OBJECTIVES

- 8. Module Games
- 7. Correlated Equilibrium
- 6. Constraint Games
- 5. Discrete Potential Games, Convergence
- 4. Replicator Dynamics
- 3. Potential Games for Infinite Player Set
- 2. Wardrop Equilibrium
- 1. Non-atomic Games

Networking:

- Introduce non-cooperative game notions that are potentially useful in wireless
Link cost: $f_l(y_l)$, strictly monotonically increasing in the link flow $y_l$.

The class of traffic $\mathcal{T}$ is a class of directed links $\mathcal{L}$ is a set of nodes $\Lambda$.

Network: a graph $G = (\Lambda, \mathcal{L})$, where $\mathcal{L}$ is a set of links.

Example 1.1

Non-atomic Games
The paths of class $I$, etc.

$x$ is a column vector (size $\nu$). Let the first $p(1)$ elements correspond to the flow over

$$|p| = \sum_{\nu} = \nu$$

$I_L$ has $\nu$ rows and $\nu$ columns, where

$$\phi = \prod_{L} x$$

Matrix form of flow constraints $I_L x$.

Let $d \in I$ if $i = I$ and $d \in d$ and $d = \sum_{\nu} = \nu$ and $0$ otherwise.

Flow constraints: For each class $i$ of $d$

$$\phi = \sum_{d \in d} \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i$$

Link traffic: Where

$$d \in d \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i = \sum_{d \in d} \sum_{\nu} i$$

The amount of traffic of class $i$ is sent over path $d$.

Definitions
\[
\setcounter{equation}{4}
\begin{align*}
\sum_i \mathbf{x}_i &\leq \mathbf{0} \\
0 &\leq \mathbf{x} - \mathbf{1}^T \mathbf{z} \\
0 &\leq \mathbf{z} \mathbf{1} - (\mathbf{x})^T \\
0 &\leq \mathbf{x} \cdot [\mathbf{z} \mathbf{1} - (\mathbf{x})^T]
\end{align*}
\]

\text{Characterization of optimal solution through complementarity:}

\[\nabla \Phi \text{ is the gradient vector of the function } \Phi.
\]

\text{Define } \Gamma \in \partial \phi \text{ class marginal cost of } \phi \text{ in terms of } \mathbf{x}.

\text{1.2 Objective:}

[Equations 1-5]
\textbf{9) Alternative characterization: Variational Inequalities}

\[ \exists \ x \ & \text{and} \ \phi = x^t I x \ \text{such that} \ F(t) = 0, \ \text{for all} \ \forall t, \ x - x_t \geq (x - x_t) \cdot (x - x_t) \]
Matrix notation:

(6) \[
0 < (\gamma)^d x \quad (\gamma)^d V = (\gamma)^d L
\]

(8) \[
0 = (\gamma)^d x \quad (\gamma)^d V \geq (\gamma)^d L
\]

Thus \( x \) is a Wardrop equilibrium if

\[
(\gamma)^d V := (\gamma)^d L \quad \exists \gamma \min = (\gamma)^d L
\]

∀ type \( k \) user chooses a path that satisfies \( d \) a type \( k \) user.

Each player has a negligible influence on others' performance.

Non-atomic setting: Large number of non-cooperative players.

Wardrop equilibrium

1.3
• Conclusion: $x$ is a Wardrop equilibrium if it solves the global optimization problem:

$$\min G(x)$$

s.t. (12)-(13).

We get the same conditions for optimality of $x$ in the global optimization problem where link costs are replaced by their integral.

$$T^p(k)(x) = \frac{\partial}{\partial x_p} (DG(x)).$$

• Define the potential $G(x) = \frac{1}{b} \sum \phi_i \int_0^{y_i} f_i(s) ds$. Then

$$[T(x) - GA] \cdot x = 0,$$

$$\Gamma^T x - \phi = 0,$$

$$\forall \phi \geq 0, \forall \geq 0.$$

where $A = [A(1), A(2), \ldots, A(k)]^T$.
The gradient of a potential, $\nabla \phi(x)$, is interpreted as again the same form of the global optimization problem that $\nabla \phi(x)$ is provided.

