
State-dependent M/G/1 type queuing analysis for congestion control in data networks

Eitan Altman *Kostya Avrachenkov* ***Chadi Barakat***

Rudesindo Nunez Queija

INRIA - Sophia Antipolis, France

CWI - The Netherlands

IEEE INFOCOM

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Outline

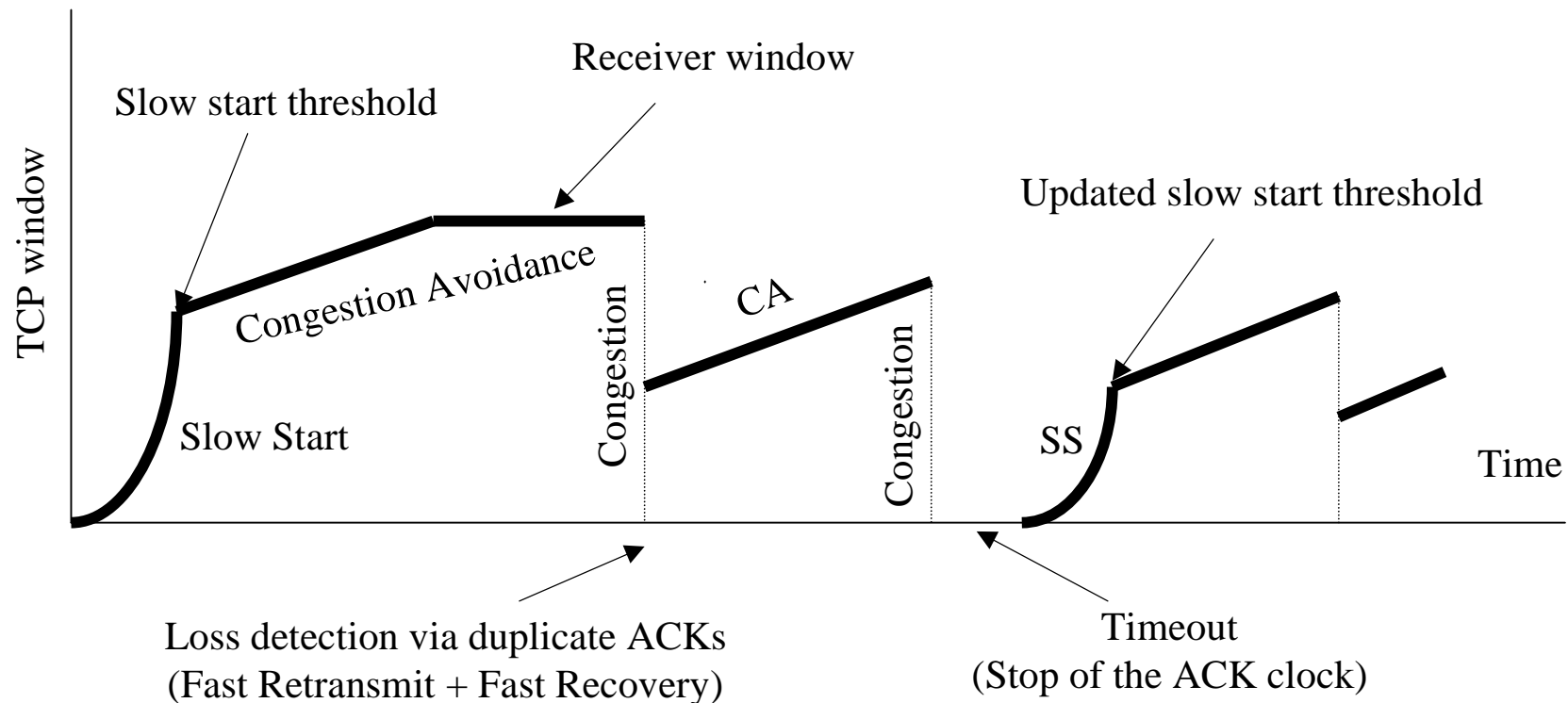
- Congestion control modeling.
- Our model and its analysis.
- Numerical and experimental results.
- Concluding remarks.

Congestion control in data networks

- ➡ **Objective:** Efficient use and fair sharing of network resources.
- ➡ Widely used algorithm:
 - ➡ Probe the network by linearly increasing the transmission rate.
 - ➡ Divide the transmission rate by a constant factor (*typically 2*) when the network becomes congested.
 - ➡ Congestion is detected either by an explicit signal sent by the network, or by inferring mechanisms at the sources.
- ➡ Henceforth, consider as a reference the TCP protocol, widely used for congestion control in the Internet ...

TCP mechanisms

- ☞ A window-based flow control protocol.
- ☞ Use the loss of packets to detect network congestion.

