Matita NG: reduction and type-checking

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Outline

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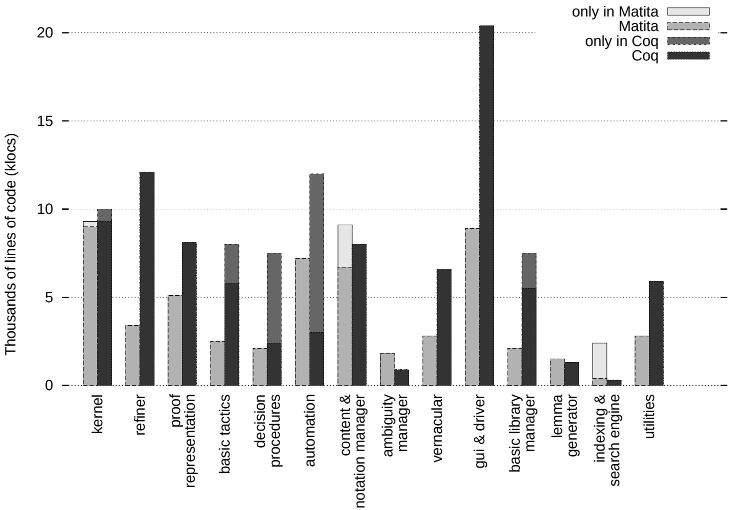
1. Matita NG
2. Reduction and Type Checking
3. Conclusions



only in Matita
Matita
only in Coq
Coq
Thousands of lines of code (klocs)
0
5
10
15
20
kernel
refiner
proof representation
basic tactics
decision procedures
automation
content & notation manager
ambiguity manager
vernacular
gui & driver
basic library manager
lemma generator
indexing & search engine
utilities

Towards Matita 1.xx (Matita NG)

Motivations:

- smaller code size
- simpler code size, easier to maintain and debug
- fix wrong design decisions; improve design decisions; try new design decisions
- experiment with new features (e.g. proof irrelevance)
- completely change the look&feel

Plan:

- entirely re-implement the system from inside out
- provide back&forth translation towards the old components (for immediate testing)
- side effect: generalize all logic independent components

Towards Matita 1.xx (Matita NG)

Already done:

- Reduction and Type Checking
  code: 2,783/7,783 = 36%, functions: 62/100 = 62%
  Special Issue on Interactive Proving and Proof Checking, Sadhana

Almost done:

- Unification and Refinement (Type Inference)
  code: 2,572/4,885 = 53%, functions: 23/41 = 56%

Work in progress:

- Tactic engine and tactics (huge improvement, but. . .)

Future work:

- Library management, consistency management, session management
- User interface

Outline

The Calculus of (Co)Inductive Constructions

$$
\begin{array}{rcl}
t & ::= & \lambda v : t.t \mid (t\ t) \mid \Pi v : t.t \mid \texttt{Let}\ v : t := t\ \texttt{in}\ t \mid x \mid s \mid c \\
 & \mid & \overline{\{i : t := \overline{k : t}\}} \mid co\overline{\{i : t := \overline{k : t}\}} \\
 & \mid & t.i \mid t.k \mid t.\texttt{Match}\ t\ \texttt{return}\ t\ \texttt{with}\ \vec{t} \\
 & \mid & \overline{\{f : t := t\}}.f_i \mid co\overline{\{g : t := t\}}.g_i \\
 & \mid & ?_i[\bar{t}] \\
s & ::= & \textbf{Prop} \mid \textbf{Set} \mid \textbf{Type}_i \\
d & ::= & c : t := t \mid c : t
\end{array}
$$

Universes: checked

Reduction: $\beta + \zeta + \delta + \iota +$ unfold $+$ co-unfold $+$ meta-subst

Conversion:

structural for (co)inductive types and (co)recursive functions (up to permutation); nominal for declarations; none for definitions

The Calculus of (Co)Inductive Constructions in Coq

$$
\begin{array}{lcl}
t & ::= & \lambda v : t.t \mid (t\ t) \mid \Pi v : t.t \mid \texttt{Let}\ v : t := t\ \texttt{in}\ t \mid x \mid s \mid c \\
 & \mid & i \mid k \mid \texttt{Match}\ t\ \texttt{in}\ i\ \texttt{return}\ t\ \texttt{with}\ \vec{t} \\
 & \mid & \{\overline{f : t := t}\}.f_i \mid co\{\overline{g : t := t}\}.g_i \\
 & \mid & ?_i[\bar{t}] \\
s & ::= & \textbf{Prop} \mid \textbf{Set} \mid \textbf{Max}\{\overline{\textbf{Type}_q \mid \textbf{Succ}(\textbf{Type}_q)}\} \\
d & ::= & c : t := t \mid c : t \mid \overline{\lambda x : t}.\{\overline{i : t := \overline{k : t}}\} \mid \overline{\lambda x : t}.co\{\overline{i : t := \overline{k : t}}\}
\end{array}
$$

