Mesh and Fluid Dynamics

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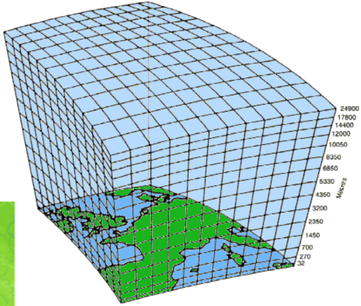
Motivations

- **Aim : Numerical Simulations for Fluid Dynamics.**

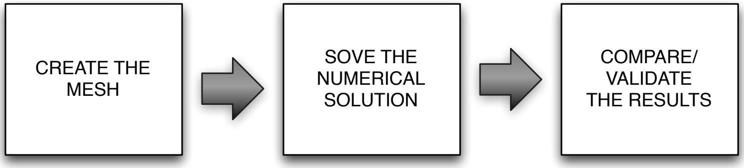
Contents

Principle of the Numerical Simulation :

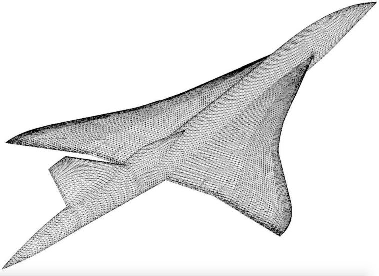


Numerical Simulation

Principle of the Numerical Simulation :



Mesh : Definition



Let $\mathcal{S}$ be a finite collection of points.
A **Simplicial Recovering** from $\mathcal{S}$ is a set $\mathcal{T}_r$ of elements K (triangles or tetrahedra), such that :

Definition (Simplical Recovery)

1. *The collection of vertices of $\mathcal{T}_r$ is $\mathcal{S}$.*
2. $\forall K \in \mathcal{T}_r, \mathring{K} \neq \emptyset.$
3. $\cup_{K \in \mathcal{T}_r} K \subset \mathrm{Conv}(\mathcal{S})$

A such recovering is conform. This is a **Triangulation** if in addition

Definition (Triangulation)

$1 + 2 + 3 +$

- $\forall K_1, K_2 \in \mathcal{T}_r, K_1 \cap K_2 =$ *either* $\emptyset$ *or the joint vertex, edge, face.*

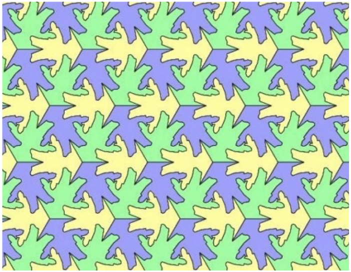
Let Ω be a domain we want to mesh. $\overline{\Omega}$ is a given constraint.
A conform **Mesh** for Ω is a set $\mathcal{M}$ of elements K (triangles or tetrahedra) which satisfies :

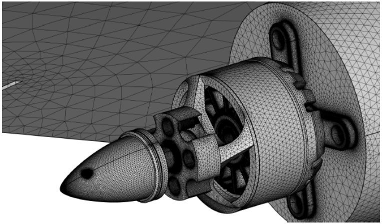
Definition (Mesh)

1. $\forall K \in \mathcal{M}, \mathring{K} \neq \emptyset$.
2. $\forall K_1, K_2 \in \mathcal{M}, K_1 \cap K_2 =$ *either* $\emptyset$ *or the joint vertex, edge, face.*
3. $\overline{\Omega} \subset \mathcal{M}$.

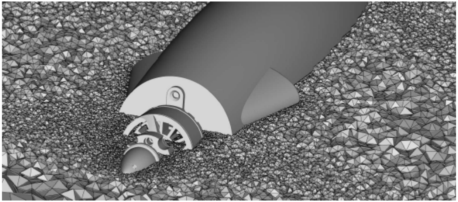
Remark : In practise, $\mathcal{M}$ is a geometric approximation of Ω.