For all $x$ and $\phi = \nabla \phi(x)$, $x \leq \mathbf{1} \cdot \phi$ for all $0 \leq (x - x) \cdot (x)$.

$x$ is a Wardrop equilibrium if and only if it is feasible and

Alternative characterization: Variational inequalities.
Delay

Reason: to decrease number of out-of-order packets and to minimize resequencing

Forwarded so as to minimize its delay.

This is a shortest path (delay) protocol: each packet that arrives at a mobile is

No real players, no Game.

IEEE CDC, 1997

P. Gupta and P. R. Kumar propose a new routing algorithm for Adhoc networks in

Applications to adhoc networks

1.4
Wireless context: Links are radio channels. They can have mutual interference.

Data network traffic: A congestion of TCP connections in one direction impacts the flow of ACKs in the opposite direction.

Road traffic examples: (i) Two way traffic. Congestion in one direction could contribute to congestion in the other direction. [Daremos, Transportation SC, 1971].

Link correlations: The cost over link \( L \) may depend on the flow over other links.

1.5 Limitations of the model
Limitations of the model (III): Multiclass traffic

• Example: Differences provide priority to some traffic over others. The priority traffic congestion.

Data networks example: Packets of different size contribute differently to assignment problem for multiclass-user transportation networks. Transp ScI 1972 [1]

Moreover, the link cost may differ from one class to another.

Total link flow, \( \sum_{i} f_i \) rather than on the flow of each user.

E. Altman, Networking Games
The Wardrop equilibrium is obtained by maximizing the potential

\[ (x)^{2f} = \frac{x_\theta}{(x)\theta} \]

differentiable function \( \phi \) such that for all \( x \) and \( \theta \), the networked game is said to be a potential game if there is a continuously

Potential Game with Continuous players set
Major research problem: what to do when there is no potential.

There is a "unique" Wardrop equilibrium (if the link costs are monotone).

Which is path independent:

\[(s)p(s) \int_0^{f'} \sum_l = (x) \mathcal{C}\]

Assume that the matrix is symmetric and positive definite.

\[
\begin{bmatrix}
\frac{\partial^2 \mathcal{C}}{\partial x \partial x} & \int_l \\
\int_l & \int_l \mathcal{C}
\end{bmatrix}
\]

Conditions for existence of a Potential.
Denote the class \( u \) workload.

- Route \( g \subseteq u \)
- Bandwidth requirement, \( w \) integer units.
- Arrival rate \( \lambda \), and an average holding time \( h \).
  Associated with class \( u \) are
  \[
  \left\{ \{1, 2, \ldots, N\} \right\} = N \text{ classes of calls, (integer)}.
  \]
- Resource \( c \) has \( R \) capacity units (integer).

Example: loss networks work with E-Azouzi and Abramov.

Limitations of the model (III): Non-additive costs.
\[(\ref{15}) \quad \left\{ u_t \in e \in uq - uq \geq u q^3 \sigma \sum \colon \chi \in u \right\} = u^\chi \]

Arrival of a class-\(u\) call:

- Let \(u^\chi\) be the subset of states for which there is an available bandwidth for another class \(C\),

\[\left\{ o \in \sigma \in u \geq u^\chi u^\sigma \sigma \sum \colon u \right\} = \chi \]

- The state space is \(\mathcal{N}\) for the class \(u\) in the system, and

\[ (\mathcal{N} u \cdots u \mathcal{N} ) = u \]

- Let \(\mathcal{N}\) be the number of calls of class \(u\) in the system, and

\[(\ref{14}) \quad \left\{ u_t \in e \in u \in \mathcal{N} \right\} = \mathcal{N}\]

