Eitan Altman

The Theory of

NETWORK ENGINEERING GAMES

Network Engineering Games

Eitan Altman,[1]

[1]INRIA Méditerranée, France, Email: altman@sophia.inria.fr

Todo list

Contents

Part I

Part 1: Background

Chapter 1

Introduction

1.1 Impact of game theory on publications in networking

We searched in Google scholar the number of documents found when searching for the following types of games: (1) Power control, (2) Flow control, (3) Rate control, (4) Access control, (5) Jamming and (6) Routing games. We searched for documents containing all of the words "XXX wireless networks GT" where GT stands for game theory and where XXX stands for one of games (1)-(6) above. All figures correspond to the date of October 15th, 2010. The number of the occurrences corresponding to each of the cases are summarized in the first row of Table 1.1. The second row of the table was obtained by repeating the above without the word "games theory" (i.e. searching for all the words "XXX wireless networks"). The last row in the matrix was obtained by searching for the words "XXX networks, game theory".

	Power control	Flow control	Rate control	Access control	Jamming	Routing
Wirelesss Networks, GT	27600	19400	25600	27400	3520	16100
Wirelesss Networks	555000	303000	59000	609000	19000	342000
Networks, GT	391000	174000	264000	298000	17000	38000

Table 1.1: The number of citations

If we consider wireless networks, then in each type of gae, the number of documents that fall within game theory forms around than in 5%. The game that was the most studied in the wireless context is the power control one. (This is also the case if we do not specify to wireless, but then it may cover also games that arise in supplying electricity).

Using the software "publish or perish", we find in 2009, 189 documents with all the words "jamming wireless networks" in google, of which 39 further contain "game" and 22 contain "game theory". In 2008 there are 123 documents containing "jamming wireless networks" and only six containing further "game theory". Thus the ratio of papers that use game theory for studying jammig seems to be around 5%.

Next we consider Figure 1.1 that shows how the number of documents that contain all of the words "Routing games Nash equilibrium game theory" vary as a function of the year. The precise numbers observed each year by searching with scholar google on October 15th, 2010, are given in Table ??. The total number of documents published on 1999 and earlier has been found to be 217. There is a steady linear growth. Within 10 years, this number is seen to have increased by more than 10.

Next we use Publish or perish, under cathegory Engineering, Computer Science and Mathematicis and check the popularity of the set of words "game theory Nash equilibrium". We make

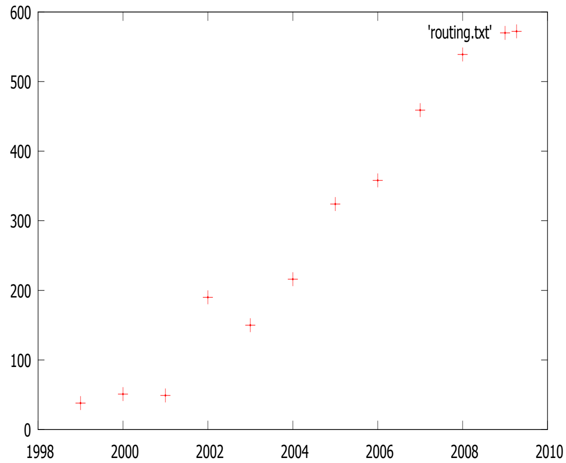


Figure 1.1: The number of documents each year containing all the words "Routing games Nash equilibrium game theory" as a function of the year.

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
No. of docs	38	51	49	190	150	216	324	358	459	539	570

Table 1.2: The evolution of the number of documents on routing games

no particular referene to networking applications. Within the documents published by the IEEE on 2009, 764 were found in 2009 containing the words "Wireless networks". In 1999 in contrasnt we find only 2.

We conclude that there is a significative increase in the impact of game theory as we observe it over documents on the Internet.

1.2 Is game theory an appropriate tool for designing networks

The word "game" may have conotations to "toys" or of "playing" (as opposed to decision making). But in fact it stands for decision making by several decision makers, each having her (or his) own individual objectives. In the special case that there is a common objectives that all players maximize, this is called a team problem. When there is only a single decision maker we speak of optimization rather than team. In a team problem we search for a maximizer of the common objective. In a non-cooperative we search for a typically for a solution at which each player is at a (local) maximum - it cannot do better by a unilateral deviation. This is called an equilibrium.

Remark 1. *Is a game where all players have a common objective to maximize, equivalent to a team problem? Is the equilibrium of the game the same as the solution of the team problem? The answer is no. Any solution of the team problem is an equilibrium to the game problem but the converse need not hold. As an example, let there be two players, where player i has to choose either $x_i = 0$ or $x_i = 1$. Consider Then $x_1 = x_2 = 1$ is the team optimal solution. It is also an equilibrium. On the other hand, $x_1 = x_2 = 0$ is an equilibrium but is not an optimal team solution.*

When should one use a game theoretic framework and when should one use a team framework?

Game theory searches for stable solutions to this problem. Is a user of a mobile phone indeed constantly trying to improve his performance? Experience shows that users tend to be coopoerative. In fact, many Internet protocols are very cooperative: TCP that controls the rate of transmission of packets, is an example for a cooperative behavior.

When are the users cooperative? The power control game seems to be the most studied one in wireless networking (according to the figures that we extracted from google). Yet in practice the user does not have access to control the power. The equipment obliges us to be cooperative.

In Section 1.4 we present an example where, in contrast, the equipment provider leaves the decision making to the user and even provides the user the appropriate tools to make and take the decisions.

1.3 Business models of jammers

According to [61], The US military routinely uses jammers to protect secure military areas from electronic surveillance. Jammers can also be used to protect traveling convoys from cell phone triggered roadside bombs in places like Iraq.

Interestingly, the business model of jammer phones include jamming one's own telephone. This allows one to avoid being disturbed. Typical prices of a jammer vary between 100 and 300 USA $.

Leading electronic companies have introduced cellular phone jammers based on the denial of service approach: they simply create noise which interferes with the communications [85]. More efficient techniques have been designed later [85].

Jammers are actually manufactured and sold over the Internet by several companies. Selling jammers in the USA and in Europe is not legal, but it generally is legal in Asia [61]. According to [61], the FCC (Federal Communications Commission) in the United States has outlawed the sale and use of jammers because they can in theory interfere with emergency communications between police and rescue personnel, aid in criminal activity as well as disrupt medical equipment like pacemakers; Using a cell phone jammer may result in fines of upto $ 11,000.

Figure 1.2: Small jammer

Figure 1.3: Big jammer

Figures 1.2 and 1.3 show a a small and a big jammer, respectively. The small one is of the size of an average wireless telephone terminal (photos taken from antennasystems.com and from Globalgadgetuk.com respectively). Most jammers only have a range of about 50 to 80 feet and will only effectively jam their immediate surroundings. Stronger jammers can cover larger structures like office buildings, are also sold. Examples of sites that sell jammers are
www.methodshop.com/gadgets/reviews/celljammers/index.shtml
and www.covert-supply.co.nz/products/Wifi,-Bluetooth,-Wireless-Video-Jammer-%252d-Portable-Wireless-Blocker.html

Most cell phone jammers come in 2 versions, one for Europe, North Africa and the Gulf states GSM networks (900 & 1800) and one for the Americas & Canada (800 & 1900 mhz) networks [61].

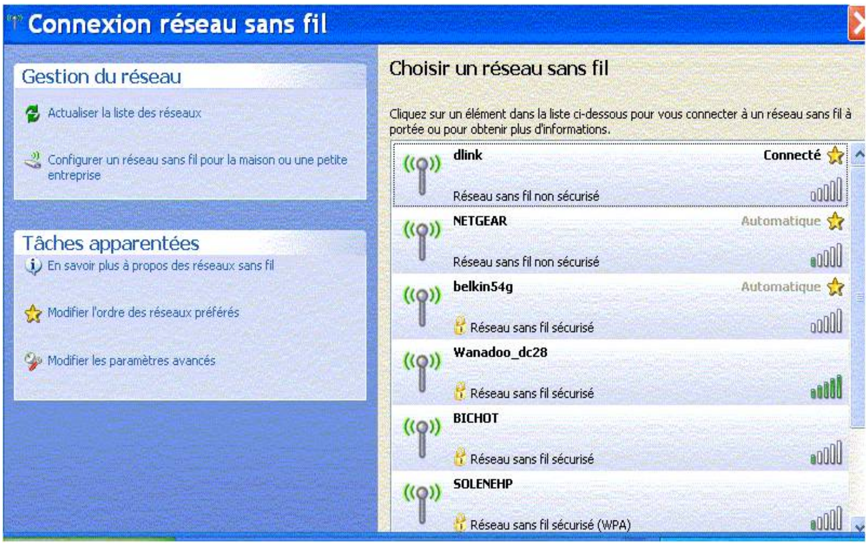


Figure 1.4: The access point association problem: Information available when taking a decision

1.4 The Association problem

There are several types of association games that one is frequently faced with.

Choosing an access point

When atempting to connect to the Internet, one may have the option of choosing between several access points that use wireless local area networks (WLANS). The decision, of which access point to connect to, is typically left to the user, The driver for the wireless card typically gives some information concerning the channel state at each one of the access points. Fig 1.4 is an example of the information presented for a user when the opportunity of taking a decision is offered to him. It is easily seen that the user is indeed put in a situation of a game.

This is a complex stochastic game as each user comes at random points, its decision will be affected by the state of the channel not only at the present (i.e. the one it has available) but also at the future, and the latter will be determined by the decisions of future users and a user is not aware of when future arrival will occur and what the decisions will be.

This game has an unusual information: it is partial and **missleading**. Missleading - because, although the the channel state indeed can gives information on the transmission rate, it is known that the actual throughput of a user is a function of not only his channel state but also of that of the other connected users. (The throughput is known to be lower bounded by the harmonic means of the rates available to each user). The real utility of a user is the throughput he would get and the user may not be aware that it is possible that an access point with a better channel may have a lower throughput because more terminals are connected to it.

Choosing between technologies

We may have to choose between several technologies: say between 3G, WIFI, bluetooth and Ad-Hoc. Figure 1.5 is an example of the iniformation available to a user in a game where one has the option of connecting to an Ad-hoc network or to an access point.

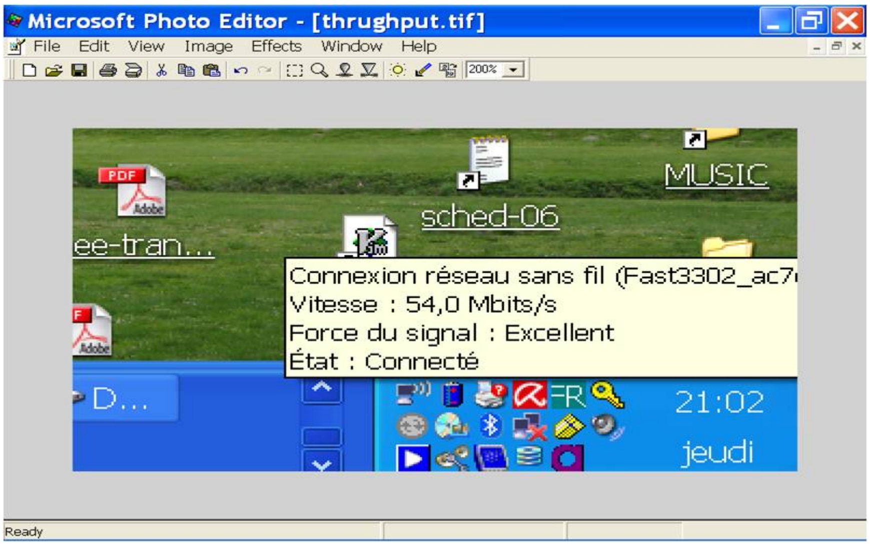


Figure 1.5: The association problem of choice of technology: Information available when taking a decision

Refined modeling of the above games

So far we considered a game where the each user takes one decision: where to connect upon arrival. However, once the user is connected he may get more information about his throughput. An example of the grafical form that such extra information is presented to us is given in Figure ??. This information too may be missleading. The one we see in Figure ?? is the physical channel rate. Again, the throughpuut of the user is not this channel rate but some function of the channel rates of all usuers; this function is bounded by the harmonic mean of the channel rates of all users connected to the access point.

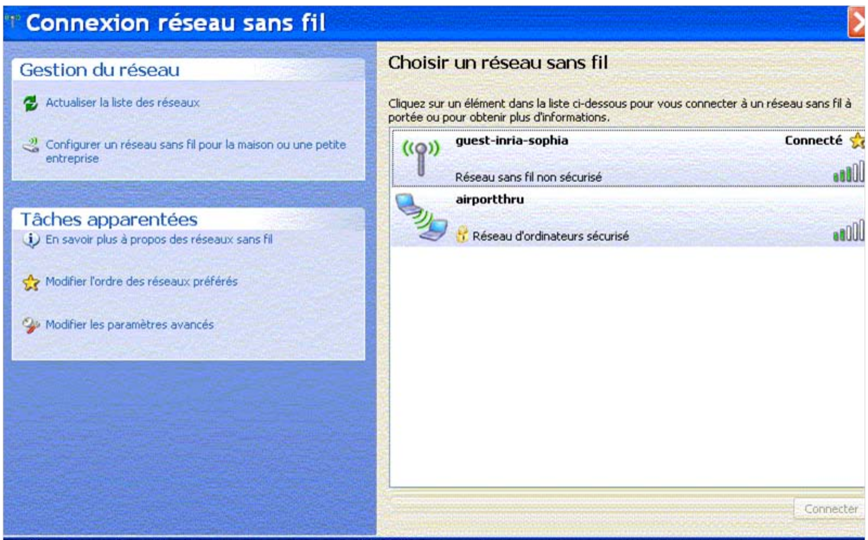


Figure 1.6: The access point association problem: Information available after taking the initial decision

The new available information could be used to reconsider the connection decision. In that case the game becomes more complex as the strategy of the users is a more complex object.

A hierarchical game of association

In some cases, each of the access points corresponds to that of another operator. In other cases, the choice of operator is offered to a user only once it cnnects to an access point. This is again a game. An example of the way that the choices are presented to the user is presented in Figure ??.

Figure 1.7: The association problem of choice of technology: Information available when taking a decision

This game may also be a hierarchical one. It may involve a preliminary decision to which service provider to attempt connection. Once an attempt is made then the user gets an information on the pricing policy of the provider (note that the user may know the pricing in formation of one or more providers before making the decision since this pricing usually remains the same for a long period. it may discover the quality of service offered by an operated only after taking the decision of whicih of the service providers to connect to.

In this game the decisions may depend on the pricing strategy of each service provider as well as on the quality of its service. The latter may be unknown, and become available only after taking where as the former may become available

1.5 Communities in Game Theory

Game theory is useful in many fields. Economy is probably the most known one. But we find game theory also in mathematics, computer science, electrical engineering, civil engieering, biology, political nt and social science, psychology, ecology, law.

Parts of game theory have been developed within different communities.

We describe below various communitities that are mostly concerned with networks. We shall focus in particular on the approaches to routing games: these are games where there are various traffic classes, each characterized by a source-destination pair and a demand. The network is given as a graph through which the demand is to be shipped. We shall mention the different concepts, the problems and the solution approaches in various communities. We shall describe the position of the communities with respect to various classifications of routing problems wihch we mention next.

- **The object to be routed** The traffic may be represented as a finite set of objects whose route is to be determined, or as a fluid of a continuum set of infinitesimal or "atomless" objects.

- **The Decision Maker** A decision maker may correspond to an object that is to be routed. But a decision maker may also correspond to a set of players. For example, a bus or a taxi company may take routing decisions for all their fleet. In computer networks it may be the Internet service provider (ISP) that would determine the routes for all its subscribers.
- **The Nature of decisions** There is a distinction between non splitable flows, where all the demand of a class of traffic has use the same common route, and splitable ones, where the demand for each class can be split among various paths.
- Solution concepts.

We next describe different communities and their positionning with respect to raouting games.

1.5.1 Road traffic

A central problem in road traffic engineering is to predict congestion levels at the road network. Drivers are non-cooperative players, and each driver determines its own path from the source to its destination. The setting is of a very large car populations so that one can model the decisions of a single car as having a negligible impact of the travel time of other cars. We are given the demand between each source and destination, the possible routes, and the congestion cost over links. The game is called the "traffic assignment problem", and the solution notion is called Wardrop equilibrium.

1.5.2 Econometrics

We should note that the economist Pigou, had stated principles analogous to Wardrop's in his 1920 *Economics of Welfare* [79] and had studied their properties. His work did not reach the road traffic community. On the other hand, we find many references to Pigou's work in the within the community of algorithmic game theory see Roughgarden [].

The fact that Wardrop equilibrium can be obtained using an equivalent optimization problem with a single player having some cost $f(x)$ is a feature common to a whole class of games known as potential games. This class of games was formally introduced by Monderer and Shapley [67] for the case of finitely many players. It was extended in [84] to the case of population games, which includes the setting of Wardrop equilibrium.

In developing the concept of potential games, game theorists seem not to have been aware of the huge literature on road traffic equilibria starting from Wardrop [100] and [19]. Monderer and Shapley write in [67]: "To our knowledge, the first to use potential functions for games in strategic form was Rosenthal [81]". Interestingly enough, this reference (see also [82]) includes a discrete version of Wardrop equilibrium with finitely many players, called "congestion games". The road traffic community, in contnrast, is well aware of the congestion games, see [76].

1.5.3 Algorithmic Game Theory

This is a young community in game theory that has emmerged computer science. This community is interested in algorithmic aspects of game theory, in the complexity (amount of computation) needed to obtain a given solution concept (such as an equilibrium) or to obtain an approximation of it.

Wikipedia (January 2011) states: "We can see Algorithmic Game Theory from two perspectives:

- Analysis: look at the current implemented algorithms and analyze them using Game Theory tools: calculate and prove properties on their Nash equilibria, Price of Anarchy, best-response dynamics ...
- Design: design games that have both good game-theoretical and algorithmic properties. This area is called Algorithmic Mechanism Design

The field was started when Nisan and Ronen in STOC'99[1] " [have] "drawn the attention of the Theoretical Computer Science community to designing algorithms for selfish (strategic) users. As they claim in the abstract".

Many issues in the center of interest to that community had been studied much earlier. In particular, algorithms to compute equilibria and their complexity had long been studied by game theorits in mathematics department, in operations research departments, in econometrics and others. Many references can be found in the survey of Bernard von Stengle in [99]. An example in the area of routing games is [55].

This community is very active in mechanism design as well as in the study of the "Price of Anarchy", which is the name they gave to the ratio between the sum of utilities under the worst possible equilibrium and that under social optimality, see Papadimitriou [56, 83].

We note that comparisons between social and individual (equilibrium) behavior had been studied systematically before including in the context of routing games and load balancing [49, 41].

The areas of research of algorithmic games that are stated by Wikipedia are

- Algorithmic mechanism design
- Inefficiency of equilibria (Price of Anarchy)
- Complexity of finding equilibria
- Market equilibrium
- Multi agent systems
- Computational social choice

Wikipedia then states as area of applications Routing, P2P systems and AdAuctions.

Routing games are frequently called "Selfish Routing" in this community.

This community is well aware of the early work on routing games by Pigou [79], but seems little interested or aware by the literature on road traffic or the telecom community.

1.5.4 The engineering community

In addition to civil Engineering in which road traffic has traditionally been investigated, networking games have been studied in electrical engineering departments since more than fifty years, although they have not attempted to be recognizied as a separate community (in contrast to the community of algorithmic games).

In the context of telecom, the objects that are routed are either packets that flow through wireline or wireless channels, or the whole file that is transferred. Unlike road traffic, it is not theh routed object that decides on the path it takes. Instead, it could often be the ISP (Internet Service Provider) that takes routing decisions for the traffic to or from its subscribers. The number of players (decision makers) is then small, and the solution concept is the Nash equilibrium rather than the Wardrop equilibrium. An intensive research on the Nash equilibrium in routing games started in this community with the pioneering paper by Orda, Shimkin and Rom [75] in 1993, and which can be viewed as the starting point of an activity of a separate telecom community.

A second application that characterizes this community is games arising in wireless communication. An important activity in that area started at around 1998 [31, 47, 6].

Another class of networking games problem is flow control. In flow control, decision makers adapt the demand to the congestion state of the network. This application, quite specific to telecom, has been studied in [43, **?**].

On the cooperative game theoretical side, a central contribution that came from this community is the notion of α-fairness introduced by Mo and Walrand [66] which we shall see later on.

Some of the leading scientists in this community are N. Shimkin, A. Orda, R. Rom, A. Lazar, I. Korilis, R. Mazumdar, L. Mason, J. Walrand, H. Kameda, P. Caines, Mung Chian, V. Poor,

[1] see [73, 72]

MacKenzie, M. Debbah, R. El-Azouzi, T. Hamidou, Mandayam, D. Goodman, T. Basar and R. Srikant, N. E. Stier-Moses,

1.5.5 More on Characteristics of Communities

Communities may be differ according to what they are interested in. They could also differ in their concepts: they may use different words to the same objects. Communities may read and publish in different scientific journals.

Below we did some statistics on the use of different names for the same concept, and on how this phenomenon appear in diferent communities.

Nash bargaining and Proportional Fairness.

The Nash bargaining concept that dates from 1950 [48], has been used in the telecom community in networks with general topologies to allocate resources fairly [102, 65].

A very related fairness concept, the proportional fairness, was introduced by F. Kelly and coauthors in [53, 52]. It is a special case of the Nash bargaining solution.

Its use in cellular communication has been patented and it plays a cruicial role in HSDPA (downlink schedulling in 3G).

Although the term Nash bargaining appears in [52], the telecom community has adopted the notion of "proporotional fairness", and quite ignore the term Nash bargaining.

In January 2011 we searched with the help of the "publish or perish" software for the number of citations of papers in which proportional fairness or Nash bargaining appear. We considered the period 2000-2010.

The above software allows us to search the number of citations including all words in some set that we define, within a given scientific area. We thus made two sets of experiments. The first for publications in the areas of Engineering, Mathematics and Computer Science, and then second in Business, Administration, Finance and Econoics. The results are summarized in tables 1.5.5 and 1.4.

No. of Docs in	Proportional Fairness Kelly,	Proportional Fairness Kelly Nash bargaining	Nash bargaining
Elsevier	319	6	136
Springer	443	11	433
IEEE	466+663+748	98	845

Table 1.3: Citation Analysis Among documents within the areas of Engineering, Computer Science and Mathematics

No. of Docs in	Proportional Fairness, Kelly,	Proportional Fairness, Kelly, Nash bargaining	Nash bargaining
Elsevier	107	3	714+573+825
Springer	87	6	808+349
IEEE	25	1	152

Table 1.4: Citation Analysis Among documents within the areas of "Business, Administration, Finance and Economics"

Note that the software "publish or perish" cannot handle more than 1000 citations in one enquiry. So whenever we were above 1000 entries at an enquiry, We split the entry to several disjoint time periods. In particular, we met this situation while examining the number of citations

- of "proportional - fairness, Kelly" in IEEE publications in Table 1.5.5. We thus separated the experiment to three periods: 2000-2004 (466 citationis), 2005-2007 (663 citationis) and 2008-2010 (748 citations).
- "Nash bargaining" in publications of Springer in Table 1.4. We thus considered the measurements during the disjoint periods 2000-2007 (808 citations) and 2008-2010 (349 citations).

- The same term in the same table we split the publications of Elsevier to three periods: 2000-2004 (714 citations), 2005-2007 (573 citations) and 2008-2010 (825 citations)

There are around 3500 documents that appeared in journals in areas related to economics and that use the terms Nash bargaining. On the other hand, there are around 2400 documents published in computer science and math journals that use the term proportional fairness. Thus, each of the communities has a different term for the same concept, inspite of the fact that the F P Kelly (who introduced the term of proportional fairness) cited the original term due to Nash.

1.6 Objectives and Organization of the book

The number of books on game theory applied to networks has has been quickly growing. This book is different in the sense that it is not meant to teach game theory to network engineers, but rather to present the area of network engineering games, to those who are already familiar with optimisation and game theory.

Networks, and networking games, appear in many fields. This book focuses on those occurring in telecommunications, and we call these Telenets game theory. The structure of the book is therefore related to the structure of these networks, and in particular to their layered structure (see the OSI standard) which we briefly state.

1.6.1 The OSI layer standard for telecommunication networks

We briefly present the seven layers of the OSI standard:

Layer 1: The Physical Layer

Defines the electrical and physical specifications for devices, their connection to the channel (wired or wireless). Modulation, power control, some coding and decoding.

Layer 2: The link and Medium Access (MAC) layer

Link and Medium Access layer: take care of communication over a link, i.e. a local connection between two neighboring network nodes, or to coordinate the access to a common channel. Corrects errors introduced in the physical layer. Takes care of flow control and schedulling decisions that concern a link.

Layer 3: the Network Layer

concerns with routing - deciding how to route a packets.

Layer 4: the Transport Layer

This layer takes care of the end-to-end connectivity, of retransmission if needed.

Layer 5: Session Layer

This layer takes care of opening and closing sessions as well as of initiating dialogues between computers.

Layers 6-7 The Presentation and Applicatioin Layers

These layers are concerned with networking aspects that are related to applications, such as the downloading files, the way to connect to the World Wide Web along with the use of HTTP, peer to peer communication etc.

1.6.2 A cross layer design

1.6.3 Network Economy

1.6.4 The structure of the book

Game issues: placement of BS. ASsociaition power ocontrol routing games rate control access control - aloha

1.7 Bibliographic notes

The book [76] (available for free download on the author's home page) is an excellent textbook on routing games. It has more than a thousand references that cover the topic.

Surveys on networking games: In [13], the authors present a survey that focuses on routing games with a special emphesis on the relation between games arising in road traffic networks and those in telecommunication networks. This survey appears in a spetial issue of the journal "Networks and Spatial Economics", devoted to "Crossovers between Transportation Planning and Telecommunications".

A more general overview on networking games can be found in [8][2].

A large part of the book [24] on power control in wireless networks is devoted to non-coopeprative games arising in that field. The book is available for free download see e.g.
http://worid-of-books.com/?id=IfgnyvTAw1YC A recent book on game theory for network security is [5]. Other books on game theory in wireless communications are [64, 27, 35]. For a survey focusing on wireless networks, see [58]. A recent survey in French can be found in [9]. An overview on algorithmic game theory can be found in [74].

[89] is a book that on cooperative game aspects in networking, including a large part on network formation games.

Our [16] is complementary to this one. [16] presents the networking games to a public with engineering background interested in learning about game theory and its application to Engineering. In contrast in this book, we present networking games to a public that already has the basis in game theory but is not expert in networking.

[2]see a more recent update in
http://www-sop.inria.fr/members/Eitan.Altman/PAPERS/srvUpdate.pdf

Chapter 2

Utilities and quality of service in Networks

The utilities are function of some performance measures. The performance measures may be predetermined by proper resource allocation (reservation), i.e. by a centralized procedure. They may also be determined by choices of users in a decentralized way.

For some services, performance bounds are pre-defined by the standard, and the service provider has to deliver them. For most of the traffic, the service is elastic or "best-effort".

2.1 Subjective performance measures

Audio quality may be perceived differently by different persons. It is thus considered to be a subjective performance measure. A population of individuals are thus needed in order to provide some statistically meaningful estimation of the audio quality. The ITU (International Telecommunication Union) has standardized the way to conduct such experiments, in terms of acoustic conditions, number of participiants, evaluation scale etc [93]. The evaluation scale goes from 1 to 5 (5 is the best), and the average grade over the population is called the Mean Opinion Score (MOS).

Concerning the number of number of participants in the experiments, we find in [93] the following recommendation: "The possible number of subjects in a viewing and listening test (as well as in usability tests on terminals or services) is from 6 to 40. Four is the absolute minimum for statistical reasons, while there is rarely any point in going beyond 40. The actual number in a specific test should really depend on the required validity and the need to generalize from a sample to a larger population. In general, at least 15 subjects should participate in the experiment. They should not be directly involved either in picture or audio quality evaluation as part of their work and should not be experienced assessors."

This number should be even larger if higher precision is sought. The standard [92] requests that "To maintain a high degree of precision a total of at least 100 interviews per condition is required."

We note that the above standards do not define "degree of precision", do not display confidence intervals nor other measures of precision,

2.2 Objective measures

Objective measures are measures that can be onbtained using some reliable measurement device. For example, if we wish to test the efficiency of a medication in decreasing the feaver or the blood preassure, then we can measure the feaver or the blood preassure before and after taking the medication using some appropriate equipment. In contrast, if we are interested in the impact of some treatment on the quality of sleeping, or more generally, on the quality of life, we will not be able to assess these using measurements only, but will need to obtain the (subjective) opinion of people who would participate in an experiment. Measures such as the quality of sleep are thus called "subjective measures".

Subjective measures require extensive experimentations with a large population. Once the quality is assessed under various conditions, one can estimate the subjective measures using objective measures. In the context of perceived audio quality, we may use indicators such as the losses and the delay in the network, the spectrum of received signal or the ratio between the received signal power and the sum of powers of noise and interferences. If we can find some function f that approximates well the subjective measure as a function of the above measurable quantities then f defines an objective measure.

2.2.1 Algorithms for Objective measurement

The following are some standardized algorithms to assess speech quality:

- PSQM ITU P.861 Perceptual Speech Quality Measure
- PESQ ITU P.862 Perceptual Evaluation of Speech Quality
- PAMS (Bristish Telecom) Perceptual Anallysis Measurement System.

