Analytical Model for Connectivity in Vehicular Ad Hoc Networks

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***Abstract*—We investigate connectivity in the ad hoc network formed between vehicles that move on a typical highway. We use a common model in vehicular traffic theory in which a fixed point on the highway sees cars passing it that are separated by times with an exponentially distributed duration. We obtain the distribution of the distances between the cars, which allows us to use techniques from queuing theory to study connectivity. We obtain the Laplace transform of the probability distribution of the connectivity distance, explicit expressions for the expected connectivity distance, and the probability distribution and expectation of the number of cars in a platoon. Then, we conduct extensive simulation studies to evaluate the obtained results. The analytical model that we present is able to describe the effects of various system parameters, including road traffic parameters (i.e., speed distribution and traffic flow) and the transmission range of vehicles, on the connectivity. To more precisely study the effect of speed on connectivity, we provide bounds obtained using stochastic ordering techniques. Our approach is based on the work of Miorandi and Altman, which transformed the problem of connectivity distance distribution into that of the distribution of the busy period of an equivalent infinite server queue. We use our analytical results, along with common road traffic statistical data, to understand connectivity in vehicular ad hoc networks.**

***Index Terms*—Connectivity distance, highway, infinite server queue, platoon size, stochastic ordering, vehicular ad hoc networks (VANETs).**

I. Introduction

Vehicular ad hoc networks (VANETs) are special types of mobile ad hoc networks (MANETs), where wireless-equipped vehicles spontaneously form a network while traveling along the road. Direct wireless transmission from vehicle to vehicle makes it possible to communicate, even where there is no telecommunication infrastructure, such as the base stations of cellular phone systems or the access points of wireless dedicated access networks.

This new way of communication has been attracting much interest in recent years in the academic and industrial communities. The U.S. Federal Communications Commission (FCC) has allocated seven 10-MHz channels in the 5.9-GHz band for dedicated short-range communication (DSRC) to enhance the safety and productivity of the transportation system [1]. The FCC's DSRC ruling has permitted both safety and nonsafety (commercial) applications, provided that safety is assigned priority. The IEEE has taken up working on a new standard for VANETs, which is called the IEEE 802.11p [2]. In addition, some other projects outside the U.S., such as the PReVENT project [3] in Europe, InternetITS [4] in Japan, and Network on Wheels [5] in Germany, aim to solve the challenges. So, in the near future, vehicles can benefit from spontaneous wireless communications.

VANETs have many distinctive characteristics and communication challenges, as described in [6]. According to the FCC frequency allocation, one can categorize two main classes of applications for VANETs. The first category aims to improve the safety level in roads, i.e., safety applications. In this case, VANETs can be seen as complementary to current intelligent transportation systems [7], [8] to enhance coverage and performance. The second class of applications, which is predicted to grow very fast in the near future, is commercial services, i.e., comfort applications. Applications in this class offer commercial services, such as Internet access on roads and music downloads, to vehicles. In both the aforementioned categories of applications, related (i.e., safety and comfort) messages should be exchanged between vehicles.

To clarify the challenge that we address in this paper, we invoke some basics from traffic theory. From the theory of traffic [9], we know that there are three macroscopic parameters that describe the traffic state on a road, i.e., speed (in kilometers per hour), density (in vehicles per kilometer per lane), and flow (in vehicles per hour per lane). Over the years, many models have been proposed for speed–flow–density relationships (see [9]). Simply, the values of these parameters are related by the so-called fundamental traffic theory equation, which is given by

$$F = S \times K \tag{1}$$

where F, S, and K are the traffic flow, average speed, and traffic density, respectively. As shown in Fig. 1, the general relationship between the aforementioned basic parameters can be studied in two different phases: First, when the density is low, the flows entering and leaving a section of the highway are the same, and no queues of vehicles are forming within the section. This state holds until the density reaches a threshold called the critical value. This phase is called stable flow and is shown by the solid lines in the figure. The peak of the flow–density curves is the maximum rate of flow or the capacity

Manuscript received November 20, 2007; revised June 25, 2008. First published July 29, 2008; current version published November 12, 2008. The work of E. Altman was supported by the BIONETS European project. The review of this paper was coordinated by Dr. T. Zhang.

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Digital Object Identifier 10.1109/TVT.2008.2002957

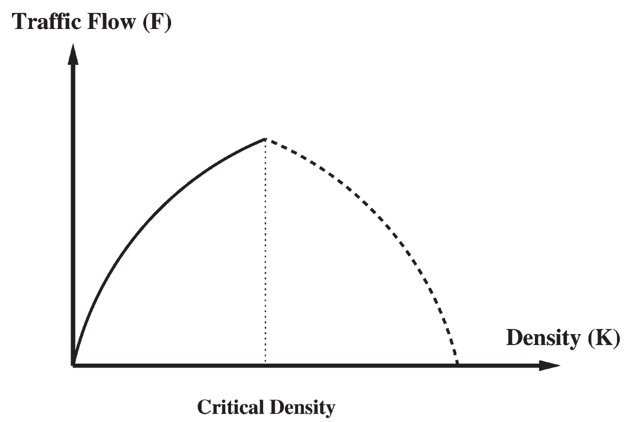


Fig. 1. Relationship between the basic parameters in the traffic theory.

of the highway. Beyond this density, some breakdown locations appear on the highway, which lead to the formation of some queues of vehicles. This phase is called forced flow and is shown by the dashed line. If the density increases more, the traffic reaches the jam state, at which vehicles have to completely stop. In the stable-flow phase, when the density is sufficiently low, the speed of the vehicles and the flow are independent; thus, drivers can drive as fast as they want. This state is called the free-flow state.

From a communication point of view, which we peruse in VANETs, different challenges should be addressed in each traffic state. Obviously, connectivity is satisfactory in the forced-flow state, whereas it deteriorates at light load, corresponding to the free-flow state, in which it might not be possible to transfer messages to other vehicles because of disconnections. However, since the network is sparse, collision between simultaneous transmissions is trivial in the free-flow state, whereas it is one of the main communication challenges that should be addressed in the forced-flow traffic state.

In this paper, we study the connectivity of VANETs operating in the free-flow regime. We use the common model in vehicular traffic theory in which any given point in space (i.e., on the highway) sees cars passing it that are separated by times with random duration exponentially distributed. We obtain the distribution of the distances between cars at steady state, which allows us to use techniques for studying connectivity that originated from the queuing theory. We obtain the Laplace transform of the probability distribution of the connectivity distance, explicit expressions for the expected connectivity distance, and the probability distribution and expectation of the number of cars in a platoon. Moreover, we conduct an extensive simulation study to evaluate the obtained results. The analytical model that we present is able to describe the effects of various system parameters, including road traffic parameters (i.e., speed distribution and traffic flow) and the transmission range of vehicles, on the connectivity. To more precisely study the effects of speed on connectivity, we provide bounds obtained using stochastic ordering techniques. The approach that we use for our analysis is based on the work of Miorandi and Altman [10], which transformed the problem of connectivity distance distribution into that of the distribution of the busy period of an equivalent infinite server queue. We use our analytical results, along with common road traffic statistics, to understand the connectivity in VANETs.

II. Related Work

Connectivity in MANETs and 1-D networks has a mature body of research, and many works investigated it through simulation and/or analytical evaluation [10]–[13]. Most of these works study the problem in static networks and are thus more suitable for sensor networks. However, some of them also tried to tackle the problem of connectivity in the presence of mobility, but the attempts have mostly been restricted to low-mobility networks and/or well-known mobility models. Our work is different from this body of research in that we consider vehicular movements, which are a special type of mobility with many distinctive characteristics.

Recently, some authors studied connectivity and related topics, specifically, in vehicular networks. The lifetimes of a link and a path (including multiple consecutive links) have been investigated in a few works. Wang [14], [15] conducted several simulation studies to analyze the lifetime of repairable unicast paths in VANETs. Rudack *et al.* [16] performed analytical and simulation studies for finding the distribution function of the link's lifetime between vehicles in different highway scenarios. Moreover, the effects of traffic parameters on the connectivity are also addressed in some recent works through extensive simulation studies. The work in [17] studies the effect of traffic parameters (i.e., speed and density) on connectivity through simulation. Furthermore, in [18], Artimy *et al.* conducted a simulation to study the effect of transmission range on the connectivity for different types of scenarios and freeways. Afterward, they presented an algorithm for the dynamic assignment of the minimum transmission range in [19]. To the best of our knowledge, there is no analytical model in the literature that addresses the effects of traffic parameters on the connectivity in VANETs. In this paper, we present an analytical model that is able to explain the effects of traffic parameters (including speed distribution and traffic flow) and the transmission range of vehicles on the connectivity for a general uninterrupted highway.

Moreover, connectivity has also been studied in the area of information propagation in multihop networks. In one of the earliest efforts, Cheng and Robertazzi [20], starting from the assumption of static uniform distribution of nodes, investigated the probability distribution of the connectivity distance (i.e., the probability of success for information to travel beyond a given distance) for 1-D MANETs and obtained a closed-form expression. In [21], Jin and Recker studied the process of information propagation for VANETs, in which they modeled a discrete case where the street is divided into cells (based on the vehicle's transmission range). Then, they proposed stochastic models for instantaneous information propagation to a given cell under uniform and general traffic streams. However, no closed-form expression is provided, and thus, the model leverages on numerical methods to provide the probability distribution of the connectivity distance. In a complementary effort and as a continuous counterpart, Wang [22], starting from the assumption of spatial Poisson distribution of vehicles, obtained closed-form expressions for the average connectivity distance and the average number of hops in the information propagation path. In addition, they have presented a recursive formula for the probability of the connectivity distance. Our work is different from this body of research mainly in two aspects: First, in addition to connectivity distance statistics, we obtain closed-form expressions for the probability distribution and average number of vehicles in a cluster (platoon) of vehicles. These results are practically beneficial to study the feasibility of deployment for both safety and comfort applications. Second, all previous works, even those addressing VANETs (i.e., [21] and [22]), start from a given vehicle distribution on the road, and thus, the effect of the vehicle's speed distribution and the road's traffic flow is not studied and remains unclear. Nevertheless, our analytical model presents a framework that gives us the possibility of studying the effects of road traffic flow and the distribution of vehicle speed on connectivity. The investigation of the speed's effect is followed by stochastic bounds obtained by using stochastic ordering techniques. To the best of our knowledge, our work is the first study in this regard.

The rest of this paper is organized as follows: In Section III, we first find the distribution function of intervehicle distances and then analyze the connectivity based on an equivalent infinite server queuing model. In Section IV, we conduct an extensive simulation study to evaluate the obtained results. In Section V, we extend the model for more general highways and traffic. In Section VI, the effects of the traffic parameters and the vehicles' transmission range on the connectivity are studied, and some stochastic bounds are provided. Finally, this paper will be concluded in Section VII.

III. Analytical Model

To study the connectivity, we invoke an equivalent queuing system model. It was observed in [10] that the busy period of an infinite server queueing system has the same distribution as the connectivity distance (see the definition provided hereinafter) in an ad hoc network. Moreover, the number of customers served during a busy period has the same distribution as the number of mobiles in a connected cluster in the ad hoc network. The equivalence is obtained when the interarrival times in the infinite server queue have the same distribution as the distance between successive nodes and when the service times have the same distribution as the transmission range of the nodes. We study the connectivity in VANETs using the corresponding infinite server queuing model; thus, we need to determine the distribution of the vehicles' transmission range as well as that of the intervehicle distances. In this paper, we assume a fixed transmission rate and a fixed transmission power for all vehicles. Thus, all vehicles have the same fixed transmission range, which is denoted as R. One can always increase the transmission range by increasing the power or decreasing the transmission rate. An obvious extension of the model is possible in the case of a variable range: A car could use a lower range if there is another car within the lower range; the connectivity distance would be the same as that if it used its maximum range. Then, we need to determine the distribution of the distances between two consecutive vehicles, which is obtained in the following.