- Let \(\mathcal{N}\) be the subset of classes that use resource \(e\).
\[ i^{\nu_{\nu}} \delta_{i=1}^{N} \prod_{\nu} \chi_{\nu}^{\nu} \supseteq \subseteq - 1 = uB \]

The probability of blocking of a class-\(u\) call is

\[ i^{\nu_{\nu}} \delta_{i=1}^{N} \prod_{\nu} \chi_{\nu}^{\nu} \supseteq \subseteq = C \]

where

\( C \)

The steady state distribution is

\[ \mathcal{X} \in \mathbb{W} \]

\[ i^{\nu_{\nu}} \delta_{i=1}^{N} \prod_{\nu} \mathcal{O} \supseteq \subseteq = \{ \mathbb{W} = \mathbb{X} \} \mathbb{P} \]
Strategy: Group $l$ sends a fraction $\pi_l$ of its demand via the route $\nu_l$. Parameters $\pi_l$, $\nu_l$ and $\lambda_l$.

characterized by a common source and destination as well as a common

Group $l$ can use any one of subset $\mathcal{N}_l \subset \mathcal{N}$ of classes. The set $\mathcal{N}_l$ is

Consider $I$ groups which split their demands via the networks.

Group of users
The second one, \( q \), has a capacity of 3 bandwidth units.
The first one, \( a \), has a capacity of 2 bandwidth units.

There are two parallel links:

Consider the following example:

Non-uniqueness of Wardrop equilibrium
where \( l = I, II \) is the group and \( j = a, b \) is the link.

\[
N = \{(I, a), (I, b), (II, a), (II, b)\},
\]

\[
N^2 = \{(II, a), (II, b)\},
\]

\[
b_{(II, a)} = b_{(II, b)} = 2.
\]

- Two groups can send traffic through both links.

- We have then 4 classes with the same source and destination:

- \( N_1 = \{(I, a), (I, b)\} \)

- \( b_{(I, a)} = b_{(I, b)} = 1 \).

- The calls of group 1 require 1 bandwidth unit.

- The calls of group 11 require 2 bandwidth units.

- There are 2 groups.
In general, no potential

We obtained three different Wardrop equilibria.

Results
We can interpret the Wardrop equilibrium as the proportional fair assignment.

\[
\min_{\gamma \in \mathcal{N}} \sum_{(i,j) \in \mathcal{A}} \frac{N_{ij}^\gamma}{\gamma(i)}
\]

where

\[
\gamma(i,j) = \frac{\gamma_{ij}}{\gamma(i)}
\]

by solving

\[
\begin{align*}
\int_0^\gamma \sum_{j \in \mathcal{N}} \gamma_{ij} & = (x)^{\gamma(i)} \\
\log \gamma & = z \cdot p(z) \cdot \int_0^\gamma \sum_{j \in \mathcal{N}} \\
\text{Let } \gamma & = \frac{1}{T} \sum_{j \in \mathcal{N}} \\
\text{Erlang B formula} & = \frac{i^i \gamma^0}{i^i \gamma^0} = ((i) \gamma)^B
\end{align*}
\]

The blocking probability over link \(i\) is given by the Erlang loss formula:

Parallel links with equal bandwidth requirements.
If $\mathcal{A}$ satisfies PC then all Wardrop equilibria are stationary points.

$0 \neq (\mathcal{A}) \Lambda \Rightarrow 0 < \frac{d}{dL} \lambda (\mathcal{A}) \sum_{\mathcal{N} \ni \mathcal{U}}$ 

is said to be PC (positively Correlated) if

$(\mathcal{A}) \Lambda = \frac{\lambda}{\lambda P}$

**Definition [Sandholm]: The Dynamic**

For a strategy $\mathcal{V}$ of class $u$, define $\mathcal{G}^{u}(\mathcal{A})$ all the pure strategies in its support.

1.6 Convergence [A. Kumar, S. Shakkottai, E.A. [F. Allman, Networking Games