PSQM and PAMS send reference signal through the telephony network and then compare the reference signal with the signal that is received on the other side of the network using Signal Processing algorithms.

2.2.2 The E-model

The outcome of many experients may give sufficient data for us to construct empirical models. The ITU has standardized such an objective model and it is known as ITU G.107, "The E-model, a computational model for use in transmission planning", [32].

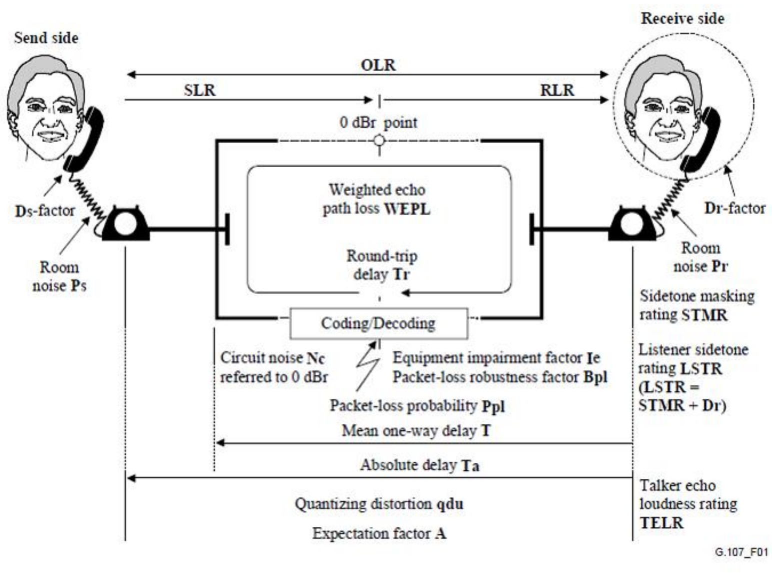


Figure 2.1: The e-momdel

The reference sais: "This Recommendation gives the algorithm for the so-called E-model as the common ITU-T Transmission Rating Model. This computational model can be useful to transmission planners, to help ensure that users will be satisfied with end-to-end transmission performance. The primary output of the model is a scalar rating of transmission quality."

The E-model introduces a scalar R that ranges from 100 (excellent) to 0 (poor).

$$R = R_0 - I_d - I_e.$$

I_d delay I_e equipment

Influenced by one way delay, jitter, losses, codec. R is mapped to MOS.

The voice may be transmitted after compression or after other processing. Larger compression means that the throughput that will be used is lower. In a network with low capacity channels, decreasing the transmission throughput may be necessary to avoid large amount of errors or losses. Higher compression may require larger delays. The network element responsible to the compression is the Codec, and the Codec's description and operation are standardized. The impact of the codec is summariized in Figure 2.2.

Codec	Bit Rate (kbps)	Packetization Delay (ms)	Codec Impairment
G.711	64.0	1.0	0
G.729	8.0	25.0	11
G.723m	6.3	67.5	15
G.723a	5.3	67.5	19

Figure 2.2: The impact of the Codec

2.3 QoE

So far we saw how performance measures impact the utility. Next, we describe what are the relevant performance measures and how they are determined at different network layers.

2.4 Performance measures at the Physical layer

2.4.1 The througuhput

Information theory has focused on computing the capacity Shannonn showed that for a Gaussian channel, the capacity is given by

$$\Theta = \log(1 + SINR)$$

This formula is often used as a utility related to the throughput of a channel.

Shannon capacity is rarely achieved. The following approximations are often used for the throughput. The first uses

$$\Theta = \log(1 + \alpha SINR)$$

where α is a constant smaller than 1. The second approximation is based upon an observation that some Codecs, are able to provide variable rates[1] and for these Codecs, the rate they provide is proportional to the SINR. More precisely, PPPP

[1] For example the AMR Codec

Shannon capacity is the throughput that can be achieved asymptotically while keeping the error probability arbitrarily small. Another approach for throughput is simply to use a Codec designed for a given rate. The rate is then taken to be the throughput, which is then independent of the SINR. The SINR then determines the error rate.

2.4.2 Loss and Error rate probabilities

How to compute capture probability: we have the following expressions for the bit error probability as a function of the modulation [80] (numerical examples based on these formulas can be found in [17, 34, 80]):

$$p_e(SINR) = \begin{cases} \frac{1}{2} erfc(\sqrt{\kappa \cdot SINR}) & \text{for GMSK} \\ \frac{1}{2}\exp(-SINR) & \text{for DBPSK} \\ \frac{1}{2}\exp(-\frac{1}{2} \cdot SINR) & \text{for GFSK} \\ \frac{1}{2} erfc(\sqrt{SINR}) & \text{for QPSK} \\ \frac{3}{8} erfc(\sqrt{\frac{2}{5} \cdot SINR}) & \text{for 16-QAM} \\ \frac{7}{32} erfc(\sqrt{\frac{4}{21} \cdot SINR}) & \text{for 64-QAM} \end{cases}$$

where κ is a constant (that depends on the amount of redundancy in the coding and on the frequency band), and where $erfc$ is the complementary error function given by

$$erfc(x) = \frac{2}{\pi} \int_x^{\infty} e^{-\zeta^2} d\zeta.$$

In the absence of redundancy this gives the following expression for f of a packet of N bits provided that the bit loss process is independent

$$f(SINR) = (1 - p_e(SINR))^N$$

2.4.3 Link and MAC layer

Queueing models are often used to model the delay introduced in that layer. Consider a queue with infinite storage capacity. The packets are served according to the order of their arrival. Here are some basic performance results of queues. Let (S_n, T_n) denote the nth service time and the nth inter-arrival time. Assume that this sequence is stationary ergodic and that $E[S_0] < E[T_0]$. Then

- The fraction of the time that the server is busy is given by $\rho := E[S]/E[T]$.
- Little's Law: Let N be the expected number of packets in the queue, including those in service, Let W be the waiting time of a customer (from arrival till it depatts). Then $\lambda E[W] = E[N]$.

Delay with packets arriving to it according to a Poisson point process with parameter λ. Let b and $b^{(2)}$ be the two first moments of the service duration of a packet. (Service times are assumede to be i.i.d. and inidependent on the arrival process. Then the expected packet delay is given by Kinchin Pollacek formula:

....

Throughput

The mean packet delay Khynchin Polacek

2.4.4 Network layer

product form of packet layer
Network calculus

2.4.5 Transport layer

TCP throughpupt formula, Kelly, Low
jitter? Playout buffer

2.4.6 Session layer

session delay (time to download a file)
PS models
Neuts
blocking probability: not additive. formulaes, the knapsack problem

2.5 Simplified measures for utility

Service classes differ from each other in their utilities. They are so far limited to pricing purposes and the undeyrlying idea is usually to give priority to some packets at the buffers. In networks context, the bandwidth does not have the same value for different users. For instance, a user consulting his or her emails does not have the same needs that another one using phone over IP. The utility functions represent the impact of the bandwidth allocation on the perceived quality. We show in the following the shapes of the utility functions for different types of application. Our discussion is qualitative and inspired from the work of Shenker [?]. For numerical results, for instance on audio communications, the reader may refer to [?, ?]. We illustrate the different shapes of the utility functions on Figure 2.3.

Elastic applications have no real-time requirements and no rate constraints. Typical examples are file transfer or email. Their utility function is concave increasing without a minimum required rate.

"Delay adaptive" or "rate adaptive" applications have soft real-time requirements. Typical examples are voice or video over IP. In such applications, the compression rate of data is computed as a function of the quantity of available resource. The utility functions that we use to represent these applications are slightly different than those in [?]. In [?], the utility is strictly positive for any non zero bandwidth and tends to zero when the bandwidth does. We consider in contrast that the utility equals zero below a certain value, as in [102]. Indeed, in many voice applications, one can select the transmission rate by choosing an appropriate compression mechanism and existing compression software have an upper bound on the compression, which implies a lower bound on the transmission rate for which a communication can be initiated, which we denote *MR*. Thus, a maximum compression rate is associated with the lower acceptable quality for the user. If there is no sufficient bandwidth, the connection is not initiated. This kind of behavior generates utility functions that are null for bandwidth below *MR* and which are not differentiable at the point $(MR, 0)$. Similarly, it is useless to allocate a bandwidth greater than a certain threshold *PR* because the perceived gain for a human being will not be noticeable. As an example, for voice transmission, we usually consider throughputs in the range $[16, 40]$ kb/s. A user to whom we would allocate a throughput of 200 kb/s would not have a better quality feeling than that if its throughput was halved.

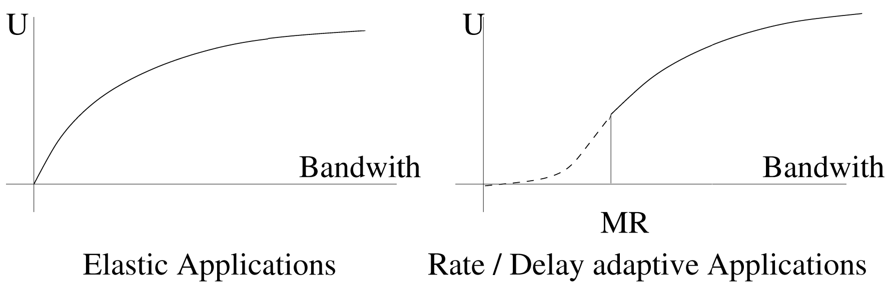


Figure 2.3: Utility function in networking

2.5.1 Other aspects

Network's utility. Fairness.

Computing queueing delay

2.6 Hybrid utilities

We already saw that determining utilities is a complex task. In practice it could be even harder, since aspects other than the quality of service could also influence: the price of the service, its availability, how much resources the service requires. For example, in wireless communications, larger transmission power could improve some performance measures but could shorten the battery life and therefore the availability of the service.

The following are examples of hybrid utilities related to power control:

- Bits per joul: this is a measure of energy efficiency in which one is interested in maximizing the amount of information that can be transmitted using a given aount of energy.
- The transport capacity (in Bits/meter/sec): we are interested here in maximizing the distance that a given amount of energy allows to transmit information per time unit.
- The word power has several meanings.
  p1: the rate of energy spending;
  p2: the arithmetic operation.
  p3: The ratio between some [p2-type] power of the throughput and the expected delay is a frequently used performance measure which too, has been called *power*.

Part II

Part 2: Physical, Link and MAC layers

Chapter 3

MAC and Power Control Games

3.1 Multiple-access game over a collision channel, Transmit or Wait

Consider two players having a packet to transmit over a collision channel. Each player may transmit or wait. If both transmit simultaneously then the packets are both lost. The payoff for each player is one if the transmission is successful. We obtain the matrix game in Figure 3.4(a).

Transmitting is a dominant strategy for both player. It leads to zero throughput and zero utility.

Player 1 \ Player 2	H	D
H	0	1
D	0	0

(a) Original Version

Player 1 \ Player 2	H	D
H	$-E$	$1-E$
D	0	0

(b) Version with energy cost

Figure 3.1: The Multiple-Access Game

We add next an energy cost E if a player transmits. We obtain the matrix game in Figure 3.4(b).

If $E \geq 1$ then $r = 0$ is the unique equilibrium.

Assume $0 < E < 1$. We see that the game is of the third type in Remark 8. It is thus a H-D game with a single mixed equilibrium given by

$$r = 1 - E$$

This also leads to zero utility but now the equilibrium throughput is

$$Thp = r(1-r) = E(1-E)$$

This is maximizied at $E = 1/2$ which gives an equilibrium throughput of $1/4$.

We note that a correlated equilibrium would lead a better throughput. The arbitrator sends to the mobiles the outcome of a randomization that decides who of them will transmit. A total throughput of 1 can be achieved.

Why did the utility at equilibrium remain zero? Due to the indifference property: at equilibrium, player 1 is indifferent between 1st and 2nd row. The utility for the 2nd row is always 0 independently of other players.

Note: The utility at equilibrium can decrease or increase when decrease the entries of the matrix. This is not the case in optimization, nor in zero-sum games.

		Player 2	
		H	D
Player 1	H	$p-E$	$1-E$
	D	0	q

Figure 3.2: High power (H) versus low power (D)

We add a cost E for using the high power. H remains a dominant strategy as long as $E < 1-q$. For such E, the uhtility is $p-E$.

3.2 MAC game: Coordination games over a collision channel

Simple Motivating Example Consider the following basic example. There are two mobiles $i = 1, 2$ and two independent channels $j = 1, 2$. Each mobile transmits at the same time one packet: Mobile i transmits a packet over channel i with probability p_i and with probability $1 - p_i$ over the other channel. A packet is successfully transmitted if it is the only one that uses the channel. Thus the transmission success probability of mobile i is

$$U_i(p) = p_i p_j + (1 - p_i)(1 - p_j), \qquad j \neq i$$

Mobile i wishes to maximize the probability U_i of successful transmission of its packet.

The policy that assigns a dedicated channel to each mobile (i.e. $p_1 = p_2 = 1$ or $p_1 = p_2 = 0$) is obviously optimal: it involves no collisions and the success probability is one. It is also a Nash equilibrium. However it requires coordination or synchronization in order to assign each channel to a different mobile.

The symmetric policy $p_1 = p_2 = 0.5$ turns out to be an equilibrium; if mobile i uses $p_i = 0.5$ then no matter what p_j mobile j ($i \neq j$) chooses, it will have the same success probability of $1/2$. Thus no mobile can benefit by unilaterally deviating from $p = 1/2$, so it is an equilibrium.

Note that if mobile i had only one option, that of choosing channel i, then the innefficient equilibrium would not occur. This is a feature similar to the inefficiency we have in the prisoner's dilemma or in the Braess' paradox in which eliminating some options for the players can result in better performance to every one. Yet if we wanted to implement this idea in our context and create mobiles with only one channel, then we would face again a synchronization problem. If half of the mobiles have built in technology for accessing one channel and the other half can only access the other channel, then two randomly selected mobiles will still be using the same channel with probability half.

The model and main result Consider 2 mobiles and 2 base stations. The base stations use, each one, an independent channel (for example, each one uses another frequency). We shall assume that mobile i has a good radio channel with base station i and a bad one with station $j \neq i$. More precisely, let h_{ij} be the gain between mobile i and base station j.

Let $SINR_i$ denote the Signal to Interference and Noise Ratio corresponding to the signal received from mobile i at the base station to which it transmits. Each mobile has two pure strategies: γ, β where γ means transmitting on its good channel and β on its bad one. Then

$$SINR_1(u) = \begin{cases} \dfrac{h_{11}P_1}{N_o} & \text{if } u = (\gamma, \gamma) \\[2ex] \dfrac{h_{12}P_1}{N_o} & \text{if } u = (\beta, \beta) \\[2ex] \dfrac{h_{11}P_1}{N_o + h_{21}P_2} & \text{if } u = (\gamma, \beta) \\[2ex] \dfrac{h_{12}P_1}{N_o + h_{22}P_2} & \text{if } u = (\beta, \gamma) \end{cases}$$

		Player 2	
		β	γ
Player 1	β	$\left(\frac{h_{12}P_1}{N_o}, \frac{h_{21}P_2}{N_o}\right)$	$\left(\frac{h_{12}P_1}{N_o + h_{22}P_2}, \frac{h_{22}P_2}{(N_o + h_{12}P_1)}\right)$
	γ	$\left(\frac{h_{11}P_1}{N_o + h_{21}P_2}, \frac{h_{21}P_2}{N_o + h_{11}P_1}\right)$	$\left(\frac{h_{11}P_1}{N_o}, \frac{h_{22}P_2}{N_o}\right)$

Figure 3.3: SINR values of the two mobiles

$$SINR_2(u) = \begin{cases} \dfrac{h_{22}P_2}{N_o} & \text{if } u = (\gamma, \gamma) \\[2ex] \dfrac{h_{21}P_2}{N_o} & \text{if } u = (\beta, \beta) \\[2ex] \dfrac{h_{22}P_2}{N_o + h_{12}P_1} & \text{if } u = (\beta, \gamma) \\[2ex] \dfrac{h_{21}P_2}{N_o + h_{11}P_1} & \text{if } u = (\gamma, \beta) \end{cases}$$

Here N_o is the thermal noise at each base station and P_i is the fixed transmission power of mobile i.

Under many modulation schemes the probability of a successful transmission of a packet is known to be a monotone increasing function of the $SINR$ [90]. We thus assume that mobile i has a success probability given by $f_i(SINR_i)$. Define

$$A := f_1\left(\frac{h_{11}P_1}{N_o},\right) \quad B := f_1\left(\frac{h_{11}P_1}{N_o + h_{21}P_2}\right)$$

$$C := f_1\left(\frac{h_{12}P_1}{N_o + h_{22}P_2}\right) \quad D := f_1\left(\frac{h_{12}P_1}{N_o}\right).$$

$$a := f_2\left(\frac{h_{22}P_2}{N_o},\right) \quad b := f_2\left(\frac{h_{22}P_2}{N_o + h_{12}P_1}\right)$$

$$c := f_2\left(\frac{h_{21}P_2}{N_o + h_{11}P_1}\right) \quad d := f_2\left(\frac{h_{21}P_2}{N_o}\right).$$

The mobiles are thus faced with the following matrix game:

	action γ	action β
action γ	A, a	B, c
action β	C, b	D, d

Theorem 1. *There are exactly three equilibria; the two pure equilibria:* (γ, γ) *and* (β, β)*, and a mixed one in which player 1 and 2 select* γ *with probabilities:*

$$X^* = \frac{D - B}{A + D - B - C}, \quad Y^* = \frac{d - b}{a + d - b - c}.$$

Proof. We note that $B < D$, $b < d$, $C < A$ and $c < a$. The game is thus a standard coordination game (see *http://en.wikipedia.org/wiki/Coordination_game*) [26] for which the result is well known. ■

The mixed equilibrium is characterized by the indifference property: when a mobile uses its mixed equilibrium policy then the other player is indifferent between γ and β.

The utility of mobile 1 and 2 at the mixed equilibrium are given by

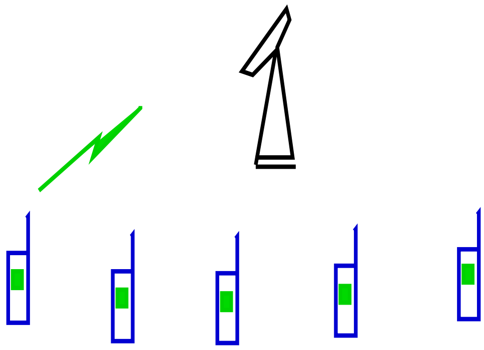
$$U_1^* = AY^* + B(1 - Y^*), \quad U_2^* = aX^* + b(1 - X^*).$$

Consider now the symmetric case $(A = a, B = b, C = c, D = d)$. Then we get at the mixed equilibrium:

$$U^* = \frac{(a-c)(d-c) + c(a+d-b-c)}{a+d-b-c} = \frac{ad - cb}{a+d-b-c}$$

3.2.1 Power control: general constraints

N mobile terminals. Each minimizes its transmission power P_i.



The transmissions of all mobiles are received at a base station (BS).

The received power of mobile i is $h_i P_i$. ξ is the power of the thermal noise at BS.

3.2.2 Multiuser detection game

PPPP

3.3 Multiple-access game over a collision channel, Transmit or Wait

Consider two players having a packet to transmit over a collision channel. Each player may transmit or wait. If both transmit simultaneously then the packets are both lost. The payoff for each player is one if the transmission is successful. We obtain the matrix game in Figure 3.4(a).

Transmitting is a dominant strategy for both player. It leads to zero throughput and zero utility.

		Player 2	
		H	D
Player 1	H	0	1
	D	0	0

(a) Original Version

		Player 2	
		H	D
Player 1	H	$-E$	$1-E$
	D	0	0

(b) Version with energy cost

Figure 3.4: The Multiple-Access Game

We add next an energy cost E if a player transmits. We obtain the matrix game in Figure 3.4(b).

If $E \geq 1$ then $r = 0$ is the unique equilibrium.

Assume $0 < E < 1$. We see that the game is of the third type in Remark 8. It is thus a H-D game with a single mixed equilibrium given by

$$r = 1 - E$$

This also leads to zero utility but now the equilibrium throughput is

$$Thp = r(1-r) = E(1-E)$$

This is maximizied at $E = 1/2$ which gives an equilibrium throughput of $1/4$.

We note that a correlated equilibrium would lead a better throughput. The arbitrator sends to the mobiles the outcome of a randomization that decides who of them will transmit. A total throughput of 1 can be achieved.

Why did the utility at equilibrium remain zero? Due to the indifference property: at equilibrium, player 1 is indifferent between 1st and 2nd row. The utility for the 2nd row is always 0 independently of other players.

3.4 At what Power to Transmit? The Capture Phenomenon

		Player 2	
		H	D
Player 1	H	Q_{11}	Q_{12}
	D	Q_{21}	Q_{22}

Figure 3.5: High power (H) versus low power (D)

Player i can transmit with power $p_i \in \mathcal{A} := \{P_H, P_L\}$. We now consider capture. Let Q_{ij} be the probability that the transmission of player 1 is successful when using power p_i and the other one uses p_j. Example: $Q_{HH} = p$, $Q_{DD} = q < p$, $Q_{HD} = 1$, $Q_{DH} = 0$. Dominating strategy: H. Gives a throughput of p to each mobile.

How to compute capture probability: we have the following expressions for the bit error probability as a function of the modulation [80] (numerical examples based on these formulas can be found in [17, 34, 80]):

$$p_e(SINR) = \begin{cases} \frac{1}{2} erfc(\sqrt{\kappa \cdot SINR}) & \text{for GMSK} \\ \frac{1}{2}\exp\left(-SINR\right) & \text{for DBPSK} \\ \frac{1}{2}\exp(-\frac{1}{2} \cdot SINR) & \text{for GFSK} \\ \frac{1}{2} erfc(\sqrt{SINR}) & \text{for QPSK} \\ \frac{3}{8} erfc(\sqrt{\frac{2}{5} \cdot SINR}) & \text{for 16-QAM} \\ \frac{7}{32} erfc(\sqrt{\frac{4}{21} \cdot SINR}) & \text{for 64-QAM} \end{cases}$$

where κ is a constant (that depends on the amount of redundancy in the coding and on the frequency band), and where $erfc$ is the complementary error function given by

$$erfc(x) = \frac{2}{\pi} \int_x^{\infty} e^{-\zeta^2} d\zeta.$$

In the absence of redundancy this gives the following expression for f of a packet of N bits provided that the bit loss process is independent

$$f(SINR) = (1 - p_e(SINR))^N$$

		Player 2: H	Player 2: D
Player 1	H	$p-E$	$1-E$
	D	0	q

Figure 3.6: High power (H) versus low power (D)

We add a cost E for using the high power. H remains a dominant strategy as long as $E < 1-q$. For such E, the uhtility is $p-E$.

Note: The utility at equilibrium can decrease or increase when decrease the entries of the matrix. This is not the case in optimization, nor in zero-sum games.

3.5 MAC game: Coordination games over a collision channel

Simple Motivating Example Consider the following basic example. There are two mobiles $i = 1, 2$ and two independent channels $j = 1, 2$. Each mobile transmits at the same time one packet: Mobile i transmits a packet over channel i with probability p_i and with probability $1 - p_i$ over the other channel. A packet is successfully transmitted if it is the only one that uses the channel. Thus the transmission success probability of mobile i is

$$U_i(p) = p_i p_j + (1 - p_i)(1 - p_j), \qquad j \neq i$$

Mobile i wishes to maximize the probability U_i of successful transmission of its packet.

The policy that assigns a dedicated channel to each mobile (i.e. $p_1 = p_2 = 1$ or $p_1 = p_2 = 0$) is obviously optimal: it involves no collisions and the success probability is one. It is also a Nash equilibrium. However it requires coordination or synchronization in order to assign each channel to a different mobile.

The symmetric policy $p_1 = p_2 = 0.5$ turns out to be an equilibrium; if mobile i uses $p_i = 0.5$ then no matter what p_j mobile j ($i \neq j$) chooses, it will have the same success probability of $1/2$. Thus no mobile can benefit by unilaterally deviating from $p = 1/2$, so it is an equilibrium.

Note that if mobile i had only one option, that of choosing channel i, then the innefficient equilibrium would not occur. This is a feature similar to the inefficiency we have in the prisoner's dilemma or in the Braess' paradox in which eliminating some options for the players can result in better performance to every one. Yet if we wanted to implement this idea in our context and create mobiles with only one channel, then we would face again a synchronization problem. If half of the mobiles have built in technology for accessing one channel and the other half can only access the other channel, then two randomly selected mobiles will still be using the same channel with probability half.

The model and main result Consider 2 mobiles and 2 base stations. The base stations use, each one, an independent channel (for example, each one uses another frequency). We shall assume that mobile i has a good radio channel with base station i and a bad one with station $j \neq i$. More precisely, let h_{ij} be the gain between mobile i and base station j.

Let $SINR_i$ denote the Signal to Interference and Noise Ratio corresponding to the signal received from mobile i at the base station to which it transmits. Each mobile has two pure

		Player 2	
		β	γ
Player 1	β	$\left(\frac{h_{12}P_1}{N_o}, \frac{h_{21}P_2}{N_o}\right)$	$\left(\frac{h_{12}P_1}{N_o + h_{22}P_2}, \frac{h_{22}P_2}{(N_o + h_{12}P_1)}\right)$
	γ	$\left(\frac{h_{11}P_1}{N_o + h_{21}P_2}, \frac{h_{21}P_2}{N_o + h_{11}P_1}\right)$	$\left(\frac{h_{11}P_1}{N_o}, \frac{h_{22}P_2}{N_o}\right)$

Figure 3.7: SINR values of the two mobiles

strategies: γ, β where γ means transmitting on its good channel and β on its bad one. Then

$$SINR_1(u) = \begin{cases} \dfrac{h_{11}P_1}{N_o} & \text{if } u = (\gamma, \gamma) \\[2ex] \dfrac{h_{12}P_1}{N_o} & \text{if } u = (\beta, \beta) \\[2ex] \dfrac{h_{11}P_1}{N_o + h_{21}P_2} & \text{if } u = (\gamma, \beta) \\[2ex] \dfrac{h_{12}P_1}{N_o + h_{22}P_2} & \text{if } u = (\beta, \gamma) \end{cases}$$

$$SINR_2(u) = \begin{cases} \dfrac{h_{22}P_2}{N_o} & \text{if } u = (\gamma, \gamma) \\[2ex] \dfrac{h_{21}P_2}{N_o} & \text{if } u = (\beta, \beta) \\[2ex] \dfrac{h_{22}P_2}{N_o + h_{12}P_1} & \text{if } u = (\beta, \gamma) \\[2ex] \dfrac{h_{21}P_2}{N_o + h_{11}P_1} & \text{if } u = (\gamma, \beta) \end{cases}$$

Here N_o is the thermal noise at each base station and P_i is the fixed transmission power of mobile i.

Under many modulation schemes the probability of a successful transmission of a packet is known to be a monotone increasing function of the $SINR$ [90]. We thus assume that mobile i has a success probability given by $f_i(SINR_i)$. Define

$$A := f_1\left(\frac{h_{11}P_1}{N_o},\right) \quad B := f_1\left(\frac{h_{11}P_1}{N_o + h_{21}P_2}\right)$$
$$C := f_1\left(\frac{h_{12}P_1}{N_o + h_{22}P_2}\right) \quad D := f_1\left(\frac{h_{12}P_1}{N_o}\right).$$
$$a := f_2\left(\frac{h_{22}P_2}{N_o},\right) \quad b := f_2\left(\frac{h_{22}P_2}{N_o + h_{12}P_1}\right)$$
$$c := f_2\left(\frac{h_{21}P_2}{N_o + h_{11}P_1}\right) \quad d := f_2\left(\frac{h_{21}P_2}{N_o}\right).$$

The mobiles are thus faced with the following matrix game:

Theorem 2. *There are exactly three equilibria; the two pure equilibria: (γ, γ) and (β, β), and a mixed one in which player 1 and 2 select γ with probabilities:*

$$X^* = \frac{D - B}{A + D - B - C}, \quad Y^* = \frac{d - b}{a + d - b - c}.$$

	action γ	action β
action γ	A, a	B, c
action β	C, b	D, d

Proof. We note that $B < D$, $b < d$, $C < A$ and $c < a$. The game is thus a standard coordination game (see *http://en.wikipedia.org/wiki/Coordination_game*) [26] for which the result is well known. ■

The mixed equilibrium is characterized by the indifference property: when a mobile uses its mixed equilibrium policy then the other player is indifferent between γ and β.

The utility of mobile 1 and 2 at the mixed equilibrium are given by

$$U_1^* = AY^* + B(1 - Y^*), \quad U_2^* = aX^* + b(1 - X^*).$$

Consider now the symmetric case ($A = a, B = b, C = c, D = d$). Then we get at the mixed equilibrium:

$$U^* = \frac{(a-c)(d-c) + c(a+d-b-c)}{a+d-b-c} = \frac{ad - cb}{a+d-b-c}$$

Chapter 4

Flow control

Two main approaches exist for controlling the congestion at the network in high speed communication systems. The first, in which sources control their *transmission rate*, has been adopted by the ATM-forum [?] (an international standardization organism for high speed telecommunication networks) for applications that have flexibility in their requirement for bandwidth (and which then use the so called "Available Bit Rate" service). Although the general approach and signaling mechanisms have been standardized, the actual way to control the sources has been left open and is still an open area of research and development. An alternative approach is the so called "*window based*" flow control which is used at the Internet. An example of a stochastic game based on the latter approach is given in [?].