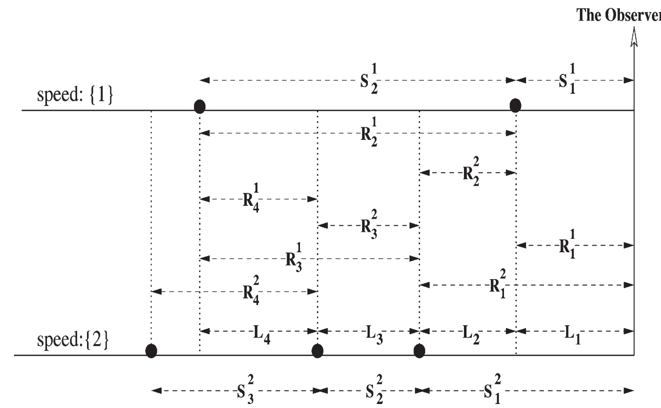


Fig. 2. Example showing the computation of the distribution of the intervehicle distances with two speed levels.

A. Distribution of the Intervehicle Distances

Assume that an observer stands at an arbitrary point of an uninterrupted highway (i.e., without traffic lights, etc.). The number of vehicles passing the observer per unit of time (e.g., an hour) is a Poisson process with mean $1/\lambda$; thus, the traffic flow is λ (in vehicles per hour). Therefore, the interarrival times are exponentially distributed with parameter λ. In addition, assume that there are N discrete levels of constant speed v_i $(i = 1, \ldots, N)$ on the highway, where the speeds are independent identically distributed (i.i.d.) and independent of the interarrival times. Denote the rate of arrivals of cars at each level of speed as λ_i $(i = 1, \ldots, N)$, where $\sum_{i=1}^{N} \lambda_i = \lambda$; thus, the occurrence probability of each speed level is $P_i = \lambda_i/\lambda$. To obtain the distribution of the intervehicle distances, we introduce four definitions.

- t_n^i is the time when the nth car $(n \geqslant 1)$ of speed v_i arrives at point 0 (of the observer).
- $T_n^i := t_n^i - t_{n-1}^i$, and $t_0^i := 0$. For each i, $\mathcal{T}^i = \{T_n^i\}$ is the interarrival time sequence (of cars with speed v_i) exponentially distributed, where $E[T_n^i] = 1/\lambda_i$. $\mathcal{T}^i$ are assumed to be independent.
- $S_n^i := T_n^i v^i$. For each i, $\{S_n^i\}$ is the intervehicle distance sequence (of cars with speed v_i) exponentially distributed, where $E[S_n^i] = v_i/\lambda_i$.
- L_n is the distance between the nth closest car to the observer and the $(n-1)$th closest car to the observer at time 0.

Hereinafter, we show that $\{L_n\}$ are i.i.d. and exponentially distributed with parameter $\sum_{i=1}^{N} \lambda_i/v_i$. Define the following:

- $R_1^j := S_1^j$.

As shown in Fig. 2, the first intervehicle distance can be obtained from $L_1 = \min_j(\{R_1^j, j = 1, \ldots, N\})$, where N is the number of speed levels. Therefore, they have the same distribution as $\{S_1^j, j = 1, \ldots, N\}$ and are thus independent of each other. As a result, L_1 is exponentially distributed with parameter $\sum_{i=1}^{N} \lambda_i/v_i$. To find the distribution of L_n for $\{n = 2, 3, \ldots\}$, define

$$\begin{cases} R_n^j := \sum_{m=1}^{k_{n-1}^j} S_m^j - \sum_{k=1}^{n-1} L_k, & j \neq \alpha(n-1) \\ R_n^j := S_{k_{n-1}^j}^j, & j = \alpha(n-1) \end{cases} \quad (2)$$

TABLE I
NORMAL-SPEED STATISTICS

$\mu[km/h]$	$\sigma[km/h]$
70	21
90	27
110	33
130	39
150	45

where we have the following.

- $\alpha(n) := \arg\min_i R_n^i$ represents the speed index of the car that is closest to the observer at time 0, excluding the first $(n-1)$th closest cars.
- $k_n^j := 1 + \sum_{l=1}^{n} \mathbf{I}_{\{\alpha(l)\}}(j)$, where $\sum_{l=1}^{n} \mathbf{I}_{\{\alpha(l)\}}(j)$ is the number of cars with speed index j among the nth closest car to the observer. $\mathbf{I}_{\{l\}}(x)$ is the indicator function; it is equal to one when $l = x$ and equal to zero otherwise.

The second intervehicle distance can be obtained from $L_2 = \min_j(\{R_2^j, j = 1, \ldots, N\})$ (see an example hereinafter). From the memoryless property of the exponential distribution, we know that, if X_2 is exponentially distributed and $X_2 > X_1$, then the distribution of the excess of X_2 over X_1 does not depend on the value of X_1. Hence

$$P(X_2 > t) = P(X_2 > X_1 + t | X_2 > X_1).$$

Therefore, from definition (2), we conclude that $\{R_2^j, j = 1, \ldots, N\}$ are independent of each other and L_1. Moreover, R_2^j is exponentially distributed with parameter λ_j/v_j, for $j = 1, \ldots, N$. As a result, L_2 is exponentially distributed with parameter $\sum_{i=1}^{N} \lambda_i/v_i$ and is independent of L_1. Generally, the nth intervehicle distance can be obtained from $L_n = \min_j(\{R_n^j, j = 1, \ldots, N\})$. By using the same procedure, we can see that L_n has the same distribution as $L_1, L_2, \ldots, L_{n-1}$ and is also independent of them. Consequently, the intervehicle distances are i.i.d. and exponentially distributed as follows:

$$P(L > x) = 1 - F_L(x) = e^{-\sum_{i=1}^{N} \frac{\lambda_i}{v_i} x} = e^{-\lambda \sum_{i=1}^{N} \frac{P_i}{v_i} x}. \quad (3)$$

Fig. 2 shows an example for the preceding proof. As shown in the figure, four conditions hold.

1) $R_1^1 = S_1^1$, and $R_1^2 = S_1^2$. Then, $L_1 = \min(R_1^1, R_1^2)$.
2) $R_2^1 = S_2^1$, and $R_2^2 = S_1^2 - L_1$. Then, $L_2 = \min(R_2^1, R_2^2)$.
3) $R_3^1 = S_1^1 + S_2^1 - L_1 - L_2$, and $R_3^2 = S_2^2$. Then, $L_3 = \min(R_3^1, R_3^2)$.
4) $R_4^1 = S_1^1 + S_2^1 - L_1 - L_2 - L_3$, and $R_4^2 = S_3^2$. Then, $L_4 = \min(R_4^1, R_4^2)$.

We next show how to set the parameters P_i and v_i. For a specific highway, these parameters can be obtained from experimental observations. However, it has widely been accepted in vehicle traffic theory that the speeds in the free-flow traffic state are normally distributed [9]. Some typical values for the mean and variance of the distribution of the vehicles' speed are provided as shown in Table I [16]. Thus, speeds are distributed according to the following probability density function (pdf):

$$f_V(v) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}} \quad (4)$$

where μ is the average speed, and σ is the standard deviation of the vehicles' speeds. We shall use a truncated version of this distribution to avoid dealing with negative speeds or even getting close to zero speed. (The latter would otherwise cause problems in (3) and elsewhere; in fact, it can be seen that a speed of zero does not make sense since a car cannot cross the observer if it has zero speed.) We thus define two limits for the speeds, i.e., $v_{\min}$ and $v_{\max}$, representing the minimum and maximum speeds in the highway, respectively. Hence, by replacing P_i and v_i in (3), we can obtain the distribution of the intervehicle distance as

$$P(L > x) = e^{-\lambda x \int_{v_{\min}}^{v_{\max}} \frac{\hat{f}_V(v)}{v} dv} \quad (5)$$

where $v_{\min} < v < v_{\max}$, and

$$\hat{f}_V(v) = \frac{f_V(v)}{\int_{v_{\min}}^{v_{\max}} f_V(s) ds} = \frac{2 f_V(v)}{\text{erf}\left(\frac{V_{\max}-\mu}{\sigma\sqrt{2}}\right) - \text{erf}\left(\frac{V_{\min}-\mu}{\sigma\sqrt{2}}\right)} \quad (6)$$

and by definition, $\text{erf}(x) = (2/\sqrt{2\pi}) \int_0^x e^{-t^2} dt$ is the error function. The important parameters in (5) that need to be determined are the minimum and maximum speeds. We know that the area under the normal curve of the speeds in range $(\mu - 3\sigma, \mu + 3\sigma)$ is about 99.7% of the whole area. Therefore, to take into account almost all speeds, we take $v_{\min} = \mu - 3\sigma$ and $v_{\max} = \mu + 3\sigma$. Clearly, we do not lose generality by the aforementioned parameter setting, and one can set the parameters differently for a specific highway based on fully experimental data.

B. Connectivity Analysis

To study the connectivity, we use the equivalent infinite server queueing model [10]. Liu and Shi [23] provided general expressions for the busy period in the $G/G/\infty$ queuing system and some closed-form equations for some special cases. As previously discussed, the intervehicle distances are exponentially distributed as in (3), and by assumption, there is one fixed transmission range; thus, we use the equivalent $M/D/\infty$ to investigate the connectivity. From now on, we shall use VANET terms instead of the queuing terms. Hereinafter, we study the connectivity in VANETs by evaluating the probability distribution and expectations of the following metrics:

1) platoon size, which is defined as the number of vehicles in each spatial connected cluster (platoon) or, equivalently, the number of vehicles in the connected path from any given vehicle;
2) connectivity distance, which is defined as the length of the connected path from any given vehicle.

The former is important, because it shows how many vehicles can hear a vehicle in the safety applications and can have data exchange in the comfort applications. The latter metric is important, because a larger connectivity distance leads to a larger announcement area for the safety applications and better accessibility to roadside equipment (e.g., Internet gateways) for the comfort applications.

