Robust Adaptive Flow Control in High Speed Telecommunication Networks

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Abstract

We consider a single controlled traffic source, sharing a bottleneck node with other (higher priority) sources, whose traffic is modeled by an ARMA process. The controlled source adapts dynamically its transmission rate to the remaining available bandwidth. We focus in this paper on the case when the ARMA parameters are unknown. We use an H^∞ approach, and design a controller which combines identification (of the ARMA parameters) with the control in a novel way.

1 Introduction

We consider "best-effort" type control of the flow of traffic into a telecommunication network. In the best-effort approach there is not a fixed bandwidth allocated to the source, but instead, the resources allocated to the source at each moment depend on those currently available. This is the main type of service provided by the Internet (in particular, by the TCP/IP congestion control [10]), and so is the Available Bit Rate (ABR) service-type in ATM (see [1]).

A primary goal of the design of flow control is to achieve some overall performance objectives for the different traffic sources, such as guaranteeing some bounds on the throughput, delay, loss probabilities. In particular, the flow control should be designed so as to protect the network; any single source should thus adapt to the changing network conditions. Another goal for flow controller design is to *optimize* the use of the network resources. This has also been spelled out in [1] p. 2: "The primary role of the parameters and procedures used for traffic control and congestion control is to protect the network and the-end-system in order to achieve network performance objectives. An additional role is to optimize the use of network resources... The design of ATM layer traffic and congestion controls should minimize network and end-system complexity while maximizing network utilization".

In this paper we consider a single controlled source, sharing a bottleneck node with other (higher priority) sources. In order to protect the high-priority traffic, the source has to adapt dynamically its transmission rate to the remaining available bandwidth. The traffic from the high priority sources is modeled by an ARMA process. Such a formulation allows for long-range correlated traffic (see [2] for a related model), and is amenable to queueing analysis, which is especially desirable in order to compute performance measures (see [2, 3, 5]). In addition, the extraction of the parameters of the ARMA process can be done in real time, with fairly low complexity (see [6]).

A most natural framework for designing control schemes that achieve the objective of optimizing the use of the network is that of optimal control. Specifically, we focus here on the H^∞ approach, which meets the objective of low-complexity of implementation, and is based on guaranteeing the best performance under worst-case conditions. We use a continuous-time version of the model introduced in [4, 5], in which optimal H^∞ flow controllers were designed, based on an ARMA model for the higher priority traffic, with known parameters. When the parameters are unknown, a natural heuristic would be to use the certainty-equivalence approach: periodically, the parameters may be estimated (using the optimal identification schemes proposed and analyzed in [8],[6]), and then use the control designed (in [4, 5]) for the interfering traffic having these as their real ARMA parameters. This approach is tested in [6], in a discrete-time setting. However, there is no theoretical justification for such a heuristic, and moreover, it may perform worse than some other scheme that combines estimation and control.

The objective of this paper is to develop an alternative control scheme that combines estimation and control in a novel way. The parameter estimation error is also included as a control objective that is to be optimized. The approach is based on and uses recent theoretical tools developed in [11]. In this framework, the adaptive flow control problem is formulated as a worst-case optimization problem. The cost function penalizes the deviation of the queue size from some desired target, the bit rate tracking error of the input rate, and the identification error of the ARMA parameters. The solution given in [11] does not directly apply here, be-

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cause of the presence of a positive weighting on the control in the cost function. In general, the solution to such (nonlinear H^∞ control) problems (with positive weight on control) is given by infinite-dimensional dynamic programming equations. This difficulty will be circumvented here by properly splitting the weighting on the disturbance, v, in the measurement equation of the service rate into two parts – one of these used in the identification phase, and the other one reserved for the control design phase. This splitting allows us to obtain explicit, albeit conservative, solutions to the worst-case adaptive control problem. In the identification design stage, we make use of the *cost-to-come* function methodology as in the recent paper [12]. The resulting identification (estimation) algorithm is robust in the sense that it admits a guaranteed attenuation level with respect to the disturbance inputs (see ϕ and v introduced in the next section) without any *a priori* stochasticity assumption. By employing such an identification scheme, we can complete the control design by using a standard H^∞ technique. It is further shown in the paper that the optimal splitting of the weighting on v is to evenly allocate it to identification and control. A simulation study of the performance of such a combined (noncertainty-equivalent) identification/control scheme is presented in the full-version of the paper, it has not been included here due to page limitations.