Folkloric Meshes

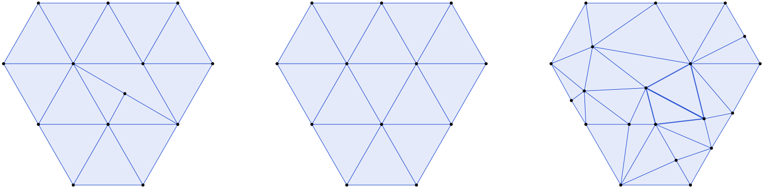




Mesh of a complex surface



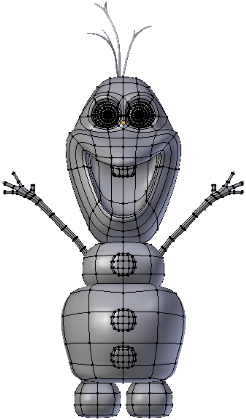
Mesh of the fluid around the body



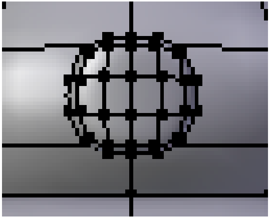
non conform *conform and structured* *conform and unstructured*

Mesh $\neq$ Synthetic image

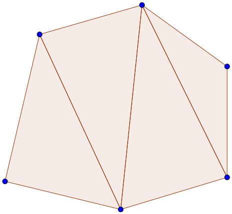
Famous character



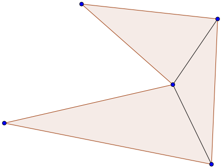
3D-Modeling



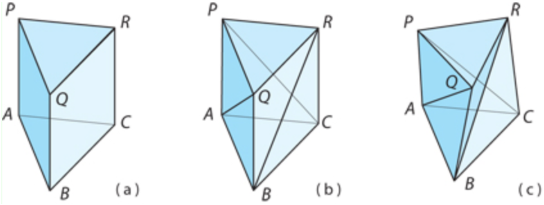
Button details

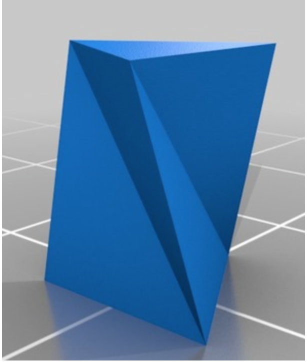


Convex polygon



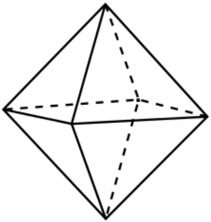
Concave polygon

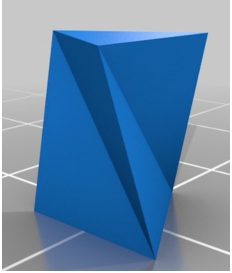


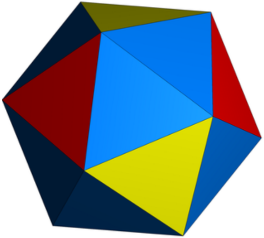


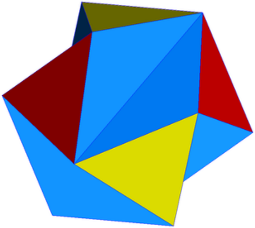
SCHÖNHARDT POLYHEDRON

Non-convex polyhedra combinatorially equiv. to regular polyhedra [Rambau 2005]