1) Platoon size: The probability-generating function of the number of customers served during a busy period in a $G/D/\infty$

queuing system is given in [23]. As stated before, we make it correspond to the number of vehicles in each platoon; thus, the probability-generating function of the platoon size is given by

$$\hat{N}(z) = z \cdot \frac{1 - F_L(R)}{1 - zF_L(R)} \tag{7}$$

where $F_L(.)$ is the distribution function of the intervehicle distances, and R is the fixed transmission range of each vehicle. Thus, the random variable N representing the number of vehicles in each platoon has the following probability mass function:

$$P(N = k) = (1 - F_L(R)) \cdot F_L(R)^{k-1}, \qquad k = 1, 2, 3, \ldots. \tag{8}$$

Hence, the tail probability of the platoon size (i.e., the probability that at least k vehicles are connected) is

$$\begin{aligned} P_N(k) = P(N \geqslant k) &= \sum_{t=k}^{\infty} P(N = t) = F_L(R)^{k-1} \\ &= \left(1 - e^{-\sum_i \frac{\lambda_i}{v_i} R}\right)^{k-1} \\ &= \left(1 - e^{-\lambda R \mathbb{E}(1/V)}\right)^{k-1}. \end{aligned} \tag{9}$$

Furthermore, the expected value of the number of vehicles in each platoon is given by

$$\begin{aligned} \mathbb{E}(N) = \lim_{z \to 1} \frac{\partial \hat{N}(z)}{\partial z} &= \frac{1}{1 - F_L(R)} \\ &= \frac{1}{e^{-\sum_i \frac{\lambda_i}{v_i} R}} \\ &= \frac{1}{e^{-\lambda R \mathbb{E}(1/V)}}. \end{aligned} \tag{10}$$

Moreover, from (8), we can easily find the probability of isolation for a given vehicle (i.e., there is just one vehicle in a cluster), which is obtained as

$$P_I = 1 - F_L(R) = e^{-\sum_i \frac{\lambda_i}{v_i} R} = e^{-\lambda R \mathbb{E}(1/V)}. \tag{11}$$

The number of hops in the connected path from a given vehicle may be studied using the obtained results of the platoon size. For a given transmission range, the number of hops depends on the position of intermediate relay nodes. On the other side, choosing relay nodes depends on the message-forwarding protocol and some other effects such as success rate. Denote the number of hops in the connected path from a vehicle as N_{hops}. Note that, in the presence of any forwarding protocol, $N_{\text{hops}} \leq N - 1$, where N is the number of vehicles in the connected path from the vehicle. Thus, the following upper bound on the expected value of the number of hops is obtained:

$$\mathbb{E}(N_{\text{hops}}) \leq \mathbb{E}(N) - 1. \tag{12}$$

In addition, we can find an upper bound for the tail probability of the number of hops as follows:

$$\begin{aligned} &P(N_{\text{hops}} \geqslant h) \leq P(N - 1 \geqslant h) \\ \Longrightarrow\ &P(N_{\text{hops}} \geqslant h) \leq P(N \geqslant h + 1) \\ \Longrightarrow\ &P(N_{\text{hops}} \geqslant h) \leq \left(1 - e^{-\lambda R \mathbb{E}(1/V)}\right)^h. \end{aligned} \tag{13}$$

2) Connectivity distance: In [23], the Laplace transform of the busy period for a $G/D/\infty$ queuing system is provided. As stated before, we make it correspond to the connectivity distance previously defined. Let d be a random variable that represents the connectivity distance, and denote the Laplace transform of the pdf of the connectivity distance as $f_d(s)$, which is given as follows:

$$\begin{aligned} f_d(s) &= \frac{e^{-sR}\left(1 - F_L(R)\right)}{1 - \int_0^R e^{-sx} dF_L(x)} \\ &= \frac{\left(s + \sum_i \frac{\lambda_i}{v_i}\right) e^{-\left(s + \sum_i \frac{\lambda_i}{v_i}\right) R}}{s + \left(\sum_i \frac{\lambda_i}{v_i}\right) e^{-\left(s + \sum_i \frac{\lambda_i}{v_i}\right) R}}. \end{aligned} \tag{14}$$

Then, the tail probability of the connectivity distance $P_d(\alpha) = P(d > \alpha) = 1 - F_d(\alpha)$ can be obtained by inverting its complementary cumulative distribution function (cdf), which is defined as

$$P_d^*(s) = \frac{1 - f_d(s)}{s} \tag{15}$$

where $f_d(s)$ is as given in (14). Since this expression may not explicitly be inverted, we resort to numerical inverting [24], [25] in the next sections. Moreover, using the derivative property of the Laplace transform, we are able to obtain the explicit form for the average connectivity distance as

$$\begin{aligned} \mathbb{E}(d) = -\left.\frac{\partial f_d(s)}{\partial s}\right|_{s=0} &= \frac{1 - e^{-\sum_i \frac{\lambda_i}{v_i} R}}{\left(\sum_i \frac{\lambda_i}{v_i}\right) e^{-\left(\sum_i \frac{\lambda_i}{v_i}\right) R}} \\ &= \frac{1 - e^{-\lambda R \mathbb{E}(1/V)}}{\lambda \mathbb{E}(1/V) e^{-\lambda R \mathbb{E}(1/V)}}. \end{aligned} \tag{16}$$

IV. Simulation

In this section, we evaluate the model through extensive simulation studies. As mentioned before, in the free-flow traffic state, there are no significant interactions between individual vehicles. Therefore, generating traffic patterns is possible without using microscopic commercial traffic simulators (e.g., CORSIM [25]). We simulate 20 km of an uninterrupted highway using Matlab. Four levels of constant speed are taken into account, which can be considered as the speed levels in different lanes. The speed levels are 80, 110, 130, and 180 km/h, with a probability of occurrence of 20%, 30%, 30%, and 20% respectively. Vehicles are generated from a Poisson process with a parameter of 500 veh/h. Note that vehicles move at their desired speed and that overtaking is allowed. In the various

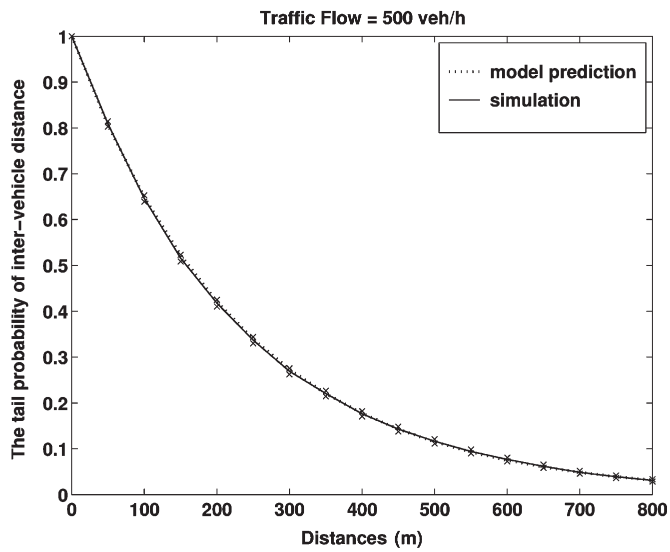


Fig. 3. Evaluating the theoretical tail probability of the intervehicle distances through simulation.

plots in this section, the measurements are plotted together, with crosses $(\times)$ delimiting the 95% confidence interval.

The tail probability of the intervehicle distances, which is given in (3), is evaluated against the results of the simulation. For this purpose, we ran the simulation model 100 times with different random seeds and measured the empirical tail probability of the distance between consecutive vehicles in each run. The measured results can be fitted to an exponential distribution with an estimated mean of 231.95, where the 95% confidence interval is [227.45, 236.51]. Note that the theoretical mean is 231.32. As shown in Fig. 3, the theoretical tail probability of the intervehicle distances sufficiently lies within the 95% confidence interval of the simulation measurement.

The model's expressions for the tail probability of the platoon size and the average platoon size given in (9) and (10), respectively, is evaluated against simulation measurements. We ran the simulation 1000 times with different random seeds and measured the platoon sizes in each run for $R = 500$ m. Then, we computed the empirical tail probability of the platoon size from all the data. Moreover, to measure the average connectivity distance, we ran the simulation for different traffic flows. The obtained empirical curves for the tail probability of the platoon size and the average platoon size are shown in Fig. 4, where the model's predictions sufficiently lie in the 95% confidence interval of the simulation measurements.

We evaluate the model for the tail probability of the connectivity distance obtained from numerical inversion of (15) and the average connectivity distance given in (16). Note that the connectivity distances for vehicles in the same platoon are not independent. Therefore, we ran the simulation 3000 times, and during each run, we measured the empirical connectivity distance for nodes belonging to different platoons. The transmission range of vehicles is 500 m. Fig. 5 shows the results of the simulation in comparison to the model's prediction. As one can conclude, the model's expressions closely match the simulation results.

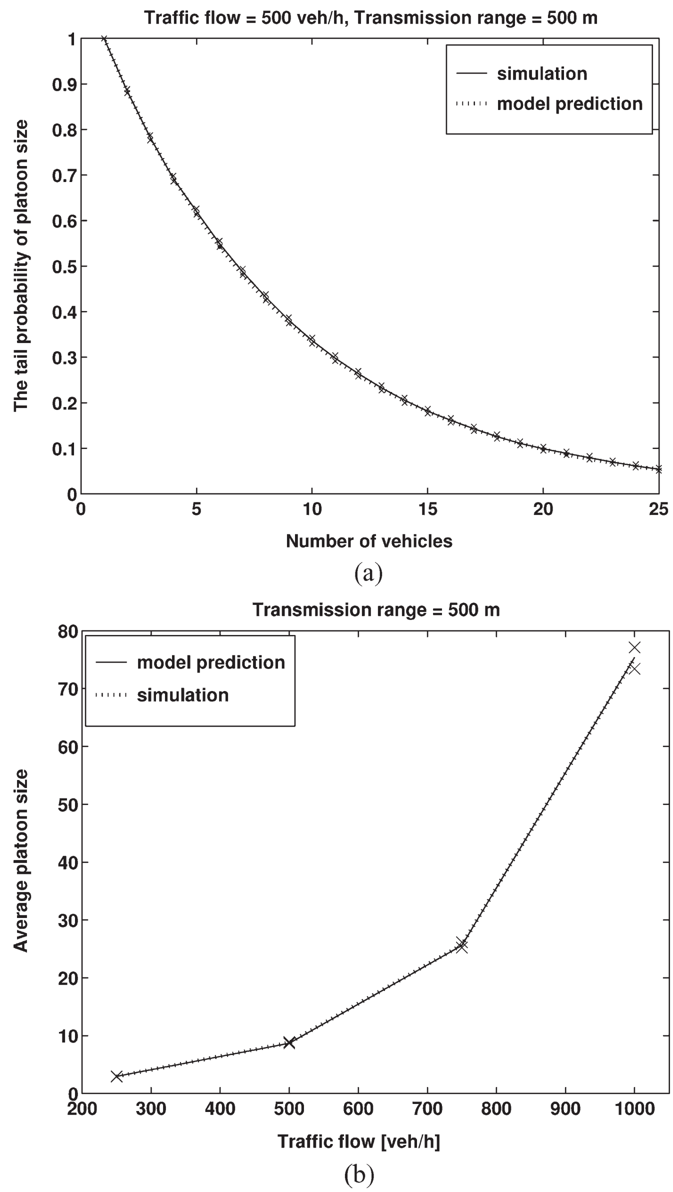


Fig. 4. Comparison of the simulation measurement and the model prediction for (a) the tail probability of the platoon size and (b) the average platoon size.

V. Model Extension

As mentioned in the previous section, as long as we are able to obtain the distribution function of the intervehicle distances and that of the transmission range of vehicles, the connectivity analysis performed in this paper can be extended to more general scenarios. In the following, we briefly introduce some extensions to the basic model.

