamma_t\,\mathrm{d}t + \pi_t\sigma\,\mathrm{d}\overline{B}_t,$$

$$\overline{W}_t^{\nu,\pi} = x\exp\left(\int_0^t \left(\pi_s\gamma_s - \frac{1}{2}\sigma^2\pi_s^2\right)\mathrm{d}s + \int_0^t \sigma\pi_s\,\mathrm{d}\overline{B}_s\right).$$

We now search an exponential martingale M_t (independent of π) such that $\widetilde{W}_t^{\nu,\pi} = \overline{W}_t^{\nu,\pi}M_t$ is an exponential martingale. Set

$$M_t = \exp\left(\int_0^t \phi_s\mathrm{d}\overline{B}_s - \frac{1}{2}\int_0^t \phi_s^2\,\mathrm{d}s\right).$$

Then ϕ needs to satisfy

$$-\frac{1}{2}(\phi_s + \pi_s\sigma)^2 = \pi_s\gamma_s - \frac{1}{2}\phi_s^2 - \frac{1}{2}\sigma^2\pi_s^2,$$

from which $\phi_s = -\frac{\gamma_s}{\sigma}$ and $M_t = H_t^\nu$. Thus, for all π,

$$\frac{\mathrm{d}\widetilde{W}_t^{\nu,\pi}}{\widetilde{W}_t^{\nu,\pi}} = \left(\sigma\pi_t - \frac{\gamma_t}{\sigma}\right)\mathrm{d}\overline{B}_t. \tag{A.9}$$

Therefore the process $(\widetilde{W}_t^{\nu,\pi}, 0 \leqslant t \leqslant T)$ is a non-negative $(\mathscr{F}_t^S, \mathbb{P})$-local martingale and so a supermartingale. Consequently,

$$\mathbb{E}\left[H_\mathrm{T}^\nu W_\mathrm{T}^{\nu,\pi}\exp\left(-rT - \int_0^T \nu^-(t)\,\mathrm{d}t\right)\right] \leqslant x.$$

We now introduce the *convex dual* of $U(\cdot)$:

$$\widetilde{U}(y) := \max_{0<x<\infty}[U(x) - xy], y > 0.$$

Using a duality method, we obtain: for all $\nu > 0$, $\pi \in \mathscr{A}(x,\nu)$, $y \geqslant 0$,

$$\begin{aligned}
\mathbb{E}[U(W_\mathrm{T}^{\nu,\pi})] &\leqslant \mathbb{E}\left[\widetilde{U}\left(yH_\mathrm{T}^\nu\exp\left(-rT - \int_0^T \nu^-(t)\,\mathrm{d}t\right)\right)\right] \\
&\quad + y\mathbb{E}\left(H_\mathrm{T}^\nu W_\mathrm{T}^{\nu,\pi}\exp\left(-rT - \int_0^T \nu^-(t)\,\mathrm{d}t\right)\right) \\
&\leqslant \mathbb{E}\left[\widetilde{U}\left(yH_\mathrm{T}^\nu\exp\left(-rT - \int_0^T \nu^-(t)\,\mathrm{d}t\right)\right)\right] + yx.
\end{aligned}$$

This inequality is an equality if and only if

$$\begin{cases} W_{\mathrm{T}}^{v,\pi} = (U')^{-1}\left(yH_{\mathrm{T}}^{v}\exp(-rT - \int_0^T v^-(t)\,\mathrm{d}t)\right), \\ \mathbb{E}\left[H_{\mathrm{T}}^{v}W_{\mathrm{T}}^{v,\pi}\exp\left(-rT - \int_0^T v^-(t)\,\mathrm{d}t\right)\right] = x. \end{cases}$$

The coefficient y is introduced to satisfy the constraint.

Now, we need to verify that there exists a portfolio such that the process

$$X_t := \frac{\mathbb{E}\left[H_{\mathrm{T}}^{v}\exp(-rT - \int_0^T v^-(s)\,\mathrm{d}s)(U')^{-1}\left(yH_{\mathrm{T}}^{v}\exp(-rT - \int_0^T v^-(s)\,\mathrm{d}s)\right)\big|\mathscr{F}_t^S\right]}{H_t^v\exp(-rt - \int_0^t v^-(s)\,\mathrm{d}s)}$$

is its wealth process. We use the martingale representation property of the Brownian filtration in order to find the optimal strategy π^*. Indeed, there exists a predictable process ϕ such that

$$\mathbb{E}\left(H_{\mathrm{T}}^{v}\mathrm{e}^{-rT-\int_0^T v^-(t)\,\mathrm{d}t}(U')^{-1}\left(yH_{\mathrm{T}}^{v}\mathrm{e}^{-rT-\int_0^T v^-(t)\,\mathrm{d}t}\right)/\mathscr{F}_t^S\right) = x + \int_0^t \phi_s\,\mathrm{d}\overline{B}_s.$$

In particular, with the notation $\widetilde{X}_t = X_tH_t^v\exp(-rt - \int_0^t v^-(s)\,\mathrm{d}s)$, we obtain

$$\mathrm{d}\widetilde{X}_t = \phi_t\,\mathrm{d}\overline{B}_t.$$

Consider the strategy

$$\pi_t^* = \sigma^{-1}\left(\frac{\mu_1 - r + (\mu_2 - \mu_1)F_t + v(t)}{\sigma} + \frac{\phi_t}{X_tH_t^v\mathrm{e}^{-rt-\int_0^t v^-(u)du}}\right).$$

In view of (A.9), we have

$$\frac{\mathrm{d}\widetilde{W}_t^{v,\pi^*}}{\widetilde{W}_t^{v,\pi^*}} = \frac{\phi_t}{\widetilde{X}_t}\,\mathrm{d}\overline{B}_t = \frac{\mathrm{d}\widetilde{X}_t}{\widetilde{X}_t}.$$

By uniqueness arguments, we obtain $X_t = W_t^{v,\pi^*}$.

A.3. Joint law of geometric Brownian motion and its integral

Lemma A.3. *Let B be a real Brownian motion. Let $\sigma > 0$ and v be in $\mathbb{R}$. Let G be a geometric Brownian motion*:

$$G_s = \exp(\sigma^2 vs + \sigma B_s).$$

It holds that

$$\mathbb{P}\left(\int_0^t G_s\,\mathrm{d}s \in \mathrm{d}y; G_t \in \mathrm{d}z\right) = \frac{z^{v-1}}{2y}\mathrm{e}^{-\frac{v^2\sigma^2 t}{2} - \frac{(1+z^2)}{2\sigma^2 y}}i_{\frac{\sigma^2 t}{2}}\left(\frac{z}{\sigma^2 y}\right)\mathrm{d}y\,\mathrm{d}z, \tag{A.10}$$

where

$$i_y(z) := \frac{z\mathrm{e}^{\pi^2/4y}}{\pi\sqrt{\pi y}}\int_0^\infty \exp\left(-z\cosh u - u^2/4y\right)\sinh u\sin(\pi u/2y)\,\mathrm{d}u. \tag{A.11}$$

Yor has obtained this last result in Yor (1992) (see also Borodin and Salminen, 2002).

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