We present in this chapter a linear quadratic differential game model for flow control based on the first approach, which has been introduced and analyzed in [?], in which M users control their transmission rate into a single bottleneck queue. Thus, each user has to answer dynamically the question of "how much to queue", see Fig. 4.1.

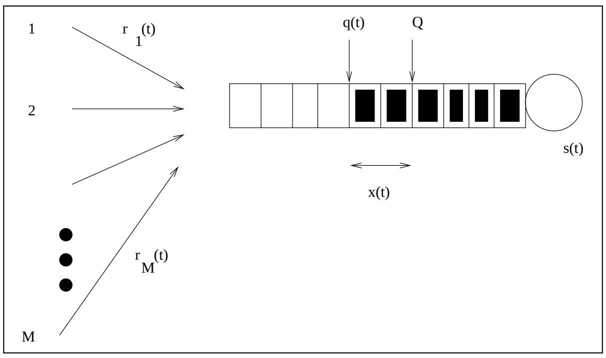


Figure 4.1: A competitive flow control model

The output rate of the queue (i.e. the server's rate) is given by $s(t)$. The *controlled* input rate of user m is denoted by $r_m(t)$. Let $q(t)$ denote the instantaneous queue length.

It is desirable for telecommunication applications to keep the instantaneous size of the queue close to some target: Q. Indeed, if the queue is too large then information packets might be lost due to overflow, and moreover, packets suffer large delays. If the queue is empty, on the other hand, then the output rate is constrained by the input rate, which might be lower than $s(t)$. We thus may have underutilization of the available service rate and we loose in throughput.

Define $x(t) := q(t) - Q$ to be the shifted instantaneous queue length. It will serve as the state of the queue. We assume that there is some desirable share of bandwidth available for user m, given by $a_m s(t)$, where $\sum_m a_{m=1}^M = 1$.

Define $u_m(t) := r_m(t) - a_m s(t)$ to be the shifted control.

Ignoring the nonlinearity in the dynamics that are due to boundary effects (at an empty queue or at a full queue), we obtain the following idealized dynamics:

$$\frac{dx}{dt} = \sum_{m=1}^{M} (r_m - a_m s) = \sum_{m=1}^{M} u_m \,, \tag{4.1}$$

Justification for using the linearized model are presented in [?].

Policies and information: We consider the following class of history dependent policies for all users:

$$u_m(t) = \mu_m(t, x_t), \qquad t \in [0, \infty) \,.$$

μ_m is piecewise continuous in its first argument, piecewise Lipschitz continuous in its second argument. The class of all such policies for user m is $\mathcal{U}_m$.

Objectives. The cost per user is a linear combination of the cost related to deviation from the desirable queue length value Q and deviating from the desirable share of the bandwidth. We consider two versions that differ by some scaling factor:

- N1: the individual cost to be minimized by user m $(m \in \mathcal{M} = \{1, ..., M\})$ is

$$J_m^{N1}(u) = \int_0^\infty \left(|x(t)|^2 + \frac{1}{c_m} |u_m(t)|^2 \right) dt \,. \tag{4.2}$$

- N2: the individual cost to be minimized by controller m $(m \in \mathcal{M})$ is

$$J_m^{N2}(u) = \int_0^\infty \left(\frac{1}{M} |x(t)|^2 + \frac{1}{c_m} |u_m(t)|^2 \right) dt \,. \tag{4.3}$$

In case N2 the "effort" for keeping the deviations of the queue length from the desired value is split equally between the users.

Nash equilibria. We seek a multi-policy $\mu^* := (\mu_1^*, ..., \mu_M^*)$ such that no user has an incentive to deviate from, i.e.

$$J_m^{N1}(\mu^*) = \inf_{\mu_m \in \mathcal{U}_m} J_m^{N1}([\mu_m | \mu_{-m}^*]) \tag{4.4}$$

where $[\mu_m|\mu_{-m}^*]$ is the policy obtained when for each $j \neq m$, player j uses policy μ_j^*, and player m uses μ_m. We define similarly the problem with the cost J^{N_2}.

4.1 Main results

We shall show that the flow control game has a simple computable equilibrium and value. We further show a uniqueness result.
<u>The equilibrium:</u> For case Ni $(i = 1, 2)$, there exists an equilibrium given by

$$\mu_{Ni,m}^*(x) = -\beta_m^{Ni} x \,, \; m = 1, ..., M,$$

where β_m^{Ni} is given by

$$\beta_m^{N1} = \overline{\beta}^{(N1)} - \sqrt{\overline{\beta}^{(N1)^2} - c_m}$$

where $\overline{\beta}^{(N1)} := \sum_{m=1}^{M} \beta_m^{N1}$, $i = 1, 2$, are the unique solutions of

$$\overline{\beta}^{(N1)} = \frac{1}{M-1} \sum_{m=1}^{M} \sqrt{(\overline{\beta}^{(N1)})^2 - c_m}$$

and for the case N2:

$$\beta_m^{N2} = \overline{\beta}^{(N2)} - \sqrt{\overline{\beta}^{(N2)^2} - \frac{c_m}{M}},$$

where $\overline{\beta}^{(N2)} := \sum_{m=1}^{M} \beta_m^{N2}$, $i = 1, 2$, are the unique solutions of

$$\overline{\beta}^{(N2)} = \frac{1}{M-1} \sum_{m=1}^{M} \sqrt{(\overline{\beta}^{(N2)})^2 - \frac{c_m}{M}} = \frac{\overline{\beta}^{(N1)}}{\sqrt{M}}.$$

Moreover,

$$\beta_m^{N1} = \beta_m^{N2}\sqrt{M}.$$

For each case, this is the unique equilibrium among stationary policies and is time-consistent.

<u>The value.</u> The costs accruing to user m, under the two Nash equilibria above, are given by

$$J_m^{N1}(\mu_{N1}^*) = \frac{\beta_m^{N1}}{c_m} x^2$$

and

$$J_m^{N2}(\mu_{N2}^*) = \frac{\beta_m^{N2}}{c_m} x^2 = \frac{1}{\sqrt{M}} J_m^{N1}(u_{N1}^*).$$

<u>The symmetric case.</u> In the case $c_m = c_j =: c$ for all $m, j \in \mathcal{M}$ we obtain:

$$\beta_m^{N1} = \sqrt{\frac{c}{2M-1}}, \qquad \text{and} \qquad \beta_m^{N2} = \sqrt{\frac{c}{M(2M-1)}}, \quad \forall m \in \mathcal{M}\,;$$

<u>The case of $M = 2$.</u> We assume general values of c_m's. we have for $m = 1, 2,\ j \neq m$,

$$\beta_m^{N1} = \left[-\frac{2c_j - c_m}{3} + 2\frac{\sqrt{c_1^2 - c_1 c_2 + c_2^2}}{3} \right]^{1/2}, \qquad \beta_m^{N2} = \frac{\beta_m^{N1}}{\sqrt{2}}.$$

If moreover, $c_1 = c_2 = c$ then $\beta_m^{N1} = \sqrt{c/3}$, $\beta_m^{N2} = \sqrt{c/6}$.

We now sketch the proof for N1. The one for N2 is similar.

Proof for (N1): Choose a candidate solution

$$u_m^*(x) = -\beta_m x\,,\ m = 1, ..., M, \quad \text{where } \beta_m = \overline{\beta} - \sqrt{\overline{\beta}^2 - c_m}$$

where $\overline{\beta} := \sum_{m=1}^{M} \beta_m$, are the unique solution of

$$\overline{\beta} = \frac{1}{M-1} \sum_{m=1}^{M} \sqrt{\overline{\beta}^2 - c_m}.$$

Fix u_j for $j \neq m$. Player m is faced with a linear quadratic optimal control problem with the dynamics

$$dx/dt = u_m - \beta_{-m} x\,,\ \beta_{-m} = \sum_{j \neq m} \beta_j$$

and cost $J_m^{N1}(u)$ that is strictly convex in u_m. Her optimal response is $u_m = -c_m P_m x$, where P_m is the unique positive solution of the Riccati equation

$$-2\beta_{-m} P_m - P_m^2 c_m + 1 = 0. \tag{4.5}$$

Denoting $\beta'_m = c_m P_m$, we obtain from (4.5)

$$\beta'_m = f_m(\beta_{-m}) := -\beta_{-m} + \sqrt{\beta_{-m}^2 + c_m}.$$

u is in equilibrium if and only if $\beta' = \beta$, or

$$\overline{\beta}^2 = \beta_{-m}^2 + c_m. \tag{4.6}$$

Hence

$$\beta_m = \overline{\beta} - \sqrt{\overline{\beta}^2 - c_m}.$$

Summing over $m \in \mathcal{M}$ we obtain

$$\Delta := \overline{\beta} - \frac{1}{M-1}\sum_{m=1}^{M}\sqrt{\overline{\beta}^2 - c_m} = 0$$

Uniqueness follows since

- Δ is strictly decreasing in $\overline{\beta}$ over the interval $[\max_m \sqrt{c_m}, \infty)$,
- it is positive at $\overline{\beta} = \max_m \sqrt{c_m}$ and
- it tends to $-\infty$ as $\beta \to \infty$.

4.2 Greedy decentralized algorithms

The Nash equilibrium requires some coordination, i.e. a (non-binding) agreement according to which all user follow the Nash equilibrium. Moreover, to compute the equilibrium, a player needs to have the knowledge of other individual utilities (c_m). Both the coordination as well as the knowledge of others' utilities are restrictive and non-realistic assumptions. This motivated the authors in [?] to propose several greedy decentralized "best response" algorithms.

ect greedy "best response" algorithm is defined by the following four conditions [?]:

- (i) Each user updates from time to time its policy by computing the best response against the most recently announced policies of the other users.
- (ii) The time between updates is sufficiently large, so that the control problem faced by a user when it updates its policy is well approximated by the original infinite horizon problem.
- (iii) The order of updates is arbitrary, but each user performs updates infinitely often.
- (iv) When the nth update occurs, a subset $K_n \subset \{1, \ldots, M\}$ of users simultaneously update their policies.

4.3 Proposed algorithms

- **Parallel update algorithm (PUA):** $K_n = \{1, ..., M\}$ for all n.
- **Round robin algorithm (RRA):** K_n is a singleton for all n and equals $(n+k) \bmod M + 1$, where k is an arbitrary integer.
- **Asynchronous algorithm (AA):** K_n is a singleton for all n and is chosen at random.

The initial policy used by each user is linear.
$\beta^{(n)}$:= value of the linear coefficient defining the policies corresponding to the end of the nth iteration.
The optimal response at each step n:

$$\beta_m^{(n)} = \begin{cases} f_m(\beta_{-m}^{(n-1)}) & \text{if } m \in K_n \\ \beta_m^{(n-1)} & \text{otherwise} \end{cases}, \tag{4.7}$$

where

$$f_m(\beta_{-m}) := -\beta_{-m} + \sqrt{\beta_{-m}^2 + c_m}. \tag{4.8}$$

4.4 Convergence results

We briefly mention the convergence results obtained in [?].

Theorem 3. *Consider PUA.*
(i.a) Let $\beta_k^{(1)} = 0$ *for all* k. *Then* $\beta_k^{(2n)}$ *monotonically decrease in* n *and* $\beta_k^{(2n+1)}$ *monotonically increase in* n, *for every player* k, *and thus, the following limits exist:* $\hat{\beta}_k := \lim_{n\to\infty} \beta_k^{(2n)}$, $\tilde{\beta}_k := \lim_{n\to\infty} \beta_k^{(2n+1)}$.
(i.b) Assume that $\hat{\beta}_k = \tilde{\beta}_k$ *(defined as above, with* $\beta_k^{(1)} = 0$ *for all* k*). Consider now a different initial condition satisfying either* $\beta_k^{(1)} \le \beta_k$ *for all* k, *(where* β_k *is the unique Nash) or* $\beta_k^{(1)} \ge \beta_k$ *for all* k. *Then for all* k, $\lim_{n\to\infty} \beta_k^{(n)} = \beta_k$.

4.4.1 Global convergence.

A global convergence result is obtained for $M = 2$:

Theorem 4. *If*
– (ii.a) $M = 2$, *and either* $\beta_k^{(1)} \le \beta_k$ *for all* k, *or* $\beta_k^{(1)} \ge \beta_k$ *for all* k; *or if*
– (ii.b) $\beta_k^{(1)}$ *and* $c := c_k$ *are the same for all* k,
then $\beta^{(n)}$ *converges to the unique equilibrium* β^*.

4.4.2 Local convergence.

Theorem 5. *For arbitrary* c_k, *there exists some neighborhood* V *of the unique equilibrium* β^* *such that if* $\beta_k^{(1)} \in V$ *then* $\beta^{(n)}$ *converges to the unique equilibrium* β^*.

4.4.3 Numerical examples

Some examples from [?] on the convergence of greedy algorithm are presented below. The parallel updates are seen to converge slowest with an oscillating behavior.

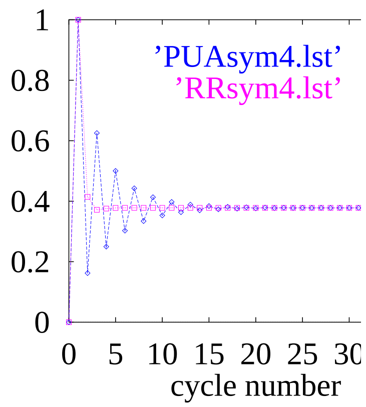


Figure 4.2: PUA versus RRA for $M = 4$

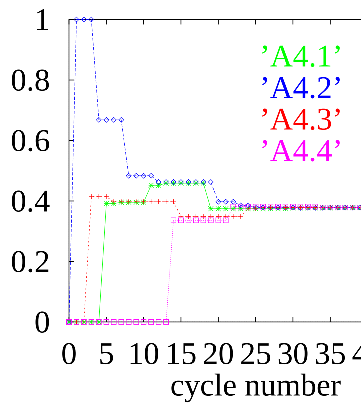


Figure 4.3: AA for $M = 4$

Part III

Part 3: Network and Session Layers

Chapter 5

Routing Games

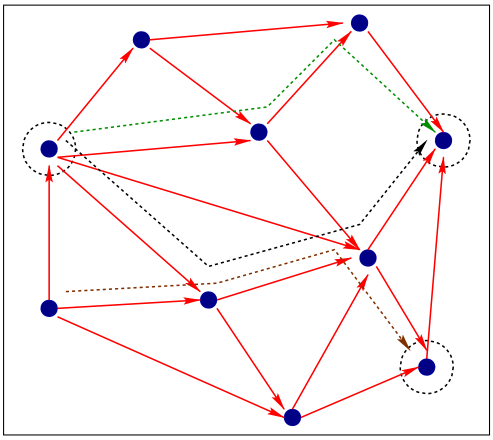


Figure 5.1: The assignment problem and competitive routing

Define the directed graph as $G = (\mathcal{N}, \mathcal{L})$. where $\mathcal{N}$ is the set of nodes and where $\mathcal{L}$ is the set of directed arcs. Let W be a set of source destination pairs. Consider a set $\mathcal{I} = \{1, ..., I\}$ of traffic classes, each represented by
(i) a source destiniation pair $w \in W$,
(ii) the traffic demand d_w betwreen the source-destinationn pair w,
(iii) a set R_w of available paths between the source destination pair w.

Define the following flows:
h^i_{wr} := the flow of player i over path r.
h_{rw} := the total flow over path r.
x^i_l := the flow of player i from class i on link l. Let $x_l = \sum_{i \in N} x^i_l$ be the flow over link l.

The following relations hold (flow conservation):

$$\sum_{r \in R_w} h^i_{wr} = d^i_w, \quad w \in W, \tag{5.1}$$

$$\sum_{w \in W} \sum_{r \in R_w} h^i_{wr} \delta^l_{wr} = x^i_l, l \in \mathcal{L}, \tag{5.2}$$

$$x^i_l \geq 0, l \in \mathcal{L}, \tag{5.3}$$

where δ^l_{wr} is a $0-1$ indicator function that takes the value 1 when link l is present on route $r \in R_w$.

Defeine $\mathbf{x}_l$ to be the vector of flows over link l of all players, and $\mathbf{x}$ to be the set of all $\{\mathbf{x}_l, l \in \mathcal{L}\}$.

Link routing case: We shall study in particular the case where at each node we can split the incoming traffic among the outgoing links.

In the link routing framework, we describe the system with respect to the variables x_l^i which are restricted by the non-negativity constraints for each link l and player i: $x_{il} \geq 0$ and by the conservation constraints for each player i and each node v:

$$r_v^i + \sum_{j \in In(v)} x_j^i = \sum_{j \in Out(v)} x_j^i \tag{5.4}$$

where $r_v^i = d_i$ if v is the source node for player i, $r_v^i = -d_i$ if v is its destination node, and $r_v^i = 0$ otherwise; $In(v)$ and $Out(v)$ are respectively all ingoing and outgoing links of node v. (d_i is the total demand of player i).

5.1 The Nash-Cournot game

A player i determines the the routing decisions for all the traffic that corresponds to the corresponding class i. The cost of player i is assumed to be additive over links

$$J^i(\mathbf{x}) = \sum_l J_l^i(\mathbf{x}_l).,\tag{5.5}$$

We shall assume that
(i) $K_l^i := \frac{\partial J_l^i(\mathbf{x})}{\partial x_l^i}$ exist and are continuous in x_l^i (for all i and l),
(ii) J_l^i are convex in x_l^i (for all i and l),
We shall often make the following assumption for each link l and player i:

A1: J_l^i depends on $\mathbf{x}_l$ only through the total flow x_l and the flow of x_l^i of player i over the link.
A2: J_l^i is incnreasing in both argumentns
A3: Whenever J_l^i is finite, $K_l^i(X_l, x_l^i)$ is strictly incnreasing in both argumentns.

Sometimes we further restrict the cost to satisfy the following:

B1: For each link l there is a nonnegative cost density $t_l(x_l)$
t_l is a function of the total flow through the link and $J_l^i = x_l^i t_l(x_l)$.
B2: t_l is positive, strictly increasing and convex, and is continuously differentiable

5.1.1 Link routing framework:

The Lagrangian with respect to the constraints on the conservation of flow is

$$L_i(\mathbf{x}, \lambda) = \sum_{l \in \mathcal{L}} J_l^i(x_l, x) + \sum_{v \in \mathcal{N}} \lambda_v^i \left(r_v^i + \sum_{j \in In(v)} x_j^i - \sum_{j \in Out(v)} x_j^i \right),$$

for each player i.

Thus a vector $\mathbf{x}$ with nonnegative components satisfying (5.4) for all i and v is an equilibrium if and only if the following Karush-Kuhn-Tucker (KKT) condition holds:

Below we shall use uv to denote the link defined by node pair u, v. There exist Lagrange multipliers λ_u^i for all nodes u and all players, i, such that for each pair of nodes u, v connected by a directed link (u, v),

$$K_{uv}^i(x_{uv}^i, x_{uv}) \geq \lambda_u^i - \lambda_v^i,\tag{5.6}$$

with equality if $x_{uv} > 0$.

Assume cost structure B. Then, the Lagrangian is given by

$$L_i(\mathbf{x}, \lambda) = \sum_{l \in \mathcal{L}} \left[t_l(x_l) + x_l^i \frac{\partial t_l(x_l)}{\partial x_l} \right] + \sum_{v \in \mathcal{N}} \lambda_v^i \left(r_v^i + \sum_{j \in In(v)} x_j^i - \sum_{j \in Out(v)} x_j^i \right),$$

for each player i.e

(5.6) can be written as

$$t_{uv}(x_{uv}) + x_{uv}^i \frac{\partial t_{uv}(x_{uv})}{\partial x_{uv}} \geq \lambda_u^i - \lambda_v^i. \tag{5.7}$$

5.1.2 The global optimal solution

Consider a single player. If K_l is strictly increasing, then there exists a unique optimal solution $\mathbf{x}^*$. There exists Lagrange multipliers λ such that $(\mathbf{x}^*, \lambda)$ is a saddle point. This follows from Sion maxmin theorem.

Note: λ need not be unique.

5.2 The case of atomless players: Wardrop

Given the fundamental nature of equilibria in many large-scale systems, it is of no surprise that researchers studying transportation networks have been preoccupied with developing models that reproduce this equilibrium, as a function of network characteristics and user demand levels. Typically, transport equilibrium models consider vehicles to be the fundamental units seeking an equilibrium, or, in the case of public transport, the individual traveler. In both of these cases, since the number of users is generally very large, the Wardrop concept, that treats individual user contributions to the costs as infinitesimal, is preferred to the (in this respect, more general) Nash paradigm.

In the context of telecommunication networks, the Wardrop equilibrium is used most often to model the situation in which the routed entities are packets, and routing decisions are taken at the nodes of the networks (rather than by the users) so as to minimize the (per-packet) delay. In many actual networks, the routers at the nodes seek to minimize the per-packet delay in terms of the number of "hops," or nodes, to the destination. There are, however, situations in which it is more advantageous to work with actual delays as cost metrics, rather than the number of hops (see [20] Gupta and Kumar, [36]), and it is in these cases that the Wardrop equilibrium has been used to describe the resulting flow patterns. This is the case, for example, in ad-hoc networks in which both users as well as base stations are mobile, or where there are no base stations so that users are responsible to relay messages of other users. For these type of networks a Wardrop type equilibrium has been advocated in [20].

Wardrop equilibria have also been used in telecommunication networks to model a large number of users that can determine individually their route and in which the routed object is a whole session, see Korilis and Orda [55] (whose model includes in addition some side constraints on the quality of service).

A third context in which Wardrop equilibrium has been used outside of transportation is in distributed computer networks, in which the routed objects are jobs. An individual job can be processed in any of several interconnected nodes (computers) and the routing decision is taken so as to minimize its expected delay in the system (composed of both communication as well as processing delay). Much material on that application can be found in [49].

The definition of the steady state equilibrium of a traffic network was put forth by J.G. Wardrop in his 1952 treatise [100] which provided two different definitions of traffic assignment concepts. The first is commonly referred to as the Wardrop, or traffic equilibrium, principle and can be understood as a variant of Nash equilibrium for networks having a contimuum of players, where a single player is negligible. It states that "*The journey time on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route*"[1]..

The Nash equilibrium with finitely many decision makers is also relevant in the above road traffic framework. A decision maker could be a driver but could also correspond to a group of drivers for whom the trajectory is determined by some common decision omaker. When the decision makers in a game are discrete and finite in number, a Nash equilibrium can be achieved without the costs of all used routes being equal, contrary to Wardrop's equilibrium principle. In some cases, Wardrop's principle represents a limiting case of the Nash equilibrium principle, as the number of players becomes very large [42].

Wardrop principle, stated above, can be expressed mathematically to state that the flow on every route r serving a commodity, or origin-destination (OD) pair, w, is either zero, or its cost is equal to the minimum cost on that OD pair. Along with the fact that the cost on any route serving an OD pair is at least as high as the minimum cost on that OD pair, and the satisfaction of demand for each OD pair, we obtain the following system:

$$h_{wr}(c_{wr} - \lambda_w) = 0, r \in R_w, w \in W, \tag{5.8}$$

$$c_{wr} - \lambda_w \geq 0, r \in R_w, w \in W, \tag{5.9}$$

$$\sum_{r \in R_w} h_{wr} = d_w, w \in W \tag{5.10}$$

Here c_{wr} is the total cost over the path $r \in R_w$.

In the link cost framework, we get instead:

$$t_{uv}(x_{uv}) \geq \lambda_u^i - \lambda_v^i, \tag{5.11}$$

with equality if $x_{uv} > 0$.

5.3 Wardrop equilibrium and Potential games

Adding non-negativity restrictions $h_{wr} \geq 0$ and $\lambda_w \geq 0$, the resulting system of equalities and inequalities can be seen as the Karush-Kuhn-Tucker (KKT) optimality conditions of the following optimization problem, known as the Beckmann transformation [19]).

$$\min f(x) = \sum_{l \in A} \int_0^{x_l} t_l(x_l) dx = \sum_{l \in A} \int_0^{\sum_{i \in N} x_{il}} t_l(x) dx$$

subject to (5.1 - 5.3).

The fact that Wardrop equilibrium can be obtained using an equivalent optimization problem with a single player having some cost $f(x)$ is a feature common to a whole class of games known as potential games. This class of games was formally introduced by Monderer and Shapley [67] for the case of finitely many players. The original definition of a potential for a game is as follows. Introduce the following N-player game $G = (N; (S^i)_{i \in N}; (\eta^i)_{i \in N})$ where S^i is the action set of player i, $S = \times_{i \in N} S^i$ and $\eta^i(s)$ is the payoff for player i when the multistrategy $s \in S$ is used. For $s \in S$, let $(s|t^i)$ denote the multistrategy in which player i uses t^i instead of s^i and other players $j \neq i$ use s^j. A potential for the game is defined in [67] as a real valued function P on S s.t. for each i, every $s \in S$ and every $t^i \in S^i$, $P(s|t^i) - P(s) = \eta^i(s|t^i) - \eta^i(s)$. Existence and uniqueness of equilibria of potential games in that setting has been established in [67] and Neyman [71][2].

An adaptation of this definition is needed for population games, see [76, Chap. 3] and [84], in which there are N classes of populations of "infinitesimal" players, where the "mass" of players of type i is given by some constants d_i. Let $\alpha(j, t)$ be the fraction of members of population type j that use action $t \in S^i$. A multistrategy is the collection $\alpha = (\alpha(j, t))$. We assume that the payoff

[1] Wardrop also considers an alternative social optimal framework in which, according to Wardrop, "*The average journey time is a minimum*

[2] Uniqueness is in fact established among the class of correlated equilibria, of which Nash equilibria is a subset.

η^i for a player of class i is a function of his own action as well as of the multistrategy α. Let S^i be the set of actions available to a player of population i, $i = 1, ..., N$. We say that α^* is an equilibrium if for any i, any $s \in S^i$ and any $t \in S^i$ such that $\alpha^*(i,t) > 0$, $\eta^i(t;\alpha^*) \geq \eta^i(s;\alpha^*)$. Equivalently, letting $f = -\eta$, we say that flow α^* is in equilibrium if the following variational inequality problem (VIP) holds for all $\alpha \in S$: $f(\alpha^*)(\alpha - \alpha^*) \geq 0$.

We then define P to be a potential for the population game if for each s, the vector of payoffs $\eta(s)$ is the gradient of $P(s)$. Under mild conditions on the payoff functions and strategy sets, one can thus establish the existence and uniqueness of equilibria in potential population games, see [76, Chap 3].

5.4 A potential for the Nash Cournot game

Define $\lambda_u = \sum_i \lambda_u^i$. where λ_u^i are defined in (5.6). Taking the sum of (5.6) Taking the sum of over all players we get the following necessary conditions for x to be an equilibrium. For each link (u, v):

$$It_{uv}(x_{uv}) + x_{uv}\frac{\partial t_{uv}(x_{uv})}{\partial x_{uv}} \geq \lambda_u - \lambda_v, \tag{5.12}$$

with equality if $x_{uv}^i > 0$ for all i, where I is the total number of players.

Assume that all players have the same source and destination, and let d be the sum of commodity demands. Then (5.17) are the KKT conditions for optimality of the vector $\{x_l\}$ with nonnegative components satisfying the conservation of flow constraints in the routing problem (single commodity) where the cost to be minimized is given by

$$\sum_{l \in A} x_l t_l(x_l) + (I-1)\int_0^{x_l} t_l(y)dy, \tag{5.13}$$

and where the total demand to be shipped from the common source to the common destination is d; in particular, (5.17) holds with equality if $x_{uv} > 0$. Assume further that t_l are strictly convex, or more generally that expression (5.18) is strictly convex. Then this problem has a unique solution in total link flows, which we denote (x_l^*).

Now, let $\{x_l^i\}$ be a Nash equilibrium for the original problem having costs (5.5) with the property:

$$\text{A1. Whenever } x_{il} > 0 \text{ for some } i \text{ and } l \text{ then } x_{jl} > 0 \text{ for all players } j. \tag{5.14}$$

A1 describes a property of the equilibrium: if (at equilibrium) one player sends positive flow through a link, then so do all other players. Under assumption A1, it follows that for all l, $\sum_i x_{il} = x_l^*$. Note however, that this is not true in general if A1 does not hold, since x_l^* need not be expressible as the sum over I of some nonnegative x_{il} that satisfy (5.4).

Remark 2. *The above is an alternative proof to the one in Orda et al [75] of the uniqueness of the total link flows at all Nash equilibria satisfying A1.*

Taking the limit in (5.18) as the number of players $I \to \infty$, the second term in (5.18) dominates, and by continuity of the functions and compactness of the feasible set, we observe that both the objective function and the solution approach that of the Wardrop equilibrium.

5.5 Uniqueness of Nash Cournot equilibrium

Lemma 1. *Assume that there are two players, say i and jJ, in a routing game which have the same common input (source) s, output (destination) o, and the same cost functions which are of type* **A**.
Consider an equilibrium flow $\mathbf{x}$ *and let $\{\lambda_u^i\}$ be associated Lagrange multipliers. Then*
(i) Assume that there is a link l at which $x_l^i > x_l^j$. Then $\lambda_s^i > \lambda_o^j$
(ii) Assume that $d_i \geq d_j$. at equilibrium, for every link l, $x_l^i \geq x_l^j$.