A. *Two Direction Traffic and Forward/Backward Communications*

We did not assume any special direction to obtain the distribution of the intervehicle distances in (3); thus, our model is able to study the connectivity in backward and forward data communications. This issue is practically important, because, in the safety applications, the messages are usually disseminated backward (e.g., when an in-danger vehicle informs the

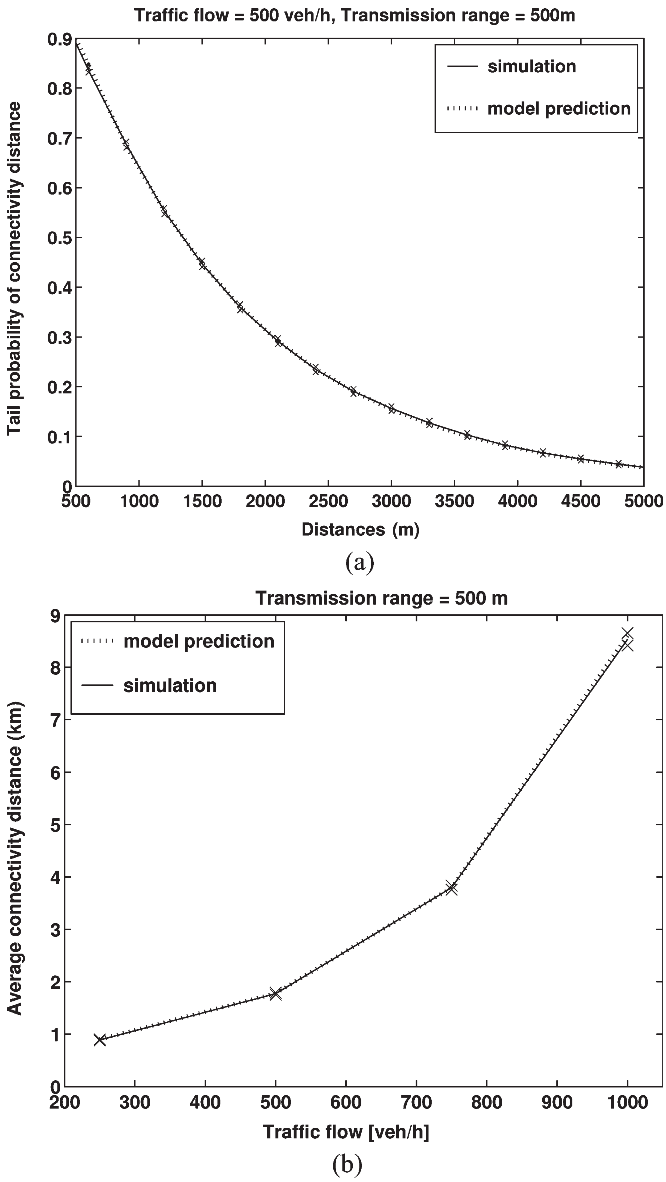


Fig. 5. Comparison of the simulation measurement and the model prediction for (a) the tail probability of the connectivity distance and (b) the average connectivity distance.

approaching vehicles about some events), whereas, in the comfort applications, communication in both directions is demanded.

Furthermore, in a two-direction highway, when the connectivity in one direction is poor, vehicles can use the traffic going the opposite direction to propagate their message. The proposed model can also cover this case. Let λ_{LR} and λ_{RL} be the traffic flows in the left-to-right and right-to-left directions, respectively. Then, we still can use (3) and all the studies done in the previous section with the following considerations: The traffic flow will be $\lambda = \lambda_{\mathrm{LR}} + \lambda_{\mathrm{RL}}$, and to obtain P_i, all speeds in both directions should be taken into account.

B. Multilane Highways

The equivalent infinite server queuing system can be used to study the connectivity in VANETs if each vehicle is able to

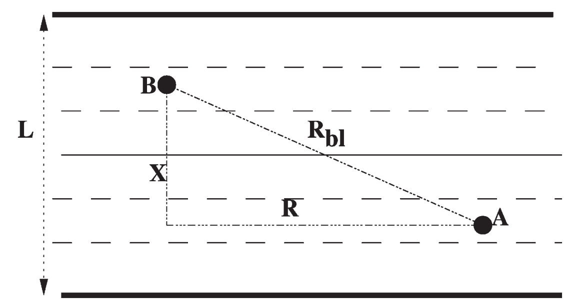


Fig. 6. Extending the model for multilane highways.

communicate with all the vehicles within its transmission range (along the direction of the highway). Therefore, the proposed model can also study the connectivity in multilane highways. Assume, in a possible scenario, as shown in Fig. 6, that two vehicles denoted A and B move in a multilane highway with width L (including all lanes and physical barrier separating the two directions of traffic). As shown in the figure, if vehicle A intends to transmit a message to a distance R along the direction of the highway, it must use a larger transmission range $R_{\mathrm{bl}} = \sqrt{R^2 + X^2} < \sqrt{R^2 + L^2}$. In other words, to propagate the message to distance R, for intralane transmissions, range R is used, whereas, for interlane transmissions, range R_{bl} should be used, which is slightly larger. However, the standard highway's lane width is 12 ft (i.e., 3.6 m) [9], and the vehicle transmission range can be increased to 1000 m, as suggested by the DSRC Standard [1], [2]. (Currently, some field tests show transmission ranges of about 500 m; see [27].) Therefore, the difference between R and R_{bl} is negligible, and our model is able to study the connectivity in multilane highways for the most practical cases.

C. Market Penetration of Wireless-Equipped Vehicles

Until now, we have implicitly assumed that all the vehicles on the road participate in message dissemination. However, the important factor that affects connectivity in VANETs is the market penetration of wireless-equipped vehicles. It is predicted that it will take a relatively long time before all vehicles on roads are equipped with wireless transceivers (e.g., DSRC-enabled systems). Studying the effects of market penetration is interesting, because it influences the density of participant vehicles. Although the basic proposed model assumes that all vehicles will participate in message dissemination, it can easily take into account market penetration. Assume market penetration to be $P_{\mathrm{mp}}\%$. Obviously, we can assume that the type of the nth vehicle, in the sense of whether it is equipped or not with wireless capabilities, is independent of the type that other vehicles will have. As obtained in Section III, the distribution of the intervehicle distances (including wireless-equipped and nonwireless-equipped vehicles) is exponential with parameter $\xi = \sum_{i=1}^{N}(\lambda_i/v_i)$. Then, we can describe the distribution of the number of participant vehicles as a thinned Poisson process with parameter ξP_{mp}. As a result, the intervehicle distances for the participant vehicles (wireless-equipped vehicles) are exponentially distributed with parameter ξP_{mp}. Thus, using the new distribution of the intervehicle distances, we can study connectivity, as was performed in Section III.

D. Heterogeneous Vehicle Network

In this paper, we assume that the transmission range of all vehicles are equal, which is represented by a fixed number R. However, an equivalent infinite server queuing model is also able to study the heterogeneous case, i.e., different vehicles have different transmission ranges. In practice, this may happen because of the difference in vehicle height (e.g., cars, trucks, and buses) or the difference in their wireless equipment (e.g., antenna gains). In this case, the transmission range of vehicles is not a deterministic value but may be expressed as a random variable with a probability distribution function. Thus, to study connectivity, we should use an equivalent $M/G/\infty$ [28] model. A special case of this problem is studied in [29].

VI. Effects of Traffic Parameters on Connectivity

To have a better understanding of the connectivity in VANETs, in this section, we investigate the connectivity by numerically evaluating the model. As noted in Section I, the connectivity problem is worth studying in low-density traffic, which corresponds to the free-flow state shown in Fig. 1. In the free-flow state, the traffic flow is usually considered to be below 1000 veh/h/lane for freeways and below 500 veh/h/lane for other roads [9]. Moreover, although the proposed transmission range for the DSRC standard is 1000 m [1], [2], the current feasible range is about 300 m [27]. In this paper, we provide results by taking the traffic flow values below 1000 veh/h and the transmission range values of up to 800 m. Furthermore, we assume that the vehicles' speed is normally distributed, which also holds in the free-flow state [9], [16], and use some values reported in Table I. In Figs. 7–16 , speed distribution is denoted as $N(\mu, \sigma)$, where μ and σ are the mean and standard deviation values, respectively.

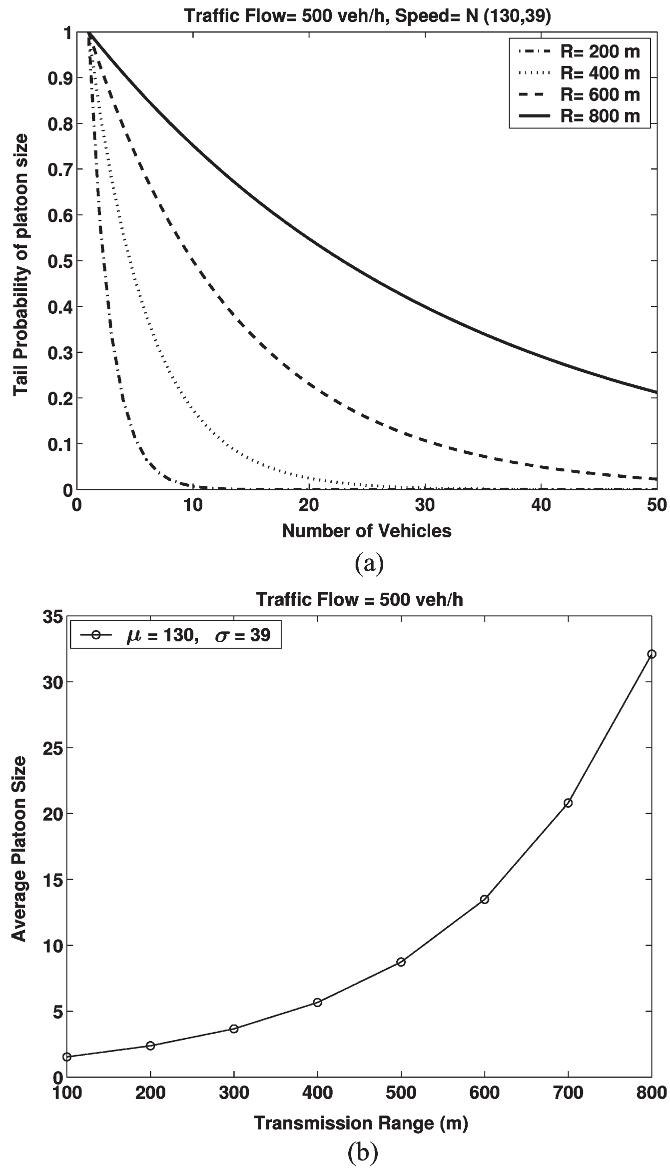


Fig. 7. Effect of the vehicle's transmission range on (a) the tail probability of the platoon size and (b) the average platoon size.

A. Effect of the Transmission Range

The transmission range is a parameter that can be tuned by the network and application designers. From (9) and (10), it follows that, by increasing the transmission range, the tail probability of the platoon size and the expected value of the platoon size increase. This fact is shown in Fig. 7(a) and (b). Furthermore, (15) gives the Laplace transform of the tail probability of the connectivity distance. Although it may be difficult to find the explicit form for its inversion, we are able to numerically invert it. For this purpose, we use the Gaver–Stehfest method [24], [25], which permits recovery of the tail probability of the connectivity distance, approximately, from its Laplace transform sampled at a few points on the positive real axis. Note that the tail probability is a continuous (for distances larger than R [12]) and nonperiodic function of distance; hence, the Gaver–Stehfest method is appropriate for our purpose. Moreover, (16) presents the explicit form for the expected value of the connectivity distance. Fig. 8(a) and (b) shows that, when the transmission range increases, the tail probability and expected value of the connectivity distance increase.

As shown in Figs. 7 and 8, when the transmission range is sufficiently large (for example, $R > 500$ m with the specific parameter settings in the figure), even if the transmission range slightly increases, the expected values of the platoon size and connectivity distance noticeably increase. Furthermore, larger platoon sizes and larger connectivity distances are more probable. This observation shows the promise for connectivity problems in VANETs, i.e., if the higher transmission ranges are achievable (from a technical point of view), then the connectivity becomes quite satisfactory, even in the free-flow traffic state.