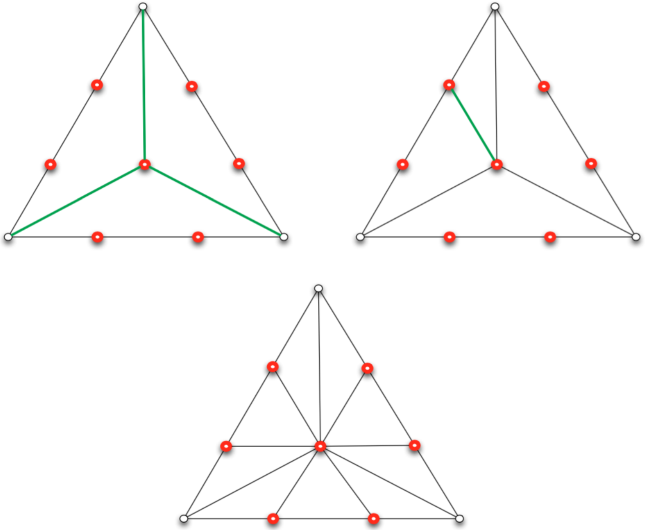




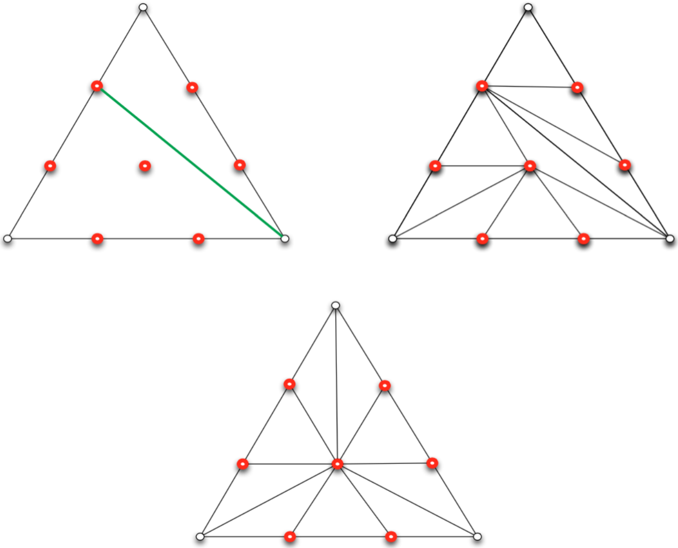




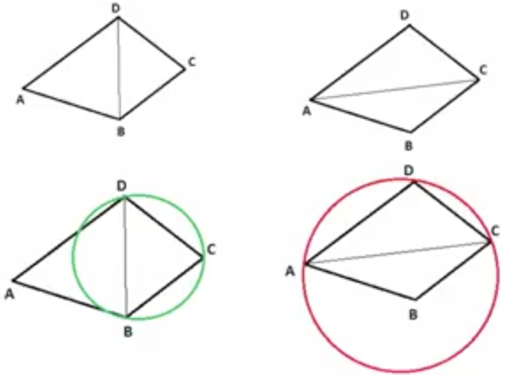
Top : *Octahedron/Schönhardt polyhedron* **Bottom** : *Icosahedron/Jessen polyhedron*



Mesh : Creation



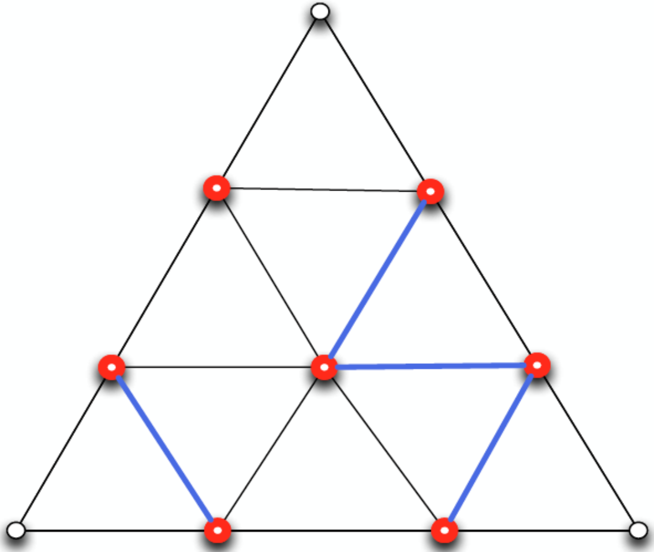
Mesh : Quality



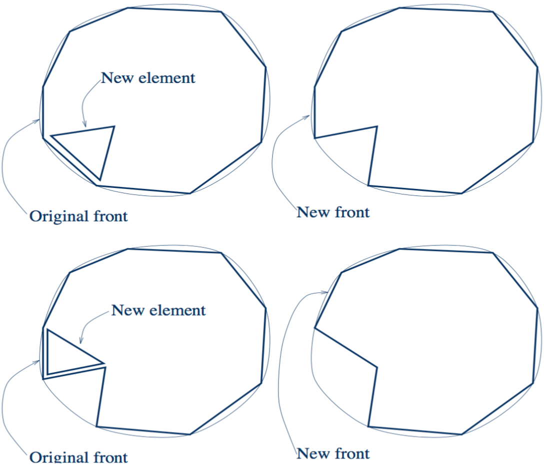
the less distorted option:
$\angle A + \angle C < 180°$
Vertex A lies outside the circumcircle passing through through vertices B, C and D