Proof. (i) Due to flow conservation for each player, there is some path from the source to the destination that includes the link l such that the relation $x^i_\ell > x^j_\ell$ for all ℓ in the path. Then holds for all links on the path. For every link $l = (uv)$ on this path we have

$$\lambda^i_u - \lambda^i_v = K^i_{uv}(x^j_{uv}, x_{uv}) = K^j_{uv}(x^j_{uv}, x_{uv}) > K^j_{uv}(x^j_{uv}, x_{uv}) \geq \lambda^j_u - \lambda^j_v$$

By summing this inequality over all links along the path we obtain the statement.
(ii) Assume that the statement does not hold. Then there is a some link for which $x^j_l > x^i_l$. Hence by the first part,

$$\lambda^j_s - \lambda^j_s < \lambda^j_o - \lambda^j_o \tag{5.15}$$

Since the $d_j < d_i$ there is another link over which the flow of player i is larger than that of j. so we get the opposit inequality in (**??**). The statement is thus established by contradiction.

The Lemma implies

Corollary 1. *Consider a symmetric game: all players have the same source, destination, demand. and cost functions assumed to be of type B. Then*
(i) there is a unique equilibrium,
(ii) the game has a potenetial given in (5.18).

5.6 Convergence to Wardrop equilibrium

This section is based on a joint work [14].

Assume that every player i in the original game is replaced by a set of m identical players, denoted by (i,k) where $i = 1, ..., m$, with the total demand of the new set beinig equal to the one of the original game. The demand of any player in the group that replaced player i is thus given by $d^{(i,k)}[m] = d^i/m$. We shall denote the flows for a given m by and $x^{(i,k)}_l[m]$.

Define $x^i_{uv}[m] = \sum_{k=1}^m x^{(i,k)}_{uv}[m]$ and $x_{uv}[m] = \sum_i x^i_{uv}[m]$.

We rewrite (5.6) fot the new game: There exist Lagrange multipliers $\lambda^{(i,k)}_u[m]$ for all nodes u and all players, (i,k), such that for each pair of nodes u, v connected by a directed link (u,v),

$$t_{uv}(x_{uv}[m]) + x^{(i,k)}_{uv}[m] \frac{\partial t_{uv}(x_{uv}[m])}{\partial x_{uv}} \geq \lambda^{(i,k)}_u[m] - \lambda^{(i,k)}_v[m], \tag{5.16}$$

with equality if $x^{(i,k)}_{uv}[m] > 0$.

Define $\lambda^i_u = m^{-1} \sum_{k=1}^m \lambda^{i,k}_u[m]$.

Taking the sum over all k subplayers in (5.16) and dividing by m, we get the following necessary conditions for x to be an equilibrium for each link (u,v):

$$t_{uv}(x_{uv}[m]) + \frac{1}{m} x^i_{uv} \frac{\partial t_{uv}(x_{uv}[m])}{\partial x_{uv}[m]} \geq \lambda^i_u[m] - \lambda^i_v[m], \tag{5.17}$$

with equality if $x^i_{uv}[m] > 0$.

It follows from the previous section that any equilibrium is symmetric: for each i, all subplayers (i,k) have the same equilibrium flows. (This is needed because this allows us to conclude that if $x^i_{uv}[m] > 0$ then also $x^{(i,k)}_{uv}[m] > 0$ for **all** $k = 1, ..., m$. And then (5.17) indeed holds with equality. The existence of symmetric solutions follows as in [75].)

Then (5.17) are the KKT conditions for the best response for player i of the vector $\{x^i_l\}$ in the original game, with nonnegative components satisfying the conservation of flow constraints in the routing problem where the cost to be minimized by player i is given by

$$\sum_{l \in \mathcal{L}} \left(\frac{1}{m} x^i_l t_l(x_l) + \frac{m-1}{m} \int_0^{x_l} t_l(y) dy \right) \tag{5.18}$$

We note that this converges to the potential of the Wardrop equilibrium, uniformly in $\mathbf{x}$. We can then conclude from [12] that the equilibrium converges to the Wardrop one (convergence of equilibrium is defined as in [12]).

5.7 Braess paradox

5.7.1 Background

The service providers or the network administrator may often be faced with decisions related to upgrading of the network. For example, where should one add capacity? Where should one add new links?

A frequently-used heuristic approach for upgrading a network is through *bottleneck analysis*, where a system bottleneck is defined as "a resource or service facility whose capacity seriously limits the performance of the entire system" (see p. 13 of [?]). Bottleneck analysis consists of adding capacity to identified bottlenecks until they cease to be bottlenecks. In a non-cooperative framework, however, this heuristic approach may have devastating effects; adding capacity to a link (and in particular, to a bottleneck link) may cause delays of all users to increase; in an economic context in which users pay the service provider, this may further cause a decrease in the revenues of the provider. This problem was identified by Braess [?] in the transportation context, and has become known as the *Braess paradox*. See also [?], [?]. The Braess paradox has been studied as well in the context of queuing networks [18], [?], [?], [?], [?].

In the latter references both queuing delay as well as rejection probabilities were considered as performance measures. The impact of the Braess paradox on the bottleneck link in a queuing context as well as the paradoxical impact on the service provider have been studied in [?]. In all the above references, the paradoxical behavior occurs in models in which the number of users is infinitely large and the equilibrium concept is that of Wardrop equilibrium, see [?].

It has been shown, however, in [?], [54], that the problem may occur also in models involving a finite number of players (e.g. service providers) for which the Nash framework is used. The Braess paradox has further been identified and studied in the context of distributed computing [?], [?], [?] where arrivals of jobs may be routed and performed on different processors. Interestingly, in those applications, the paradox often does not occur in the context of Wardrop equilibria; see [?].

In [83] (see also [56]), it was shown that the decrease in performance due to the Braess paradox can be arbitrarily larger than the best possible network performance, but the authors showed also that the performance decrease is no more than that which occurs if twice as much traffic is routed. The result was extended and elaborated upon in more recent papers by the same authors. In [?], a comment on the results of [83] was made in which it is shown that if TCP or other congestion control is used, rather than agents choosing their own transmission rates, then the Braess phenomenon is reduced considerably. Indeed, this conclusion can be reached intuitively by considering (as is well known in the study of transportation equilibria) that the system optimal equilibrium model (in which the sum of all delays are minimized) does not exhibit the Braess paradox; congestion control serves to force transmission rates to such a system optimal operating point.

An updated list of references on the Braess paradox is kept in Braess' home page at
http://homepage.ruhr-uni-bochum.de/Dietrich.Braess/#paradox

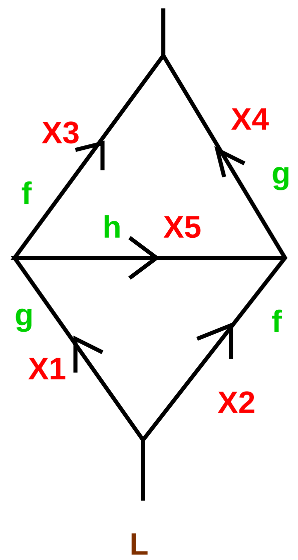
5.7.2 Architecting equilibria and network upgrade

The Braess paradox illustrates that the network designer, the service provider, or, more generally, whoever is responsible for setting the network topology and link capacities, should take into consideration the reaction of (non-cooperative) users to her or his decisions. Some guidelines for upgrading networks in light of this have been proposed in [?], [?], [?], [?], [54], so as to avoid the Braess paradox, or so as to obtain a better performance. Another approach to dealing with the Braess paradox is to answer the question of which link in a network should be upgraded; see, for example, [?] who computes the gradient of the performance with respect to link capacities.

A more ambitious aim is to drive the equilibrium to a socially optimal solution. In [?] this is carried out under the assumption that a central manager of the network has some small amount of his or her own flow to be shipped in the network. It is then shown that the manager's routing decision concerning his own flow can be taken in a way so that the equilibrium corresponding to the remaining flows attain a socially optimal solution.

5.7.3 Quick computation of the equilibria in Braess paradox

Consider the network in Figure 5.7.3. We assume a total demand of 6 units.



In the absence of link 5, the equilibrium is obviously to send half of the traffic on each one of the two paths 1-3 and 2-4.

Next we consider the network with the link 5.

Assume: route 1-3, as well as 2-4 are used.

$$g(x_1) + f(x_3) = f(x_2) + g(x_4). \tag{5.19}$$

We now express x_5:

$$x_1 - x_3 = -x_2 + x_4.$$

If f is linear then this implies

$$f(x_1) - f(x_3) = -f(x_2) + f(x_4).$$

Summing with (5.19), we get

$$f(x_1) + g(x_1) = f(x_4) + g(x_4).$$

If $f + g$ is strictly increasing then $x_1 = x_4$. Hence also $x_2 = x_3$.

Now, $x_2 = L - x_1$. Hence

$$x_5 = x_1 - x_3 = x_1 - x_2 = 2x_1 - L.$$

If route 1-5-4 is also used then

$$g(x_1) + h(x_5) = f(x_2).$$

We conclude that

$$g(x_1) + h(2x_1 - L) = f(L - x_1).$$

This gives x_1.

Choose $L = 6$, $f(x) = 50 + x$, $g(x) = 10x$, $h(x) = \infty$.

Then $x_1 = x_2 = 3$,

$$D_{13} = D_{24} = 83.$$

Take $h = 10 + x$.
With $x_1 = x_3 = 3$, $D_{154} = 70 < D_{13}$. Not Wardrop Equilibrium!
Suppose 1 unit moves from 2-4 to 1-5-4.
Then

$$D_{1-3} = 40 + 53 = 93, \quad D_{2-4} = 52 + 30 = 82,$$

$$D_{1-5-4} = 40 + 11 + 40 = 91.$$

Suppose 1 unit moves from 1-3 to 1-5-4. We have $x_1 = 4, x_2 = 2, x_5 = 2$.
Then

$$D_{1-3} = 40 + 52 = 92, \quad D_{2-4} = 52 + 40 = 92,$$

$$D_{1-5-4} = 40 + 12 + 40 = 92.$$

This satisfies Wardrop conditions!
We now check the equation:

$$g(x_1) + h(2x_1 - L) = f(L - x_1).$$

We get:

$$10x_1 + (10 + 2x_1 - 6) = (50 + 6 - x_1)$$

Hence $x_1 = 4$, $x_2 = L - x_1 = 2$, $x_5 = x_1 - x_2 = 2$.

5.8 Exercise

1. Extend the result in [75] on symmetric equilibria to allow symmetry under other operators [exact formulation of the problem will be given later].

Chapter 6

Routing Games: General models

6.1 General models and variational inequalities

The basic equilibrium model imposes a number of simplifications on the model of the traffic flow phenomenon, and in particular, on the travel time, or impedance, functions.

Most notably, for the potential function to exist, the travel time function, t_l, defined for each link of the network, $l \in A$, must be integrable. The most common way for this to occur is that the travel time on a link l depends only upon the flow present on the link l, that is, $t_l(x) = t_l(x_l)$. This simplification, in the traffic context, means that interactions between different traffic streams at junctions cannot be modeled within this paradigm (even if we use a virtual link to model the node), since then, the travel time on a link l that reaches the junction is a function of flows on some or all links meeting link l at the junction, that is, $t_l(x) = t_l(x_1, \ldots, x_l, \ldots, x_m)$ (and is not just a function of the sum of flows). The simplification also imposes that only a single class of users is modeled, since multiple classes of users would interact on each link, resulting once again in multivariate link travel time functions. In short, when the identities (in terms of multiple user classes, or the multiple links they use) is needed in the cost function of a single link, then the single-class model is no longer applicable.

When the link travel time functions are multivariate, it is usually the case that no potential that can be obtained by integrating the travel time functions as in the basic model. Examples of multivariate link cost functions can be found in the literature on modeling signalizes junctions on a road network Heydecker (1983) [?]. Other examples can be found in the modeling of multimodal networks, such as networks on which buses and cars share the road space or trucks and light vehicles, since each traffic class effects the traffic differently, and each class has its own travel time function, depending on all classes present on the link. Some characteristics of this type of multivariate cost functions can be found in Toint and Wynter (1995) [?].

Although the potential function approach cannot be used to describe the multivariate equilibrium, the Wardrop equilibrium conditions are valid regardless of whether the cost functions are univariate or multivariate, and they can be expressed for both types of cost functions in a compact variational framework.

In telecommunication network planning, link impedance functions can often be quite complex, due to the underlying probabilistic phenomena as well as the interacting cost components of delay, packet loss, jitter, etc... A typical form of the commodity-link cost functions (see, for example, [75]) is

$$t_{il}(x_l^i, x_l) = x_l^i t_l(x_l) = \frac{x_l^i}{C_l - x_l}, \tag{6.1}$$

where, as before, $x_l = \sum_{i=1..I} x_l^i$ is the flow of all classes i on link l. The constant C_l is the capacity of link l. The user classes in this case correspond to commodities, or origin-destination demands. For more justification on this type of delay models, see Baskett et al. (1975), [?] Kameda and Zhang [50]. In some simple settings, such as a network of parallel arcs, when the sum of the demands of all classes is less than link capacity, or on some one-commodity networks, Orda et al. [75] show uniqueness of the Nash equilibrium. Also under "diagonal strict convexity", the authors show uniqueness of the Nash equilibrium, yet this condition only holds under quite restrictive conditions on the cost functions or on the topology (Altman et al., 2002 [?]). In [75] and Altman and Kameda (2001), uniqueness of the link class flows is shown for costs of the form (6.1) for general networks under assumption A1 (see eq. (5.14).

We can analyze the *Wardrop* equilibrium for this system easily by expressing it as the solution $\{x_i^*\}$ of the following variational inequality:

$$t_i(x^*)^T(x_i^* - x_i) \leq 0, \tag{6.2}$$

for all feasible class-flow vectors, x_i, where, as above, the vector $x = \sum_{i=1..I} x_i$ and $t_i = t$ for all classes $i = 1..I$. Since the Jacobian of the mapping t is clearly singular ($\partial t_i(x)/\partial x_j = \partial t_j(x)/\partial x_i$ for all i, j), we recover the nonuniqueness of the equilibrium (in the variable x_i) that was observed by [75] to occur on general networks.

We can also contrast the Wardrop and Nash equilibria through this example. The Nash equilibrium x_i^* satisfies

$$\sum_{l \in R_i} x_{il}^* t(x_{il}^*, x_{\neq il}^*) \leq \sum_{l \in R_i} x_{il} t(x_{il}, x_{\neq il}^*)$$

for each user class $i = 1..I$, where the index $\neq i$ includes all classes not equal to i. Rewriting, we obtain that $\sum_{l \in R_i} x_{il}^* t(x_{il}^*, x_{\neq il}^*) - \sum_{l \in R_i} x_{il} t(x_{il}, x_{\neq il}^*) \leq 0$ for each $i \in I$ and therefore when

$$t(x_{il}^*, x_{\neq il}^*) - t(x_{il}, x_{\neq il}^*) = 0, \tag{6.3}$$

the above reduces to $\sum_{l \in R_i} (x_{il}^* - x_{il}) t(x_{il}^*, x_{\neq il}^*) \leq 0$, for each $i \in I$, which is equivalent to the (Wardrop) VIP with cost operator t and classes given by users $i \in I$. Indeed, (6.3) occurs precisely when the influence of an additional user on the cost, t, is 0.

Another example of general costs in telecommunications can be found in Altman, El-Azouzi, and Abramov [?] where the network model includes dropping of calls if capacity is exceeded. The cost criterion for each user class i is the probability that his message is rejected along its path, given by

$$B_i(x) = 1 - \frac{\sum_{\mathbf{m} \in S_i} \prod_{j=1}^{I} (x_j^{m_j}/m_j!)}{\sum_{\mathbf{m} \in S} \prod_{j=1}^{I} (x_j^{m_j}/m_j!)}.$$

S is the set of feasible "states"; S_i is the set of states for which another call of user i can still be accepted (without violating capacity constraints), $\mathbf{m}$ is the system state whose jth component m_j is the number of class j calls in the system. Hence the number of terms in the cost function depends upon the configuration and capacity constraints of the network. (The larger the number of feasible paths for a user class, the more terms present.) When each class, which can be represented by an origin-destination pair, seeks a Wardrop equilibrium, the corresponding variational inequality is to find x_i^* such that $B_i(x^*)^T(x_i^* - x_i) \leq 0$, for all x_i, for all classes i. Even in the simplest topology of parallel links, it has been shown in Altman, El-Azouzi, and Abramov (2002) [?], that there may exist several equilibria with different total link flows.

Depending on the form of monotonicity satisfied by the cost operator in the variational inequality, one can choose convergent algorithms for its solution, as well as determine whether or not the solution will be unique. Weaker forms of monotonicity for which the mathematical properties and a number of convergent algorithms are known include pseudo-monotonicity and strong nested monotonicity. (See Marcotte and Wynter, 2001 [?], Cohen and Chaplais [25], for the latter.)

6.2 Additive versus non-additive models

The most widely studied performance measure investigated to date in transportation, computer and telecommunication networks has been the expected delay. In transportation networks, this cost metric leads naturally to models in which the route costs are additive functions of link costs along the route, that is $c_r(x) = \sum_{l \in R} t_l(x_l)$.

In telecommunication networks, exogenous arrivals of jobs or of packets are modeled as Poisson processes. Delays at links are modeled by infinite buffer queues with i.i.d. service times, independent of the interarrival times. In the particular case in which the service time at a queue is exponentially distributed, then whenever the input process has a Poisson distribution so will the output stream. This makes the modeling of service times through exponential distributions quite appealing, and makes the cost along a path of tandem queues additive. However, the expected delay turns out to be additive over constituent links in a general topology under a much more general setting, known as BCMP networks (named after its authors Baskett et al., 1975). [?]

Indeed, as long as the exogenous arrivals have Poisson distributions and under fairly general assumptions on the service order and service distribution, the expected delay over each link is given by the expected service time divided by (1-ρ), where ρ is the product of total average traffic flow at the queue and the expected service time. This general framework also allows for the modeling of multiclass systems, i.e. where different traffic classes require different expected service times at a queue, see Kameda and Zhang [50].

Other more general types of separable additive cost functions have been used in telecommunication networks which can represent physical link costs due to congestion pricing, see e.g. [75].

However, in both transportation and telecommunication networks, equilibrium models in which path costs are not the sum of link costs do arise.

In the transport sector, when environmental concerns are taken into account, such as the pollution associated with trips, non-additive terms arise. Path costs due to tolls or public transport costs are generally non-additive as well, since they are calculated over entire paths, and cannot be decomposed into a sum of the costs on component links.

In Gabriel and Bernstein (1997) [?] and Bernstein and L. Wynter (2000), [?] the authors discussed properties of a bicriteria equilibrium problem in which both time delay and prices are modeled on the links. Non-additivity arises from the nonlinear valuation of the tradeoff between time and money, known as a nonlinear *value of time*. For example, users are willing to pay more (or less) per minute for longer trips than for shorter trips. That is, a route cost function may be expressed as $c_r(x,p) = V\left(\sum_{l \in R_i} t_l(x_l)\right) + \sum_{l \in R_i} p_l$, where $V : \Re^m_+ \mapsto \Re_+$ is the nonlinear value of time function.

In telecommunication networks, many important performance measures are neither additive nor separable. The first example is that of loss probabilities when the network contains finite buffer queues. We note that in this case there is no flow conservation at the nodes[1] This model has been dealt with by light traffic approximations which are additive and separable, see [28].

Another performance measure (already mentioned in the previous subsection) that has been studied in the context of network equilibrium is that of rejection probabilities. The network consists of resources at each link, and requests for connections between a source and a destination. The resources are limited, and a connection can only be established if there are sufficiently many resources along each link of a route between the source and destination. For the case where connections arrive according to a Poisson process and where calls last for an exponentially distributed duration (these assumptions model telephone networks well), simple expressions for rejection probabilities (i.e. the probability that an arriving call will find the line busy) are available. These expressions are neither separable nor additive. Such networks have been studied in Altman, El-Azouzi, and Abramov (2002) [?] and in [18].

When route costs are no longer the sum of constituent link costs, algorithms for solving the network equilibrium problems must be modified; indeed, underlying most algorithms for the network equilibrium problem is a shortest path search, and standard searches all suppose additive path costs. One algorithm for the non-additive route cost model can be found in [33] while another is provided in the article by M. Patriksson [?]

[1] Note that flow conservation fails even in the case of infinite queues when one considers multicast applications in which packets are duplicated at some nodes, see [22].

6.3 Routing in Loss networks

6.4 Cooperation and Altrouism in Routing games

Chapter 7

Stochastic Routing games:

7.1 The gas station game

The example we present here illustrates the dynamic routing choices between two paths. When a routing decision is made, the decision maker knows the congestion state of only one of the routes; the congestion state in the second route is unknown to the decision maker. The problem originates from the context of two gas stations on a highway [39]. A driver arriving at the first station sees the amount of other cars already queued there and has to decide whether to join that queue, or to proceed to the next gas station. The state of the next gas station (i.e. the number of cars there) is not available when making the decision. The situation is illustrated in Fig. 7.1. The exact mathematical solution of the model was obtained in [11] and we describe it below.

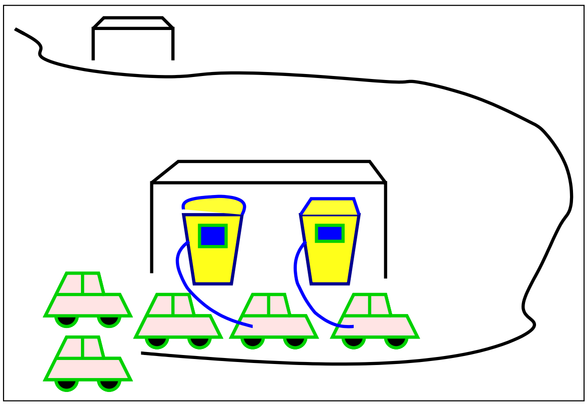


Figure 7.1: The gas station problem

This problem has natural applications in telecommunication networks: when making routing decisions for packets in a network, the state in a down stream node may become available after a considerable delay, which makes that information irrelevant when taking the routing decisions.

Although the precise congestion state of the second route is unknown, its probability distribution, which depends on the routing policy, can be computed by the router.

We assume that the times that corresponds to the arrivals instants of individuals is a Poisson process with rate λ. Each arrival is a player, so there is a countable number of players, each of whom takes one routing decision, of whether to join the first or the second service station. A player eventually leaves the system and does not affect it anymore, once it receives service.

To obtain an equilibrium, we need to compute the joint distribution of the congestion state in both routes as a function of the routing policy.

We restrict to random threshold policies (n, r):

- if the the number of packets in the first path is less than or equal to $n-1$ at the instance of an arrival, the arriving packet is sent to path 1.
- If the number is n then it is routed to path 1 with probability r.
- If the number of packets is greater than n then it is routed to path 2.

The delay in each path is modeled by a state dependent queue:

- Service time at queue i is exponentially distributed with parameter μ_i
- Global inter-arrival times are exponential i.i.d. with parameter λ.

When all arrivals use policy (n, r), the steady state distribution is obtained by solving the steady state probabilities of the continuous time Markov chain [11].

If an arrival finds i customers at queue 1, it computes

$$E_i[X_2] = E[X_2|X_1 = i]$$

and takes a routing decision according to whether

$$T^{n,r}(i,1) := \frac{i+1}{\mu_1} \leq_? \frac{E_i[X_2]+1}{\mu_2} =: T^{n,r}(i,2).$$

To compute it, the arrival should know the policy (n, r) used by all previous arrivals.

If the decisions of the arrival as a function of i coincide with (n, r) then (n, r) is a Nash equilibrium.

The optimal response against $[g] = (n, r)$ is monotone decreasing in g. This is the **Avoid The Crowd** behavior.

Computing the conditional distributions, one can show [11] that there are parameters $(\mu_1, \mu_2, \lambda, n, r)$ for which the optimal response to (n, r) is indeed a threshold policy.

Denote

$$\rho := \frac{\lambda}{\mu_1}, \qquad s := \frac{\mu_2}{\mu_1}$$

There are other parameters for which the optimal response to (n, r) is a two-threshold policy characterized by $t^-(n, \rho, s)$ and $t^+(n, \rho, s)$ as follows.

It is optimal to route a packet to queue 2 **if** $t^-(n, \rho, s) \leq X_1 \leq t^+(n, \rho, s)$ **and to queue** 1 **otherwise.**

At the boundaries t^- and t^+ routing to queue 1 or randomizing is also optimal if $T^{n,r}(i,1) = T^{n,r}(i,2)$. For parameters in which the best response does not have a single threshold, we cannot conclude anymore what is the structure of a Nash equilibrium.

Example [11]. Consider $n = 3,\ r = 1,\ \rho = \lambda/\mu_1 = 1$ and $s = \mu_2/\mu_1 = 0.56$. We plot in Fig. 7.2 $T^{n,r}(i,1)$ and $T^{n,r}(i,2)$ for $i = 0, 1, \ldots, 4$.

Conclusions: As opposed to the example of the choice of players between a PC and a MF in Sec. **??**, we saw in this section an example where for some parameters there may be no threshold type (n, r) equilibria. The form of the Nash equilibrium in these cases remains an open problem. Moreover, even the question of existence of a Nash equilibrium is then an open question.

Yet, in the case of equal service rates in both stations, a threshold equilibrium does exist, and it turns out to have the same type of behavior as in the PC-MF game, i.e. it is unique, and the best response has a tendency of "Avoiding the Crowd" [39].

By actually analyzing (numerically) the equilibrium for equal service rates, it was noted in [39] that when players use the equilibrium strategy, then the revenue of the first station is higher than the second one. Thus the additional information that the users have on the state of the first station produces an extra profit to that station. An interesting open problem is whether the second station can increase its profits by using a different pricing than the first station, so that users will have an extra incentive to go to that station. Determining an optimal pricing is also an interesting problem.

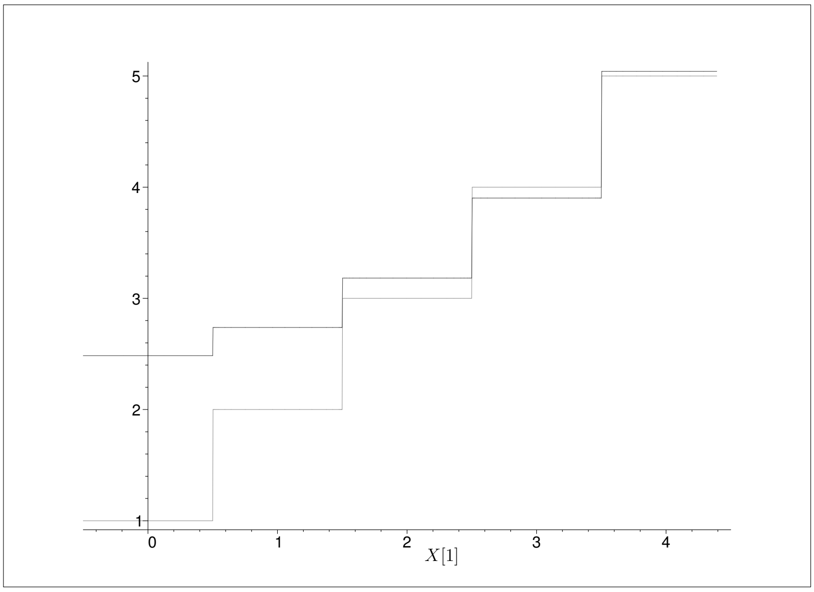


Figure 7.2: $T^{n,r}(i,1)$ and $T^{n,r}(i,2)$

7.2 Queues with priority

We present a second queueing problem modeled as a stochastic game with infinitely many individual players due to [3, 40]. We assume again that players arrive at the system according to a Poisson process with intensity λ, and have to take a decision of whether to join a low priority (second class) or a high priority (first class) queue, as illustrated in Fig. 7.2. There is a single server that serves both queues but gives strict priority to first class customers. Thus a customer in the second class queue gets served only when the first class queue is empty. We assume exponentially distributed service time with parameter μ and define $\rho := \lambda/\mu$.

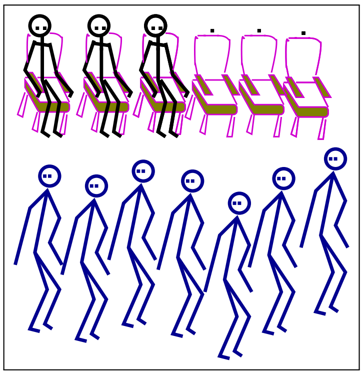


Figure 7.3: Choice between first and second class priorities

The game model is then as follows:

The actions Upon arrival, a customer (player) observes the two queues and may purchase the high priority for a payment of an amount θ, or join the low priority queue.

The state: The state is a pair of integers (i,j) corresponding to the number of customers in each queue; i is the number of high priority customers and j, the number of low priority ones.

The analysis of this problem can be considerably simplified by using the following **monotonicity property**, identified in [3]: If for some strategy adopted by everybody, it is optimal for an individual to purchase priority at (i, j), then he must purchase priority at (r, j) for $r > i$.

This implies that the problem has an effective **lower dimensional state space:** It follows that starting at $(0, 0)$ and playing optimally, there is some n such that the only reachable states are

$$(0, j),\ j \leq n, \quad \text{and } (i, n),\ i \geq 1.$$

Indeed, due to monotonicity, if at some state $(0, m)$ it is optimal not to purchase priority, it is also optimal at states $(0, i)$, for $i \leq m$. Let $n - 1$ be the largest such state. Then starting from $(0, 0)$ we go through states $(0, i)$, $i < n$, until $(0, n - 1)$ is reached. At $(0, n)$ it is optimal to purchase priority. We then move to state $(1, n)$.

