Competition and cooperation between nodes in Delay Tolerant Networks with Two Hop Routing

Eitan Altman

INRIA B.P.93, 2004 Route des Lucioles, 06902 Sophia-Antipolis Cedex, France

Abstract. This paper revisits the two-hop forwarding policy in delay tolerant networks (DTNs) using simple probabilistic arguments. Closed form expressions are derived for the main performance measures. We then study competitive and cooperative operation of DTNs and derive the structure of optimal and of equilibrium policies.

1 Introduction

Through mobility of devices that serve as relays, Delay Tolerant Networks (DTNs) allow non connected nodes to communicate with each other. Such networks have been developed in recent years and adapted both to human mobility where the contact process is between pedestrians [4], as well as to vehicle mobility [6].

The source does not know which of the nodes that it meets will reach the destination within a requested time, so it has to send many copies in order to maximize the successful delivery probability. How should it use its limited energy resources for efficient transmission? Assume that the first relay node to transfer the copy of the file to the destination will receive a reward, or that some reward is divided among the nodes that participated in forwarding the packets. With what probability should a mobile participate in the forwarding, what is the optimal population size of mobiles when taking into account energy and/or other costs that incnrease as the number of nodes increase? If it is costly to be activated, how should one control the activation periods?

We propose in this paper some answers to these questions using simple probabilistic arguments. We identify structural properties of both static and dynamic optimal policies, covering both cooperative and non cooperative scenarios.

This paper pursues the research initiated in [2] where the authors already studied the optimal static and dynamic control problems using a fluid model that represents the mean field limit as the number of mobiles becomes very large. In [3], the optimal dynamic control problem was solved in a discrete time setting. The optimality of a threshold type policy, already established in [2] for the fluid limit framework, was shown to hold in [3] for the actual discrete control problem. A game problem between two groups of DTN networks was further studied in [3].

In this paper we study competition between individual mobiles in a game theoretical setting. We obtain the structure of equilibrium policies and compare them to the cooperative case.

2 Model

Consider n mobiles, and moreover, a single static source and destination. The source has a packet generated at time 0 that it wishes to send to the destination.

Assume that any two mobiles meet each other according to a Poisson process with parameter λ. At each time a mobile meets the source, the source may forward to it a packet. We consider the two hop routing scheme [1] in which a mobile that receives a copy of the packet from the source can only forward it if it meats the destination. It cannot copy it into the memory of another mobile.

Each mobile can decide whether to be active or not, or when to be active (e.g. how long to remain active). We call the first type of decision a "static" one and the second type a "dynamic" decision.

The first mobile that delivers the packet to the destination receives one unit reward. Moreover, each mobile pays a cost of g per time unit that it is active.

Consider an arbitrary mobile. Let T_1 be the first time it meets the source and let T_2 be the first time after T_1 that it meets the destination. Denote

$$q_{_R} = \exp(-\lambda R),$$

Consider the event that the mobile relays a packet from the source to the destination within time R, i.e. $T_1 + T_2 \leq R$. $T_1 + T_2$ is an Erlang(2) random variable and therefore the probability of the above event is $1 - Q_{_R}$ where

$$Q_{_R} = Q_{_R}(\lambda) = (1 + \lambda R) \exp(-\lambda R)$$

Note that $Q_{_R} - q_{_R}$ is the probability that $T_1 < R$ but that $T_1 + T_2 > r$.

2.1 A state representation

Let X_t be the number of mobiles with a copy of the packet at time t. We call X_t the state.

Each mobile meets the destination according to a Poisson random process with parameter λ. The intensity of the process that counts the number of contacts between nodes with copies of the file and the destination at time t is λX_t. Thus the number of contacts during the interval $[0, \tau]$ between the destination and mobiles that have copies of the files is a Poisson random variable with intensity $\lambda \int_0^\tau X_s ds$.

Consider the following dynamic control policy u which is assumed to be common to all mobiles. u is a piecewise continuous function that takes values in the unit interval. If a mobile meets the source at time t then it receives the packet with probability u_t. We can view u as a decision rule taken by the source: the source transmits a copy of the packet with probability u_t. We are then concerned with the control problem faced by the source node who wishes to maximize the successful delivery probability and has cost for transmission energy. On the other hand u can be interpreted as a common control for the mobiles if the decision is taken by then on whether or not to forward a packet.