The structure of the paper is as follows. In the next section we present the model and formulate the flow control problem as a disturbance attenuation problem. The controller, which combines estimation and control, is designed in Section 3. The paper ends with the concluding remarks of Section 4.

2 Problem Formulation

We consider an adaptive flow control problem for High Speed Networks, using the continuous-time counterpart of the model of [4]. We let $q(t)$ denote the length of the queue at time $t \geq 0$, with its desired length being Q. At time $t \geq 0$, we denote the effective service rate by $\mu(t)$ and the source rate by $x(t)$, with the average service rate being μ_a.

To facilitate our discussion and the analysis, we introduce the shifted variables:

$$\tilde{q}(t) := q(t) - Q, \quad \tilde{\mu}(t) := \mu(t) - \mu_a, \quad \tilde{x}(t) := x(t) - \mu_a.$$

Then, the queue size $\tilde{q}$ satisfies the following differential equation:

$$\dot{\tilde{q}} = \tilde{x} - \tilde{\mu} \tag{1}$$

(Note that we have ignored the boundary effects at zero and at full buffer. Such an approximation has been justified in [3, 4]; as have been illustrated in the full version of the paper by simulations, if the target queue length is chosen properly, then the controller can indeed ensure that the queue size does not hit the boundaries.) At any time $t \geq 0$, we have a noisy measurement of the queue size $\hat{q}$, and a noisy measurement of the effective service rate $\hat{\mu}$. Using the shifted variables $\hat{\tilde{q}} := \hat{q} - Q$ and $\hat{\tilde{\mu}} := \hat{\mu} - \mu_a$, we can write the measurement equations as

$$\begin{aligned} \hat{\tilde{q}} &= \tilde{q} + aw & (2)\\ \hat{\tilde{\mu}} &= \tilde{\mu} + cv & (3) \end{aligned}$$

where a and c are some positive scalar constants signifying the magnitude of the uncertainties in the measurements $\hat{\tilde{q}}$ and $\hat{\tilde{\mu}}$, respectively, and w and v are deterministic disturbances. In the H^∞ setting of this paper, the noise is allowed to be quite general, and in particular, to depend on the state of the system. This allows the model to include quantization noise, so that information on queue length and available bandwidth may each be given at a level of precision that is determined by the number of bits dedicated to transmit this information. The noise may also account for imprecisions due to delay phenomena, as well as losses of information packets.

Deviation of the effective service rate from the average rate, $\tilde{\mu}$, is in general due to the presence of cross traffic in the network. As in [4], we assume that this interference can be adequately modeled as the output of a continuous-time ARMA process of order d, driven by some disturbance input $\tilde{\phi}$. This ARMA process is given by:

$$\tilde{\mu}^{(d)} - l_1\tilde{\mu}^{(d-1)} - \cdots - l_d\tilde{\mu} = k\tilde{\phi}$$

where the parameters $l_1, \ldots, l_d$ are unknown, which will be identified on line using an identification scheme, and the parameter k, which quantifies the magnitude of the disturbance $\tilde{\phi}$, is assumed to be known. For our ensuing analysis, we will represent this ARMA process using a state space model, with the state variable η being a d-dimensional vector:

$$\begin{aligned} \dot{\eta} &= A_D\eta + \theta\tilde{\mu} + \tilde{D}\tilde{\phi} & (4)\\ \tilde{\mu} &= e_1'\eta & (5) \end{aligned}$$

where

$$A_D = \begin{bmatrix} \mathbf{0}_{(d-1)\times 1} & I_{d-1} \\ 0 & \mathbf{0}_{1\times(d-1)} \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} \mathbf{0}_{(d-1)\times 1} \\ k \end{bmatrix}$$
$$\theta = \begin{bmatrix} l_1 & \cdots & l_d \end{bmatrix}'.$$