the more distorted option
$\angle B + \angle D > 180°$.
Vertex B lies inside the circumcircle passing through vertices A, C and D

Mesh : Quality



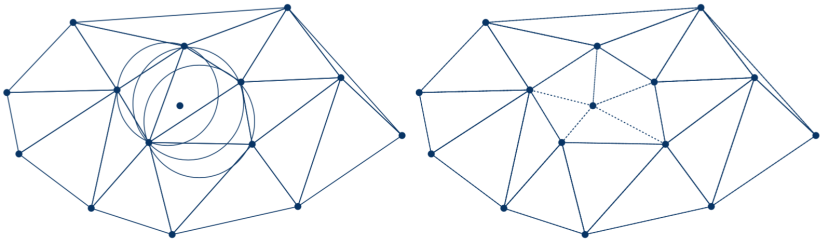
- Advancing Front Mesh Generation Method



- Delaunay Mesh Generation Method

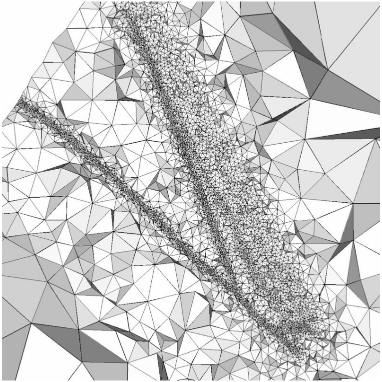
Definition (Delaunay triangulation)

A Delaunay triangulation for a set $\mathcal{S}$ of points in a plane is a triangulation $\mathcal{T}_r$ such that no point in $\mathcal{S}$ is inside the circumcircle of any triangle in $\mathcal{T}_r$.