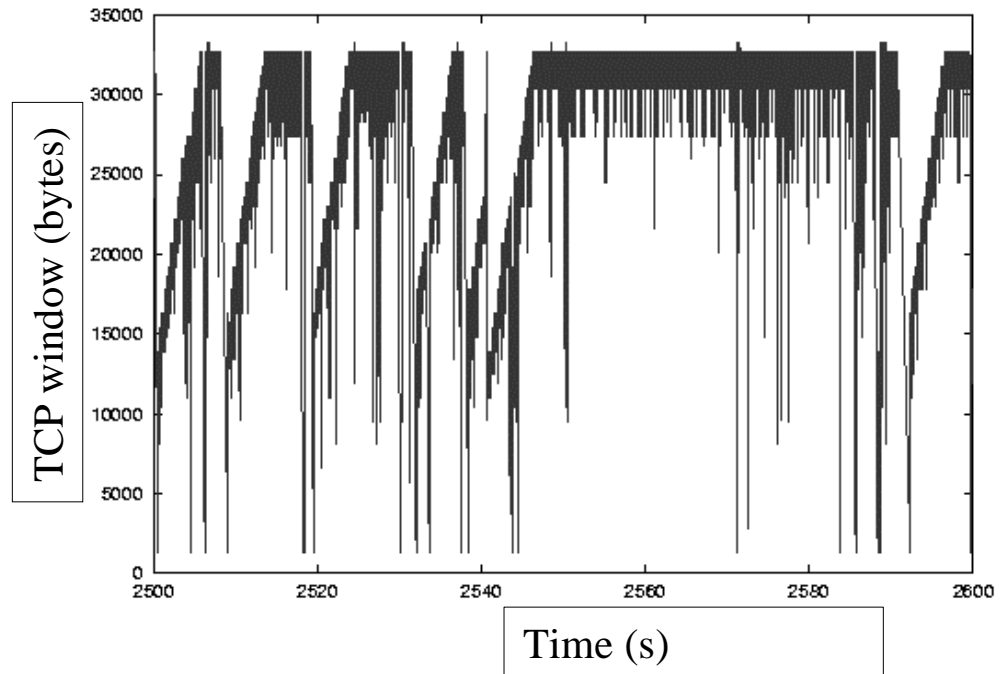
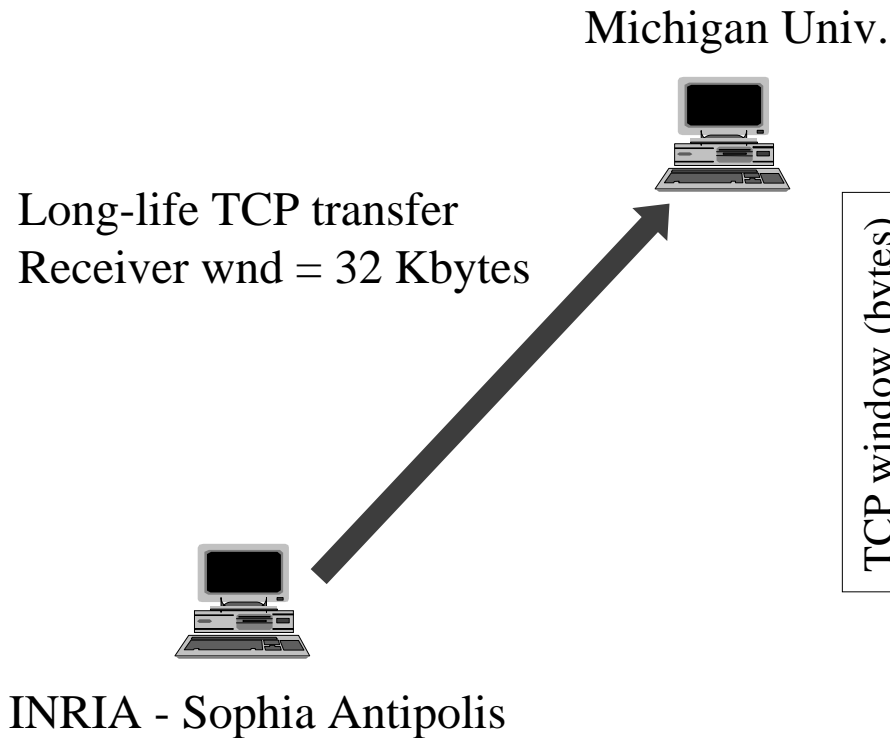


Congestion control modeling

- ☞ **Main objective:** Calculation of the average transmission rate (*the throughput*).
 - ☞ Useful for understanding the behavior of the congestion control and for the design of new applications and protocols (e.g., mechanisms for routers).
- ☞ **Requirements:**
 - ☞ Model for the variation of the transmission rate between congestion events and during congestion.
 - ☞ Model for the appearance of congestion events.
- ☞ **Literature:**
 - ☞ General models for congestion events, but
 - ☞ No exact expressions of the throughput when the rate is limited ...