Here, I_{d-1} is the identity matrix of dimension $d-1$, e_1 is a d-dimensional unit vector with the first element being 1 (and other elements 0). Because of the fact that $\tilde{\mu}$ is measured in additive noise (see (3)), the dynamical equation (4) involves the product $\theta\tilde{\mu}$ of two unknown quantities. Replacing $\tilde{\mu}$ by $\hat{\tilde{\mu}}$ in (4), we arrive at the following expression:

$$\dot{\eta} = A_D\eta + \theta\hat{\tilde{\mu}} + \tilde{D}\tilde{\phi} - c\theta v \tag{6}$$

For a rigorous analytical study, we will consider the following general dynamical model for η:

$$\dot{\eta} = A_D\eta + \theta\hat{\tilde{\mu}} + D\phi \tag{7}$$

where $D := kI_d$, and ϕ is a d-dimensional disturbance input. Clearly, the dynamics (6) is a special case of the dynamics (7) with the disturbance ϕ set to

$$\phi = \tilde{D}\tilde{\phi} - c\theta v/k$$

Henceforth, we will study the flow control design problem using the dynamics (7), which will result in some extra degree of conservatism than the designs that would use (6), but would still lead to a guaranteed level of performance.

Flow controller design for the case where the parameter vector θ is completely known has been extensively studied before in [4] in a discrete-time framework. In this paper, our main objective is to design flow controllers that make use of the measurements $\hat{\tilde{\mu}}$ to identify the parameter θ on line, and to achieve high level of flow control performance, i.e. small queue length variation and accurate bit rate tracking. The amount of queue length variation and the value of the target level Q are directly related to loss probabilities and to the throughput (see [4, 9]).

Associated with the above system, we introduce the standard quadratic cost function:

$$L^t = \int_0^t (g|\tilde{x}-\tilde{\mu}|^2+|\tilde{q}|^2+|\theta-\hat{\theta}|^2_{M(s)})\, ds; \qquad t \geq 0 \quad (8)$$

where the parameter $g > 0$ is a constant weighting on the tracking error of the input rate to the available service rate, and $M(\cdot)$ is a time-varying weighting matrix for the identification error of the unknown parameter θ. A larger value of g will result in a smaller rate tracking error for the optimal system, as also corroborated by the simulations results of [4]. The third term in the cost function reflects our objective of parameter identification for θ. As it will turn out, the nonnegative-definite weighting matrix $M(\cdot)$ will have to be chosen in a specific form for the problem to admit a solution. We will see later that a particular choice for this weighting matrix will lead to major simplifications in the structure of the adaptive flow controller.

The objective of the flow control design is to regulate the queue length to the desired value Q (i.e., to keep $\tilde{q}$ small), achieve good tracking of the available service rate (i.e., make $|\tilde{x}-\tilde{\mu}|$ small), and attenuate the effect of exogenous disturbance input on the three quantities $\tilde{q}$, $\tilde{x}-\tilde{\mu}$, and $\theta-\hat{\theta}$, as measured by the cost function L^t. In this paper, the disturbance attenuation property for the controller is specified by a desired attenuation level $\gamma > 0$, and the attenuation is with respect to the energy (square of the $\mathcal{L}_2$ norm) of the disturbance inputs. This objective can then be captured by worst-case optimization of a single cost function as described in the following. For any given disturbance attenuation level $\gamma > 0$, the flow controller is to guarantee for the closed-loop system the following worst-case attenuation property for all $t \geq 0$:

$$\sup_{\phi_{[0,t]}, v_{[0,t]}, w_{[0,t]}} \{L^t - \gamma^2 \int_0^t (|\phi|^2+|v|^2+|w|^2)\, ds - \gamma^2|\theta-\bar{\theta}|^2 - C\} \leq 0 \qquad (9)$$

where $\bar{\theta}$ is some initial estimate of the unknown parameter vector θ; C is some constant that depends only on the initial state of the system $\tilde{q}(0)$, $\eta(0)$, $\hat{\tilde{q}}(0)$, $\hat{\eta}(0)$; and $\hat{\eta}(0)$ is an initial guess for the value of the state vector η at $t=0$.