The low priority queue does not decrease as long as there are high-priority customers. Due to monotonicity, it also does not increase as long as there are high-priority customers since at (i, n), $i \geq 1$ arrivals purchase priority! Therefore we remain at (i, n), as long as $i \geq 1$.

The Equilibrium. Suppose that the customers in the population, except for a given individual, adopt a common threshold policy $[g]$. Then the optimal threshold for the individual is non-decreasing in g.

This property is called **"Follow The Crowd" Behavior**

This property clearly implies *Existence* of an equilibrium, that can be obtained by a monotone best response argument.

However, it turns out that there is no uniqueness of the equilibrium! Indeed, Hassin and Haviv have shown in [40] that there may be up to

$$\left\lfloor \frac{1}{1-\rho} \right\rfloor$$

pure threshold Nash equilibria, as well as other mixed equilibria! They further present numerical examples of multiple equilibria.

We conclude that in this problem we have definitely a different behavior of the equilibria than in the previous stochastic games in which we had threshold equilibria (the PC-MF game and the gas station game).

Chapter 8

Coalition games and Multicast in cellular networks

8.1 Introduction

This Chapter is based on my joint work with Chandramani Singh [87].

8.1.1 The Multicast Problem

Consider a network with one source or one base station (BS). N users participate in a multicast session in which each one receives the same content from the source. Users are non-homogeneous: Transmission of the content is associated with some costly resource (power, bandwidth etc). There is some amount of resource that a user requests associated with the quality of the content it wishes to receive as well as to the network conditions of that user. It is assumed that if a given amount of resource is used for transmission, then all users with a request not exceeding this amount are satisfied. In order to satisfy all users, the source needs to allocate the amount that corresponds to the largest request. Some examples are:

- Power control: consider a cell with N mobiles. A mobile uses a service that needs some given power level at the reception. As the locations of mobiles differ, the channel gain differs from one mobile to the other. Thus the BS has to transmit at a power level that ensures that the mobile with the worst channel conditions receives the signal with the requested power. For example, if the channel gain between the base station located at X_{bs} and mobile i, located at X_i, is determined by the distance and the path loss constant α, i.e. $p_r = Pd(X_{bs}, X_i)^{-\alpha}$ then the BS has to transmit at a power of $P = \theta \max_i d(X_{bs}, X_i)^{\alpha}$ in order to ensure that all mobiles receive at a power of at least θ.

- Power control with quality of service constraints: We consider again a BS and N mobiles. The bit error rate (BER) at the reception is known to be a function of the signal to noise ratio and of the modulation type of the signal. Each user may wish to receive at a different quality in terms of BER. Therefore the power needed to satisfy all users is not necessarily the one corresponding to the mobile who is the furthest away.

- Hierarchical multicast. Assume that a hierarchical coding is used. The content is coded and transmitted over several carriers. A user can decide what level of details he wishes to have; this translates to the decision of how many carriers the user wishes to receive. The more carriers one receives, the better is the quality of the received signal after decoding. If mobile i subscribes to j_i carriers, then in order to satisfy all users, the source has to use $\max_{i=1}^{N} j_i$ carriers.

8.1.2 The structure of the chapter

The first question we address in this chapter is how to split the cost for establishing the multicast session among the users. In the situations described above it seems clear that the global benefit increases in the size of the multicast group. Nonetheless, depending on the splitting rule, users may join the multicast session or may prefer not to join it. Indeed, it is assumed that a user or a set of users that are not satisfied with the way the cost is split, can form an alternative multicast group that would perhaps require together less resources and may thus be more beneficial for these. We look for rules for sharing the multicast cost that will make it advantageous for all participants to form one large multicast group, which is called then the "grand coalition". We use tools from cooperative game theory where multicast groups are viewed as coalitions. The set of such rules is called **the core**.

We next consider explicitly an alternative unicast source that each mobile can connect to at some constant cost. We loose the property that we had before where we always had splitting rules that made the grand coalition appealing to everyone. One may view this problem as a hierarchical non-cooperative association problem: given the leader's rules (the rules for splitting the cost among the participants of the multicast group), each player has the choice to join the multicast group or a dedicated unicast one. We study the impact of the cost sharing rule in the multicast group on the number of participants in it (which we call capacity) and on its geographical size (which we call coverage). (Note: One can formulate this problem as one with multiple coalition structure, as defined in [77, p. 44, section 3.8].) We study the impact of information on the performance. We discover a paradoxical behavior in which the performances improves by providing less information to mobiles.

8.1.3 Related Work

The cost structure in our problem is identical to that proposed by Littlechild and Owen [60, 59] in the context of *Aircraft landing fees*. Thomson [94] provides a survey on cost allocation for the airport problem.

Myerson [70] developed the theory of large Poisson games, and in particular, proved the existence of equilibria in such games. Under the setup there, utility of a player depends on the aggregate action profile of the whole population, and not on the type-wise action profile. In our setup the cost of a user depends on the type-wise action profile of the population, where type of a user can be identified as its location.

Penna and Ventre [78] and Bilo et al. [97] study the problem of sharing the cost of multicast transmission in a wireless network. In another paper, Bilo et al. [98] frame the selfish nature of users as a noncooperative game among them, for several given cost allocation methods.

8.2 Coalition Game Preliminaries

We begin by defining the coalition game. A cooperative cost game [95] is a pair $(\mathcal{N}, c)$ where $\mathcal{N} := \{1, \dots, N\}$ denotes the set of players and $c : 2^{\mathcal{N}} \to \mathbb{R}$ is the cost function For any nonempty coalition $\mathcal{S}$, $c(\mathcal{S}\mathcal{S})$ is the minimal cost incurred if players in $\mathcal{S}$ work together to serve their purposes; $c(\emptyset) = 0$. A cooperative game is called *concave* if the cost function is *sub-modular* (the precise definition is delayed to Eqs (8.3)).

A cost allocation $\mathbf{q} \in \mathbb{R}^{\mathcal{N}}$ charges cost q_i to player i. An allocation $\mathbf{q}$ is called *efficient* if $\sum_{i \in \mathcal{N}} q_i = c(\mathcal{N})$. An efficient allocation $\mathbf{q}$ is called an *imputation* if $q_i \leq c(\{i\})$ for all $i \in \mathcal{N}$.

The core: The core, $\mathcal{C}$, of the game is defined as follows

$$\begin{aligned}\mathcal{C} = \{\mathbf{q} \in \mathbb{R}^{\mathcal{N}} \quad : \quad & \sum_{i \in \mathcal{N}} q_i = c(\mathcal{N}), \\ & \sum_{i \in \mathcal{S}} q_i \leq c(\mathcal{S}), \ \forall \ \mathcal{S} \subset \mathcal{N}\} \end{aligned} \tag{8.1}$$

The core of a concave cooperative game is nonempty [86].

Next we state a number of appealing rules for cost allocation.

- **Shapley value:** For any i, and $\mathcal{S} \subset \mathcal{N}$ such that $i \notin \mathcal{S}$, let $\Delta_i(\mathcal{S}) = c(\mathcal{S} \cup \{i\}) - c(\mathcal{S})$. The Shapley value is the cost allocation $\mathbf{q}$ for which

$$q_i = \frac{1}{n!} \sum_{U \in \mathcal{U}} \Delta_i(\mathcal{S}_i(U)), \tag{8.2}$$

  where $\mathcal{U}$ is the set of all orderings of $\mathcal{N}$, and $\mathcal{S}_i(U)$ is the set of players preceding i in ordering U. The Shapley value of a concave cooperative game lies in the core.

- **Nucleolus:** The *excess* of a coalition $\mathcal{S}$ under an imputation $\mathbf{q}$ is $e_S(\mathbf{q}) = \sum_{i \in \mathcal{S}} q_i - v(\mathcal{S})$; this is a measure of dissatisfaction of $\mathbf{S}$ under $\mathbf{q}$. Let $E(\mathbf{q}) = (e_S(\mathbf{q}), S \in 2^{\mathcal{N}})$ be the vector of excesses arranged in monotonically increasing order. The *nucleolus* is the set of imputations $\mathbf{q}$ for which the vector $E(\mathbf{q})$ is lexicographically minimal.

  Nucleolus is a singleton and belongs to the *core* whenever the latter is nonempty.

- **Egalitarian Allocation:** The egalitarian allocation for cooperative games was introduced by Dutta and Ray [30]. It is unique whenever it exists, and it always exists and lies in the core for concave cost games. The following characterization applies to such games only.

  The egalitarian allocation is the element of core which Lorentz dominates all other core allocations.

  Remark 3. *A min-max fair allocation for the cooperative cost game is defined as follows. For $\mathbf{q} \in \mathbb{R}^{\mathcal{N}}$, define $\bar{\mathbf{q}}$ to be the vector obtained by arranging the components of $\mathbf{q}$ in decreasing order. Further define*

$$\bar{\mathcal{C}} = \{\bar{\mathbf{q}} : \mathbf{q} \in \mathcal{C}\}.$$

  Then $\mathbf{q} \in \mathcal{C}$ is a min-max fair allocation if and only if $\bar{\mathbf{q}}$ is lexicographically minimal in $\bar{\mathcal{C}}$.

  A max-min fair allocation is also defined similarly. Jain and Vazirani[46] show that for concave games, there exists a unique cost allocation which is min-max fair as well as max-min fair, and which coincides with the egalitarian allocation.

8.3 System Model

8.3.1 Network and Communication Model

We consider a wireless network with a base station (BS) and a set $\mathcal{N} = \{1, \ldots, N\}$ of users. We shall assume N to be either a known constant (in the case of perfect information), or a Poisson random variable of rate λ (in the framework of imperfect information).

Both the radio channel varies from one mobile to another, as well as the required QoS level. Mobile i requires transmission at power p_i in order to meet its QoS needs. We assume p_is to be independently identically distributed (i. i. d.) random variables with distribution $G(p)$ (density $g(p) := G'(p)$).

We assume that any subset S of users can subscribe for a multicast session. The BS then broadcasts information (say, a radio channel) with the minimum power p that guarantees that all mobiles in S receive a satisfactory level of quality of service. Evidently $p = \max\{p_i : i \in S\}$.

Assume that there is a cost $f(p)$ per time unit during which the BS transmits at power p; this cost has to be shared by all the mobiles in the multicast group. f is assumed to be increasing in its argument.

We also assume that every user has yet another option, that of using a dedicated connection using some other technology, at a cost V.

8.3.2 Cost Sharing Models

Let us assume that the set of mobiles in multicast group is N. We index the mobiles such that such that p_i is increasing in i.

A cost sharing mechanism is a map $\mathbf{q} : \mathbb{R}^{\mathcal{N}} \to \mathbb{R}^{\mathcal{N}}$ with elements $q_j, j \in \mathcal{N}$; q_j is the cost share of user j. We study the cost sharing mechanisms that satisfy the following economical constraints.

Budget-balance A cost sharing mechanism is called *budget-balanced* if users pay exactly the total cost of the service.

$$\sum_{j=1}^{N} q_j = f(p_N).$$

cross-monotonicity A cost sharing mechanism is called *cross-monotonic* if each user's cost decreases as the service set expands.

efficiency An *efficient* cost sharing mechanism is one that maximizes net social utility.

Remark 4. *A cost sharing mechanism is called strategy-proof if revealing true utilities is a dominant strategy for each user. However, in the cost sharing problem studied here all the users are assumed to have equal utility which is known.*

The cost sharing problem can be formulated as a cooperative cost game $(\mathcal{N}, c)$. Here $c : 2^{\mathcal{N}} \to \mathbb{R}$, for a coalition $\mathcal{S} \subset \mathcal{N}$, gives the cost to support communication to all the users in $\mathcal{S}$, i.e., $c(\mathcal{S}) = \max\{f(p_i) : i \in \mathcal{S}\}$.

Now consider two coalitions $S_1, S_2 \subseteq \mathcal{N}$. Observe that

$$\begin{aligned} c(S_1 \cup S_2) &= \max\{c(S_1), c(S_2)\} && (8.3)\\ \text{and } c(S_1 \cap S_2) &\leq \min\{c(S_1), c(S_2)\}. && (8.4) \end{aligned}$$

Hence,

$$c(S_1 \cup S_2) + c(S_1 \cap S_2) \leq c(S_1) + c(S_2),$$

i.e., the cost function is submodular. This implies the following.

Theorem 6. *(i) The core of the cost allocation game is nonempty.*
(ii) The Shapley value lies in the core.
(iii) The egalitarian allocation lies in the core and is min-max (also max-min) fair.

The core of the game can be expressed as

$$\Big\{\mathbf{q} \in \mathbb{R}^{\mathcal{N}} : \sum_{i \in \mathcal{S}} q_i \leq f(p_{\max \mathcal{S}}), \mathcal{S} \subset \mathcal{N}, \sum_{i \in \mathcal{N}} q_i \leq f(p_N)\Big\},$$

where $\max(S) := \max i : i \in (S)$. We make the following observations

1. All the cost allocation in the core are nonnegative; if $q_i < 0$, $\mathbf{q}$ can not satisfy the constraint corresponding to subset $\mathcal{N} \backslash \{i\}$.
2. The constraint $\sum_{i=1}^{j} q_i \leq f(p_j)$ makes the constraints corresponding to the subsets $S \subsetneq \{1, \ldots, j\}$ redundant.

In view of these, the core can be rewritten as

$$\Big\{q \in \mathbb{R}_+^N : \sum_{i=1}^{j} q_i \leq f(p_j),\ 1 \leq j \leq N, \sum_{i=1}^{N} q_i = f(p_N)\Big\}$$

A budget-balanced cost sharing mechanism is cross-monotonic only if it belongs to the core of the associated cooperative cost game. Hence we focus on cost allocations from the core. Following criteria can be used.

1. **Highest cost allocation (HCA):** The user requiring the highest power, N, pays the whole cost. Of course $(0, \ldots, f(p_N))$ is in the core.

2. **Incremental cost allocation (ICA):** A more fair cost allocation is where user i pays $f(p_i) - f(p_{i-1})$; $f(p_0) := 0$. We call it incremental cost allocation.

3. **Shapley value (SV):** Following [60], the cost q_i for player i is given by

$$q_i = \sum_{j=1}^{i} \frac{f(p_j) - f(p_{j-1})}{N + 1 - j}$$

4. **Nucleolus (NS):** The following algorithm for calculating the nucleolus is given by Littlechild [59] in the context of the *airport cost game*.

   Define $i_0 = r_0 = 0$. For $k \geq 1$, iteratively define

$$r_k = \min_{i_{k-1}+1,\ldots,n-1} \left\{ \frac{f(p_i) - f(p_{i_{k-1}}) + r_{k-1}}{i - i_{k-1} + 1} \right\},$$

   and i_k as the largest value of i for which the minimum is attained in the above expression. Continue this until $k = k'$ where $i_{k'} = N - 1$. The nucleolus of the game, q, is given by

$$\begin{aligned} q_i &= r_k, \ i_{k-1} < i \leq i_k, \ k = 1, \ldots, k' \\ q_N &= f(p_N) - f(p_{N-1}) + r_{k'} \end{aligned}$$

5. **Egalitarian allocation (EA):** The egalitarian allocation for our cost sharing problem can be computed by applying the following algorithm [104].

   Define $i_0 = r_0 = 0$. For $k \geq 1$, iteratively define

$$r_k = \min_{i_{k-1}+1,\ldots,n} \left\{ \frac{f(p_i) - f(p_{i_{k-1}})}{i - i_{k-1}} \right\},$$

   and i_k as the largest value of i for which the minimum is attained in the above expression. Continue this until $k = k'$ where $i_{k'} = N$. The max-min fair allocation, q, is given by

$$q_i = r_k, \ i_{k-1} < i \leq i_k, \ k = 1, \ldots, k'$$

Cross-monotonicity of the above cost allocations: Evidently *HCA* is cross-monotonic. Shapley values is well known to be cross-monotonic (see Moulin[68]). Dutta [29] showed that the egalitarian allocation in concave games is also cross-monotonic. Sonmez [91] showed that the nucleolus of a generic concave cost allocation game need not be cross-monotonic; however he also proved the cross-monotonicity of nucleolus for the airport game which has identical formulation as ours.

We observe also the following monotonicity property of cost allocations *HCA, SV, NS* and *EA*. *"For any two users i, j such that $p_j > p_i$, $q_j \geq q_i$."*

The expression for nucleolus has the similar form as that of max-min fair allocation. Hence, in the following we analyze max-min fair allocation but do not discuss nucleolus.

8.4 Non-cooperative Coalition Formation game: perfect information

Each player independently decides whether to join the multicast group or not. Recall that, a player if it does not use the multicast service, bears a cost V. We formulate the decision problem as a noncooperative game. Assume that all users know about the resources requested by all other users in the network.

Definition 1. *An equilibrium is a multicast subset $\mathcal{M} \subset \mathcal{N}$ of users, such that the cost share of each one in that set is not greater than V, and the cost of each user not in the set would not be smaller than V if it joined the set.*

Let p_i be the amount of resources required by player i. Let $p_{\mathcal{M}}$ denote the set $p_i, i \in \mathcal{M}$ and let q_i be the cost share of player i. It is given as some function h_i that depends on the set $\mathcal{M}$ and on the resource requests of each player in the set $\mathcal{M}$. With this notation, the equilibrium is characterized by the following conditions: $h_i(\mathcal{M}, p_{\mathcal{M}}) \leq V$ for all $i \in \mathcal{M}$, and $h_i(\mathcal{M} \cup i, p_{\mathcal{M} \cup i}) \geq V$ for all $i \in \mathcal{N} \backslash \mathcal{M}$.

Remark 5. *We shall assume that the amount of resources requested by a mobile is a random realization of some probability distribution that has no mass at isolated points. For simplicity we shall focus on the case where the resource is power, and the amount required is a strictly monotone continuous increasing function of the distance of the mobile from the base station. The distance between the users and the base station are independent from user to user and are assumed to be drawn from a probability distribution that has a density. Therefore the probability of two mobiles having the same distance from the base station is zero. We shall thus assume below that all distances are different from each other. Similarly, we shall ignore the possibility (which has probability zero) that $f(p_i) = V$ for some i.*

We next provide the Nash equilibrium (NE) for the various cost sharing policies.

1. Under the rule that the user requiring highest power pays the whole cost $\mathcal{M} = \{i : f(p_i) < V\}$. This is in fact a *strong equilibrium*: it is robust not only to any deviation by a single user but to any deviation by any number of users. No coordination between users is needed for reaching this equilibrium since the best response of a user (as specified by this strategy) is independent of what other users do.

2. When users pay incremental costs,

$$\mathcal{M} = \begin{cases} \emptyset & \text{if } p_1 \geq V, \\ \{1, \ldots, j-1\} & \text{otherwise}, \end{cases} \tag{8.5}$$

where $j = \arg\min_i \{i : f(p_i) - f(p_{i-1}) \geq V\}$.

3. Shapley value: Let us define

$$b_j = \sum_{i=1}^{j} \frac{f(p_j) - f(p_{j-1})}{j + 1 - 1},$$

Then the unique NE is $\mathcal{M} = \{1, \ldots, j\}$ where j is the largest index such that $b_j \leq V$.

4. Max-min fair allocation: Define $i_0 = r_0 = 0$. For $k \geq 1$, iteratively define

$$r_k = \min_{i_{k-1}+1, \ldots, n} \left\{ \frac{f(p_i) - f(p_{i_{k-1}})}{i - i_{k-1}} \right\},$$

and i_k as the largest value of i for which the minimum is attained in the above expression. Continue this until $r_k > V$. The unique NE is $\mathcal{M} = \{1, \ldots, i_{k-1}\}$.

8.5 Static problem with incomplete information

Here we assume that any user does not know the number of other users in the network and their resource (say, power) requirements. However, the users know their own requirements. What is the equilibrium policy?

We restrict ourselves to symmetric strategies for all the users (see Myerson [70] for a discussion on this). The number of players is Poisson distributed with mean λ. *Environmental equivalence* property of Poisson games [70] ensures that from the perspective of any player, the number of other players in the game is also a Poisson random variable with the same mean λ. These arguments imply that a user's decision is a function of its power requirement, and a pure strategy equilibrium is characterized by a set such that users with power requirements in that set join the multicast group and others do not.

Let $Q \subset \mathbb{R}_+$ be such that users with power requirements in Q join the multicast group. Consider user i with power requirement p_i and let ω_{-i} denote a realization for the rest of the network. The cost share of user i is given as

$$q_i(p_i, Q, \omega_{-i}) = h_i(\mathcal{M}_Q(\omega) \cup i, p_{\mathcal{M}_Q \cup i}(\omega)),$$

where $\mathcal{M}_Q = \{i : p_i \leq Q\}$. The expected cost share of user i is $\mathbb{E}q_i(p_i, Q)$.

Definition 2. *An NE in this case is characterized by a set $Q \subset \mathbb{R}_+$ such that users with power requirements $p_i \in Q$ join the multicast group. More precisely $\mathbb{E}q_i(p_i, Q) \leq V$ if $p_i \in Q$, and $\mathbb{E}q_i(p_i, Q) > V$ if $p_i \notin Q$.*

Again consider user i, and a fixed realization of the network. We can view the function $h_i(\mathcal{M}, p_{\mathcal{M}\setminus i}, \cdot) : p_i \mapsto q_i$ as parametrized by power requirements of other users. We use $\hat{h}_i : \mathbb{R}_+ \to \mathbb{R}_+$ to denote it, i.e.,

$$\hat{h}_i(p_i) := h_i(\mathcal{M}, p_{\mathcal{M}}).$$

Following is an important observation.

Lemma 2. *Under cost allocation policies HCA, SV and EA, $\hat{h}_i$ is a monotone increasing function.*

Proof. Clearly the claim is true for *HCA*.
SV: Let us consider user i with required power p_i. Assume that its power requirement is increased to p_i'. If $p_i' \leq p_{i+1}$ then the cost share increases by

$$\hat{h}(p_i') - \hat{h}(p_i) = \frac{f(p_i') - f(p_i)}{N+1-i}.$$

Let us consider the case when $p_{i+1} < p_i' < p_{i+2}$. Other cases can be analyzed with a repeated application of this procedure. Now the new cost share of player i is

$$\begin{aligned}
\hat{h}(p_i') &= \sum_{j=1}^{i-1} \frac{f(p_j) - f(p_{j-1})}{N+1-j} \\
&\quad + \frac{f(p_{i+1}) - f(p_{i-1})}{N+1-i} + \frac{f(p_i') - f(p_{i+1})}{N-i} \\
&\geq \sum_{j=1}^{i} \frac{f(p_j) - f(p_{j-1})}{N+1-j} \\
&= \hat{h}(p_i)
\end{aligned}$$

EA: Let us revisit the algorithm used to obtain *EA* and assume $i = i_k$ for some k. Now user i increases its power requirement to p_i'. As before first consider the case when $p_i' \leq p_{i+1}$. Clearly $\hat{h}(p_i') \geq \hat{h}(p_i)$ in this case. The same holds true if $p_{i+1} < p_i' < p_{i+2}$. Similar arguments can be made in the case when $i \neq i_k$ for any k. $\square$

Recall that we consider symmetric strategies for all the users. The following theorems show that only candidates for NEs are closed intervals containing 0.

Corollary 2. *For the cost sharing mechanisms HCA, SV and EA, under any symmetric policy Q, there exists a threshold p^* such that $\mathbb{E}q_i(p_i, Q) \leq V$ if and only if $p_i \leq p^*$.*

Corollary 3. *Under cost sharing mechanisms HCA, SV and EA, the only candidates for NEs are sets of the form $[0, p^*]$ and $[0, \infty)$.*

Following is another useful property of the above cost sharing mechanisms.

Lemma 3. *For the cost sharing mechanisms HCA, SV and EA,* $\mathbb{E}q_i(p_i, [0,p])$ *is monotonically increasing with* p_i *for* $p_i \geq p$*.*

Theorem 7. *For the cost sharing mechanism ICA, the only candidates for NEs are sets of the form* $[0, p^*]$ *and* $[0, \infty)$*.*

Proof. We prove the claim via contradiction. Assume that $Q = \cup_{k=1}^{K}[a_k, b_k]$ is an NE where $b_{k-1} < a_k$ for all k ($b_0 := 0$). For $0 \leq p_i \leq a_1$, user i's expected cost $f(p_i)$ will be increasing in p_i. For $b_1 \leq p_i \leq a_2$, user i's expected cost $\mathbb{E}q_i(p_i, Q)$ is

$$f(p_i) - \int_{a_1}^{b_1} \lambda f(p)g(p) \exp\Big(- \lambda \int_p^{b_1} g(s)\ ds\Big)\ dp.$$

In writing the above expression we have used the *decomposition property* of Poisson distribution: The number of users with power requirements in the range $[p, b_1]$ is a Poisson random variable with rate $\lambda \int_p^{b_1} g(s)\ ds$. Finally it is seen that the above expression is increasing in p_i. Similarly it can be shown that $\mathbb{E}\hat{h}_i(p_i), b_{k-1} \leq p_i \leq a_k$ is increasing for all $1 \leq k \leq K$.

Hence if all other users are using the strategy Q, Q can not be player i's best response. Thus only candidates for symmetric NEs are the threshold strategies $[0, p^*]$.

On the other hand if $\mathbb{E}q_i(p_i, [0, \infty)) \leq V$ for all p_i then $[0, \infty)$ is also an NE. □

Expressions for the NEs

We have shown that for each of the cost allocation strategies the NEs are characterized by thresholds p^*. In this section we derive expressions for the thresholds.

Theorem 8. *For the cost sharing mechanisms HCA, SV and EA, a symmetric multi-strategy* $[0, p^*]$ *is an NE if and only if* $\mathbb{E}q_i(p^*, [0, p^*]) = V$*.* $[0, \infty)$ *is also an NE provided* $\mathbb{E}q_i(p_i, [0, \infty)) \leq V$ *for all* p_i*.*

Proof. Suppose $\mathbb{E}q_i(p^*, [0, p^*]) = V$. Then $\mathbb{E}q_i(p_i, [0, p^*]) \leq V$ for all $p_i \leq p^*$. Also from Lemma 3, $\mathbb{E}q_i(p_i, [0, p^*]) > V$ for all $p_i > p^*$. Thus $[0, p^*]$ is indeed an NE.

Now assume $\mathbb{E}q_i(p^*, [0, p^*]) > V$. Consider user i with $p_i = p^*$. Then $[0, p^*]$ can not be an equilibrium strategy of user i. Finally assume $\mathbb{E}q_i(p^*, [0, p^*]) < V$ and denote $\epsilon := V - \mathbb{E}q_i(p^*, [0, p^*])$. Since $\mathbb{E}q_i(p_i, [0, p^*])$ is continuous and increasing in p_i for $p \geq p^*$, there exists a $\delta > 0$ such that $\mathbb{E}q_i(p_i, [0, p^*]) \leq V$ for $p_i = p^* + \delta$. Thus $[0, p^*]$ can not be an equilibrium strategy of user i.

If $\mathbb{E}q_i(p_i, [0, \infty)) \leq V$, user i's best response is to join the multicast group given that all others have joined. Hence $[0, \infty)$ is also an NE. □

HCA:

Corollary 4. $[0, f^{-1}(V)]$ *is an NE.* $[0, \infty)$ *is also an NE provided* $f(p_i) \leq V$ *for all* p_i*.*

Proof. For *HCA* cost sharing $\mathbb{E}q_i(p_i, [0, p_i]) = f(p_i)$. Since $f(\cdot)$ in strictly increasing, the unique solution to $\mathbb{E}q_i(p^*, [0, p^*]) = V$ is $p_i = f^{-1}(V)$. Theorem 8 proves the claim. □

ICA: Consider user i with power requirement p_i. Let us assume that all other users join the multicast group. Again using the *decomposition property* of Poisson distribution, the expected cost of user i, $\mathbb{E}q_i(p_i, [0, \infty))$ is

$$f(p_i) - \int_0^{p_i} \lambda f(p)g(p) \exp\Big(- \lambda \int_p^{p_i} g(s)ds\Big)dp.$$

Lemma 4. $[0, p^*]$ *is an NE if and only if* $\mathbb{E}q_i(p_i, [0, \infty)) \leq V$ *for all* $p_i \leq p^*$ *and* $\mathbb{E}q_i(p^*, [0, \infty)) = V$*. If* $\mathbb{E}q_i(p_i, [0, \infty)) \leq V$ *for all* p_i*,* $[0, \infty)$ *is also an NE.*

Proof. if part: Recall that, under *ICA* mechanism, user i's cost share depends on only those users that have power requirements less than p_i. Hence $\mathbb{E}\hat{h}_i(p_i, [0, p])$ are same for all $p \geq p_i$. Now, from the conditions on p^*, $\mathbb{E}\hat{h}_i(p_i, [0, p^*]) \leq V$ for all $0 \leq p_i \leq p^*$. Also, following the proof of Proposition 7, $\mathbb{E}\hat{h}_i(p_i, [0, p^*]) > V$ for all $p_i > p^*$. Hence $[0, p^*]$ is indeed an NE.

only if part: Consider a symmetric strategy $[0, p']$, and assume that there exists a $0 < p \leq p'$ such that $\mathbb{E}\hat{h}_i(p, [0, p']) > V$. Clearly $[0, p']$ can not be an equilibrium strategy of user i. Finally consider the case when $[0, p']$ is a symmetric strategy while $\mathbb{E}\hat{h}_i(p', [0, \infty)) < V$. Denote $\epsilon := V - \mathbb{E}q_i(p', [0, \infty))$. Since $f(p)$ is continuous and increasing, there exists a $\delta > 0$ such that $f(p' + \delta) - f(p') \leq \epsilon$ implying $\mathbb{E}q_i(p_i, [0, \infty)) \leq V$ for $p_i = p' + \delta$. Thus $[0, p']$ can not be an equilibrium strategy of user i. □

8.6 Static problem with no information

Next we assume that the requirement of a user is not known even to that own user. Why should a user not know its own request? The amount of resource requested may depend on the access channel quality which might not be known.