B. Effect of the Traffic Flow

The traffic flow is usually given as a number measured by counting the number of vehicles passing an observer per unit of

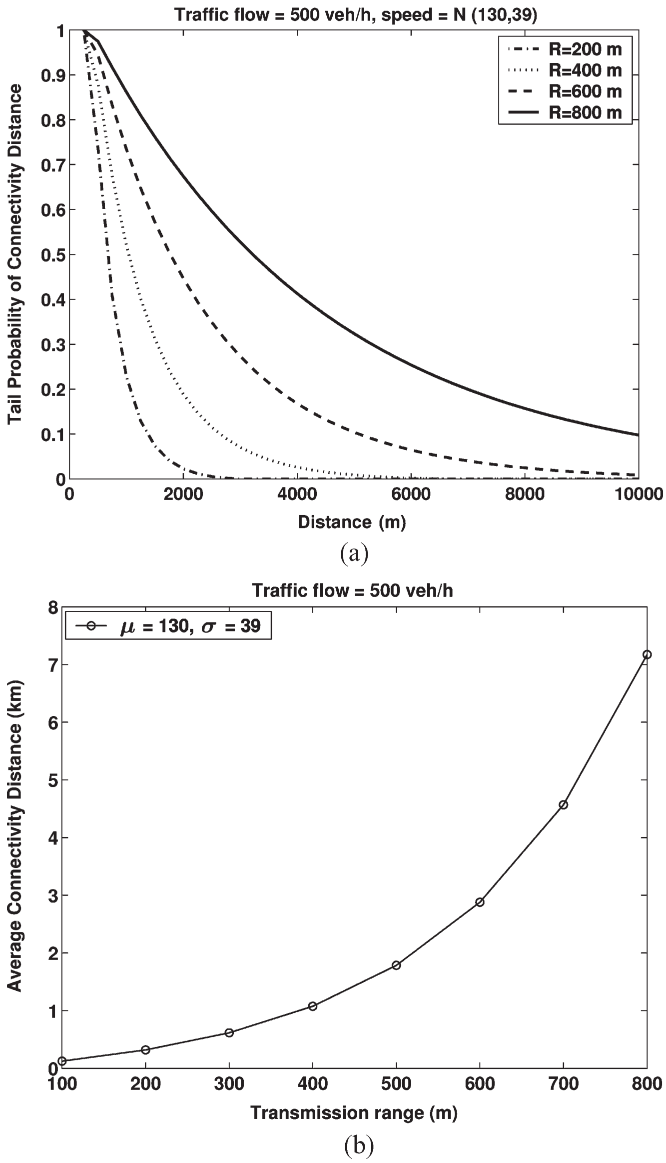


Fig. 8. Effect of the vehicle's transmission range on (a) the tail probability of the connectivity distance and (b) the average connectivity distance.

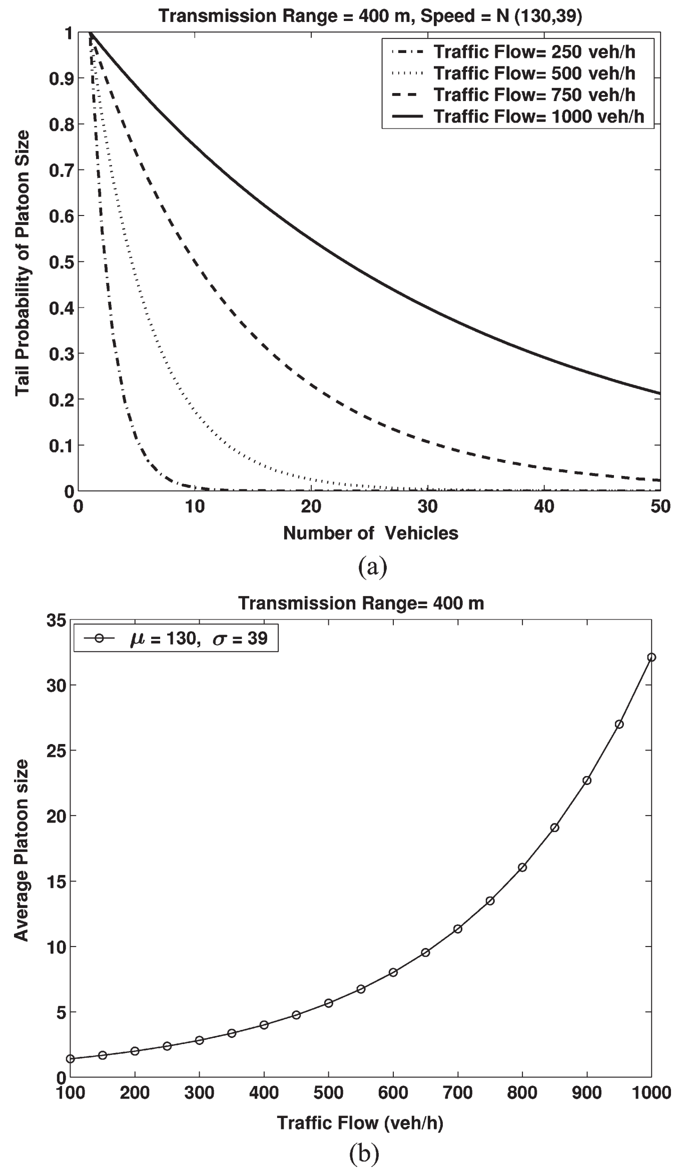


Fig. 9. Effect of traffic flow on (a) the probability of the platoon size and (b) the average platoon size.

time. It affects the connectivity by influencing the density of the vehicles on the road. It follows from (9) and (10) that, when the traffic flow increases, the tail probability of the platoon size and the expected number of vehicles in a platoon increase. Fig. 9(a) and (b) shows this phenomenon. Moreover, Fig. 10(a) and (b) shows the effect of the traffic flow on the tail probability [which is obtained from numerical inversion of (15)] and expected value of the connectivity distance, as given in (16). As one can conclude that when the traffic flow increases, the tail probability of the connectivity distance and the average connectivity distance increase.

As shown in Figs. 9 and 10, for sufficiently large traffic flows (for example, $\lambda > 600$ veh/h with the specific parameter settings in the figures), the expected values of the platoon size and connectivity distance are noticeably improved by increasing the traffic flow. Furthermore, larger platoon sizes and connectivity distances are more likely to be observed.

C. Effect of the Traffic's Speed

Speed is one of the most distinctive characteristics of VANETs in comparison with general MANETs. In the presence of high speed, the connectivity dramatically degrades. Therefore, deploying communication protocols and applications demands having a good understanding of the effects of speed on connectivity and other communication phenomena. To investigate the effect of speed on connectivity, we use stochastic ordering tools to provide bounds on the connectivity metrics. In the following, we first find general stochastic bounds; then, we numerically study the effects of speed and evaluate the obtained bounds.

1) Stochastic Bounds: Stochastic ordering is a way to compare stochastic processes and provide some bounds for their behavior. It has been applied to many different fields such as statistics, reliability, and finance. To investigate stochastic orders between two random variables, one can study their cdfs.

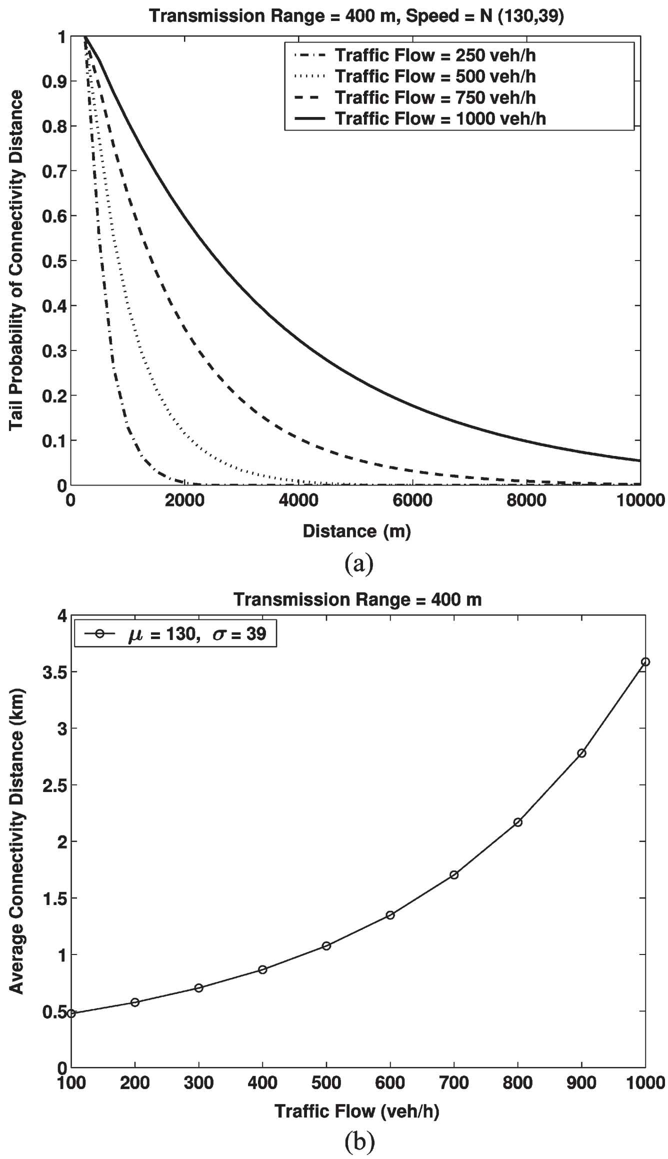


Fig. 10. Effect of traffic flow on (a) the probability of the connectivity distance and (b) the average connectivity distance.

Let $F(.)$ and $G(.)$ be the distribution function of two random variables, e.g., X and Y, respectively. Then, some stochastic orders can be defined as follows [30]–[32]:

1) Increasing (first-order) stochastic ordering is defined as follows:

$$X \preceq_{\text{inc}} Y \Longleftrightarrow F(a) \geqslant G(a) \tag{17}$$

for all values of a, with strict inequality in at least one point in the right-hand expression [33]. This definition easily leads to the following result:

$$X \preceq_{\text{inc}} Y \Longleftrightarrow \mathbb{E}\left(f(X)\right) \leqslant \mathbb{E}\left(f(Y)\right) \quad \text{for all } f: f(x) \text{ is increasing in } x. \tag{18}$$

Note that, in the preceding equation, the direction of the inequality is reversed for all decreasing functions.

2) Increasing-concave (second-order) stochastic ordering is defined as

$$X \preceq_{\text{incv}} Y \Longleftrightarrow \int_{-\infty}^{a} F(u)du \geqslant \int_{-\infty}^{a} G(u)du \tag{19}$$

for all values of a, with strict inequality in at least one point in the right-hand expression [33]. The preceding definition leads to the following result:

$$X \preceq_{\text{cnv}} Y \Longleftrightarrow \mathbb{E}(f(X)) \leqslant \mathbb{E}(f(Y)) \quad \text{for all } f: f(x) \text{ is increasing and concave in } x. \tag{20}$$

Note that, in the preceding equation, the direction of inequality is reversed for all decreasing-convex functions.

3) Concave stochastic ordering is defined as follows:

$$X \preceq_{\text{cnv}} Y \Longleftrightarrow \mathbb{E}(X) = \mathbb{E}(Y) \text{ and } X \preceq_{\text{incv}} Y. \tag{21}$$

The preceding definition is also equivalent to the following expression:

$$X \preceq_{\text{cnv}} Y \Longleftrightarrow \mathbb{E}\left(f(X)\right) \leqslant \mathbb{E}\left(f(Y)\right) \quad \text{for all } f: f(x) \text{ is concave in } x \tag{22}$$

where the direction of inequality is reversed for all convex functions. From (21), it can directly be concluded that the concave stochastic ordering implies the increasing-concave stochastic ordering, i.e., $X \preceq_{\text{cnv}} Y \Rightarrow X \preceq_{\text{incv}} Y$.