We should note that the choice for the initial estimate $\bar{\theta}$ of the unknown parameter vector is a subjective one, and may have to be based on the statistical information collected previously. The closer $\bar{\theta}$ is to the true value of θ, the better will the transient performance be. This choice will not make any difference, however, as far as the long term performance of the system is concerned, under persistent disturbances.

The constant C is needed in (9) to account for the uncertainty in the values of the initial condition of q and η. In the standard setup of an H^∞-optimal control problem under imperfect state measurements, one takes $C = |x_0 - \bar{x}_0|^2_{Q_0}$. In the present case, to avoid the additional complication that may result from the requirement of an appropriate spectral radius condition, we take C here to be arbitrary, only requiring that it be allowed to depend on the initial state of the system. The specific choice of C does not affect the long term performance of the system under persistent disturbance inputs. It does quantify, however, the worst-case transient performance. A larger value of C implies a worse transient performance of the closed loop system.

Clearly, for any flow control law that guarantees the worst-case inequality (9) for all $t \geq 0$, we can deduce the following:

(1) If the disturbances ϕ, w and v all have finite energy, then L^t is bounded from above by a constant for any $t \geq 0$. Therefore, we achieve perfect regulation of queue size and perfect tracking of the service rate as $t \to \infty$ despite the lack of knowledge of the precise values of the parameter vector θ.

(2) For any disturbance waveform and any $t \geq 0$, the finite-interval energy levels (up to time t) of the rate tracking error and queue size tracking error are less than or equal to γ^2 times the sum of the corresponding finite-interval energy levels of the disturbances and a constant that depends only on the initial state of the system. This then guarantees a good steady-state system response for disturbance inputs with infinite energy over the infinite horizon, such as white noise.

(3) Since the inequality is required to hold for any $t \geq 0$, there exists a guaranteed bound on the transient response of the closed-loop system despite the lack of knowledge on the true value of the parameter vector θ.

This completes formulation of the problem, which involves system dynamics (1), (7), with $\tilde{\mu}$ given by (5), measurement equations (2), (3), and a worst-case performance criterion (9). Now, we turn in the next section to the derivation of an optimal adaptive flow control algorithm.

3 Main Results

The solution to the adaptive flow control problem formulated in Section 2 above will be obtained in two steps. At the first step, we shall derive a parameter identification scheme for the unknown parameter θ, as in [12]. At the second step, we shall design a flow control algorithm using the parameter identification algorithm from the first step.

For the parameter identification design, only the dynamics (7) and measurement (3) are of relevance. The relevant disturbance inputs at this stage are ϕ and v. For the design at the second step to be feasible, we set our objective at this stage as attenuation of the disturbances ϕ and $\sqrt{\alpha}v$ to the level γ, where $\alpha \in (0,1)$ is a constant, fixed arbitrarily for the time being. This objective is thus more conservative than that of the attenuating the disturbances ϕ and v to the level γ. Nevertheless, this allows us to retain part of the negative weighting on v in the second step of optimization. Subsequently, we will compute an optimal value for α, by minimizing the achievable level of disturbance attenuation.