As an example, assume that receiver nodes are mobile and should decide whether to join a multicast session an hour in advance. The session is multicast by some BS. A mobile cannot predict what its distance to the BS will be an hour ahead. It only has the probability distributions of the number N of users (Poisson with mean λ), and users' distances to the BS that are i.i.d. and yield distributions $G(p)$ on the power requirements.

Consider a tagged user, say i. Given that there are n other users in contention, the expected cost share user of user i (for a given cost sharing strategy) is

$$\bar{q}_i(n) = \mathbb{E}\Big[h_i(\mathcal{M}(\omega), p_{\mathcal{M}}(\omega))\Big| |\mathcal{M}(\omega)| = n + 1\Big].$$

Using the cross-monotonicity of the cost sharing strategy and a coupling argument it can be shown that $\bar{q}_i(n)$ is decreasing in n.

Now, consider the symmetric multi-strategy where each user joins with probability s. From user i's perspective, the number of other users in the multicast group will be Poisson distributed with mean $s\lambda$. Hence the unconditional expected cost share of user i will be

$$q_i(s) = \sum_{n=0}^{\infty} \frac{(s\lambda)^n \exp(-s\lambda)\bar{q}_i(n)}{n!}$$

Since the family of Poisson distributions, Poisson($s\lambda$), is stochastically increasing, $q_i(s)$ is decreasing in s.

Lemma 5. *1. If* $q_i(0) > V$ *then* 0 *is an NE, a pure strategy equilibrium where none of the users joins the multicast group.*
2. If $q_i(1) \leq V$ *then* 1 *is an NE, a pure strategy equilibrium where all the users join the multicast group.*
3. If $q_i(0) \geq V \geq q_i(1)$*, the symmetric multi-strategy* s^* *such that* $q_i(s^*) = V$*, is the unique mixed strategy NE.*

8.6.1 Information on the number of users

We restrict to *HCA* cost sharing in this section. Let U be the random variable for the cost, and $F(\cdot)$ be its distribution. f(p) being the cost of power p,

$$F(u) = G(f^{-1}(u)).$$

We assume that the base station broadcasts, N, the number of users.

Consider user i. If there are n other users in contention, the expected cost share user i, is

$$\bar{q}_i^n = \frac{\int_V^\infty (1 - F^{n+1}(u))du}{n+1}.$$

Consider the symmetric multi-strategy where each user joins with probability s. Then the unconditional expected cost share of user i will be

$$\bar{q}_i(s) = \sum_{n=0}^{N-1} \binom{N-1}{n} s^n (1-s)^{N-1-n} \bar{q}_i^n.$$

The equilibrium policy s is the solution of the equation $\bar{q}_i(s) = V$.

8.6.2 Some more information on ones own requirement

We assume a little more information: the base station tells each user i whether $f(p_i)$ is below or above V. It further broadcasts, N, the number of users having power requirements above $f^{-1}(V)$. The conditional distribution of cost for such a user

$$\tilde{F}(u) = \frac{F(u) - F(V)}{1 - F(V)}.$$

Note that for users with requirements below $f^{-1}(V)$, joining the multicast group is the dominant strategy. They also do not affect the costs of other N users. Hence we consider a noncooperative gave with N players only.

Again consider user i. If there are n other users in contention, the expected cost share user i, is

$$\tilde{q}_i^n = \frac{\int_V^\infty (1 - \tilde{F}^{n+1}(u))du}{n+1}.$$

Also consider the symmetric multi-strategy where each user joins with probability s. From user i's perspective, the number of other users in the multicast group will have Binomial $(N-1, s)$ distribution. Hence the unconditional expected cost share of user i will be

$$\tilde{q}_i(s) = \sum_{n=0}^{N-1} \binom{N-1}{n} s^n (1-s)^{N-1-n} \tilde{q}_i^n.$$

Lemma 6. *A symmetric multi-strategy s^* such that $\tilde{q}_i(s^*) = V$ is a symmetric NE.*

Remark 6. *1. As expected $s^* = 0$ is an NE.*
2. If $\tilde{q}_i^{N-1} \le V$, $s^* = 1$ is also an NE. Thus providing less information may potentially improve the user base.

8.6.3 Properties of NEs

Theorem 9. *Assume that each user knows at least its own request. Then among all monotone coalitions achievable by some cost sharing policy, HCA achieves the smallest range.*

8.7 On the expected capacity and coverage

Consider a random realization of the network: there is a single BS, a point process that describes the location of the mobiles. All mobiles are assumed to have full knowledge. Let $\overline{S}$ be the set of all mobiles. Let there be an alternative unicast solution for any individual that costs V. Fix some cost sharing mechanism. Consider a subset $S \subset \overline{S}$ such that there is no mobile in S that can benefit from leaving the multicast group S (for the given cost sharing policy) and getting instead

V. We call such a set a "V-stable set". Assume that S is a maximal set, i.e. if we add to it another $s \in \overline{S}$ then the new set is not V-stable anymore.

We define the capacity associated with a V-stable set as the number of mobiles in S.

is this well defined? could the same problem have several different stable sets?

We define the coverage of S to be the set of locations in which if we placed another mobile, then the new set will still be V-stable.

Unless otherwize stated, coverage and capacity are defined under full information conditions.

8.7.1 Computing Capacity and Coverage

Consider a network with the BS at zero. Assume that the mobiles are located according to a stationary Poisson process with intensity λ.

HCA policy: network over the line The capacity of the system for a given value of V is a Poisson random variable with parameter $2\lambda V$. In particular, $2\lambda V$ is also the expected capacity. The coverage region is the interval $[-V, V]$.

Incremental Cost Policy: network over the plain. The capacity of the system for a given value of V is a Poisson random variable with parameter $\pi(\lambda V)^2/2$. In particular, $\pi(\lambda V)^2/2$. is also the expected capacity. The coverage region is the circle of radius V centered at zero.

The incremental cost policy over the line: Recall (8.5). This is a condition in terms of the difference of powers needed by adjacant mobiles. We find it useful to express the latter as a function of the locations of the mobiles and the BS, since coverage is understood as a geometric property. We shall use the following simple path loss attenuation model for simplicity.

If the BS transmits at power P then we have at distance d a signal of power $p = hPd^{-\alpha}$. If we have a given sensitivity threshold p that guarantees reception at reasonable quality, then the power needed to get p at a distance d is pd^{α}/h. Let $v(q)$ correspond to the price of transmitting at a power q.

Assume that there is a mobile at b and at $a < b$. There is a BS at the origin. Assume that the mobile at a participates in the multicast session. then that at b will participate in the multicast session if

$$V > v(pb^{\alpha}/h) - v(pa^{\alpha}/h)$$

or equivalently

$$v(pb^{\alpha}/h) \leq V + v(pa^{\alpha}/h)$$

Let v be linear. Then the condition becomes

$$b \leq \left(\frac{Vh}{p} + a^{\alpha}\right)^{1/\alpha} \tag{8.6}$$

Let $c := h/p$.

The number of mobiles in a multicast session: Enumerate the mobiles according to the increasing distance to the BS. Define X_0 and let X_n be the location of the nth mobile. Then the capacity N is given by

$$N = \sup\{k : (X_n)^{\alpha} - (X_{n-1})^{\alpha} < cV, \forall n = 1, ..., k\}$$

where $\sup \emptyset := 0$. The coverage is given by

$$C = \left(\frac{Vh}{p} + (X_N)^{\alpha}\right)^{1/\alpha}$$

Let $N(0) := N$ and define

$$N(m) = \sup\{k : (X_n)^{\alpha} - (X_{n-1})^{\alpha} < cV, \forall n = m+1, ..., k\}$$

$N(m)$ is the capacity of the system if the coverage satisfies $C > X_m$. Define $C(m)$ as

$$C(m) := \left(\frac{Vh}{p} + (X_{N(m)})^{\alpha}\right)^{1/\alpha}$$

8.7.2 The linear case: $\alpha = 1$

Lemma 7. *In case $\alpha = 1$ then*
(i) $N(m)$ are identically distributed, $m = 0, 1, 2, ...$
(ii) $C(m)$ are identically distributed, $m = 0, 1, 2, ...$

The expected coverage distance C can be computed as follows. Define

$$C_0 = \frac{Vh}{p}$$

(This is also the capacity reached when applying the rule that it is the furthest that pays.) The location X of the first mobile is exponentially distriubted with parameter λ. With probability $\exp(-\lambda C_0)$ we have $X > C_0$ and then $C = C_0$. With the complementary probability, $X < C_0$ so that $C > X$. In that case, $C = X + C(1)$ where $C(1)$ has the same distribution as C (due to Lemma 7). We conclude that

$$\begin{aligned} E[C] &= E[C1\{X > C_0\}] + E[(X + C)1\{X \le C_0\}] \\ &= C_0 \exp(-\lambda C_0) + E[X + C] - E[(X + C)1\{X > C_0\}] \\ &= C_0 \exp(-\lambda C_0) + C(1 - \exp(-\lambda C_0)) + E[X] - E[(X1\{X > C_0\}] \\ &= C_0 \exp(-\lambda C_0) + E[C](1 - \exp(-\lambda C_0)) + \frac{1}{\lambda} - \exp(-\lambda C_0)(C_0 + \frac{1}{\lambda}) \end{aligned}$$

Thus the expected capacity is given by

$$E[C] = \frac{C_0 \exp(-\lambda C_0) + \frac{1}{\lambda} - \exp(-\lambda C_0)(C_0 + \frac{1}{\lambda})}{\exp(-\lambda C_0))}$$

8.7.3 $\alpha > 1$

We have:

$$b \le \Big(\frac{Vh}{p} + a^\alpha\Big)^{1/\alpha} \le \frac{Vh}{p} + a \tag{8.7}$$

Is it more generally, decreasing in α?

Can this be used to show that the linear case gives an upper bound?

Part IV

Part 4: Transport control

Chapter 9

Transport layer protcols

A large majority of the Internet traffic is transmitted according to TCP - Transmission Control Protocol.

9.1 Description of TCP

9.1.1 Objectives of TCP

TCP has three objectives:

- Adapt the rate of transmission of packets to the available resources.
- Avoid congestion within the network.
- Provide reliability to end-to-end communications by retransmission of packets that are lost or corrupted.

9.1.2 Window based Control

In order to achieve these objectives, each packet transmitted has a sequence number. To control the speed of transmission, the source can not transmit into the network more than some given number certain packets. This number is called *window* and is denoted by W. After W packets have been transmitted, the source can not transmit more packets packets until it knows that a packet has successfully reached the destination. To find out which packets attained destination successfully, the source makes use of acknowledgements sent to it from the destination that allows it to know if a transmission failed.

9.1.3 Acknowledgements

The Acknowledgement (or ACK) has two objectives:

- Regulate the rate of transmission of the TCP, ensuring that packets can be transmitted only if other ones have left the network,
- Render the communication reliable by returning information on packets that are received well by the destination.

The ACK tells the source which is the packet sequence number that is expected at the destination. For example, assume that packets number 1,2,..., 6 reached successfully the destination (in order). When the packet 6 arrives, the destination sends an ACK for saying it expects to receive package 7. Then when packet 7 arrives, the destination sais it expects 8. Now, if packet 8 is lost, then the next packet that arrives is 9. In this moment, the destination sends an ACK saying it is waiting for Package 8. This type of ACK is called "repeated ACK" it informs again the reception of packat 8.

Not only the packets can be lost but also the ACKs. Assume that an ACK saying that the destination expects packat 5, is lost. The next ACK that arrives to the sourceu says that the destination expects packet 6. Then the source knows that the packet 5 was also received.

A TCP packet is considered lost if TCP! losses

- Three repeated ACKs for the same packet arrive at the source, or
- Timeout: When a packet is transmitted, there is a timer start counting. If the ACK does not arrive during a per of t_0, there is a "Time-Out" and the packet is considered lost.

TCP! timeout How to choose T_0? The source has an estimate on (i) the average time RTT, which is the time a packet needs to arrive at the destination and for its ACK to return back to the source, and (ii) the variability of T_0 T_0 is Dermined by the estimates:

$$T_0 = \overline{RTT} + 4D$$

Where $\overline{RTT}$ is the estimate of RTT, and D is the estimate of its variability. To estimate the RTT, we measure the difference M, between the transmission time of a packet and the time it takes for its ACK to return. We then compute

$$\overline{RTT} \leftarrow a \times \ overlineRTT + (1-a)M,$$

$$D \leftarrow aD + (1-a)|\ overlineRTT - M|.$$

To reduce the number of ACKs in the system, TCP uses frequently a method called "delayed ACK" where an ACK is transmitted after two packets arrive and not after each arrival.

9.1.4 Dynamic Window

TCP! window

Since the beginning of the eighties, for several years, the TCP had a fixed window. The networks were unstable, and had long periods of severe congestion, during which there were many retransmissions and many losses. To solve this problem, Van Jacobson proposed [45] proposed to use a dynamic window: its size varies according to the state of the network. When the window is small, it can grow quickly, and when it is large, it has to grow very slowly. When there is congestion, the size window size is decreased drastrically. This allows to get rid quickly of congestion and then to use well the system resources.

More precisely, define a threshold W_{th} called "slow start threshold" which represents our estimation of the network capacity. The window starts with the value of one. With each arriving ACK, the window increases by one. Thus after transmitting a single packet, when its ACK returns, we can transmit two packets. When two ACKs arrive, the window grows to 4, and can transmit 4 packets. We see that there is an exponential growth. This period of growth is called "slow start". It is so called because, although its growth is rapid, it is slower than if it had started directly with $W = W_{th}$.

When $W = W_{th}$, we move to a per ' i heard that is called " congestion avoidance, "where the window W grows $1/\ lfloorW\ rfloor$ with each ACK that arrives. Then After transmitting packets W, W increases from 1. If transmit packets of W in t, then t + RTT transmit $W+1$, and t transmit RTT $+2$ $+2$ W , etc ... We see that growth is linear.

9.1.5 Losses and a dynamic threshold W_{th}

Not only is W dynamic, but also its transmission rate is. We set W_{th} to half the value of W when a loss is detected.

There are several variants of TCP: In the first variant called "Tahoe", if a loss is detected, the window is always reduced to 1 and starts a period of Slow-start. It is an extreme drop in performance.

In the variants that are used today, called Reno or New Reno, the window reacts to a loss by decreasssing to 1 only if a loss is detected by a time-out. Otherwize, the window size decreases to only half of its value and does not start the "slow-start", we remain in the congestion avoidance regime.

9.2 Modeling TCP

9.2.1 Flow model of TCP with independent random losses

TCP! random losses

When there is a large network with many connections, the contribution of a single connection to the congestion is negligible. The congestion is caused by the aggregate effect of all other connections. This is represented as a random process. Let T_n be the time of the nth loss and set $S_n = T_{n+1} - T_n$ be the time between packet losses from a TCP connection. We assume that

- S_n are independent with the same distribution. Let $s = E[S_n]$ and $s^{(2)} = E[S_n^2]$.
- We use the version Reno or TCP New-Reno, and all losses are assumed to be detected by duplicate ACKs.
- RTT is constant.

We thus consider only the Congestion-Avoidance phase where the window grows linearly. Let X_n the window just before the n-th loss. Then

$$X_{n+1} = \nu X_n + \alpha s_n, \tag{9.1}$$

where $nu = 1/2$ and where $alpha = RTT^{-1}$.

Let $x = E[X_n]$ and $x^{(2)} = E[X_n^2]$. In the stationary regime, x does not depend on n. We take expectation in (**??**). So, $x = nux\ alphas$, then

$$x = \frac{\alpha s}{1-\nu}$$

This is not the expected average size, but rather its size averaged at the epochs just prior to losses.

We now take the square of each side of (**??**). We get

$$x^{(2)} = \nu^2 x^{(2)} + \alpha^2 s^{(2)} + 2\nu\alpha x s$$

Hence

$$x^{(2)} = \frac{\alpha^2 s^{(2)} + 2\nu\alpha x s}{1-\nu^2} = \frac{\alpha^2}{1-\nu^2}\left(s^{(2)} + \frac{2\nu s^2}{1-\nu}\right)$$

The mean window size is then obtained by

$$\begin{aligned} \overline{W} &= \frac{1}{E[S_1]} E\left[\int_{T_0}^{T_1} X(t)dt\right] = \frac{1}{s} E\left[\int_{T_0}^{T_1} (\nu X_0 + \alpha t)dt\right] \\ &= \frac{1}{s} E[\nu X_0 S_0 + \frac{\alpha}{2} S_0^2] = \alpha\left(\frac{\nu s}{1-\nu} + \frac{s^{(2)}}{2s}\right) \end{aligned} \tag{9.2}$$

We used here the independence of X_0 and S_0, which gives $E[X_0 S_0] = E[X_0]E[S_0]$. This independence follows since X_0 is a function of S_i with $i < 0$, and not of $i \geq 0$.

We see that if s is ia constant, then $\overline{W}$ grows with $s^{(2)}$.

9.2.2 Flow level model of TCP with general random losses

Measures on the network have shown that packet losses of connections that have long RTTs, are independent. On the other hand, for connections over distances of a few kilometres, the times between losses are dependent [7]. We use the method from [7].

(**??**) is a difference equation and its solution is

$$X_n = \alpha \sum_{k=0}^{\infty} \nu^k S_{n-k-1}. \tag{9.3}$$

Since S_n is stationary, we can assume that X_n is also stationary. Taking the expectation, we can see that the solution of $E[X_n]$ does not change.

However, the calculus of $\overline{W}$ change as now $E[x_0 s_0] \neq E[x_0]E[s_0]$. We can use again (**??**) where only $E[x_0 s_0]$ change. We have from (9.3):

$$E[X_0 S_0] = \alpha \sum_{k=0}^{\infty} \nu^k E[S_{-k-1} S_0] = \alpha \sum_{k=0}^{\infty} \nu^k R(k+1)$$

where we define $R(k) = E[X_n X_{n+k}]$. $R(k)$ does not depend on n since X_n is stationary. Using (9.2) with this expression, we get

$$\overline{W} = \frac{\alpha}{s}\left[\frac{R(0)}{2} + \sum_{k=1}^{\infty} \nu^k R(k)\right].$$

The probability to lose a packet is

$$p = \frac{RTT}{s\overline{W}} = \frac{1}{\alpha s \overline{W}}$$

since the rate of losses is $1/s$ and the throughput is $\overline{W}/RTT$. Denote the normalized correlacions by $\hat{R}(k) = R(k)/s^2$. Then

$$\overline{W} = \alpha s\left[\frac{\hat{R}(0)}{2} + \sum_{k=1}^{\infty} \nu^k \hat{R}(k)\right] = \frac{1}{p\overline{W}}\left[\frac{\hat{R}(0)}{2} + \sum_{k=1}^{\infty} \nu^k \hat{R}(k)\right].$$

Hence,

$$\overline{W} = \frac{1}{\sqrt{p}}\sqrt{\frac{\hat{R}(0)}{2} + \sum_{k=1}^{\infty} \nu^k \hat{R}(k)},$$

and the throughput is

$$Thp = \frac{\overline{W}}{RTT} = \frac{1}{RTT\sqrt{p}}\sqrt{\frac{\hat{R}(0)}{2} + \sum_{k=1}^{\infty} \nu^k \hat{R}(k)},$$

We see that

- The mean throughput of TCP is invenrsly proportional to RTT and to $\sqrt{p}$.

Denote the normalized covariance as

$$\hat{C}(k) = \hat{R}(k) - 1 = \frac{R(k) - s^2}{s^2}.$$

Then

$$Thp = \frac{\overline{W}}{RTT} = \frac{1}{RTT\sqrt{p}}\sqrt{\frac{1+\nu}{2(1-\nu)} + \frac{\hat{C}(0)}{2} + \sum_{k=1}^{\infty} \nu^k \hat{C}(k)}.$$

If S_n are independient then $\hat{C}_k = 0$ for all $k \neq 0$, so

$$Thp = \frac{\overline{W}}{RTT} = \frac{1}{RTT\sqrt{p}}\sqrt{\frac{1+\nu}{2(1-\nu)} + \frac{\hat{C}(0)}{2}}.$$

If S_n are constant then $\hat{C}_k = 0$ para todo k.

9.3 Competition between protocols, the indifference property

Non-cooperative Game theory tries to predict stable outcomes of competition. There may be various ways to understand a prediction of the outcome of competition, and the standard concept of Nash equilibrium is one of them. Before presenting the definition of the Nash equilibrium we mention other alternative concepts that or tests that have been used to predict future evolution of competition. We then intnroduce concepts from game Theory that predict the evolution of competition and provide the motivation for using them

Transport protocols are mechanims that are used for transmitting data in the Internet. We focus on TCP (Transmission Control Protocol) which is a family of protocols that carry a vast majority of the packets in the Internet. Its objectives are

- (a) Detecting packets that are lost in the network, i.e. that did not reach the destination within a given time limit. Loss of packets may occur during congestion at buffers in routers along the path of the packets from the source to the destination. A congestion is a period during which the rate of arrival of packets to a buffer exceeds the rate at which the buffer can handle them. During such periods the number of buffered packets grows until the buffer fills and overflows, which causes losses. We note that losses may also be caused by noisy links and are common in wireless communication.
- (b) Retransmitting those packets from the source to the destination.
- (c) Adapting the transmission rate to the available bandwidth so as to avoid congestion. All variants of TCP are based on increasing gradually the transmission rate until congestion indications are received (in general these indications are simply the detection of a packet loss). This then triggers a decrease in the transmission rate.

Consider two types of protocols, A and B. We would like to project from measurements how will the future Internet look like, assuming that consumers will prefer to use protocols that perform better. Will it be composed only of aggressive or of friendly protocols? Or a mixture of them? In the latter case, what will be the fraction of aggressive protocols?

9.4 Predicting the protocoles that will dominatne Future Internet

There have been various approaches to answer the above questions.

A1(u) The isolation test See how well the protocol performs if everyone uses the friendly protocol only. Then imagine the world with the aggressive TCP only. Compare the two worlds. The version u for which users are happier is the candidate for the future Internet. Example for this prediction approach is [21][1]

[1]The author of this references writes in the concluding Section "The last issue, which was not addressed in this paper, concerns the deploying of TCP Vegas in the Internet. It may be argued that due to its conservative strategy, a TCP Vegas user will be severely disadvantaged compared to TCP Reno users, ... it is likely that TCP Vegas, which improves both the individual utility of the users and the global utility of the network, will gradually replace TCP Reno."

A2(u) The Confrontation test Consider interactions between agressive and pieceful sessions that share a common congested link. The future Intrnet is predicted to belong to the transport protocol of version u if u performs better in the interaction with v.

A3. Combined approach: Assume that a version u does better than v under both the isolation test as well as the confrontation test. We then call this version a strong TCP version and predicn that it will dominate future Internet. If there is no strong of TCP, we predict that both versions will co-exist. A weak TCP version is one that fails in both tests.

A4. Comparative test: Assume that everyone uses a version u of TCP and that one session starts using a version v instead of u. The comparative approach would choose u as a candidate for dominating the future internet if the performance of the deviating TCP (that uses version v) is strictly inferior to that of u. We then call u a strict Nash equilibrium in pure strategies. It is called a Nash Equilibrium in pure strategies if the inequality is non-strict.

A5. Indifference approach: Assume that some fraction p of the population uses version $T1$ and a fraction $1-p$ uses version $T2$. Assume that p is such that **the average performance of a protocol is the same under both u and v.** This is called the indifference test. When it holds then we say that the g ame has a Nash equilibrium $(p, 1-p)$ in mixed strategies.

9.5 The TCP Game: General Examples

Let us introduce a two player game with Sally, the first player, and Van, the second one. We assume that a bottleneck is sharead by two TCP coonnections. Each player controls one of these connections, and has to choose the version of the protocole among the two versions T1 and T2. We write the utilities in a matrix form where Sally chooses a row (indexed by T1 and T2) and Van chooses a column (indexed by T1 or T2).

Sally \ Van	$T1$	$T2$
$T1$	$2, 2$	$10, 0$
$T2$	$0, 10$	$3, 3$

Figure 9.1: Game in Example 1

Sally \ Van	$T1$	$T2$
$T1$	$3, 3$	$0, 1$
$T2$	$1, 0$	$2, 2$

Figure 9.2: Game in Example 2

Sally \ Van	$T1$	$T2$
$T1$	a, a	b, c
$T2$	c, b	d, d

Figure 9.3: General symmetric two by two matrix game

Example 1. *Consider the payoffs as given in Figure 9.1. Version $T1$ is seen to dominate version $T2$ in the Isolation Test, providing a utility of 3 units where as version $T2$ gets only 2 units in isolation. In contrast, the confrontation test as well as the comparative test select version $T1$.*

Example 2. *Consider next the situation in Figure 9.2. T1 is a strong strategy - it satisfies both the Isolation and Confrontation Tests, where as T2 is a weak strategy - it does not satisfy any of these. Yet both satisfy the comparative test and are thus Nash equilibria in pure strategies.*

Remark 7. *A word on the notation. The examples that we encounter in this Section are two by two symmetric matrix game. In the matrix representations that we had in the previous examples, the first chooses a row and the second chooses the column. The first of the two numbers in the correspondingn entry of the matrix is then the utility for player 1 and the second - that of player 2. Of course, row i (respectively, column j) corresponds to choosing T_i (respectively, T_j) by player 1 (resp. player 2).*

In two by two matrix games we shalll use sometime for simplicity T (Top) and B (bottom) to describe choices of rows, and L (left) and R (right) for the choice of column.

We restrict here to symmetric games. It is thus sufficient to have only one number in each of entries of the matrix G that represents the game. In that case, we define G_{ij}, the ij-th entry of the matrix G, to be the utility of player 1 when it uses action i and the other player uses action j. Due to symmetry, the utility of player 2 is then given by G_{ji}.

Remark 8. *The generic symmetric two by two matrix game form is given in Figure 9.3. If all parameters (a, b, c) are different, then one classifies the generic game into three classes.*

- *The case $(a - c)(d - b) < 0$ is called the Prisoner's Dilema game. It has one single pure equilibrium which is said to be dominating. Example 9.1 falls within this category.*
- *The case $a > c, d > b$ is called a coordination game. It has two pure equilibria: a, a and d, d, and one mixed equilibrium. Example 9.2 falls into this category.*
- *The case $a < c, d < b$. The game has one single equilibrium which is mixed. This game can be seen to satisfy the indifference criterion. The game is known as the Hawk and Dove game.*

9.6 The TCP Game: New Reno Vs Scalable TCP

Consider two types of protocols:

- (i) *aggressive*, which try to rapidly grab as much bandwidth as possible, and
- (ii) *friendly*, which are much slower to grab extra bandwidth.

More specifically, we shall consider the NR (New Reno) and Sc (Scalable) versions of TCP. Sally and Van have to choose which of the two TCP protocols they will use. They first simulate in ns-2 [44] (and repeat their experiment several times) and discover that when a bottleneck is shared by two TCP connections then

- NR vs NR: share fairly the link capacity. If the buffers are well dimensioned then each one's throughput is close to half the link's capacity.
- Sc vs Sc: Again, if the buffers are well dimensioned then the sum of throughputs of the connections will be the available bandwidth (speed at which packets leave the buffer). By symmetry, each one will receive half the bandwidth. However, unless the connections start at the same time, it will take a very long time till they share the buffer fairly, unlike NR vs NR where fairness is achieved very fast. We conclude that in Sc vs Sc there is a short term unfairness. We assume that users are unhappy when treated unfairly, and represent this with some cost δ.
- Sc vs NR: The share of NR denoted by α is smaller than that of Sc denoted by $1 - \alpha$. Thus $\alpha < 0.5$. There are no fairness issues here other than the fact that the shares are shared unfairly.

We summarize this in Fig. 9.4. We shall assume below that $a + d > 0.5$.

		Van H	Van D
Sally	H	$0.5 - \delta$	$1 - \alpha$
	D	α	0.5

Figure 9.4: Aggressive (H) versus friendly (D) TCPs

According to Remark 8, this is the "Hawk-Dove" game. It has a unique mixed equilibrium between aggressive (Hawk) behavior and the peaceful (Dove) one. We shall use H for Sc TCP and D for NR.

We observe that NR wins in the isolation test and that Sc wins in the confrontation test. Neither versions wins the comparison test, but the indifference principle holds. There exists a unique equilibrium which is mixed.

Reccall that we had assumed that $0.5 < a + d$. If we assume the opposite inequality to hold then the game becomes a prisoner's dilema game with a single pure equilibrium.

9.7 Predicting the Evolution of protocols

After introducing different tests and criteria and after introducing the TCP game, we are ready to examine the question of - which of the criteria are relevant in predicting the structure of the future Internet.

We adopt in this book game theoretic answers to the above question.