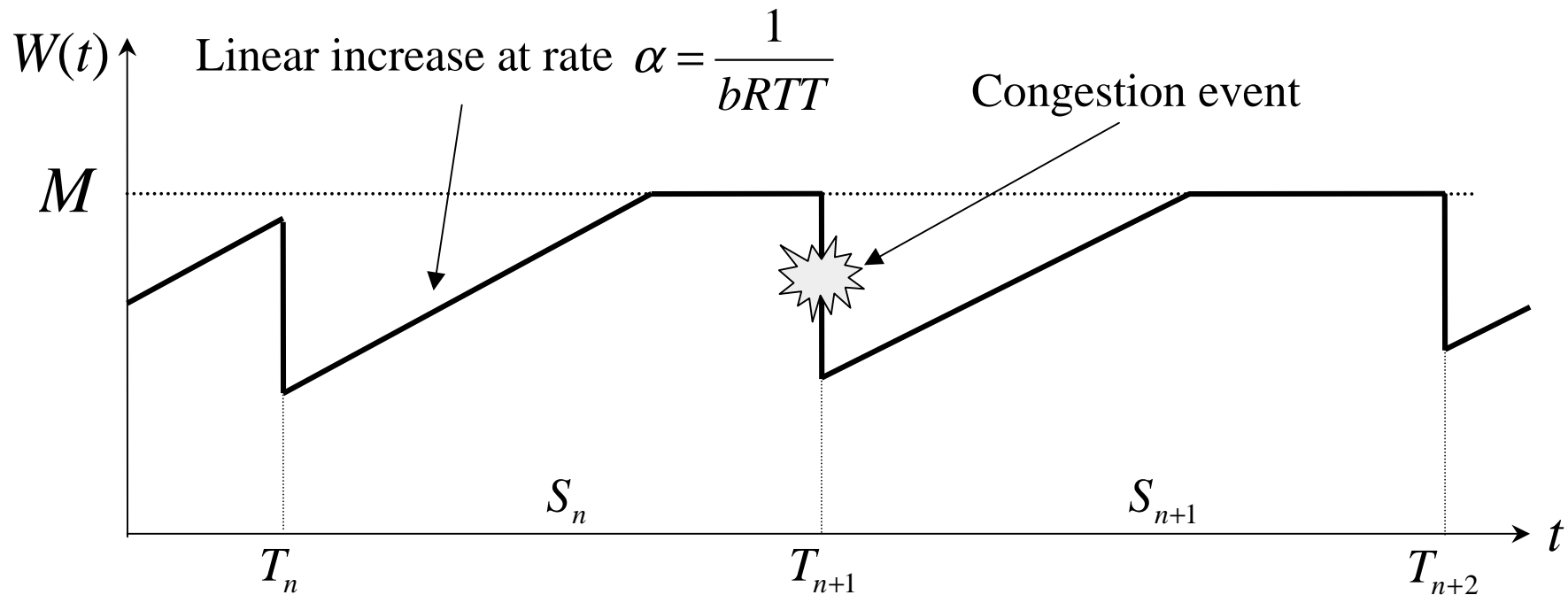
Limitation of the transmission rate

Caused by the receiver window in case of TCP ...



Our model for congestion control

☞ A fluid model for the variation of TCP window:



☞ Use the techniques in [Altman,Avrachenkov,Barakat,SIGCOMM'00] to account for timeouts and the discrete nature of TCP.

Our model for congestion control (Ctd.)

⇒ At the n -th congestion event, N_n divisions of the window by a factor $\gamma > 1$

$$W(T_n +) = \frac{W(T_n -)}{\gamma^{N_n}} \quad \{N_n\} \text{ form an i.i.d. sequence with,}$$
$$Q(z) = \sum_{k=1}^{\infty} z^k \mathbf{P}\{N_n = k\}$$

⇒ Useful when the congestion lasts for multiple consecutive RTTs, or

⇒ For future versions of TCP that reduce the window as a function of the congestion level (e.g., number of packets dropped during congestion).

⇒ Congestion events (i.e., the instants $\{T_n\}$) occur according to a homogenous Poisson process of intensity λ .

The analysis

☞ **Theorem:** The window of the TCP connection converges to the same stationary regime for any initial state.

☞ **Outputs of the analysis:**

☞ $F(x)$ = PDF of $W(t)$ in the stationary regime.

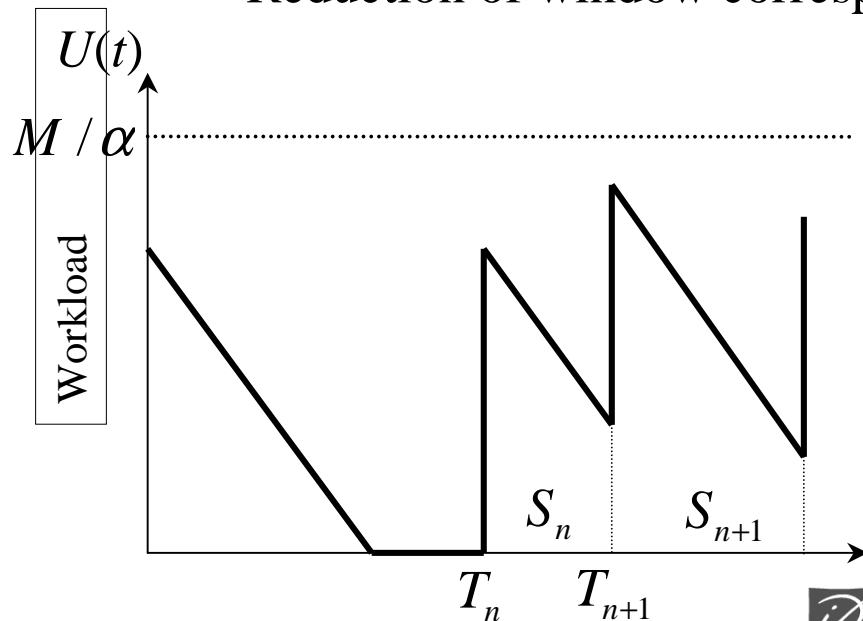
☞ $E[W^k(t)]$, $k \geq 1$ = Moments of $W(t)$ in the stationary regime.