Universes: inferred (constraint programming), algebraic

Reduction: $\beta + \zeta + \delta + \iota +$ unfold $+$ co-unfold $+$ meta-substitution

Conversion (ignoring modules):

structural for (co)recursive functions (up to permutation)

nominal for (co)inductive types and declarations;

(none for definitions);

The Calculus of (Co)Inductive Constructions in Matita

$$
\begin{array}{rcl}
t & ::= & \lambda v : t.t \mid (t\ t) \mid \Pi v : t.t \mid \texttt{Let}\ v : t := t\ \texttt{in}\ t \mid x \mid s \mid c \\
 & \mid & i \mid k \mid \texttt{Match}\ t\ \texttt{in}\ i\ \texttt{return}\ t\ \texttt{with}\ \vec{t} \\
 & \mid & f_i \mid g_i \\
 & \mid & ?_i[s, \bar{t}] \mid ?_i(s, n) \\
s & ::= & \textbf{Prop} \mid \textbf{Set} \mid \textbf{Max}\{\overline{\textbf{Type}_u \mid \textbf{Succ}(\textbf{Type}_u)}\} \\
d & ::= & c : t := t \mid c : t \mid \overline{\lambda x : t}.\{\overline{i : t := \overline{k : t}}\} \mid \overline{\lambda x : t}.co\{\overline{i : t := \overline{k : t}}\} \\
 & \mid & \overline{\{f : t := t\}} \mid co\overline{\{f : t := t\}}
\end{array}
$$

Universes: checked, user declared, algebraic

Reduction: $\beta + \zeta + \delta + \iota +$ unfold $+$ co-unfold $+$ meta-substitution

Conversion:

nominal for declarations, (co)inductive types, (co)-recursive functions; (none for definitions)

Non first-order (co)recursive definitions

First-order recursive definitions:

$$(\lambda x.\overline{\{f : T := t\}}.f_i)\ M \triangleright \overline{\{f : T[M/x] := t[M/x]\}}.f_i$$

Non first-order recursive definitions:

$$(\lambda x.f_i\ t[x])\ M \triangleright f_i\ t[M]$$

I.e. together with nominal conversion, this makes a closure!

Non first-order (co)recursive definitions

First-order recursive definitions:

$$\begin{array}{lll} \overline{\{f : T := t\}}.f_i\ k & \rhd & t_i[\overline{\{f : T := t\}}/\overline{f}]\ k \\ & \rhd & M[\overline{\{f : T[N[k]] := t[N[k]]\}}] \end{array}$$

Non first-order recursive definitions:

$$f_i\ M\ k \rhd t_i\ M\ k \rhd L[t_j\ P[k]]$$

Non first-order (co)recursive definitions

Pros (so far):

- Reduction machine with recursive environments (major speed up, help the GC)
- Greatly simplified conversion checks
- No `simplify` tactic (URRAH!)
- No artificial duplication of top-level mutual recursive definitions (i.e. for all i, $f_i : T_i := \overline{\{f : T := t\}}.f_i$)

Cons (so far):

- Less conversion (seem useless)
- No nested definitions (but difficult to reason on)

Non first-order (co)recursive definitions

λ-lifting at work:

```
let rec f n :=
 match n with
    O => O
  | S m =>
     let rec g x :=
      match x with
         O => n
       | S k => g k + f m
     in
      g m
```

```
let rec g n f x :=
 match x with
    O => n
  | S k => g k + f n
```

```
let rec f n :=
 match n with
    O => O
  | S m => g n f m
```

But the r.h.s. is NOT accepted by Coq's guardedness conditions $\Rightarrow$ in Matita recursive definitions can be passed around to other recursive definitions

Non first-order (co)recursive definitions

can λ-lifting do this?

```
(f:(let rec g n :=
     match n with O => O | S m => g m in g y) -> T)
 (x: let rec g n :=
      match n with O => O+O | S m => g m in g y)
```

$$\Downarrow$$

```
  let rec g k n :=
   match n with O => k | S m => g n m
  ...
  (f : g O y -> T) (x: g (O+O) y)
```

Non first-order (co)recursive definitions

can λ-lifting do this (up to conversion)?

i.e.