Now, let $f(x) = 1/x$. Obviously, this function is decreasing and convex for $x > 0$. Hence, applying (18), we can have the following:

$$V \preceq_{\text{inc}} V' \Longrightarrow \mathbb{E}(1/V) \geqslant \mathbb{E}(1/V'). \tag{23}$$

Furthermore, from (22), the following inequality is obtained:

$$V \preceq_{\text{cnv}} V' \Longrightarrow \mathbb{E}(1/V) \geqslant \mathbb{E}(1/V'). \tag{24}$$

The preceding expressions enable us to find some bounds on our metrics of connectivity to compare different speed scenarios. Let V and V' be two random variables representing two different speed scenarios, and let N and N' be the corresponding platoon size (for a fixed traffic flow and transmission range). Since $f(x) = 1 - e^{-\lambda R x}$ is an increasing function in x, from (9) and using (23) and (24), we reach the following results:

$$\begin{aligned} V \preceq_{\text{inc}} V' &\Longrightarrow \mathbb{E}(1/V) \geqslant \mathbb{E}(1/V') \\ &\Longrightarrow \left(1 - e^{-\lambda R \mathbb{E}(1/V)}\right) \geqslant \left(1 - e^{-\lambda R \mathbb{E}(1/V')}\right) \\ &\Longrightarrow \left(1 - e^{-\lambda R \mathbb{E}(1/V)}\right)^{k-1} \geqslant \left(1 - e^{-\lambda R \mathbb{E}(1/V')}\right)^{k-1} \\ &\Longrightarrow P_N(k) \geqslant P_{N'}(k) \end{aligned} \tag{25}$$

and similarly

$$\begin{aligned} V \preceq_{\mathrm{cnv}} V' &\Longrightarrow \mathbb{E}(1/V) \geqslant \mathbb{E}(1/V') \\ &\Longrightarrow \left(1 - e^{-\lambda R \mathbb{E}(1/V)}\right) \geqslant \left(1 - e^{-\lambda R \mathbb{E}(1/V')}\right) \\ &\Longrightarrow \left(1 - e^{-\lambda R \mathbb{E}(1/V)}\right)^{k-1} \geqslant \left(1 - e^{-\lambda R \mathbb{E}(1/V')}\right)^{k-1} \\ &\Longrightarrow P_N(k) \geqslant P_{N'}(k). \end{aligned} \tag{26}$$

Moreover, $f(x) = 1/e^{-\lambda R x}$ is an increasing function of x. Thus, from (10) and using (23) and (24), we reach the following results:

$$\begin{aligned} V \preceq_{\mathrm{inc}} V' &\Longrightarrow \mathbb{E}(1/V) \geqslant \mathbb{E}(1/V') \\ &\Longrightarrow \frac{1}{e^{-\lambda R \mathbb{E}(1/V)}} \geqslant \frac{1}{e^{-\lambda R \mathbb{E}(1/V')}} \\ &\Longrightarrow \mathbb{E}(N) \geqslant \mathbb{E}(N') \end{aligned} \tag{27}$$

and likewise

$$\begin{aligned} V \preceq_{\mathrm{cnv}} V' &\Longrightarrow \mathbb{E}(1/V) \geqslant \mathbb{E}(1/V') \\ &\Longrightarrow \frac{1}{e^{-\lambda R \mathbb{E}(1/V)}} \geqslant \frac{1}{e^{-\lambda R \mathbb{E}(1/V')}} \\ &\Longrightarrow \mathbb{E}(N) \geqslant \mathbb{E}(N'). \end{aligned} \tag{28}$$

We conclude that, for a fixed traffic flow (i.e., λ) and the vehicles' transmission range (i.e., R), the tail probability of the platoon size and the expected value of the platoon size decrease if the velocity increases in the sense of the increasing stochastic ordering as well as in the sense of the concave stochastic ordering.

Now, let V and V' be two random variables representing two different speed scenarios, and let d and d' be the corresponding connectivity distance (for a fixed traffic flow and transmission range). Considering the practically significant values of traffic flow, the vehicles' transmission range, and the vehicles' speed, we can easily see that $f(\mathbb{E}(1/V)) = (1 - e^{-\lambda R \mathbb{E}(1/V)})/(\lambda \mathbb{E}(1/V) e^{-\lambda R \mathbb{E}(1/V)})$ is an increasing function. Therefore, from (16) and applying (23) and (24), we reach the following results:

$$\begin{aligned} V \preceq_{\mathrm{inc}} V' &\Longrightarrow \mathbb{E}(1/V) \geqslant \mathbb{E}(1/V') \\ &\Longrightarrow \frac{1 - e^{-\lambda R \mathbb{E}(1/V)}}{\lambda \mathbb{E}(1/V) e^{-\lambda R \mathbb{E}(1/V)}} \\ &\qquad \geqslant \frac{1 - e^{-\lambda R \mathbb{E}(1/V')}}{\lambda \mathbb{E}(1/V') e^{-\lambda R \mathbb{E}(1/V')}} \\ &\Longrightarrow \mathbb{E}(d) \geqslant \mathbb{E}(d') \end{aligned} \tag{29}$$

$$\begin{aligned} V \preceq_{\mathrm{cnv}} V' &\Longrightarrow \mathbb{E}(1/V) \geqslant \mathbb{E}(1/V') \\ &\Longrightarrow \frac{1 - e^{-\lambda R \mathbb{E}(1/V)}}{\lambda \mathbb{E}(1/V) e^{-\lambda R \mathbb{E}(1/V)}} \\ &\qquad \geqslant \frac{1 - e^{-\lambda R \mathbb{E}(1/V')}}{\lambda \mathbb{E}(1/V') e^{-\lambda R \mathbb{E}(1/V')}} \\ &\Longrightarrow \mathbb{E}(d) \geqslant \mathbb{E}(d'). \end{aligned} \tag{30}$$

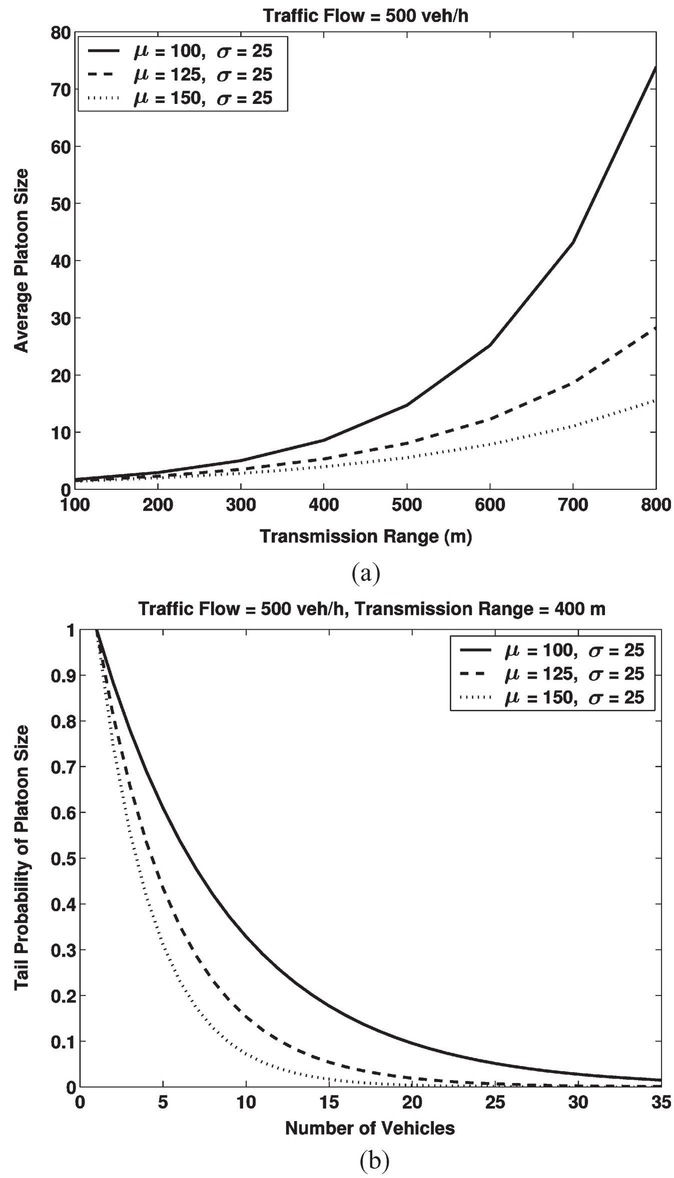


Fig. 11. Effect of speed scenarios with similar variances and different means on (a) the average platoon size and (b) the tail probability of the platoon size.

From (29) and (30), we conclude that, for a fixed traffic flow (i.e., λ) and the vehicles' transmission range (i.e., R), the expected value of the connectivity distance decreases if the velocity increases under increasing stochastic ordering and concave stochastic ordering.

The stochastic bounds presented in this section hold for general-speed distributions. Given some speed scenarios, one can investigate the stochastic ordering between them based on their distribution function and using (17) and (19). Below we evaluate the aforementioned bounds for normally distributed speeds, which are practically important in the free-flow traffic state.

2) Numerical Study: As argued in Section III, the traffic's speed distribution in the free-flow state is usually normal. Since a normal distribution is completely described by its mean and variance, we intend to study the effects of the speed's mean and variance on the connectivity. Meanwhile, we exclude the zero speed, and so, we deal with a truncated version of the speed distribution. Let F^* and G^* be two normal distributions with parameters (μ_1, σ_1) and (μ_2, σ_2), respectively, and let F and G be the corresponding truncated distributions. In the next section, we will study the following cases. Note that the effect of speed is studied under the assumption of the same traffic flow and the same transmission range.

1) If $\mu_1 > \mu_2$ and $\sigma_1 = \sigma_2$: In this case, it is easy to show that $F \succeq_{\text{inc}} G$ (see the Appendix). From (25) and (27), it follows that, for the same traffic flow and transmission range, the speed, which is larger under increasing stochastic order, leads to a lower tail probability and a lower average platoon size. Figs. 11 and 12 show the numerical evaluation of the model using three speed scenarios with similar variances but different means. As shown in Fig. 11(a) and (b), the speed scenario with higher mean leads to a lower tail probability of the platoon size and a lower expected value of the platoon size, as expected from the obtained bounds.

   The bounds in (29) for the average connectivity distance is evaluated in Fig. 12(a). Furthermore, Fig. 12(b) shows the tail probability of the connectivity distance obtained from numerical inversion of (15) [24], [25]. As one can conclude, for the same variance, the speed scenario with higher mean causes a lower tail probability of the connectivity distance and a lower expected value of the connectivity distance.
2) If $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$: In this case, it can be shown that $F \succeq_{\text{incv}} G$ and that $F \succeq_{\text{cnv}} G$ (see the Appendix). From (26) and (28), we conclude that the speed scenario, which is larger under concave stochastic ordering, leads to a lower tail probability of the platoon size and a lower average platoon size. Fig. 13(a) and (b) shows the comparison of the average platoon size and the tail probability of the platoon size for three different speed scenarios with similar means but different variances. As it follows from the figure, the traffic's speed with higher variance leads to a higher average platoon size and a higher tail probability of the platoon size. Thus, it agrees with the obtained bounds.

   Moreover, the effects of variance on the average connectivity distance, when the speed's mean remains fixed, is studied in Fig. 14(a). As shown in the figure, the speed with higher variance leads to a higher average connectivity distance; thus, the bound in (30) is confirmed. Furthermore, we depict the tail probability of the connectivity distance obtained from numerical inversion of (15) [24], [25]. As shown in Fig. 14(b), the speed scenario with higher variance results in a higher tail probability of the connectivity distance. The obtained results show that, if the variance of the speed's distribution increases, then, provided that the average speed remains fixed, the connectivity is improved. This may be a surprising finding.
3) If $\mu_1 \neq \mu_2$ and $\sigma_1 \neq \sigma_2$: In this case, the following theorem from [34] provides some stochastic orders:

Theorem 1: Let F^* and G^* be two normal distributions with parameters (μ_1, σ_1) and (μ_2, σ_2), respectively, and let F and G be the corresponding truncated distributions; then, three conditions hold.

1) If $\mu_1 > \mu_2$ and $\sigma_1 > \sigma_2$, F is larger than G under increasing stochastic order (first order) if and only if $(\mu_1 - \mu_2)/(\sigma_1 - \sigma_2) > \delta$.
2) If $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$, F is larger than G under increasing stochastic order (first order) if and only if $(\mu_1 - \mu_2)/(\sigma_2 - \sigma_1) > \delta$.
3) If $\mu_1 > \mu_2$ and $\sigma_1 < \sigma_2$, F is larger than G under of increasing concave order (second order).