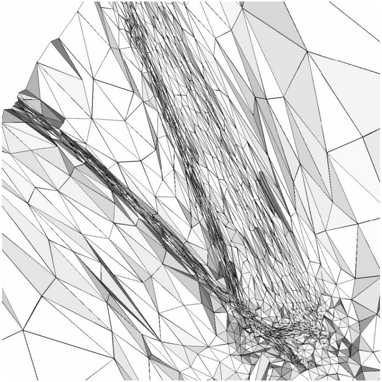


Mesh : Creation

Anisotropic Mesh

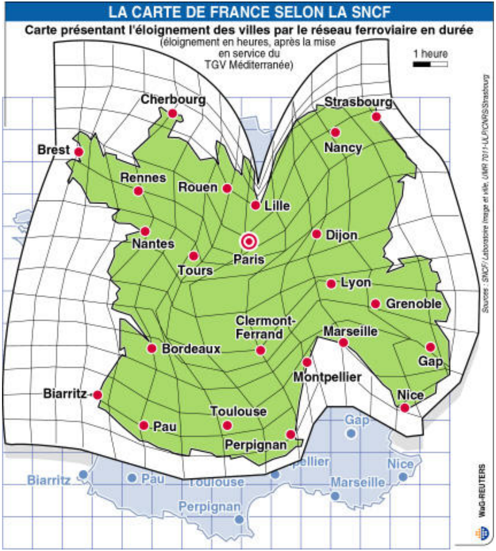


Isotropic Mesh



Anisotropic Mesh

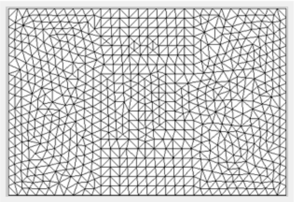
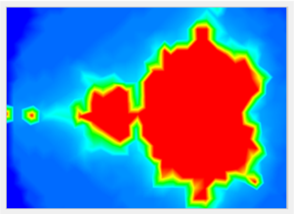
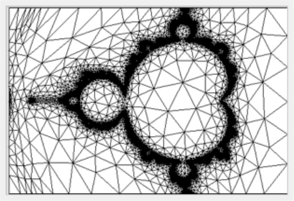
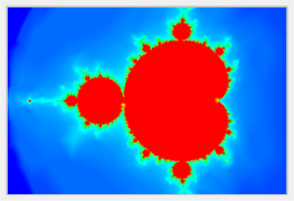
Idea of adapted mesh



Mesh

- need to capture the details of one solution

  $\Rightarrow$ **need to adapt the mesh to the solution** .

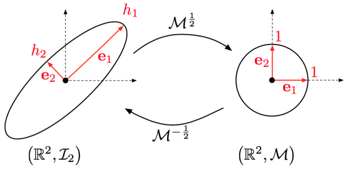
Main idea : introduce the use of metrics field, and notion of unit mesh.

[George, Hecht and Vallet., Adv Eng. Software 1991]

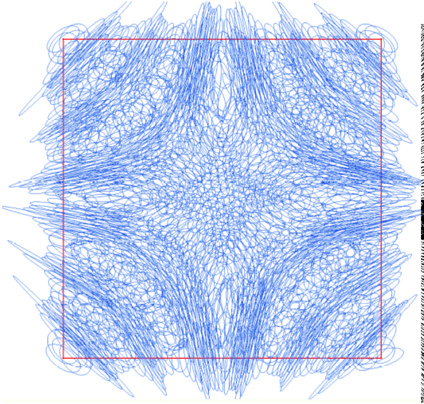
- Riemannian metric space: $\mathcal{M}$: $d \times d$ symmetric definite positive matrix

$$\langle u, v \rangle_{\mathcal{M}} = {}^t\, \mathbf{u}\mathcal{M}\mathbf{v} \Rightarrow \ell_{\mathcal{M}}(\mathbf{a}, \mathbf{b}) = \int_{\mathbf{0}}^{\mathbf{1}} \sqrt{{}^t\mathbf{ab}\ \mathcal{M}(\mathbf{a} + \mathbf{t}\mathbf{ab})\ \mathbf{ab}}\ \mathrm{d}\mathbf{t}$$

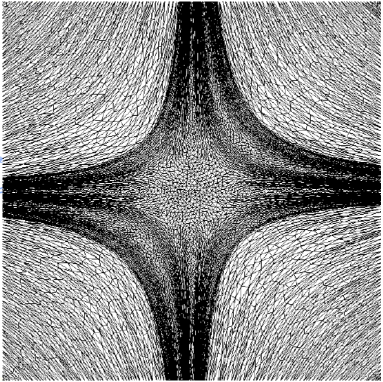
$$|K|_{\mathcal{M}} = \int_K \sqrt{\det \mathcal{M}}\ \mathrm{d}|K|$$



continuous **Metric Field**$\rightarrow$ discrete **Mesh**.



Metric Field



Corresponding Mesh

Deriving the best mesh to compute the characteristics of a given solution w in space

[Tam et al.,CMAME 2000], [Picasso, SIAMJSC 2003], [Formaggia et al, ANM 2004], [Frey and Alauzet CMAME 2005], [Gruau and Coupez, CMAME 2005], [Huang, JCP 2005], [Compere et al., 2007]

- **Discrete** mesh adaptation problem :

Find $\mathcal{H}_{L^p}^{opt}$ having N vertices such that

$$\mathcal{H}_{L^p}^{opt} = \mathrm{Argmin}_{\mathcal{H}} ||u - \Pi_h u||_{\mathcal{H},\mathbf{L}^p(\Omega)}$$