☞ In particular:

TCP throughput:
$$\overline{X} = \lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t \frac{W(\tau)}{RTT} d\tau = \frac{E[W(t)]}{RTT}$$

Dual M/G/1 queuing model *

- ➡ The problem of congestion control can be seen as a queuing model:
 - ➡ The window corresponds to system workload ($U(t) = \frac{M - W(t)}{\alpha}$).
 - ➡ Congestion events correspond to customer arrivals.
 - ➡ Reduction of window corresponds to increase in workload.



Service time of a customer in the dual model:

$$x_n = \left(\frac{M}{\alpha} - U_n \right) \left(1 - \frac{1}{\gamma^{N_n}} \right)$$

Dependent of the workload.

* [Misra, Gong, Towsley, 1999]

Recursive equations for moments

➡ **Steady state Kolmogorov equation:** Relation between $F(x+dx)$ and $F(x)$

$$\alpha \frac{dF(x)}{dx} = \lambda \sum_{k=1}^{\infty} P\{N = k\} F(\min(\gamma^k x, M)) - \lambda F(x)$$

➡ Applying Laplace Stieltjes Transform, then differentiating, we get

$$\text{For } k = 1, 2, \dots, \quad E[W^k(t)] = \frac{k\alpha(E[W^{k-1}(t)] - P_M M^{k-1})}{\lambda(1 - Q(\gamma^{-k}))}$$

which gives all the moments of $W(t)$, as a function of $P_M = P\{W(t) = M\} \dots$

Window distribution

⇒ $F(x)$ is continuous on the interval $(0, M)$ with a jump at M due to P_M .

⇒ From the Kolmogorov equation we can write:

$$\text{For } x \in \left[\frac{M}{\gamma^k}, \frac{M}{\gamma^{k-1}} \right], k \geq 1, \quad \bar{F}(x) = 1 - F(x) = P_M \sum_{i=1}^k c_i^{(k)} e^{-\frac{\lambda}{\alpha} \gamma^{i-1} x}$$

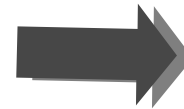
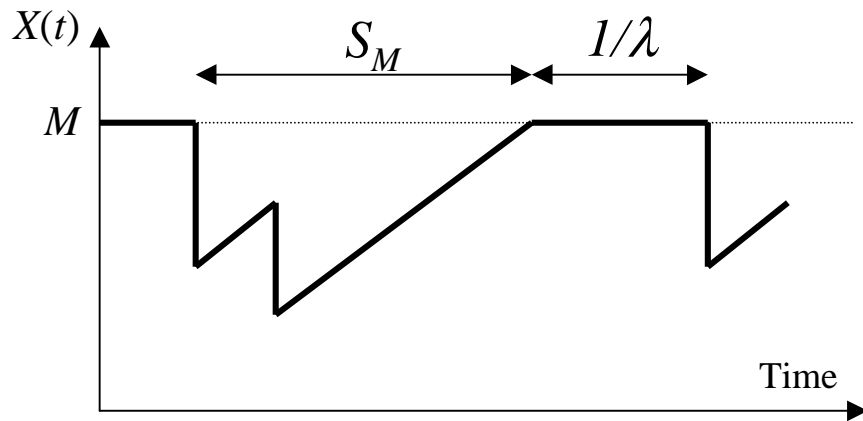
with $c_i^{(k)}$ some constants that can be recursively determined.

First method to calculate P_M :

$$\lim_{x \rightarrow 0} F(x) = 0$$

But, not efficient since the calculation of $c_i^{(k)}$ for large k is not very accurate.