A_1 and B_1 are the left and right truncation points of F, respectively. In addition, in G, A_2 and B_2 are the left and right truncation points, respectively. The left truncation points correspond to $v_{\min}$, and the right truncation points correspond to $v_{\max}$ in (5). The parameter $\delta > 0$ is the function of the truncation points defined as $(A_1 - \mu_1)/\sigma_1 = (A_2 - \mu_2)/\sigma_2 = -\delta$ and $(B_1 - \mu_1)/\sigma_1 = (B_2 - \mu_2)/\sigma_2 = \delta$. The

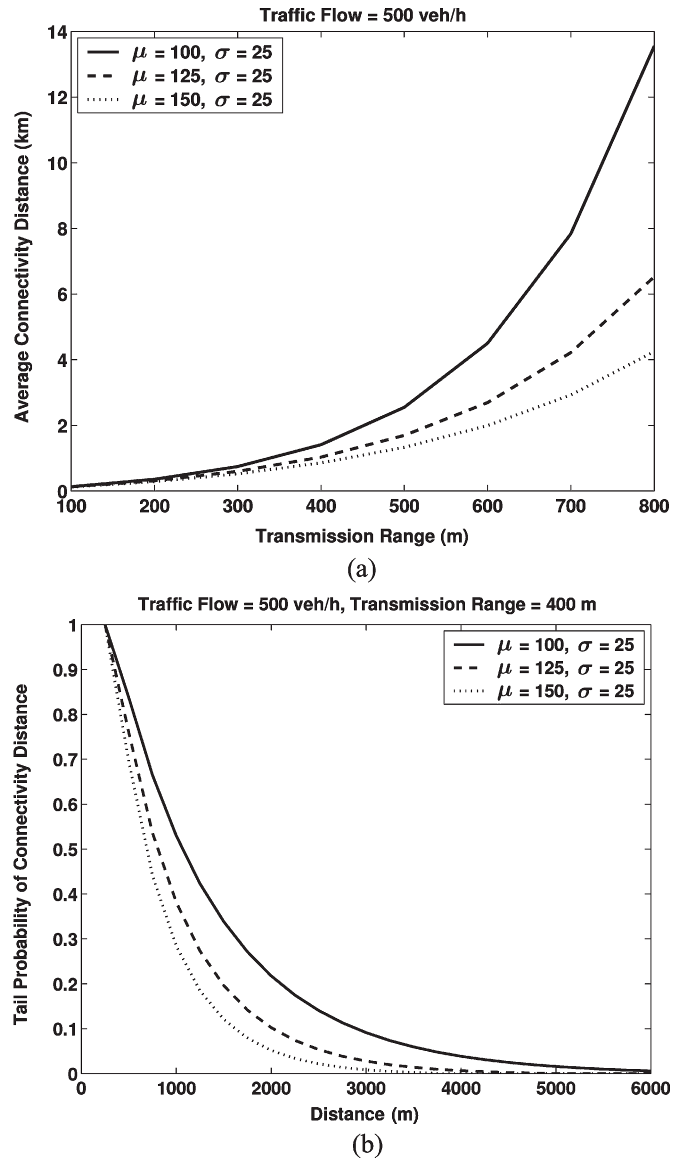


Fig. 12. Effect of speed scenarios with similar variances and different means on (a) the average connectivity distance and (b) the tail probability of the connectivity distance.

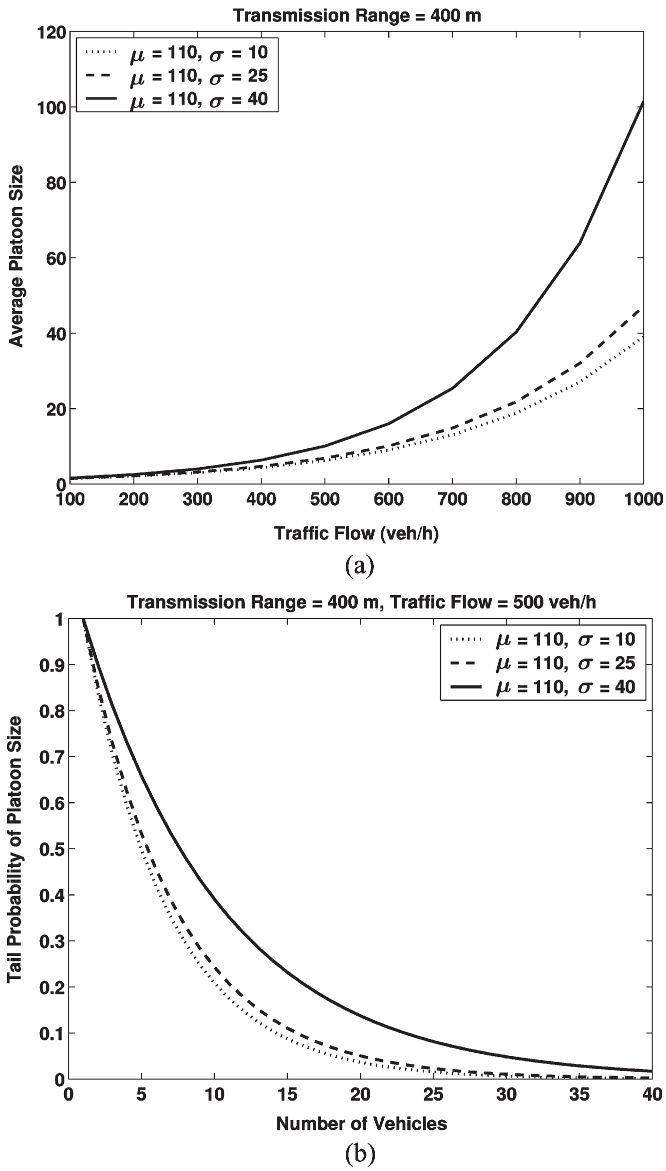


Fig. 13. Effect of speed scenarios with similar means and different variances on (a) the average platoon size and (b) the tail probability of the platoon size.

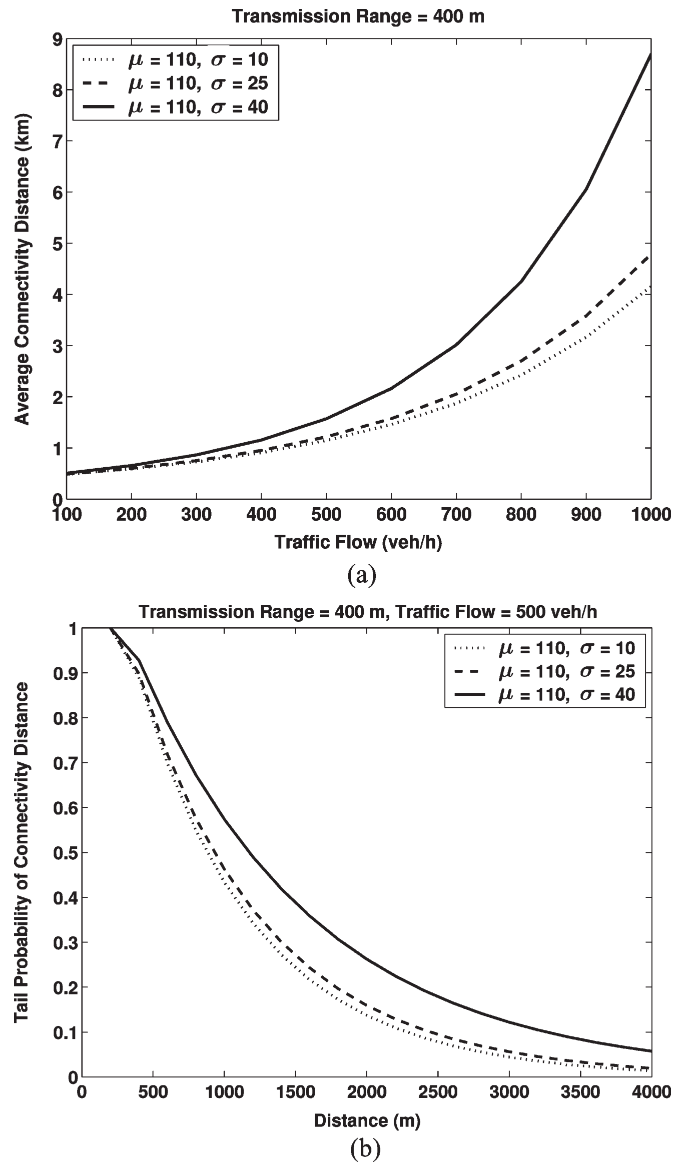


Fig. 14. Effect of speed scenarios with similar means and different variances on (a) the average connectivity distance and (b) the tail probability of the connectivity distance.

aforementioned theorem is able to define stochastic orders for many scenarios. However, in this section, we conduct some numerical investigation, along with the practically interesting speed data in Table I. Note that, considering the values in the table, the truncation condition in the aforementioned theorem holds by taking $\delta > 3$; thus, case 1 holds. Fig. 15(a) and (b) shows the tail probability of the platoon size and the average platoon size depicted by (9) and (10), respectively. As one can conclude, the figure confirms the bounds obtained in (25) and (27), i.e., the speed scenario with a larger mean and variance leads to a lower tail probability of the platoon size and a lower average platoon size. Furthermore, Fig. 16(a) shows the average connectivity distance depicted by (16). The figure agrees with the bounds obtained in (29). Moreover, Fig. 16(b) shows the tail probability of the connectivity distance obtained from numerical inversion of (15) [24], [25]. As a result, the speed scenario with a higher mean and variance, provided that the truncation condition is satisfied, results in a lower tail probability and a lower expected value of the connectivity distance.

VII. Conclusion and Future Work

We presented an analytical model for studying connectivity in VANETs. We invoked an equivalent $M/D/\infty$ queueing model to obtain the Laplace transform of the tail probability of the connectivity distance and an explicit form for the expected value of the connectivity distance. Moreover, we obtained explicit forms for the tail probability of the platoon size and the expected value of the platoon size. Then, we numerically studied the model, along with some publicly available traffic statistics. Our findings show that increasing the traffic flow and the vehicles' transmission range leads to increasing the aforementioned metrics of connectivity. However, the observed improvement is more considerable if the traffic flow and the

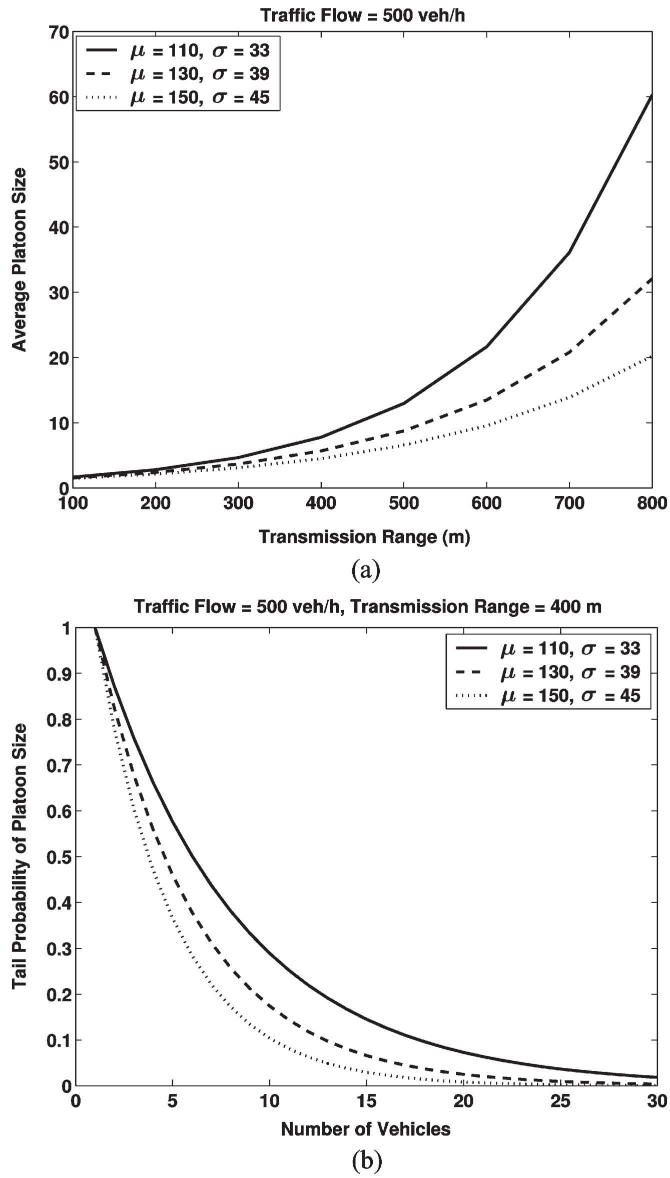


Fig. 15. Effects of the speed scenarios in Table I on (a) the average platoon size and (b) the tail probability of the platoon size.