For the identification problem, the cost function to be minimized, with respect to $\hat{\theta}$, is the following:

$$\begin{aligned}\sup_{\eta(0),\theta,\phi_{[0,t]},v_{[0,t]}|\hat{\tilde{\mu}}_{[0,t]},\tilde{x}_{[0,t]}} & \{\int_0^t (g|\tilde{x}-\tilde{\mu}|^2 \\ & +|\theta-\hat{\theta}|^2_{M(s)})\,ds - \gamma^2\int_0^t (|\phi|^2+\alpha|v|^2)\,ds \\ & -\gamma^2|\theta-\bar{\theta}|^2-\bar{C}\} \qquad (10)\end{aligned}$$

where $\bar{C}$ is some positive constant that depends only on the initial condition of η, and the sup operation is over all initial conditions $\eta(0)$, parameter value θ, and disturbance inputs $\phi_{[0,t]}$, $v_{[0,t]}$ which, along with input rate $\tilde{x}_{[0,t]}$, yield the measurements $\hat{\tilde{\mu}}_{[0,t]}$. The reason for including the constant $\bar{C}$ above is exactly the same reason cited earlier for inclusion of the constant C in (9); it quantifies the quality of the transient performance of the identification scheme.

Toward solving this worst-case identification problem, we expand the dynamics (7) to include the simple dynamics of the parameter θ, and express it in terms of the expanded state vector $\zeta := (\theta', \eta')'$:

$$\begin{aligned}\dot{\zeta} &= \begin{bmatrix} \mathbf{0}_{d\times d} & \mathbf{0}_{d\times d} \\ \hat{\tilde{\mu}} I_d & A_D \end{bmatrix}\zeta + \begin{bmatrix} \mathbf{0}_{d\times 1} \\ D \end{bmatrix}\phi \\ &=: A_F\zeta + D_F\phi \qquad (11)\end{aligned}$$

The measurement equation (3) can then be rewritten as

$$\hat{\tilde{\mu}} = \begin{bmatrix} \mathbf{0}_{1\times d} & e_1' \end{bmatrix}\zeta + cv =: C_F\zeta + cv \qquad (12)$$

We will take the constant $\bar{C}$ to be in quadratic form:

$$\bar{C} = \gamma^2|\eta(0)-\hat{\eta}(0)|^2_{P_0}$$

where the matrix P_0 is an arbitrary positive definite matrix. It will be shown later that a particular choice of P_0 leads to a simple structure for the controller.

The optimization problem (10), when expressed in terms of ζ, involves the maximization of a quadratic cost function subject to linear constraints (11) and (12). Hence, it can be solved using standard dynamic programming arguments which are similar to those used in [12]. As a result of such a dynamic programming derivation, we obtain the following set of equations describing the minimax identifier:

$$\begin{aligned}\dot{\hat{\theta}} &= -\alpha/c^2\Xi\Phi' e_1(\hat{\tilde{\mu}}-e_1'\hat{\eta}) - g/\gamma^2\Xi\Phi' e_1(e_1'\hat{\eta}-\tilde{x}), \\ & \qquad \hat{\theta}(0)=\bar{\theta} \qquad (13)\\ \dot{\hat{\eta}} &= A_D\hat{\eta} + \hat{\theta}\hat{\tilde{\mu}} + \alpha/c^2(\bar{\Pi}+\Phi\Xi\Phi')e_1(\hat{\tilde{\mu}}-e_1'\hat{\eta}) \\ & \quad + g/\gamma^2(\bar{\Pi}+\Phi\Xi\Phi')e_1(e_1'\hat{\eta}-\tilde{x}) \qquad (14)\\ \dot{\bar{\Pi}} &= A_D\bar{\Pi}+\bar{\Pi}A_D' + k^2 I_d - (\alpha/c^2 - g/\gamma^2)\bar{\Pi}e_1e_1'\bar{\Pi}; \\ & \qquad \bar{\Pi}(0) = P_0^{-1} \qquad (15)\\ \dot{\Phi} &= (A_D - (\alpha/c^2-g/\gamma^2)\bar{\Pi}e_1e_1')\Phi - \hat{\tilde{\mu}} I_d, \\ & \qquad \Phi(0) = \mathbf{0}_{d\times d} \qquad (16)\\ \dot{\Xi} &= -(\alpha/c^2-g/\gamma^2)\Xi\Phi' e_1e_1'\Phi\Xi + (1/\gamma^2)\Xi M\Xi; \\ & \qquad \Xi(0) = I \qquad (17)\end{aligned}$$

We are now in a position to state optimality of the solution $\hat{\theta}$ above, which is done in the proposition below. Its proof requires arguments similar to those used in [12] and is therefore not included here.