Game theory is concerned with predicting both the outcome (equilibrium) as well as the dynamics of competition. The Nash equilibria are the candidates for the possible stationary points of competition dynamics. This will be discussed in more details at later chapters. Nash equilibria have the property that if both players start initially at equilibrium and then react to each other by using say, at a round robin way, their optimal response to the other player, then the players remain at that equilibrium. In that sense the Nash equilibrium predicts potential stable outcomes of competition. Here we mean by Stable that no player can strictly do better by deviating unilaterally and playing another action than the equilibrium one.

Note: Nash equilibrium does not state what can or cannot happen when more than one decision maker changes their strategy (route) simultaneously.

Much later after the introduction of Nash equilibria, evolutionary equilibria notions appeared as well. The question of convergence when starting away from equilibria as well as convergence in the presence of mutations have been studied [51].

[51] showed under some technical conditions, that even if mutations continue to appear and we start away from equilibrium, we shall converge to a dominant equilibrium, if sucuh exists. Here, an equilibrium is said to be dominant if it outperforms any other equilibrium.

We saw that competition between two TCP versions can be formulated as either one of the following games: the prisoner's dilema, the Hawk-Dove game or the coordinated game. The two first ones have a unique equilibrium. Thus from the evolutionary game literature (e.g. [51]), we may predict that in the Hawk-Dove game (that arises in the competition between NR and Sc versions of TCP) both versions will coexist. We shall compute later in the chapter the proportions that each policy is used. If the game were to follow the prisoner's dilema then example then the future Internet would consist of one dominating version of TCP. In the coordination case we would expect to have convergence to the dominating equilibrium.

9.8 Background on the evolution of transport protocols

Today, NR turns out to be a very popular version of TCP. Although Sc is more aggressive than NR, it is more friendly than the TCP used more than twenty years ago, which did not at all adapt its rate to the congestion. At that time the Internet suffered from "congestion collapse" periods during which the throughput was very small and during which many packets were lost and had to be retransmitted. Van Jacobson then invented the first TCP that had an adaptive reaction to congestion, called TAHOE (TA). With TA, the congstion collapse was prevented. Thus in a game between TAHOE and the previous version, TA wins the isolation test and looses the confrontation test.

Reno version came after Tahoe. Reno is slightly more aggressive than Tahoe; in a game against Tahoe it wins both the isolation as well as the confrontation tests. It thus dominated Tahoe, which is thus not in use any more. NR is an improved version of Reno and in a game against Reno, NR dominates.

Much has been written on the comparison between NR and the Vegas version of TCP. The latter is more friendly than NR; it detects congestion not only through losses but also by measuring the end-to-end delay. In a game against NR, Vegas wins the isolation test but looses in the confrontation one.

In what follows, we shall provide more precise definitions of the equilibria and of the strategies.

9.9 Transport Layer game: TCP over wireless

During the last few years, many researchers have been studying TCP performances in terms of energy consumption and average goodput within wireless networks [88, 105]. Via simulation, the authors show that the TCP New-Reno can be considered as well performing within wireless environment among all other TCP variants and allows for greater energy savings. Indeed, a less aggressive TCP, as TCP New-Reno, may generate lower packet loss than other aggressive TCP. Thus the advantage of an aggressive TCP in terms of throughput could be compensated with energy efficiency of a more gentle TCP version. (In Section **??** we shall illustrate another consideration that affects the competition between TCP versions.) The goal of this section is to illustrate this point, as well as its possible impact on the evolution of the share of TCP versions, through a simple model of an aggressive TCP.

The model. We consider two populations of connections, all of which use AIMD TCP. A connection of population i is characterized by a linear increase rate α_i and a multiplicative decrease factor β_i. Let $\zeta_i(t)$ be the transmission rate of connection i at time t. We consider the following simple model for competition.

(i) The RTT (round trip times) are the same for all connections.

(ii) There is light traffic in the system in the sense that a connection either has all the resources it needs or it shares the resources with one other connection. (If files are large then this is a light regime in terms of number of connections but not in terms of workload).

(iii) Losses occur whenever the sum of rates reaches the capacity C: $\zeta_1(t) + \zeta_2(t) = C$.

(iv) Losses are synchronized: when the combined rates attain C, both connections suffer from a loss. This synchronization has been observed in simulations for connections with RTTs close to each other [4]. The rate of connection i is reduced by the factor $\beta_i < 1$.

(v) As long as there are no losses, the rate of connection i increases linearly by a factor α_i.

We say that a TCP connection i is more aggressive than a connection j if $\alpha_i \geq \alpha_j$ and $\beta_i \geq \beta_j$. Let $\overline{\beta}_i := 1 - \beta_i$. Let y_n and z_n be the transmission rates of connection i and j, respectively, just before a loss occurs. We have $y_n + z_n = C$. Just after the loss, the rates are $\beta_1 y_n$ and $\beta_2 z_n$. The time it takes to reach again C is

$$T_n = \frac{C - \beta_1 y_n - \beta_2 z_n}{\alpha_1 + \alpha_2}$$

which yields the difference equation:

$$y_{n+1} \quad = \quad \beta_1 y_n + \alpha_1 T_n = q y_n + \frac{\alpha_1 C \overline{\beta}_2}{\alpha_1 + \alpha_2}$$

where $q = \frac{\alpha_1 \beta_2 + \alpha_2 \beta_1}{\alpha_1 + \alpha_2}$. The solution is given by

$$y_n = q^n y_0 + \left(\frac{\alpha_1 C \overline{\beta}_2}{\alpha_1 + \alpha_2} \right) \frac{1 - q^n}{1 - q}.$$

HD game: throughput-loss tradeoff

In wireline, the utility related to file transfers is usually taken to be the throughput, or a function of the throughput (e.g. the delay). It does not explicitly depend on the loss rate. This is not the case in wireless context. Indeed, since TCP retransmits lost packets, losses present energy inefficiency. Since energy is a costly resource in wireless, the loss rate is included explicitly in the utility of a user through the term representing energy cost. We thus consider fitness of the form $J_i = Thp_i - \lambda R$ for connection i; it is the difference between the throughput Thp_i and the loss rate R weighted by the so called tradeoff parameter, λ, that allows us to model the tradeoff between the valuation of losses and throughput in the fitness. We now proceed to show that our competition model between aggressive and non-aggressive TCP connections can be formulated as a HD game. We study how the fraction of aggressive TCP in the population at (the mixed) ESS depends on the tradeoff parameter λ.

Since $|q| < 1$, we get the following limit $\overline{y}$ of y_n when $n \to \infty$:

$$\overline{y} = \frac{\alpha_1 C \overline{\beta}_2}{\alpha_1 + \alpha_2} \cdot \frac{1}{1-q} = \frac{\alpha_1 \overline{\beta}_2 C}{\alpha_1 \overline{\beta}_2 + \alpha_2 \overline{\beta}_1}.$$

It is easily seen that the share of the bandwidth (just before losses) of a user is increasing in its aggressiveness. Hence the average throughput of connection 1 is

$$Thp_1 = \frac{1+\beta_1}{2} \times \frac{\alpha_1 \overline{\beta}_2}{\alpha_1 \overline{\beta}_2 + \alpha_2 \overline{\beta}_1} \times C.$$

The average loss rate of connection 1 is the same as that of connection 2 and is given by

$$R = \frac{1}{T} = \left(\frac{\alpha_1}{\overline{\beta}_1} + \frac{\alpha_2}{\overline{\beta}_2} \right) \frac{1}{C} \text{ where } T = \frac{\overline{\beta}_1 \overline{\beta}_2 C}{\alpha_1 \overline{\beta}_2 + \alpha_2 \overline{\beta}_1}$$

with T being the limit as $n \to \infty$ of T_n.

Let H corresponds to (α_H, β_H) and D to (α_D, β_D) such that $\alpha_H \geq \alpha_D$ and $\beta_H \geq \beta_D$. Then, for $i = 1, 2$, $Thp_i(H, H) = Thp_i(D, D)$. Since the loss rate for any user is increasing in $\alpha_1, \alpha_2, \beta_1, \beta_2$ it then follows that $J(H, H) < J(D, D)$, and $J(D, H) < J(D, D)$. We conclude that the utility that describes a tradeoff between average throughput and the loss rate leads to the HD structure.

The mixed ESS is given by the following probability of using H:

$$x^*(\lambda) = \frac{\eta_1 - \eta_2 \lambda}{\eta_3} \quad \text{where}$$

$$\eta_1 = \left(\overline{\mu} \frac{1+\beta_1}{2} - \frac{1+\beta_2}{4} \right) C, \ \eta_2 = \frac{1}{C} \left(\frac{\alpha_1}{\overline{\beta}_1} - \frac{\alpha_2}{\overline{\beta}_2} \right),$$

$$\eta_3 = C(\frac{1}{2} - \mu) \frac{\beta_1 - \beta_2}{2}, \ \mu = \frac{\alpha_2(\overline{\beta}_1)}{\alpha_2(\overline{\beta}_1) + \alpha_1(\overline{\beta}_2)}.$$

where $\overline{\mu} := 1 - \mu$. Note that η_2 and η_3 are positive. Hence, the equilibrium point x^* decrease linearly on λ. We conclude that applications that are more sensitive to losses would be less aggressive at ESS (Braess type paradoxes do not occur here).

For more details on this model, including the tradeoff between transient and steady-state behavior, we refer the reader to [10].

Part V

Part 5: Economics of Networking

Chapter 10

Network Economy and the network neutrality

10.1 Definition and properties

Stackelberg game one introduces hierarchy between players. Actions are not taken anymore simultaneously. In a two level game, there are leaders that take actions first and then the others follow and take actions accordinigly. There is thus assymetry in the information: A follower decides knowing the leaders' current move.

This can be viewed as a game where the solution notion is a Nash equilibrium (so we "forget" the asymetry) but where the actions of the followers are in fact "strategies": they are functions of what the actions of the other player.

In a two player zero sum game, the two possible notions of Stackelberg give the upper and the lower value, respectively.

This remaining of the Chapter is based on [15].

10.2 Introduction to the network neutrality issue

Network neutrality is an approach to providing network access without unfair discrimination among applications, content, nor the specific source of traffic. What is discrimination and what is fair discrimination? If there are two applications or two services or two providers that require the same network resources and one is offered better quality of service (shorter delays, higher transmission capacity, etc.) then there is a discrimination. When is a discrimination "fair"[1]? A preferential treatment of traffic is considered fair as long as the preference is left for the user[2]. Internet service

[1] The recent decision on Comcast v. the FCC was expected by the general public to deal with the subject of "fair" traffic discrimination, as the FCC ordered Comcast to stop interfering with subscribers traffic generated by peer-to-peer networking applications. The Court of Appeals for the District of Columbia Circuit was asked to review this order by Comcast, arguing not only on the necessity of managing scarce network resources, but also on the non-existent jurisdiction of the FCC over network management practices. The Court decided that the FCC did not have express statutory authority over the subject, neither demonstrated that its action was "reasonably ancillary to the ... effective performance of its statutorily mandated responsibilities". The FCC was deemed, then, unable to sanction discrimatory practices on Internet's traffic carried out by american ISPs, and the underlying case on the "fairness" of their discriminatory practices was not even discussed.

[2] Nonetheless, users are just one of many actors in the net neutrality debate, which has been enliven throughout the world by several public consultations on new legislation on the subject. The first one, proposed in the USA (expired on 26/04/2010), was looking for the best means of preserving a free and open Internet. The second one, carried out in France (finishing 17/05/2010), asks for the different points of view over net neutrality. A third one is intended to be presented by the UE in the summer of 2010, looking for a balance on the parties concerned as users are entitled to an access the services they want, while ISPs and CPs should have the right incentives and opportunities to keep investing, competing and innovating. See [1, 2, 57].

providers (ISPs) may have interest in discrimination either for technological problems or for economic reasons. Traffic congestion has been a central argument for the need to discriminate traffic (for technological reasons) and moreover, for not practicing network neutrality, in particular to deal with high-volume peer-to-peer traffic. However, many ISPs have been blocked or throttled p2p traffic independently of congestion conditions.

There may be many hypothetical ways to violate the principle of network neutrality. Hahn and Wallsten wrote that network neutrality "usually means that broadband service providers charge consumers only once for Internet access, do not favor one content provider over another, and do not charge content providers for sending information over broadband lines to end users." (p. 1 of [37]) We therefore restrict our attention here to the practices of these types of network neutrality.

That net neutrality acts as a disincentive for capacity expansion of their networks, is an argument recently raised by ISPs. In [23] the validity of this claim was checked. Their main conclusion is that under net neutrality the ISPs invest to reach the social optimal level, while under-or-over investing is the result when net neutrality is dropped. In this case, ISPs stand as winners, while content providers (CP) move to a worst position. Users that rely on services that have paid the ISPs for preferential treatment will be better off, while the rest of the users will have a significantly worse service.

ISPs often justify charging content providers by their need to cover large and expensive amount of network resources. This is in particular relevant in the 3G wireless networks where huge investments were required for getting licenses for the use of radio frequencies. On the other hand, the content offered by a CP may be the most important source of the demand for Internet access; thus, the benefits of the access providers are due in part to the content of the CPs. It thus seems "fair" that benefits that ISP make of that demand would be shard by the CPs.

We find this notion of fair sharing of reveneus between economic actors in the heart of cooperative game theory. In particular, the Shapley value approach for splitting reveneus is based on several axioms and the latter fairness is on of them. Many references have advocated the use of the Shapley value approach for sharing the profits between the providers, see, *e.g.*, [62, 63]. We note however that the same reasoning used to support payments by access providers to content providers (in the context of can be used in the opposite direction. Indeed, many CPs receive third party income such as advertising revenue thanks to the user demand (eyeballs) that they create. Therefore, using a Shapley value approach would require content providers to help pay for the network access that is necessary to create this new income.

The goal is to study the impact of such side payments between providers on the utilities of all actors. More precisely, we study implications of one provider being at a dominating position so as to impose payments from the other one[3]. We examine these questions using simple game theoretic tools. We show how side payments may be harmful for all parties involved (users and providers).

Another way to favor a provider over another one is to enforce a leader-follower relation to determine pricing actions. We show how this too can be harmful for all.

10.3 Basic model: three collective actors and usage-based pricing

We consider the following simple model of three actors,

- the internauts (users) collectively,
- a network access provider for the internauts, collectively called ISP1, and
- a content provider and its ISP, collectively called CP2.

[3] In the European Union, such dominating positions in the telecommunications markets are controlled by the article 14, paragraph 3 of the Directive 2009/140/EC, considering the application of remedies to prevent the leverage of a large market power over a secondary market closely related.

In this section, the two providers are assumed peers; leader-follower dynamics are considered in Section 10.11 below. The internauts pay for service/content that requires both providers. Assume that they pay $p_i \geq 0$ to provider i (CP2 being $i = 2$ and ISP1 being $i = 1$) and that their demand is given by

$$D = D_0 - pd$$

where

$$p = p_1 + p_2 \geq 0, \ \ D \geq 0.$$

So, provider i's revenues are

$$U_i = Dp_i, \quad i = 1, 2.$$

10.4 Collaboration

The total price that the providers can obtain if they cooperate is maximized at $p_i = D_0/(4d)$. The total revenue per provider is then $U_i^{max} = D_0^2/(8d)$. The demand is then $D_0/2$.

10.5 Fair competition

If the providers do not cooperate then the utility of provider i is obtained by computing the Nash equilibrium. We get:

$$\frac{\partial U_i}{\partial p_i} = D - p_i d = 0, \ \ i = 1, 2. \tag{10.1}$$

This gives $p_1 = p_2 = D_0/(3d)$. The demand is now $D_0/3$, larger than in the cooperative case, and the revenue of each provider is $D_0^2/9$, less than before.

Next consider the competitive model and assume we install *side payments*: CP2 is requested to pay p_3 to ISP1 for "transit" costs. So, the revenues of the providers are:

$$\begin{aligned} U_1 &= [D_0 - (p_1 + p_2) \cdot d] (p_1 + p_3) \\ U_2 &= [D_0 - (p_1 + p_2) \cdot d] (p_2 - p_3) \end{aligned}$$

As the model so far is symmetric, we can in fact allow for negative value of p_3 which would model payment from the ISP1 to CP2 instead, *e.g.*, payment for copyright, as discussed below.

10.6 Discussion of side payments

At this point we render it asymmetric by assuming that p_3 is determined by ISP1 for the case $p_3 > 0$, *i.e.* additional transit revenue from the content provider in a "two sided" payment model to ISP1 [38, 69]. Then, unless $D = 0$ there is no optimal p_3: as it increases, so does U_1. Thus, at equilibrium necessarily $D = 0$, and the revenues of both service and content providers are 0. Hence p_1 and p_2 sum up to D_0/d. Then by decreasing p_1 slightly, the demand will become strictly positive, so ISP1 can increase its utility by U_1 without bound by choosing p_3 sufficiently large. Therefore, at equilibrium $p_1 = 0$ and $p_2 = D_0/d$. If $p_2 > p_3$ then by a slight decrease in p_2, U_2 strictly increases so this is not equilibrium. We conclude that at equilibrium, $p_3 \geq p_2$. To summarize, the set of equilibria is given by $\{p_1 = 0, p_2 = D_0/d$ and $p_3 \geq D_0/d\}$.

Thus by discriminating one provider over the other and letting it charge the other provider, both providers lose. Obviously the internauts do not gain anything either, as their demand is zero!

We have considered above side payment from the CP2 to ISP1. In practice, the side payment may go in the other direction. Indeed, there is a growing literature that argues that ISP1 has to pay to CP2. This conclusion is based on cooperative game theory (and in particular on Shapley values) which stipulates that if the presence of an economic actor A in a coalition creates revenue to another actor B, then actor A ought to be paid proportionally to the benefits that its presence in the coalition created. In our case, the CP2 creates a demand of users who need Internet access, and without the CP2, ISP1 would have less subscribers.

The use of Shapley value (and of a coalition game approach, rather than of a non-cooperative approach) has the advantage of achieving Pareto optimality. In particular this means that the total revenue for ISP1 and CP2 would be those computed under the cooperative approach.

Side payment to the CP2 from ISP1 may also represent payment to the copyright holders of the content being downloaded by the internauts. In particular, a new law is proposed in France, by a member of parliament of the governing party, to allow download of unauthorized copyright content and in return be charged *proportionally* to the volume of the download, with an average payment of about five euros per month. A similar law had been already proposed and rejected five years ago by the opposition in France. It suggested to apply a tax of about five euros on those who wish to be authorized to download copyrighted content. In contrast, the previously proposed laws received the support of the trade union of musicians in France. If these laws were accepted, the service providers would have been requested to collect the tax (that would be paid by the internauts as part of their subscription contract). Note that although $p_3 < 0$ in our model could represent these types of side payments, the copyright payments per user are actually not decision variables.

10.7 Revenue generated by advertising

We now go back to the basic collaborative model to consider the case where the CP2 has an additional source of revenue from advertisement that amounts to $p_4 D$. p_4 is assumed to be a constant. The total income of the providers is

$$\Pi = (D_0 - pd)(p + p_4) \tag{10.2}$$

Then

$$\frac{\partial \Pi}{\partial p} = D_0 - 2pd - dp_4 \tag{10.3}$$

Equating to zero, we obtain

$$p = \frac{D_0 - p_4 d}{2d} \tag{10.4}$$

The total demand is $(D_0 + p_4 d)/2$, and the total revenues at equilibrium are

$$U_t^{\max} = \frac{D_0^2 + 2\, d\, p_4\, D_0 + d^2\, p_4^2}{4\, d} \tag{10.5}$$

This result does not depend on the way the revenue from the internauts is split between the providers.

10.8 The case where $p_2 = 0$

In particular, the previous result covers the case where $p_2 = 0$, *i.e.*, the case where advertising is the only source of revenue for the content provider CP2. One may consider this to be the business model of the collective consisting of (i) BitTorrent permanent seeders and (ii) specialized torrent file resolvers (*e.g.*, Pirate Bay).

Note that BitTorrent permanent seeders may be *indifferent* to downloading to BitTorrent leecher clients (particularly during periods of time when the seeder workstations are not otherwise being used) because of flat-rate pricing for network access, *i.e.*, a flat-rate based on capacity without associated usage-based costs (not even as overages).

10.9 Best response

The utilities for the network access provider ISP1 and the content provider CP2 are, respectively,

$$U_1 = [D_0 - (p_1 + p_2) \cdot d]\,(p_1 + p_3) \tag{10.6}$$

and

$$U_2 = [D_0 - (p_1 + p_2) \cdot d]\,(p_2 - p_3 + p_4). \tag{10.7}$$

We first show that for any p_2, it is optimal for the ISP1 to choose $p_1 = 0$. First consider the problem of the best choice of p_1 and p_3 assuming the quantity $p_1 + p_3$ is constant; clearly, U_1 strictly decreases in p_1 so that a best response cannot have $p_1 > 0$.

Thus, if p_2 is not controlled (in particular if $p_2 = 0$ so that CP2's only revenue is from a third party and not directly from the users), then ISP1 would gain more by charging the CP2 than by charging the users. This is also consistent with the simple fact that $\partial U_1/\partial p_3 \;\geq\; \partial U_1/\partial p_1$.

10.10 Nash equilibrium

With $p_1 = 0$ and $p_3 \geq 0$, the utility of ISP1 is

$$U_1 = [D_0 - p_2 d]\,p_3 \tag{10.8}$$

Thus the condition on the best response of ISP1 for a given p_2 gives $p_2 = D_0/d$, *i.e.*, the demand is zero. On the other hand, for this p_2 to be a best response for U_2, $p_3 = p_2 + p_4$. We conclude that there is a unique Nash equilibrium given by $p_1 = 0$, $p_2 = D_0/d$, and $p_3 = D_0/d + p_4$.

10.11 Stackelberg equilibrium in network neutrality

Stackelberg equilibrium corresponds to another aspect of asymmetric competition, in which one competitor is a leader and the other a follower. Actions are no longer taken independently: here, first the leader takes an action, and then the follower reacts to this action.

Let's restrict to $p_3 \geq 0$.

We assume that the ISP1 is the leader. Given p_1 and p_3, U_2 is concave in p_2. So, a necessary and sufficient condition for p_2 to maximize this is

$$\frac{\partial U_2}{\partial p_2} = D_0 - d \cdot (p_2 - p_3 + p_4) - d \cdot (p_1 + p_2) = 0 \tag{10.9}$$

holds with equality for $p_2 > 0$. That is, to maximize U_2,

$$p_2 = \frac{1}{2}\Big(\frac{D_0}{d} + p_3 - p_1 - p_4\Big) \;> 0. \tag{10.10}$$

Substituting p_2 in U_1, we obtain:

$$\begin{aligned} U_1 &= [D_0 - (p_1 + p_2) \cdot d]\,(p_1 + p_3) \\ &= \frac{1}{2}\,[D_0 - 3p_1 d - p_3 d + p_4 d]\,(p_1 + p_3) \end{aligned}$$

We now compute the actions that maximize the utility U_1 which is concave in (p_1, p_3). We have

$$\frac{\partial U_1}{\partial p_1} = \frac{D_0 - 4dp_3 - 6dp_1 + dp_4}{2} \leq 0 \tag{10.11}$$

$$\frac{\partial U_1}{\partial p_3} = \frac{D_0 - 2dp_3 - 4dp_1 + dp_4}{2} \leq 0 \tag{10.12}$$

For $p_1 > 0$, (10.11) should hold as equality. Subtracting (10.11) from (10.12) we get $p_3 \leq -p_1$, and hence they are zero. This conclusion is in contradiction with our assumption $p_1 > 0$.

Assume that $p_1 = 0$ and $p_3 > 0$. Then U_1 is concave in p_3 and (10.12) holds with equality. Hence

$$p_3 = \frac{D_0}{2d} + \frac{p_4}{2} \tag{10.13}$$

maximizes U_1. Substituting in (10.10) we get

$$p_2 = \frac{1}{4}\Big(\frac{3D_0}{d} - p_4\Big) \tag{10.14}$$

We conclude that if $p_4 d < 3D_0$ Then the Nash equilibrium is $p_1 = 0$, and p_3 and p_2 are given, respectively, by (10.13) and (10.14).

Since we assume here that $p_2 \geq 0$, then in case $p_4 d \geq 3D_0$, we will have $p_2 = 0$ since this value maximizes (10.14).

10.12 Conclusions and on-going work

Using a simple, parsimonious model of linearly diminishing user/consumer demand as a function of price, we studied a game between collective players, the user ISP and content provider, under a variety of scenarios including: non-neutral two-sided transit pricing, copyright payments made by the ISP, the effects of flat-rate pricing, advertising revenue, cooperation, and leadership. In particular, we demonstrated under what conditions non-neutral transit pricing of content providers may result in revenue loss for all parties in play (*i.e.*, so that at least one player opts out of the game, where all players are necessary for positive outcome).

In on-going work, we are considering issues of non-monetary value and copyright. Moreover, we are including the users as active players. Finally, we are considering the effects of content-specific (not *application* neutral) pricing.

10.13 Exercise

Find all Stackelberg equilibria in the following coordination game given in Table 10.1. Are there equilibria in mixed strategies?

	action 1.a	action 1.b
action 2.i	$2, 1$	$0, 0$
action 2.ii	$0, 0$	$1, 2$

Table 10.1: A coordination game [Aumann]

Part VI

Part 6: Appendices

Chapter 11

Appendix: Nash bargaining and fair assignment

11.1 Nash Bargaining

Let a finite number of rational individuals collaborate in order to get mutual benefit. Assume they can compare their satisfaction from the possession of the objects of the bargaining. We can then associate the users to a utility function, which is, obviously, not unique: if u is such a function, then $au + b$ is an equivalent one (for $a, b \in \mathbb{R}, a > 0$). In the case where the players cannot find an agreement, the game ends at the "disagreement point", characterized by a certain utility, u^0.

Let $X \subset \mathbb{R}^n$ denote the set of possible strategies. It is a convex closed and non empty set. The utility functions, $f_i : X \to \mathbb{R}, i = 1, \ldots, k$, are supposed to be upper bounded functions. The set of achievable utilities, U, $U \subset R^k$ such that $U = \{u \in \mathbb{R}^k | x \in X, u = (f_1(x), ..., f_k(x))\}$ is non empty, convex and closed and $u^0 \in \mathbb{R}^k$ is the utility from which the players accept to bargain. Finally, we denote by U^0 the set $U^0 = \{u \in U | u^0 \leq u\}$, the subset of U in which the players achieve more than their minimum requirements. Similarly, we define $X_0 = \{x \in X | \forall i, f_i(x) \geq u_i^0\}$.

Definition 3. *A mapping $S : (U, u^0) \to \mathbb{R}^n$ is said to be an NBP (Nash bargaining point) if:*

1. *it guarantees the **minimum required performances**: $S(U, u^0) \in U^0$.*
2. *$S(U, u^0)$ is **Pareto optimal**.*
3. *It is **linearly invariant**, i.e. the bargaining point is unchanged if the performance objectives are affinely scaled. More precisely, if $\phi : \mathbb{R}^n \to \mathbb{R}^n$ is a linear map such that $\forall i, \phi_i(v) = a_i v_i + b_i$, then $S(\phi(u), \phi(u^0)) = \phi(S(U, u^0))$.*
4. *S is **symmetric**, i.e. does not depend on the specific labels. Hence, connections with the same minimum performance u_i^0 and the same utilities will have the same performances.*
5. *S is **not affected by reducing the domain** if a solution to the problem with the larger domain can be found on the restricted one. More precisely, if $V \subset U$, and $S(U, u^0) \in V$ then $S(U, u^0) = S(V, u^0)$.*

Definition 4. *The point $u^* = S(U, u^0)$ is called the Nash Bargaining Point and $f^{-1}(u^*)$ is the set of Nash Bargaining Solutions.*

We have the equivalent optimization problem:

Theorem 10. *[102, Thm. 2.1, Thm 2.2] and [?]. Let the utility functions f_i be concave, upper-bounded, defined on X which is a convex and compact subset of $\mathbb{R}^n$. Let J be the set of users able to achieve a performance strictly superior to their initial performance, i.e. $J = \{j \in \{1, ..., N\} | \exists x \in$*

X_0, s.t. $f_j(x) > u_j^0$}. Assume that $\{f_j\}_{j \in J}$ are injective. Then there exists a unique NBP as well as a unique NBS x that verifies $f_j(x) > u_j(x), j \in J$, and is the unique solution of the problem:

$$\max \prod_{j \in J} (f_j(x) - u_j^0), \quad x \in X_0. \tag{11.1}$$

Equivalently, it is the unique solution of $\max \sum_{j \in J} \ln(f_j(x) - u_j^0), \quad x \in X_0$.

In 1991, [65] adapted the Nash bargaining solution to Jackson networks. Some years after, Yaïche et al. [102] adapted it for bandwidth allocation in networks. In their works however, they restricted themselves to linear utility functions.

We can note that the NBS corresponds to proportional fairness in the case where the utility functions are linear and where the MR_i are null.

Remark 9 (Other fairness criteria defined through a set of axioms)**.** *The last axiom (5) of the Nash Bargaining Point has suffered some criticisms, as it does not take into account how much the other players have given up. Two other interesting fairness criteria were then defined when modifying this last axiom, namely the Raiffa-Kalai-Smorondinsky and the Thomson (or "utilitarian choice rule") solution [**?**]. We do not treat here in more detail these two criteria as they correspond respectively of special cases of max-min fairness (Section* **??***) and the maximization of the global throughput (Section* **??***) when considering the utility of the applications.*

Chapter 12

S-modular games

We have seen in several examples in previous Sections that equilibria best response strategies have either the "Join the Crowd" property or the the "Avoid the Crowd" property, which typically leads to threshold equilibria policies. These properties turn out to be also useful when we seek to obtain convergence to equilibria from a non-equilibrium initial point. These issues will be presented in this section within the framework of S-modular games due to Yao [103] who extends the notion of submodular games introduced by Topkis [96].