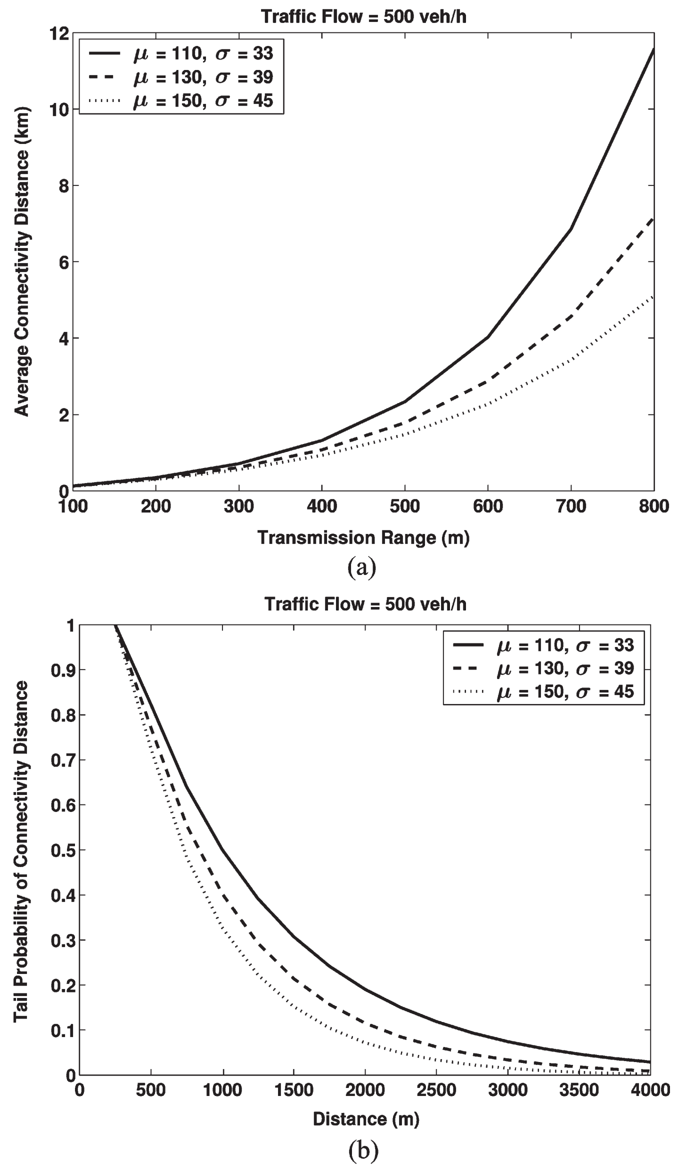


Fig. 16. Effects of the speed scenarios in Table I on (a) the average connectivity distance and (b) the tail probability of the connectivity distance.

vehicles' transmission range are sufficiently large. Moreover, we found that when the traffic's speed increases under increasing stochastic ordering and concave stochastic ordering, the metrics of connectivity decrease. For the case of normally distributed speeds (which holds in the free-flow traffic state), we studied the effects of the mean and variance of the traffic's speed distribution on connectivity. One of our findings shows that if the variance of the speed's distribution is increased, then, provided that the average speed remains fixed, the connectivity is improved.

In future work, we plan the following: 1) to study the effects of channel randomness (i.e., shadowing and fading) in a vehicular environment on the connectivity; 2) to study the connectivity in the presence of fixed and mobile base stations; 3) to evaluate the model with realistic traffic patterns; and 4) to study the duration and reliability of connectivity.

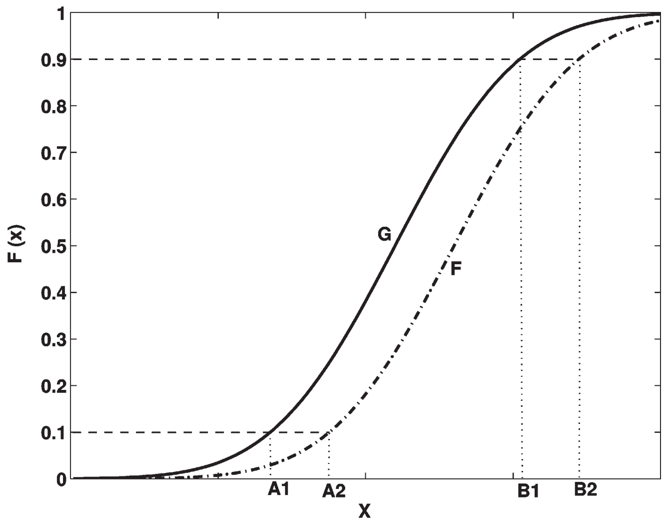
Appendix
Some Stochastic Orders for Truncated Normal Distributions

The purpose of this Appendix is to prove some stochastic orders used in this paper for truncated normal distributions. Let F^* and G^* be two normal distributions with parameters (μ_1, σ_1) and (μ_2, σ_2), respectively, and let F and G be the corresponding truncated distributions. We intent to study the stochastic orders between F and G for the following special cases:

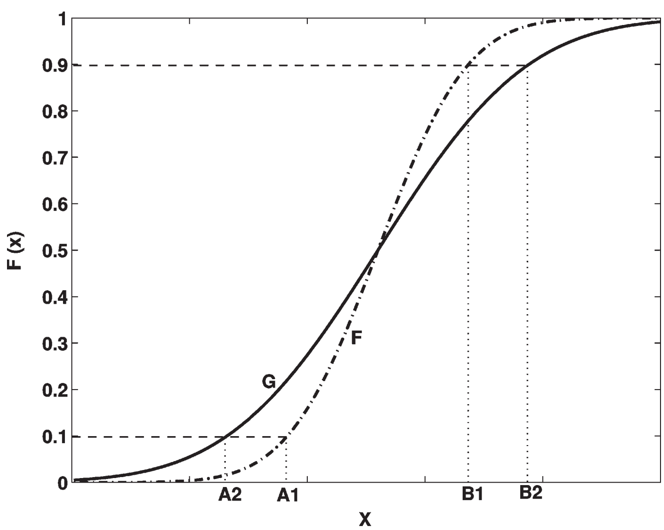
- if $\mu_1 > \mu_2$ and $\sigma_1 = \sigma_2$;
- if $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$.

Some other cases are covered by [34]. First, we know that the normal distribution has the following distribution function:

$$H_X(x) = P(X \leqslant x) = \frac{1}{2}\left(1 + \mathrm{erf}\left[\frac{x-\mu}{\sigma\sqrt{2}}\right]\right) \tag{31}$$



(a)



(b)

Fig. 17. CDF of two truncated normal distributions with (a) equal variances but different means ($\mu_F > \mu_G$) and (b) equal means but different variances ($\sigma_G > \sigma_F$).

where $\operatorname{erf}(x) = (2/\sqrt{2\pi}) \int_0^x e^{-t^2} dt$ is, by definition, the error function. When a normal distribution is truncated, the area cut from the left and right of its pdf is shifted to the center. We denote the left and right truncation points as A and B, where $H_X(x) = 0$ if $x < A$ and $H_X(x) = 1$ if $x > B$. Therefore, F^* and G^* intersect at most once, and the intersection point x_0 is given by $(x_0 - \mu_1)/\sigma_1 = (x_0 - \mu_2)/\sigma_2$. The truncation is symmetric if $(A_1 - \mu_1)/\sigma_1 = (A_2 - \mu_2)/\sigma_2 = -\delta$ and $(B_1 - \mu_1)/\sigma_1 = (B_2 - \mu_2)/\sigma_2 = \delta$, where A_i and B_i $(i = 1, 2)$ are the left and right truncation points, respectively. Hereinafter, we consider the symmetric truncation.

Some stochastic orders between two random variables can be studied based on their distribution functions [30]–[32].

- Increasing (first-order) stochastic ordering is defined as follows:

$$X \preceq_{\text{inc}} Y \Longleftrightarrow F(a) \geqslant G(a) \text{ for all values of } a. \quad (32)$$

- Increasing-concave (second-order) stochastic ordering is defined as

$$X \preceq_{\text{incv}} Y \Longleftrightarrow \int_{-\infty}^{a} F(u)du \geqslant \int_{-\infty}^{a} G(u)du \quad (33)$$

for all values of a.

- Concave stochastic ordering is defined as follows:

$$X \preceq_{\text{cnv}} Y \Longleftrightarrow \mathbb{E}(X) = \mathbb{E}(Y) \text{ and } X \preceq_{\text{incv}} Y. \quad (34)$$

- Increasing convex stochastic ordering is defined as follows:

$$X \preceq_{\text{incx}} Y \Longleftrightarrow \int_{a}^{\infty} F(u)du \geqslant \int_{a}^{\infty} G(u)du \quad (35)$$

for all values of a.

Note that, in (32), (33), and (35), at lease one strict inequality must hold [33]. Then, two conditions hold.

1) If $\mu_1 > \mu_2$ and $\sigma_1 = \sigma_2$: $F \succeq_{\text{inc}} G$, $F \succeq_{\text{incv}} G$, and $F \succeq_{\text{incx}} G$: In this case, F^* and G^* do not intersect, and neither do F and G. From Fig. 17(a), we can see that $A1 < A2$ and $B1 < B2$. Thus, from (32), it is easy to see that $F \succeq_{\text{inc}} G$. Although it is possible to investigate (33) and (35), since the increasing stochastic order is the stronger order, we can directly deduce that $F \succeq_{\text{incv}} G$ and $F \succeq_{\text{incx}} G$.
2) If $\mu_1 = \mu_2$ and $\sigma_1 < \sigma_2$: $F \succeq_{\text{incv}} G$, $F \preceq_{\text{incx}} G$ and $F \preceq_{\text{cnx}} G$: First, we know that F^* and G^* intersect at, most, one point, where $x_0 = \mu_F = \mu_G$ [see Fig. 17(b)]. Moreover, the one with lower variance F crosses the other from below. In this case, $A1 > A2$ and $B1 < B2$, and since the averages are equal, the area between the two curves is symmetric around the average. Thus, for any given point a, it can easily be shown that $\int_{-\infty}^{a}[G(u) - F(u)]du \geqslant 0$, and so, $F \succeq_{\text{incv}} G$. Furthermore, for the all values of a, we have $\int_{a}^{\infty}[F(u) - G(u)] \geqslant 0$, and so, $F \preceq_{\text{incx}} G$. Moreover, since the averages are equal, from (34), we can easily deduce that $F \preceq_{\text{cnx}} G$, and consequently, $F \succeq_{\text{cnv}} G$.

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