Define $\zeta_t(j)$ to be the indicator that the jth mobile among the n receives the file during $[0,t]$. Then

$$X_t = \sum_{j=1}^{n} \zeta_t(j)$$

$\{\zeta_t(j)\}_j$ are i.i.d. with the expectation and the Laplace Stieltjes Transform given by:

$$w_t := E[\zeta_t(j)] = 1 - \exp(-\lambda \int_0^t u_s ds)$$

$$\begin{aligned} E\left[\exp\left(-\lambda\zeta_t(1)\right)\right] &= (1-w_t)\exp(-\lambda) + w_t \\ &= \exp(-\lambda) - (1-\exp(-\lambda))\exp\left(-\lambda \int_0^t u_s ds\right) \end{aligned}$$

The Laplace Stieltjes transform of X_t satisfies

$$X_t^*(\lambda) := E\left[\exp\left(-\lambda X_t\right)\right] = E\left[\exp\left(-\lambda\left(\sum_{i=1}^{n}\zeta_t(i)\right)\right)\right] = \left(E\left[\exp\left(-\lambda\zeta_t(1)\right)\right]\right)^n$$

Define $F_D(\tau)$:= the probability that the destination receives the packet by time τ. Then

$$F_D(n) = 1 - E\left[\exp\left(-\lambda \int_0^\tau X_t dt\right)\right]$$

3 The static DTN game

It is assumed that the packet has to arrive at the destination τ units of times after it was created, otherwise it brings no utility to the destination.

Each mobile decides whether to participate or not in the forwarding. Let each mobile choose to participate with the same probability u. u is a symmetric equilibrium if no mobile can benefit from a unilateral deviation to some $v \neq u$.

Let $p(v,u)$ be the probability that the tagged mobile is the first to deliver the packet to the destination when it plays v and all others play u.

The probability that k given mobiles meet the source and afterwards meet the destination between the interval $[0,\tau]$ is given by $(Q_\tau)^k$. The probability that $k-1$ mobiles out of $n-1$ deliver a packet to the destination during the interval $[0,\tau]$ is

$$P_k = \binom{n-1}{k-1} u^{k-1}(1-u)^{n-k}(1-Q_\tau)^k$$

Hence the probability of the tagged mobile to receive the unit award is if it decides to participate is

$$\begin{aligned} P(u) &= \sum_{k=1}^{n} \binom{n-1}{k-1} \frac{(u(1-Q_\tau)^{k-1}(1-u)^{n-k}(1-Q_\tau)}{k} \\ &= \Big((1-uQ_\tau)^n - (1-u)^n\Big)\frac{1}{un} \qquad (1) \end{aligned}$$

We shall sometime write $P_{n,\tau}(u)$ in order to make explicit the dependence on n and τ.

3.1 The utility and equilibrium

A mobile that participates receives a unit of reward if it is the first to deliver a copy of the packet to the destination. It further pays some energy cost $g\tau$ where $g > 0$ is some constant. $U(1,u) = P(u) - g\tau$ is thus the (expected) utility for a tagged mobile of participating when each other mobile participates with probability u. We assume that the utility $U(0,u)$ for not participating is zero for all u. The utility for a mobile that participates with probability v when each other participates with probability u is $U(v,u) = vU(1,u)$. The following indifference property easily follows:

Lemma 1. *If there exists a policy u such that $U(1,u) = 0$ then u is a symmetric equilibrium.*

$P(u)$ is a continuous convex decreasing function, $\lim_{u\to 0} P(u) = 1 - Q_\tau$ and $P(1) = (1-Q_\tau)^n/n$. Thus the utility for choosing to participate $U(1,u) = P(u) - g\tau$ is a continuous convex decreasing function, $\lim_{u\to 0} U(1,u) = 1 - Q_\tau - g(\tau)$ and $U(1,1) = (1-Q_\tau)^n/n - g\tau$. The utility for not participating is assumed to be zero. Thus we have

Lemma 2. *If $(1-Q_\tau)^n/n < g\tau$ then there exists a unique symmetric equilibrium u which is the unique solution of $U(1,u) = U(0,u) = 0$.*

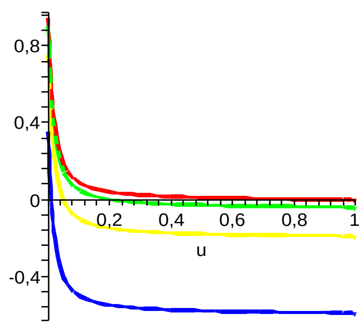

Fig. 1. The utility of participating as a function of u, for various values of the duration τ.