Proposition 3.1 *Consider the maximization problem (10) subject to the dynamical constraint (11) and the equality constraint (12). The maximum exists if there exist matrix functions $\bar{\Pi}(t)$ and $\Xi(t)$ satisfying (15)–(17), which are further positive definite for each $t \in [0,\infty)$. If such $\bar{\Pi}(\cdot)$ and $\Xi(\cdot)$ exist, then the identifier achieving the minimum worst-case cost is given by (13)–(14). Conversely, if either one of the matrix functions $\bar{\Pi}(t)$ and $\Xi(t)$ is not nonnegative definite for some $t \in [0,\infty)$, then the maximum is unbounded for any identification scheme $\hat{\theta}$.*

□

Remark 3.1 *We observe that the dynamics of $\hat{\eta}$ and $\hat{\theta}$ given by (13) and (14) depend explicitly on the input rate $\tilde{x}$, which does not enter the cross-traffic dynamics (7). This dependence implies that the adaptive flow controller to be obtained shortly is not a certainty-equivalent controller.* ◇

The identification algorithm (15)–(14) requires $2*d^2+3*d$ integrators to implement, assuming that the fact that the matrix functions Ξ and $\bar{\Pi}$ are symmetric is utilized. This may be computationally demanding in actual implementation. Significant structural simplifications can be achieved, however, by properly choosing the weighting matrices $M(t)$ and P_0. Toward this end, let

$$M(t) = (\gamma^2\alpha/c^2 - g)\Phi' e_1 e_1'\Phi.$$

The dynamics of $\Xi(t)$ are then expressed as: $\dot{\Xi} = \mathbf{0}_{d\times d}$. Therefore, $\Xi(t) \equiv I_d$ on the interval $[0,\infty)$. This reduces the required number of integrators by $d(d+1)/2$.

From standard H^∞-optimal control theory [7], the generalized Riccati differential equation (15) admits a positive-definite solution on the infinite interval $[0,\infty)$ if and only if

$$\gamma \geq c\sqrt{g/\alpha}. \tag{18}$$

Furthermore, for any $\gamma > c\sqrt{g/\alpha}$, the solution $\bar{\Pi}(t)$ converges to the unique positive definite solution of the following generalized algebraic Riccati equation (GARE):

$$A_D\Pi + \Pi A_D' + k^2 I_d - (\alpha/c^2 - g/\gamma^2)\Pi e_1 e_1' \Pi = 0 \tag{19}$$

For $\gamma = c\sqrt{g/\alpha}$, the matrix function $\bar{\Pi}(\cdot)$ is unbounded on $[0,\infty)$, and furthermore (19) does not admit any nonnegative-definite solution. Therefore, we consider here only the case $\gamma > c\sqrt{g/\alpha}$, and choose $P_0 = \Pi^{-1}$, where Π is the unique positive-definite solution of GARE (19). It is a general property of the solution to the GARE (19) that the matrix

$$A_{fo} := A_D - (\alpha/c^2 - g/\gamma^2)\Pi e_1 e_1'$$

is Hurwitz. This observation leads to a further reduction in the number of integrators, by a factor of $d(d+1)/2$.