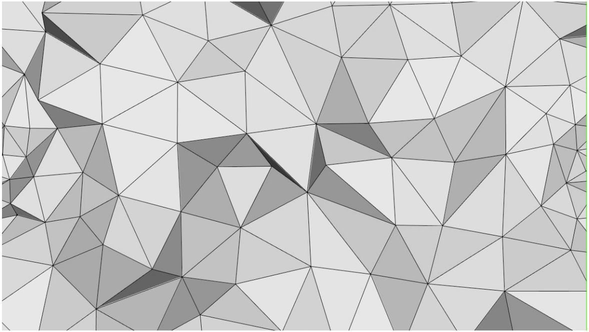
- Well-posed **Continuous** mesh adaptation problem :

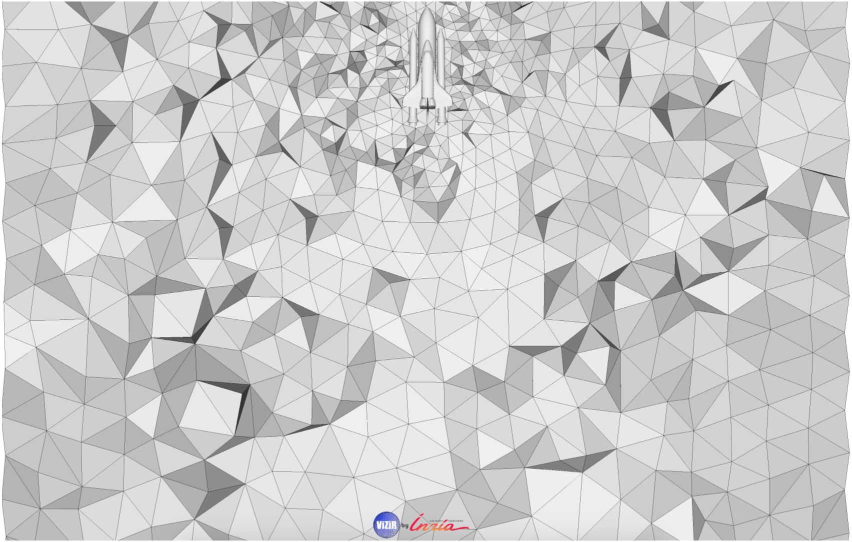
Find $\mathcal{M}_{\mathbf{L^p}}^{\mathbf{opt}}$ of complexity N such that

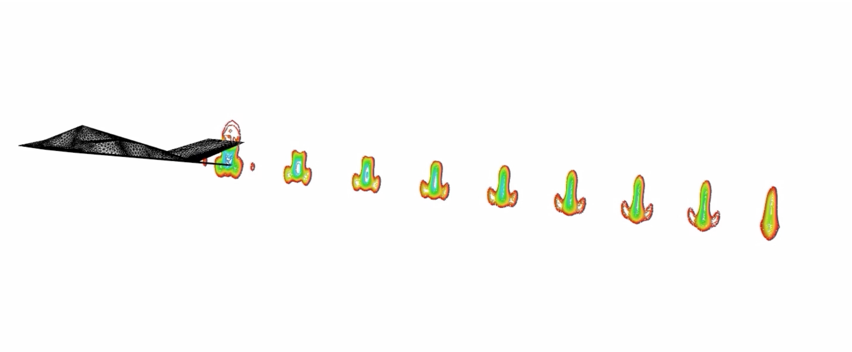
$$E_{L^p}(\mathcal{M}_{L^p}^{opt}) = \min_{\mathcal{M}} \left(\int_\Omega \mathrm{Trace}(\mathcal{M}(\mathbf{x})^{-\frac{1}{2}} |H_u(\mathbf{x})| \mathcal{M}(\mathbf{x})^{-\frac{1}{2}})^p \mathrm{d}\mathbf{x} \right)^{\frac{1}{p}}$$

$\Rightarrow$ Solved by variational calculus

- Two planes moved at Mach 0.4 inside an inert air.
- The planes are translated and rotating
- 50 sub intervals and 3 adaptation loops
- Total space time complexity: $36,000,000$ vertices, average mesh size: $732,000$ vertices, $80,000$ timesteps




VIZIR by Inria



Thank you for your attention