$$
\begin{array}{lll}
M_1 & \lhd & (\lambda x.\mathtt{let\ rec}\ f\ n := \ldots\ \mathtt{in}\ f)\ N_1 \\
M_2 & \lhd & (\lambda x.\mathtt{let\ rec}\ f\ n := \ldots\ \mathtt{in}\ f)\ N_2
\end{array}
$$

Non first-order (co)recursive definitions

Achievements:

1. Incomplete algorithm to map Coq λ-terms into new ones
   - Claim: we are functionally complete (???)
   - Is type-preserving λ-lifting a decidable problem?
2. Extended positivity checks to allow passing (co)recursive functions around
   - Something I am ashamed of (at least in public. . .)
   - Accepts a (slightly) more understandable class of definitions
   - Still some work (makes the code more complex) to accept a reasonable class of (co)-recursive definitions over non (co)-recursive types

Checked, algebraic universes

$$\frac{}{\mathbf{Max}\{\mathbf{Type}_u\} : \mathbf{Max}\{\mathbf{Succ}(\mathbf{Type}_u)\}}$$

$$\frac{S : \mathbf{Max}\{\overline{u_1}\} \qquad T : \mathbf{Max}\{\overline{u_2}\}}{\Pi x : S.T : \mathbf{Max}\{\overline{u_1 @ u_2}\}\}}$$

$$\frac{f : s_2 \rightarrow T \qquad x : s_1 \qquad s_1 \leq s_2}{f\ x : T}$$

Checked, algebraic universes

$$\frac{\forall i,j.\ u_i \preceq v_j}{\textbf{Max}\{\overline{u}\} \leq \textbf{Max}\{\overline{v}\}}$$

$$\frac{u_i \leq v_j}{\textbf{Succ}(u_i) \preceq \textbf{Succ}(v_j)} \qquad \frac{u \leq v}{u \preceq \textbf{Succ}(v)} \qquad \frac{u < v}{\textbf{Succ}(u) \preceq v}$$

$$\frac{u < w \in E \qquad w \leq v}{u \leq v} \qquad \frac{}{u \leq u} \qquad \frac{u \leq w \in E \qquad w \leq v}{u \leq v}$$

$$\frac{u < w \in E \qquad w \leq v}{u < v} \qquad \frac{u \leq w \in E \qquad w < v}{u < v}$$

Aciclicity: $\nexists u, v.\ u < v \wedge v < u$

Checked, algebraic universes

Pros:

- The universe graph is very small, aciclicity check very quick and done once
- Customazibale PTS (also w.r.t impredicativity, computational content, etc.)
- Universe errors are localized and immediately given
- Major reduction in code size, complexity and efficiency
- Makes predicative mathematicians (Sambin) happy
- Easy to lift universes (at the library level only)
- The user must care about universes

Cons:

- The user must care about universes
- Cannot take successor of non user provided universe

Compacts local contexts (explicit substitutions) for metavariables

Metasenv: $\Gamma_i \vdash ?_i : T_i$

Subst: $\Gamma_i \vdash ?_i : T_i := t_i$

Occurrences: $\Delta \vdash ?_i[\bar{t}] : T_i[\bar{t}/\bar{x}]$ where $x_i : t_i \in \Gamma_i$

Example:

$\vdash ?_1 : A \rightarrow B \rightarrow C$

```
intros (x y);
```

$x : A, y : B \vdash ?_2 : C$

$\vdash ?_1 := \lambda x : A.\lambda y : B.?_2[x, y]$

Example:

$y : A \vdash ?_1 : B$

$(\lambda x : A.?_1[x])\ M \triangleright ?_1[M]$

Example:

$y : A \vdash ?_1 : A \times A := (a, a)$

$?_1[M] \triangleright (M, M)$

Compacts local contexts (explicit substitutions) for metavariables

Canonical contexts (in the metasenv/subst) are usually large

Most of the time local contexts are [`Rel k+1`,...,`Rel k+n`]

Major space/time optimization, improved sharing:
represent them as $[k, n]$

Major drawback:
must efficient code, greater complexity and code size

Outline

Conclusions (1/2)

Changes to the calculus:

- nominal, top-level only (co)recursive definitions
- (co)recursive definitions can be passed to other (co)recursive definitions
- algebraic universes (already in Coq)

Changes to the implementation:

- checked algebraic universes
- compact representation of explicit substitutions for metavariables

Conclusions (2/2)

Top-level (co)recursive definitions:

- major reduction in code size
- reduction/conversion speed-up
- recursive environments for reduction machines
- simplification under control

Checked algebraic universes:

- major reduction in code size and simplification of data structures
- major speed up
- customizable PTS
- understandable and localized universe errors

References

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