Notice the structure of the matrix A_{fo}:

$$\begin{bmatrix} * & 1 & \cdots & 0 \\ * & \vdots & \ddots & \vdots \\ * & 0 & \cdots & 1 \\ * & 0 & \cdots & 0 \end{bmatrix}$$

where $*$ denotes possibly nonzero elements. This structure of the matrix A_{fo} permits the matrix function $\Phi(t)$ to be generated using a d-dimensional linear system:

$$\dot{\lambda} = A_{fo}\lambda - e_1\hat{\tilde{\mu}}; \quad \lambda(0) = 0 \tag{20}$$

$$\Phi = \begin{bmatrix} \lambda & \cdots & A_{fo}^{-d+1}\lambda \end{bmatrix} \tag{21}$$

This further simplifies the structure of the identification algorithm by $d^2 - d$ integrators. Using all these (three) simplifications, the dynamics of $\hat{\eta}$ and $\hat{\theta}$ can be expressed as:

$$\dot{\hat{\theta}} = -\alpha/c^2\Phi' e_1(\hat{\tilde{\mu}} - e_1'\hat{\eta}) - g/\gamma^2\Phi' e_1(e_1'\hat{\eta} - \tilde{x}), \quad \hat{\theta}(0) = \bar{\theta} \tag{22}$$

$$\dot{\hat{\eta}} = A_D\hat{\eta} + \hat{\theta}\hat{\tilde{\mu}} + \alpha/c^2(\Pi + \Phi\Phi')e_1(\hat{\tilde{\mu}} - e_1'\hat{\eta}) + g/\gamma^2(\Pi + \Phi\Phi')e_1(e_1'\hat{\eta} - \tilde{x}) \tag{23}$$

In the remainder of this paper, we will limit the analysis (and the derivation) to the simpler identification algorithm, which is given by (19)–(23), although the results to be obtained will hold true for the general identification scheme (15)–(14) as well. The existence condition, as well as the optimality condition, for this simplified identification scheme is precisely $\gamma > c\sqrt{g/\alpha}$.

To deduce disturbance attenuation properties of the parameter identification algorithm (19)–(23), we now introduce the following value (cost-to-come) function:

$$W(t) = \gamma^2|\theta - \hat{\theta}|^2 + \gamma^2|\eta - \hat{\eta} + \Phi(\theta - \hat{\theta})|^2_{\Pi^{-1}} \tag{24}$$

Using this value function, and the simple identity

$$W(0) - W(t) + \int_0^t \dot{W}(s)\,ds = 0,$$

we obtain

$$L^t - \gamma^2\int_0^t (|v|^2 + |w|^2 + |\phi|^2)\,dt - W(0) \leq \int_0^t [|\tilde{q}|^2 + g|\tilde{x} - e_1'\hat{\eta}|^2 - \gamma^2(|w|^2 + (1-\alpha)|v|^2 + \alpha|\bar{v}|^2)]\,dt$$

where

$$\bar{v} := (\hat{\tilde{\mu}} - e_1'\hat{\eta})/c.$$

We observe that the right-hand-side of the above inequality forms an H^∞ type cost function. This observation then enables us to proceed easily to the control design stage.

Let us rewrite the dynamical equation (1) as

$$\dot{\tilde{q}} = u + cv - c\bar{v} \tag{25}$$

where $u := \tilde{x} - e_1'\hat{\eta}$, to relate it to the estimates of the identifier. The queue length is measured in additive noise as in (2).

The objective at this second (control design) stage is to guarantee satisfaction of the following disturbance attenuation inequality:

$$\sup_{v_{[0,t]},\bar{v}_{[0,t]},w_{[0,t]}} \{\int_0^t (g|u|^2 + |\tilde{q}|^2)\,ds - \gamma^2\int_0^t ((1-\alpha)|v|^2 + \alpha|\bar{v}|^2 + |w|^2)\,ds - \tilde{C}\} \leq 0 \tag{26}$$

for all $t \geq 0$, and for some constant $\tilde{C}$ that depends only on the initial state of the system.