Efficient method to calculate P_M

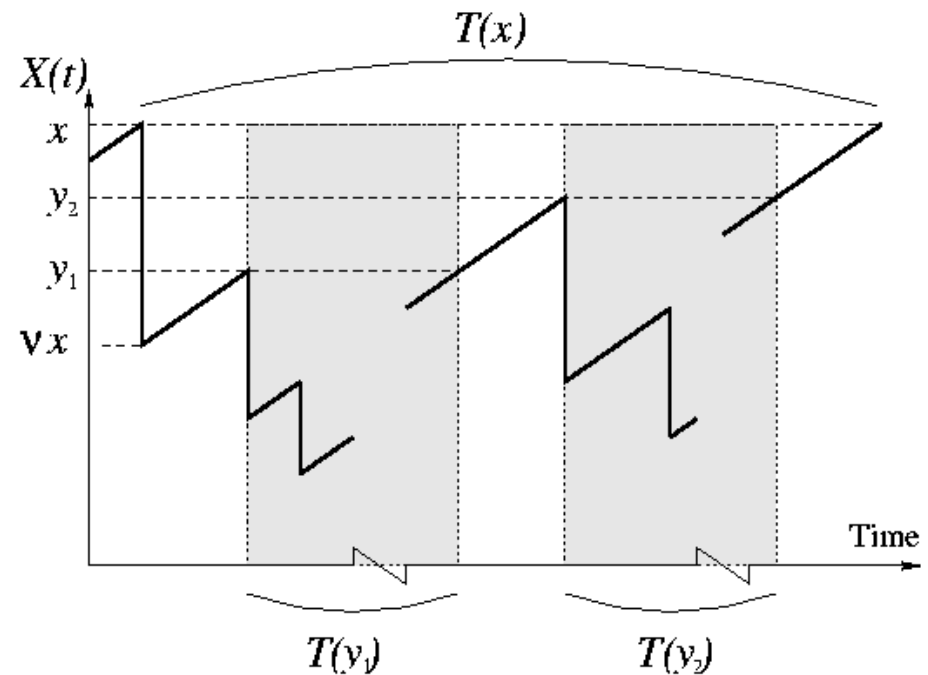


$$P_M = \frac{1/\lambda}{1/\lambda + S_M}$$

☞ $E[T(x)]$: Average time to return to x



$$S_M = E[T(M)]$$



Efficient method to calculate P_M (Ctd.)

☞ Derive an integral equation for $E[T(x)]$.

☞ Apply Laplace Transforms, solve the equation and invert back:

$$E[T(x)] = \frac{1 - Q(\gamma^{-1})}{\lambda} \sum_{i=0}^{\infty} d_i \left(e^{\gamma^{-i} \frac{\lambda}{\alpha} x} - 1 \right)$$

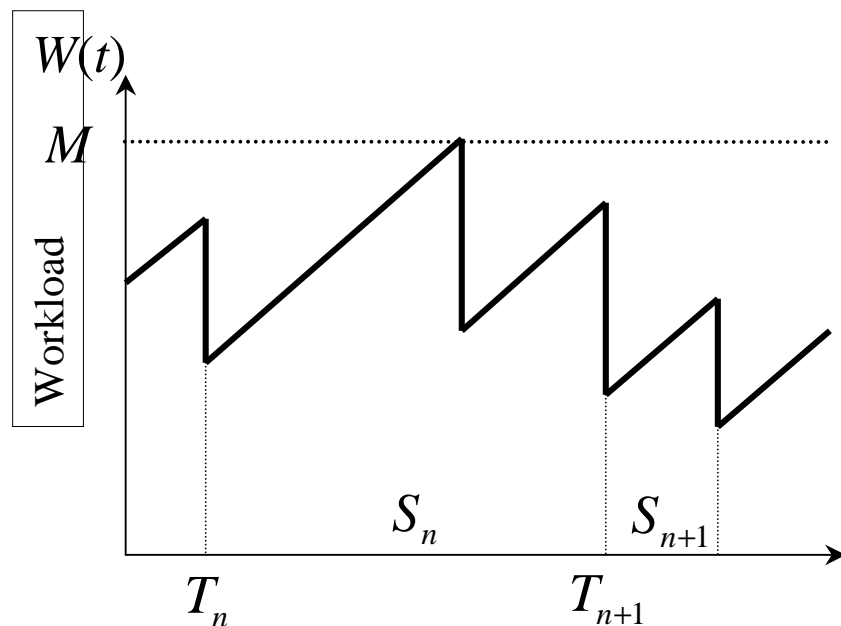
where d_i are constants that can be recursively determined.

➤ The algorithm is efficient since the infinite series converge very fast.

Extension to congestion limitation

➡ The model can be easily extended to the case when the window is reduced whenever it reaches M (*Congestion Limitation case*).