12.1 Model, definitions and assumptions

General model are developed in [96, 103] for games where the strategy space S_i of player i is a compact sublattice of $R^{\overline{m}}$. By sublattice we mean that it has the property that for any two elements x, y that are contained in S_i, also $\min(x, y)$ (denoted by $x \wedge y$) and $\max(x, y)$ (denoted by $x \vee y$) are contained there (by $max(x, y)$ we mean the componentwise max, and similarly with the min). We describe below the main results for the case that $\overline{m} = 1$.

Definition 5. *The utility f_i for player i is supermodular if and only if*

$$f_i(x \wedge y) + f_i(x \vee y) \geq f_i(x) + f_i(y).$$

It is submodular if the opposite inequality holds.

If f_i is twice differentiable then supermodularity is equivalent to

$$\frac{\partial^2 f_i(x)}{\partial x_1 \partial x_2} \geq 0.$$

Monotonicity of maximizers. The following important property was shown to hold in [96]. Let f be a supermodular function. Then the maximizer with respect to x_i is increasing in $x_j, j \neq i$.

More precisely, define the best response

$$BR_1^*(x_2) = argmax_{x_1} f(x_1, x_2);$$

if there are more than one argmax above we shall always limit ourselves to the smallest one (or always limit ourselves to the largest one). Then $x_2 \leq x_2'$ implies $BR_1^*(x_2) \leq BR_1^*(x_2')$. This monotonicity property holds also for non-independent policy sets such as (12.1), provided that they satisfy the ascending property (defined below).

Definition 6. (**Monotonicity of sublattices**) *Let A and B be sublattices. We say that $A \prec B$ if for any $a \in A$ and $b \in B$, $a \wedge b \in A$ and $a \vee b \in B$.*

Next, we introduce some properties on the policy spaces.
Consider two players. We allow S_i to depend on x_j

$$S_i = S_i(x_j), \quad i,j = 1,2, \quad i \neq j. \tag{12.1}$$

Monotonicity of policy sets We assume

$$x_j \leq x'_j \Longrightarrow S_i(x_j) \prec S_i(x'_j).$$

This is called the **Ascending Property**. We define similarly the **Descending Property**.

Lower semi continuity of policies We say that the point to set map $S_i(\cdot)$ is lower semi continuous if for any $x_j^k \to x_j^*$ and $x_i^* \in S_i(x_j^*)$ ($j \neq i$), there exist $\{x_i^k\}$ s.t. $x_i^k \in S_i(x_j^k)$ for each k, and $x_i^k \to x_i^*$.

12.2 Existence of Equilibria and Round Robin algorithms

Consider an n-player game. Yao [103, Algorithm 1] and Topkis [96, algorithm I] consider a greedy round robin scheme where at some infinite strictly increasing sequence of time instants T_k, players update their strategies using each the best response to the strategies of the others. Player l updates at times T_k with $k = mn + l$, $m = 1, 2, 3, ...$

Assume lower semi-continuity and compactness of the strategy sets. Under these conditions, Supermodularity together with the ascending property imply monotone convergence of the payoffs to an equilibrium [103]. The monotonicity is in the same direction for all players: the sequences of strategies for each player either all increase or all decrease.

The same type of result is also obtained in [103, Thm. 2.3] with submodularity instead of supermodularity for the case of two players, where the ascending property is replaced by the descending property. The monotone convergence of the round robin policies still holds but it is in opposite directions: the sequence of responses of one player increases to his equilibrium strategy, while the ones of the other player decreases.

In both cases, there need not be a unique equilibrium.

Yao [103] further extends these results to cases of costs (or utilities) that are submodular in some components and supermodular in others. The notion of s-modularity is used to describe either submodularity or supermodularity. Another extension in [103] is to vector policies (i.e. a strategy of a player is in a sublattice of R^k).

Next we present several examples for games in queues where s-modularity can be used. The first two examples are due to Yao [103].

12.3 Example of supermodularity: queues in tandem

Consider a set of queues in tandem. Each queue has a server whose speed is controlled. The utility of each server rewards the throughput and penalizes the delay. Under appropriate conditions, it is then shown in [103] that the players have compatible incentives: if one speeds up, the other also want to speed up.

More precisely, consider two queues in tandem with i.i.d. exponentially distributed service times with parameters μ_i, $i = 1, 2$. Let $\mu_i \leq u$ for some constant u. Server one has an infinite source of input jobs There is an infinite buffer between server 1 and 2. **The throughput** is given by $\mu_1 \wedge \mu_2$.

The expected number of jobs in the buffer is given [103] by

$$\frac{\mu_1}{\mu_2 - \mu_1}$$

when $\mu_1 < \mu_2$, and is otherwize infinite.

Let

- $p_i(\mu_1 \wedge \mu_2)$ be the profit of server i,
- $c_i(\mu_i)$ be the operating cost,
- $g(\cdot)$ be the inventory cost.

The utilities of the players are defined as

$$f_1(\mu_1, \mu_2) := p_1(\mu_1 \wedge \mu_2) - c_1(\mu_1) - g\left(\frac{\mu_1}{\mu_2 - \mu_1}\right)$$

$$f_2(\mu_1, \mu_2) := p_2(\mu_1 \wedge \mu_2) - c_2(\mu_2) - g\left(\frac{\mu_1}{\mu_2 - \mu_1}\right).$$

The strategy spaces are given by

$$S_1(\mu_2) = \{\mu_1 : 0 \le \mu_1 \le \mu_2\},$$

$$S_2(\mu_1) = \{\mu_2 : \mu_1 \le \mu_2 \le u\}.$$

It is shown in [103] that if g is convex increasing then f_i are supermodular. So we can apply the results of the previous subsection, and obtain (1) the property of "joining the crowd" of the best response policies, (2) existence of an equilibrium, (3) convergence to equilibrium of some round robin dynamic update schemes.

12.4 Flow control

We consider now an example for submodularity. There is a single queueing centre with two input streams with Poisson arrivals with rates λ_1 and λ_2. The rates of the streams are controlled by 2 players.

The queueing center consists of c servers and no buffers. Each server has one unit of service rate.

When all servers are occupied, an arrival is blocked and lost.

The blocking probability is given by the Erlang loss formula:

$$B(\lambda) = \frac{\lambda^c}{c!}\left[\sum_{k=0}^{c} \frac{\lambda^k}{k!}\right]^{-1}$$

where $\lambda = \lambda_1 + \lambda_2$.

Suppose user i maximizes

$$f_i = r_i(\lambda_i) - c_i(\lambda B(\lambda)).$$

c_i is assumed to be convex increasing. $\lambda B(\lambda)$ is the total loss rate.

Then it is easy to check [103] that f_i are submodular.

Two different settings can be assumed for the strategy sets. In the first, the available set for player i consists of $\lambda_i \le \overline{\lambda}$. Alternatively, we may consider that the strategy sets of the players depend on each other and the sum of input rates has to be bounded: $\lambda \le \overline{\lambda}$. Then S_i satisfy the descending property. We can thus apply again the results of the section 12.2.

12.5 A flow versus service control

Exponentially inter-arrivals as we used in previous examples are quite appealing to handle mathematically, and they can model sporadic arrivals, or alternatively, information packets that arrive one after the other but whose size can be approximated by an exponential random variable. In this example we consider, in contrast, a constant time T between arrivals of packets, which can be used for modeling ATM (Asynchronous Transfer Mode) networks in which information packet have a fixed size.

We consider a single node with a periodic arrival process, in which the first player controls the constant time period $T \in [\underline{T}, \overline{T}]$ between any two consecutive arrivals. We consider a single server with no buffer. The service time distribution is exponential with a parameter $\mu \in [\underline{\mu}, \overline{\mu}]$, which is controlled by the second player. If an arrival finds the server busy then it is lost. The loss probability is given by

$$P_l = \exp(-\mu T),$$

which is simply the probability that the random service time of (the previous) customer is greater than the constant T.

The transmission rate of packets is T^{-1}, but since a fraction P_l is lost then the goodput (the actual rate of packets that are transmitted successfully) is

$$G = \frac{1}{T}(1 - \exp(-\mu T)).$$

We assume that the utility of the first player is the goodput plus some function of the input rate T^{-1}. The server earns a reward that is also proportional to the goodput, and has some extra operation costs which is a function of the service rate μ. In other words,

$$J_1(T, \mu) = \frac{1}{T}(1 - \exp(-\mu T)) + f(T^{-1}), \qquad J_2(T, \mu) = \frac{1}{T}(1 - \exp(-\mu T)) + g(\mu).$$

We then have for $i = 1, 2$

$$\frac{\partial {J_i}^2}{\partial T \partial \mu} = -\mu \exp(-\mu T) \leq 0.$$

We conclude that the cost is submodular.

Chapter 13

Evolutionary Stable Strategies

Consider a large population of players. Each individual needs occasionally to take some action. We focus on some (arbitrary) tagged individual. The actions of some M (possibly random number of) other individuals may interact with the action of the tagged individual (e.g. some other connections share a common bottleneck). In order to make use of the wealth of tools and theory developed in the biology literature, we shall restrict here (as they do), to interactions that are limited to pairwise, i.e. to $M = 1$. This will correspond to networks operating at light loads, such as sensor networks that need to track some rare events such as the arrival at the vicinity of a sensor of some tagged animal.

We define by $J(p, q)$ the expected payoff for our tagged individual if it uses a strategy p when meeting another individual who adopts the strategy q. This payoff is called “fitness" and strategies with larger fitness are expected to propagate faster in a population.

We assume that there are N pure strategies. A strategy of an individual is a probability distribution over the pure strategies. An equivalent interpretation of strategies is obtained by assuming that individuals choose pure strategies and then the probability distribution represents the fraction of individuals in the population that choose each strategy.

Suppose that the whole population uses a strategy q and that a small fraction ϵ (called “mutations") adopts another strategy p. Evolutionary forces are expected to select q against p if

$$J(q, \epsilon p + (1-\epsilon)q) > J(p, \epsilon p + (1-\epsilon)q) \tag{13.1}$$

A strategy q is said to be ESS if for every $p \neq q$ there exists some $\hat{\epsilon}_y > 0$ such that (13.1) holds for all $\epsilon \in (0, \hat{\epsilon}_y)$.

In fact, we expect that if for all $p \neq q$,

$$J(q, q) > J(p, q) \tag{13.2}$$

then the mutations fraction in the population will tend to decrease (as it has a lower reward, meaning a lower growth rate). The strategy q is then immune to mutations. If it does not but if still the following holds,

$$J(q, q) = J(p, q) \text{ and } J(q, p) > J(p, p) \quad \forall p \neq q \tag{13.3}$$

then a population using q are “weakly” immune against a mutation using p since if the mutant’s population grows, then we shall frequently have individuals with strategy q competing with mutants; in such cases, the condition $J(q, p) > J(p, p)$ ensures that the growth rate of the original population exceeds that of the mutants. A strategy is ESS if and only if it satisfies (13.2) or (13.3), see [101, Proposition 2.1].

The conditions to be an ESS can be related to and interpreted in terms of Nash equilibrium in a matrix game. The situation in which an individual, say player 1, is faced with a member of a population in which a fraction p chooses strategy A is then translated to playing the matrix game against a second player who uses mixed strategies (randomizes) with probabilities p and $1-p$, resp. The central model that we shall use to investigate protocol evolution is introduced in the next subsection along with its matrix game representation.

Bibliography

[1] Notice of proposed rulemaking (fcc 09/93). October 2009 – April 2010.

[2] Consultation publique sur la neutralité du net, du 9 avril au 17 mai. 2010.

[3] I. Adiri and U. Yechiali. Optimal priority purchasing and pricing decisions in nonmonopoly and monopoly queues. *Operations Research*, 1974.

[4] O. Ait-Hellal, E. Altman, D. Elouadghiri, M. Erramdani, and N. Mikou. Performance of tcp/ip: the case of two controlled sources. In *ICCC'97, Cannes, France, November 19-21*, 1997.

[5] T. Alpcan and T. Basar. *Network Security: A Decision and Game Theoretic Approach*. Cambridge University Press, 2010.

[6] Tansu Alpcan, Tamer Basar, R. Srikant, and Eitan Altman. Cdma uplink power control as a noncooperative game. *Wireless Networks*, 8(6):659–670, 2002.

[7] E. Altman, K. Avrachenkov, and C. Barakat. A stochastic model of TCP/IP with stationary random losses. In *ACM Sigcomm*, Stockholm, Sweden, Aug. 28 - Sept. 1 2000.

[8] E. Altman, T. Boulogne, R. El Azouzi, T. Jimenez, and L. Wynter. A survey on networking games. *Computers and Operations Research*, 33(2):286–311, 2006.

[9] E. Altman and R. El-Azouzi. La théorie des jeux non-coopératifs appliquée aux réseaux de télécommunication. *Annales des Télécommunications*, 2007. (in French).

[10] E. Altman, R. El-Azouzi, Y. Hayel, and H. Tembine. An evolutionary game approach for the design of congestion control protocols in wireless networks. In *Physicomnet workshop, Berlin, April 4*, 2008.

[11] E. Altman, T. Jiménez, R. Nunez-Queija, and U. Yechiali. Queueing analysis for optimal routing with partial information. *Stochastici Models*, 20(2):149–172, 2004.

[12] E. Altman, O. Pourtallier, A. Haurie, and F. Moresino. Approximating nash equilibria in nonzero-sum games. *International Game Theory Review*, 2(2-3):155–172, 2000.

[13] E. Altman and L. Wynter. Equilibrium, games, and pricing in transportation and telecommunications networks. *Networks and Spatial Economics*, 4(1):7–21, March 2004. special issue on "Crossovers between Transportation Planning and Telecommunications".

[14] Eitan Altman, Zwi Altman, Richard Combes, and Sylvain Sorin. Routing games in the many players regime. 2011.

[15] Eitan Altman, Pierre Bernhard, Stephane Caron, George Kesidis, Julio Rojas-Mora, and Sulan Wong. A study of non-neutral networks. In *3rd Workshop on Economic Traffic Management (ETM) Collocated with 22nd International Teletraffic Congress (ITC 22)*, Amsterdam, The Netherlands, September 6 2010.

[16] Eitan Altman and Corinne Touati. *Networking Games*. Preliminary Draft Available at http://www-sop.inria.fr/members/Eitan.Altman/PAPERS/bookgame.pdf, 2010.

[17] D. Barman, I. Matta, E. Altman, and R. El Azouzi. Tcp optimization through fec, arq and transmission power tradeoffs. In *2nd international Conference on Wired/Wireless Internet Communications WWIC*, Frankfurt (Oder), Germany, February 2004.

[18] N. G. Beans, F. P. Kelly, and P. G. Taylor. Braess's paradox in a loss network. *J. Appl. Prob.*, 34:155–159, 1997.

[19] M. Beckmann, C. B. McGuire, and C. B. Winsten. *Studies in the Economics of Transportation*. New Haven: Yale Univ. Press, 1956.

[20] D. Bertsekas and R. Gallager. *Data Networks*. Prentice Hall, Englewood Cliffs, New Jersey, 1987.

[21] Thomas Bonald. Comparison of TCP Reno and TCP Vegas|: Efficiency and fairness. *Perform. Eval.*, pages 307–332, 1999.

[22] Thomas Boulogne and Eitan Altman. Competitive routing in multicast communications, networks. *Networks*, 46(1):22–35, 2005.

[23] K. Cheng, S. Bandyopadhyay, and H. Gon. The debate on net neutrality: A policy perspective. *Information Systems Research*, June 2008.

[24] Mung Chiang, Chee Wei Tan, Prashanth Hande, and Tian Lan. Power control in wireless cellular networks. 2(4):381–533, 2008.

[25] G. Cohen and F. Chaplais. Nested monotony for variational inequalities over product of spaces and convergence of iterative algorithms. *Journal of Optimization Theory and Applications*, 59:369–390, 1988.

[26] Russell Cooper. *Coordination Games*. Cambridge University Press, 1998. (ISBN 0-521-57896-5).

[27] Merouane Debbah and Samson Lasaulce. *Game Theory for Wireless Networks: From Fundamentals to Practice*. Elsevier Science & Technology, 2011.

[28] E. Dinan, D. O. Awduche, and B. Jabbarie. Optimal traffic partitioning in mpls networks. In *Proceedings of IFIP-TC6 Networking*, Paris, France, May 14-19 2000.

[29] Bhaskar Dutta. The egalitarian solution and reduced game properties in convex games. *International Journal of Game Theory*, 19:153–169, 1990.

[30] Bhaskar Dutta and Debraj Ray. A concept of egalitarianism under participation constraints. *Econometrica*, 57(3):615–635, May 1989.

[31] D. Falomari, N. Mandayam, and D. Goodman. A new framework for power control in wireless data networks: games utility and pricing. In *Proceedings of the Allerton Conference on Communication, Control and Computing*, pages 546–555, Champaign, Illinois, USA, Sept 1998.

[32] ITU G.107. The e-model, a computational model for use in transmission planning, series g: Transmission systems and media, digital systems and networks. 03/2005.

[33] S. A. Gabriel and D. Bernstein. Nonadditive shortest paths: Subproblems in multi-agent competitive network models. *Computational and Mathematical Organization Theory*, 6:29–45, 2000.

[34] L. Galluccio, G. Morabito, and S. Pallazzo. An analytical study of tradeoff between transmission power and fec for tcp optimization in wireless networks. In *Proceedings of IEEE INFOCOM*, 2003.

[35] Yan Zhang Mohsen Guizani. Taylor and Francis, 2010.

[36] P. Gupta and P. R. Kumar. A system and traffic dependent adaptive routing algorithm for Ad Hoc networks. In *Proceedings of the 36th IEEE Conference on Decision and Control*, pages 2375–2380, San Diego, USA, December 1997.

[37] R. Hahn and S. Wallsten. The economics of net neutrality. *Economists' Voice*, 3(6):1–7, 2006.

[38] P. Hande, M. Chiang, R. Calderbank, and S. Rangan. Network pricing and rate allocation with content provider participation. In *Proc. IEEE INFOCOM*, 2009.

[39] R. Hassin. On the advantage of being the first server. *Management Science*, 42:618–623, 1996.

[40] R. Hassin and M. Haviv. Equilibrium threshold strategies: the case of queues with priorities. *Operations Research*, 45:966–973, 1997.

[41] R. Hassin and M. Haviv. *To queue or not to queue: equilibrium behavior in queueing systems*, volume 59. Kluwer Academic Publishers, Boston, international series in operations research and management science edition, 2002.

[42] A. Haurie and P. Marcotte. On the relationship between nash-cournot and wardrop equilibria. *Networks*, 15:295–308, 1985.

[43] M. T. Hsiao and A. A. Lazar. Optimal decentralized flow control of Markovian queueing networks with multiple controllers. *Performance Evaluation*, 13:181–204, 1991.

[44] T. Issariyakul and E Hossain. *Introduction to Network Simulator NS2*. Springer, November 2008. ISBN 978-0-387-71759-3.

[45] V. Jacobson. Congestion avoidance and control. In *ACM SIGCOMM 88*, pages 273–280, 1988.

[46] Kamal Jain and Vijay V. Vazirani. Equitable cost allocations via primal-dual-type algorithms. *34th annual ACM symposium on Theory of computing*, pages 313–321, 2002.

[47] H. Ji and C. Huang. Non-cooperative uplink power control in cellular radio systems. *Wireless Networks*, 4(3):233–240, April, 1998.

[48] John F. Nash Jr. The bargaining problem. *Econometrics*, 18:155–162, 1950.

[49] H. Kameda, J. Li, C. Kim, and Y. Zhang. *Optimal Load Balancing in Distributed Computer Systems*. Springer-Verlag, London, Telecommunications Networks and Computer Systems (TNCS) Series, 1997.

[50] H. Kameda and Y. Zhang. Uniqueness of the solution for optimal static routing in open bcmp queueing networks. *Mathematical and Computer Modeling*, 22:119–130, 1995.

[51] Michihiro Kandori, George J Mailath, and Rafael Rob. Learning, mutation, and long run equilibria in games. *Econometrica*, 61(1):29–56, January 1993.

[52] F. Kelly, A. Maulloo, and D. Tan. Rate control in communication networks: shadow prices, proportional fairness and stability. *Journal of the Operational Research Society*, 49:237–252, 1998.

[53] F. P. Kelly. Charging and rate control for elastic traffic. *European Transactions on Telecommunications, special issue on Elastic Services over ATM networks*, 8(1), 1997.

[54] Y. A. Korilis, A. A. Lazar, and A. Orda. Avoiding the Braess paradox in non-cooperative networks. *Journal of Applied Probability*, 36:211–222, 1999.

[55] Y. A. Korilis and A. Orda. Incentive-compatible pricing strategies for QoS routing. In *Proceedings of IEEE INFOCOM'99*, New York, NY, USA, March 1999.

[56] E. Koutsoupias and C. Papadimitriou. Worst-case equilibria. In *16th Annual Symposium on Theoretical Aspects of Computer Science*, pages 404–413, Trier, Germany, March 1999.

[57] Neelie Kroes. vice president of the european commission commissioner for the digital agenda. *Press release RAPID*, April 2010.

[58] S. Lasaulce, M. Debbah, and E. Altman. Methodologies for analyzing equilibria in wireless games. *IEEE Signal Processing Magazine*, 26(5):51–52, 2009. Special issue on Game Theory.

[59] S. C. Littlechild. A simple expression for the nucleolus in a special case. *International Journal of Game Theory*, 3(1):21–29, March 1974.

[60] S. C. Littlechild and G. Owen. A simple expression for the shapley value in a special case. *Management Science*, 20(3):370–372, Nov. 1973.

[61] J. Love. Cell phone jammers. *methodshop.com, site createdat 09/01/2005 and updated at 04/24/2009, www.methodshop.com/gadgets/reviews/celljammers.*

[62] R.T.B. Ma, D.-M. Chiu, J.C.S. Lui, V. Misra, and D. Rubenstein. Interconnecting eyeballs to content: A shapley value perspective on isp peering and settlement. In *Proc. Int'l Workshop on Economics of Networked Systems (NetEcon)*, pages 61–66, 2008.

[63] R.T.B. Ma, D.-M. Chiu, J.C.S. Lui, V. Misra, and D. Rubenstein. On cooperative settlement between content, transit and eyeball internet service providers. In *Proc. ACM Int'l Conference on Emerging Networking EXperiments and Technologies (CoNEXT)*, 2008.

[64] Allen Mackenzie and Luiz DaSilva. *Game Theory for Wireless Engineering*. Morgan and Claypool Publishers, 2006.

[65] R. Mazumdar, L. G. Mason, and C. Douligeris. Fairness in network optimal flow control: optimality of product forms. *IEEE Trans. on Comm.*, 39:775–782, 1991.

[66] Jeonghoon Mo and Jean Walrand. Fair end-to-end window-based congestion control. *IEEE/ACM Transactions on Networking*, 8(5):556–567, 2000.

[67] D. Monderer and L. S. Shapley. Potential games. *Games and Econ. Behavior*, 14:124–143, 1996.

[68] Herve Moulin. Incremental cost sharing: Characterization by coalition strategy-proofness. *Social Choice and Welfare*, 16(2):279–320, 1999.

[69] J. Musacchio, G. Schwartz, and J. Walrand. A two-sided market analysis of provider investment incentives with an application to the net-neutrality issue. *Review of Network Economics*, 8(1), 2009.

[70] Roger B. Myerson. Large poisson games. *Journal of Economic Theory*, 94:1–45, 2000.

[71] A. Neyman. Correlated equilibrium and potential games. *Int. J. of Game Theory*, 26:223–227, 1997.

[72] N. Nisan and A. Ronen. Algorithmic mechanism design. *Games and Economic Behavior*, 35:166–196, 2001.

[73] Noam Nisan and Amir Ronen. Algorithmic mechanism design (extended abstract). In *STOC 1999: 31st Annual ACM Symposium on Theory of Computing*, pages 129–140, 1999.

[74] Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay V. Vazirani. *Algorithmic Game Theory*. Cambridge University Press, New York, NY, USA, 2007.

[75] A. Orda, R. Rom, and N. Shimkin. Competitive routing in multi-user communication networks. *IEEE/ACM Transactions on Networking*, 1(5):510–520, 1993.

[76] M. Patriksson. *The Traffic Assignment Problem: Models and Methods*. VSP BV, P.O. Box 346, 3700 AH Zeist, The Netherlands, 1994.

[77] Bezalel Peleg and Peter Sudhaulter. *Introduction to the Theory of Cooperative Games*, volume 34 of *Theory and Decision Library, Series C: Game Theory, Mathematical Programming and Operations Research*. 2nd Edition, Springer-Verlag Berlin Heidelberg, 2007.

[78] Paolo Penna and Carmine Ventre. Sharing the cost of multicast transmissions in wireless networks. In *SIROCCO 2004, LNCS*, number 3104, pages 255–266, 2004.

[79] A. C. Pigou. *The Economics of Welfare*. McMillan & Co., London, 1920.

[80] J. G. Proakis. *Communication Systems Engineering*. Prentice Hall International Editions, 1994.

[81] R. W. Rosenthal. A class of games possessing pure strategy nash equilibria. *Int. J. Game Theory*, 2:65–67, 1973.

[82] R. W. Rosenthal. The network equilibrium problem in integers. *Networks*, 3:53–59, 1973.

[83] T. Roughgarden and É. Tardos. How bad is selfish routing? *Journal of the ACM*, 49:236–259, 2002.

[84] W. H. Sandholm. Potential games with continuous player sets. *Journal of Economic Theory*, 97:81–108, 2001.

[85] S. W. Shah, M. I. Babar, and M. N. Arbab. Cell phone jammer. In *Multitopic Conference, INMIC 2008. IEEE International Issue available in Xplore*, 23-24 Dec 2008.

[86] L. S. Shapley. Cores of convex games. *International Journal of Game Theory*, 1(1):11–26, Dec. 1971.

[87] Chandramani Singh and Eitan Altman. The wireless multicast coalition game and the non-cooperative association problem. In *Proceedings of IEEE Infocom*, Beijing, China, May 2011.

[88] H. Singh and S. Singh. Energy consumption of tcp reno, new reno and sack in multi-hop wireless networks. In *ACM SIGMERTICS, Jun.*, 2002.

[89] Marco Slikker and Anne van den Nouweland. *Social and Economic Networks in Cooperative Game Theory*. Theory and Decision Library C, Vol. 27. 2001.

[90] Pablo Soldati. *Cross-Layer Optimization of Wireless Multi-hop Networks*. Royal Institute of Technology (KTH), 2007. (ISBN 978-7178-711-8).

[91] T. Sonmez. Population monotonicity of the nucleolus on a class of public good problems. *Mimeo*, 1994.

[92] Telecommunication Standardization Sector of ITU. Methods for subjective determination of transmission quality - Series P: Telephone transmission quality; methods for objective and subjective assessment of quality., 1996.

[93] Telecommunication Standardization Sector of ITU. Subjective audiovisual quality assessment methods for multimedia applications - Series P: Telephone transmission quality, telephone installations, local line networks; audiovisual quality in multimedia services., 1998.

[94] William Thomson. Cost allocation and airport problems. *Working paper*, (538), Dec. 2007.

[95] S. H. Tjis and T. S. H. Driessen. Game theory and cost allocation problems. *Management Science*, 32(8):1015–1028, August 1986.

[96] D. Topkis. Equilibrium points in nonzero-sum n-person submodular games. *SIAM J. Control and Optimization*, 17:773–787, November 1979.

[97] M. Flammini V. Bilo, G. Melideo, L. Moscardelli, and A. Navarra. Sharing the cost of multicast transmissions in wireless networks. *Theoretical Computer Science*, 369:269–284, 2006.

[98] Giovanna Melideo Vittorio Bilo, Michele Flammini and Luca Moscardelli. On nash equilibria for multicast transmissions in ad-hoc wireless networks. *Wireless Networks*, 14:147–157, 2008.

[99] B. von Stengel. Editorial to the special issue devoted to the symposium on computation of nash equilibria in finite games. *Economic Theory*, 42(1), January 2010.

[100] J. G. Wardrop. Some theoretical aspects of road traffic research communication networks. *Proc. Inst. Civ. Eng., Part 2*, 1:325–378, 1952.

[101] J. Weibull. *Evolutionary Game Theory*. M.I.T. Press, 1995.

[102] Haïkel Yaïche, Ravi R. Mazumdar, and Catherine Rosenberg. A game theoretic framework for bandwidth allocation and pricing in broadband networks. *IEEE/ACM Transactions on Networking*, 8(5):667–678, 2000.

[103] D. D. Yao. S-modular games, with queuing applications. *Queuing Systems: Theory and Applications*, 21:449–475, 1995.

[104] Chun-Hsien Yeh. An alternative characterization of the nucleolus in airport problems. *Mimeo, Institute of Economics, Academia Sinica, Taipei, Taiwan*, 2004.

[105] M. Zorzi and R. Rao. Energy efficiency of tcp in a local wireless environment. *Mobile Networks and Applications*, 6(3), 2001.