The system dynamics (25), measurement equation (2) and performance criterion (26) then form a standard linear quadratic H^∞ optimal control problem with a single state $\tilde{q}$, a single control input u, and disturbance inputs v, $\bar{v}$ and w. The set of necessary and sufficient conditions for the existence of a solution to this H^∞ control problem ([7]) involves two GARE's and a spectral radius condition as described in the following:

(i) The filtering GARE:

$$-P^2(1/a^2 - 1/\gamma^2) + c^2/\alpha + c^2/(1-\alpha) = 0,$$

whose solution is

$$P = \frac{ca\gamma}{\sqrt{\alpha(1-\alpha)(\gamma^2 - a^2)}}; \qquad \text{if } \gamma > a$$

(ii) The control GARE:

$$-Z^2(1/g - c^2/\gamma^2(1/\alpha + 1/(1-\alpha))) + 1 = 0$$

which admits the solution:

$$Z = \sqrt{\frac{g\alpha(1-\alpha)}{\alpha(1-\alpha) - gc^2/\gamma^2}}; \qquad \text{if } \gamma > c\sqrt{\frac{g}{\alpha(1-\alpha)}}$$

(iii) Under the condition that both P and Z above exist, the H^∞ optimal controller exists if and only if the following spectral radius condition is satisfied:

$$\gamma^2 - PZ > 0 \quad \Leftrightarrow \quad \gamma > \sqrt{\frac{gc^2}{(1-\alpha)\alpha} + a^2}$$

Among the three conditions above on the range of the admissible values for γ, the last one is the most restrictive. Therefore, the H^∞ optimal control problem admits a solution if

$$\gamma > \sqrt{\frac{gc^2}{(1-\alpha)\alpha} + a^2} \tag{27}$$

For this range of values of γ, the H^∞ controller is given by

$$\begin{aligned} \dot{\bar{\hat{q}}} &= \frac{c\gamma\sqrt{\gamma^2 - \frac{gc^2}{\alpha(1-\alpha)}}}{a\sqrt{(\gamma^2 - a^2)(\alpha(1-\alpha)\gamma^2 - gc^2)} - ca^2\sqrt{g}}(\hat{\bar{q}} - \bar{\tilde{q}}) \\ &\quad - \sqrt{\frac{\alpha(1-\alpha) - gc^2/\gamma^2}{g\alpha(1-\alpha)}}\bar{\tilde{q}} \end{aligned} \tag{28}$$

$$\tilde{x} = e_1'\hat{\eta} - \sqrt{\frac{\alpha(1-\alpha)}{g\alpha(1-\alpha) - g^2c^2/\gamma^2}}\bar{\tilde{q}} \tag{29}$$

For γ less than the threshold level (27), the supremum in (26) is always $+\infty$ for any flow control law one can come up with. This then completes the control design analysis.

For both the identifier and the controller to exist, the desired disturbance attenuation level γ must satisfy both constraints (18) and (27), and clearly the latter is the governing one. Then, the threshold level for any achievable attenuation is

$$\gamma^*(\alpha) = \sqrt{\frac{gc^2}{(1-\alpha)\alpha} + a^2} \tag{30}$$

Clearly, the optimal value for the parameter α, that allows for a minimal level of achievable disturbance attenuation, is

$$\alpha^* = 1/2\,, \tag{31}$$

which leads to the optimum threshold value:

$$\gamma^*(\alpha^*) = \sqrt{4gc^2 + a^2} \tag{32}$$

This now brings us to the following theorem. The proof of this theorem can be found in the full version of the paper, but is omitted here due to page limitation.

Theorem 3.1 *Consider the worst-case adaptive flow control problem formulated in Section 2. Let the desired disturbance attenuation level γ be strictly larger than $\gamma^*(\alpha^*)$, and fix the parameter α at its optimal value 1/2. The parametrized (in γ) controller, defined by (28)–(29), along with the parametrized identifier given by (19)–(23), guarantees the worst-case disturbance attenuation inequality (9).*

4 Conclusions

We have derived in this paper a rate-based flow controller that operates under unknown environmental conditions. It is a noncertainty-equivalent adaptive controller that achieves precomputable levels of disturbance attenuation. It combines identification and control functions in a unique way, and has an appealing structure where various terms can easily be computed on line in real time. In the full version of the paper, we have shown through simulations how the relative weights attached to different terms in the cost function influence the dynamic behavior of the system, and the design of the identifier and the controller.

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