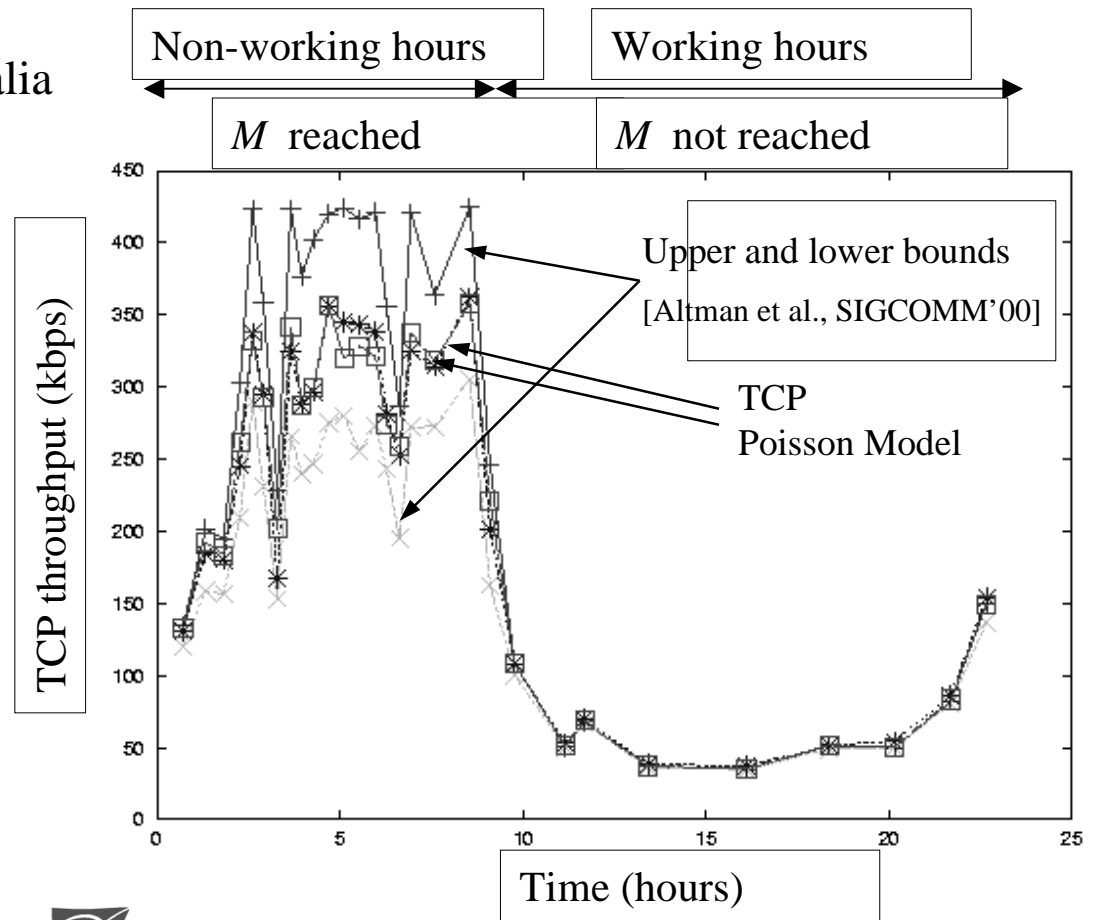
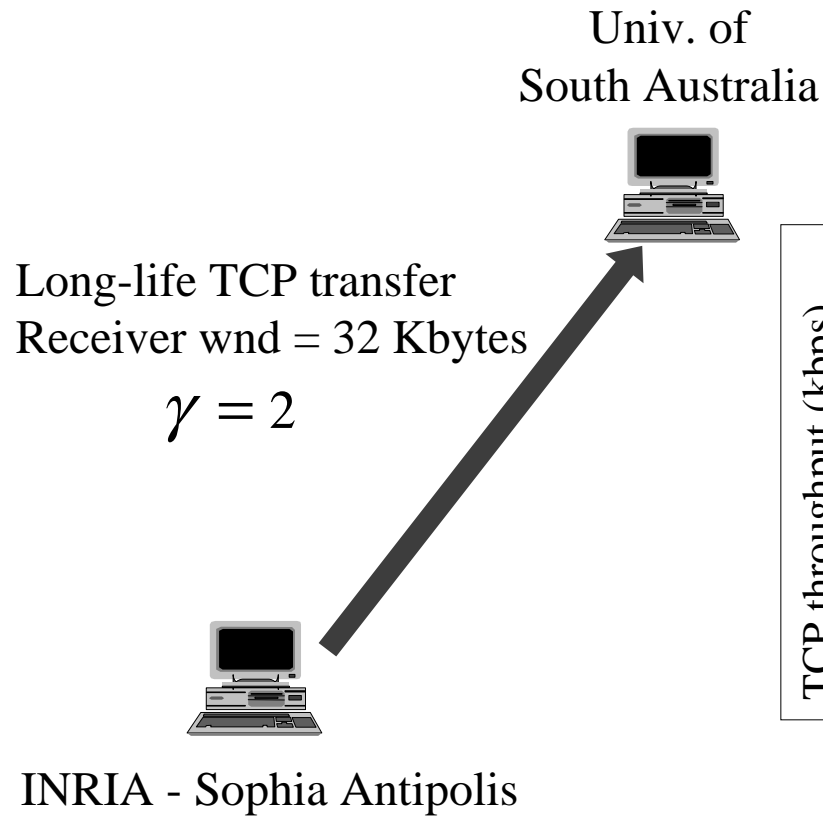
➡ **Example:** M corresponds to the available bandwidth.



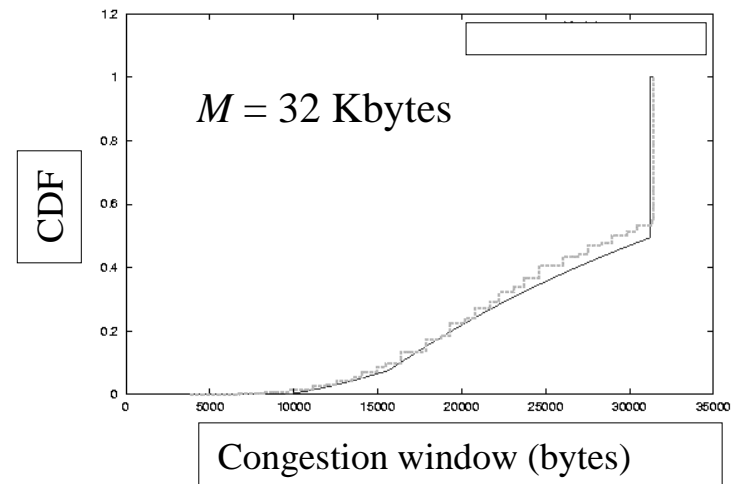
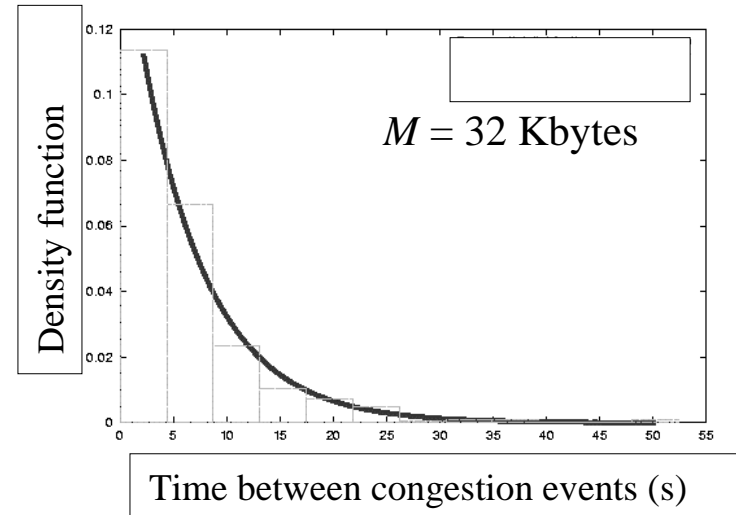
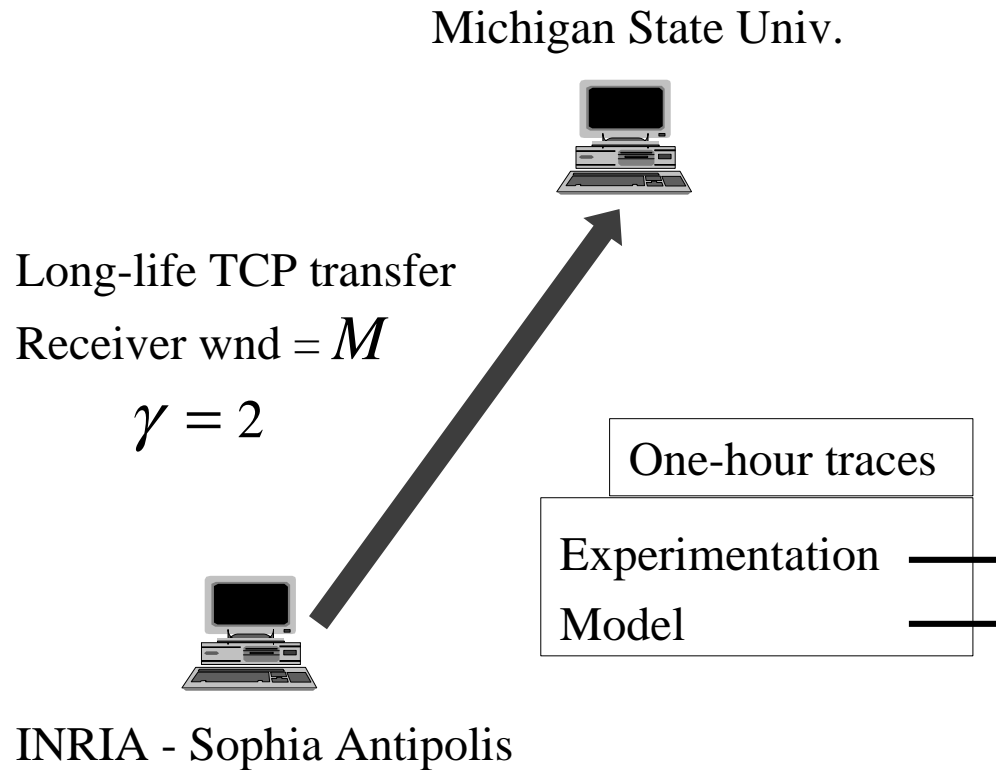
$$F^{cl}(x) = \frac{F(x)}{1 - P_M}$$

$$E[(W^{cl})^k] = \frac{E[W^k] - P_M M^k}{1 - P_M}$$

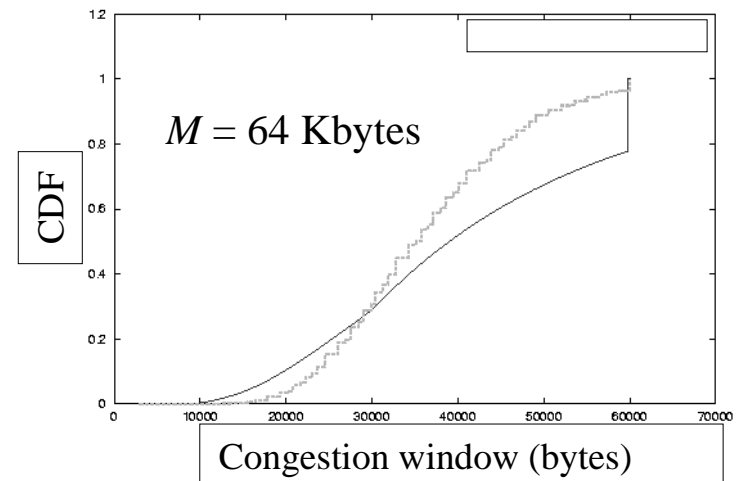
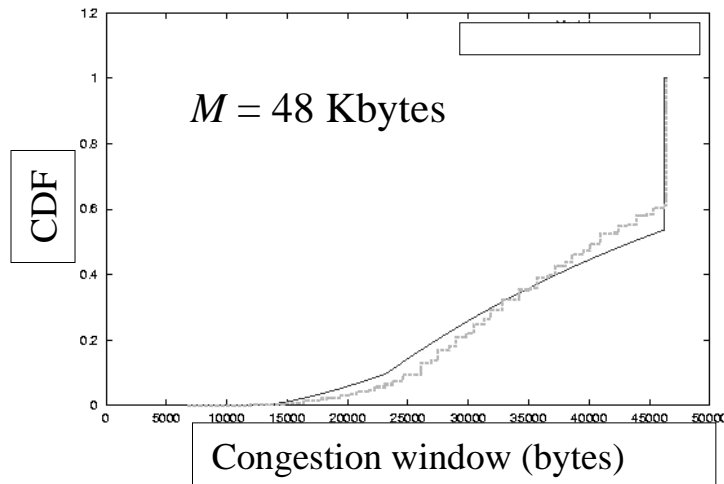
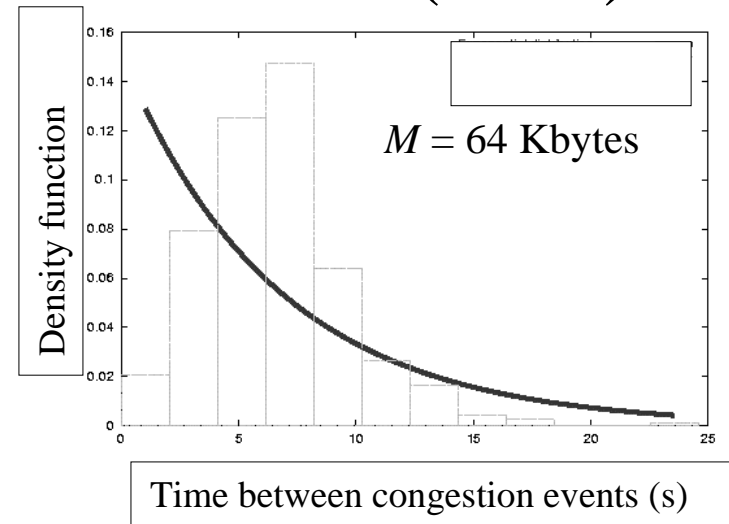
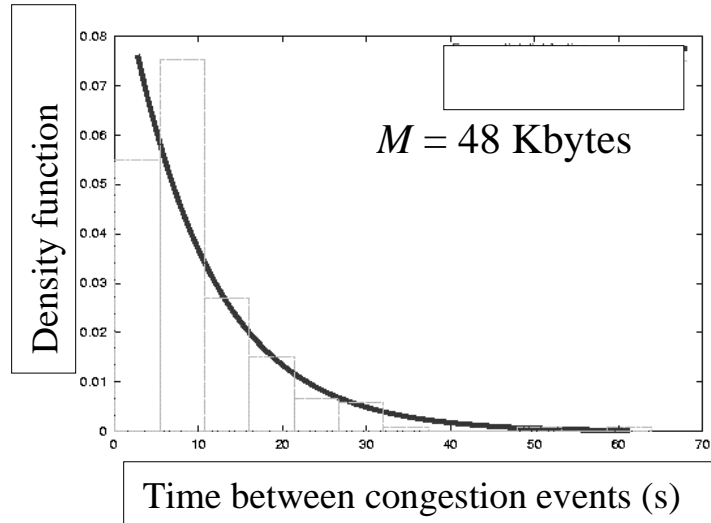
Model validation: Throughput



Model validation: Distribution



Model validation: Distribution (Ctd.)



Conclusions and perspectives

☞ **Results:** Moments and distribution of transmission rate were obtained when there is a maximum limit and when the process of congestion events is close to Poisson.

Approximations of the throughput exist in the literature (e.g., the fixed-point approach by [Padhye, Firoiu, Towsley, and Kurose, SIGCOMM'98]).

☞ **Future work:**

- ☞ Consider more general processes for congestion events (e.g., MMPP).
- ☞ Validate the model on bursty paths where the congestion of the network lasts for multiple consecutive RTTs.
- ☞ Other rate increase policies (e.g., sub-linear increase).