3.2 Numerical Examples

Figure 1 presents the utility for choosing to participate as a function of the strategy u of all other players for various values of the maximum duration τ: $\tau = 1, 5, 20, 60$. $\lambda = 10$ is taken to be a constant. The other parameters are $n = 100, g = 0.01$. We obtain four curves (one for each τ). We see that indeed for each value of R there is a unique value of u for which $U(1, u) = 0$, and this is the equilibrium. Here the curve that is the highest corresponds to $\tau = 1$, and the larger τ is, the lower the is also curve. This then implies that the equilibrium value of u increases with τ.

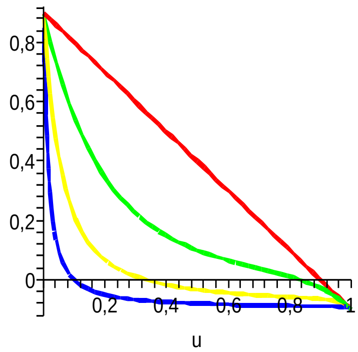

Fig. 2. The utility of participating as a function of u, for various values of the number n of users.

In Figure 2 we repeat the same but with a fixed value $R = 10$ and varying values of n: $n = 3, 10, 30$ and 100. Again, the larger n is, the larger is also the curve. The equilibrium is thus increasing in n.

4 The static team problem

We next study the static team problem. In can be interpreted as the control problem that arises when all mobiles collaborate. If mobiles are undistinguishable then this would lead us to search for an optimal symmetric policy. If there is coordination between the mobiles then one can consider also non-symmetric policies, which will be shown to out-perform the symmetric ones.

The global utility for an arbitrary (possibly non-symmetric policy) is minimized by a symmetric policy. Indeed, choose an arbitrary set of probabilities $u_1, ..., u_n$. The corresponding utility is

$$U(u) = P_s(u) - g\tau \sum_{k=1}^{n} u_k$$

where

$$P_s(u) = 1 - \prod_{k=1}^{n} (u_k Q_\tau + (1 - u_k)) = 1 - \prod_{k=1}^{n} (1 - (1 - Q_\tau) u_k)$$

is the probability of successful transmission by time τ.

Let U_d be the set of policies for which each u_k is either 0 or 1.

Lemma 3. *(i) There exists an optimal policy among U_d.*
(ii) A necessary condition for a policy u to be globally optimal is the following:

$$\boxed{\begin{array}{c}\text{Except for one mobile at most, each mobile}\\ \text{transmits at either maximum or at zero power.}\end{array}} \quad (2)$$

(iii) Any non-symmetric policy u performs strictly better than the symmetric policy v that has the same sum $\sum_{k=1}^n u_k = \sum_{k=1}^n v_k$.

Proof. (i) Let u be a policy for which for some k, $0 < u_k < 1$. We shall show that there exists a policy $v \in U_d$ that performs at least as well. Since the utility is linear in each u_k, we can change u_k to either 0 or to 1 without decreasing the utility. Repeating this procedure for all the remaining j's that are not extreme points, we obtain a policy in U_d that performs at least as well as u. This implies (i).
Choose an arbitrary $u = (u_1, ..., u_n)$. Consider now the problem of finding $v \in U(u)$ under the constraint $\sum_{i=1}^n v_k = \sum_{i=1}^n u_k$. The policy maximizes this objective if and only if it minimizes the function $f(u)$ defined as

$$f(u) := \sum_{k=1}^{n} \psi(u_k) \quad \text{where } \psi(u) = \log(1 - (1 - Q_\tau)u).$$

ψ is a concave function of its argument, which implies that f is a Schur concave function, see Appendix. For any policy u which does not satisfy (2), we can construct a policy u' which satisfies (2) and $\sum_{k=1}^n u'_k = \sum_{k=1}^n u_k$. Then u' strictly majorizes u and therefore u strictly outperforms u'. In the same way one shows that any policy performs strictly better than the symmetric policy that has the same sum of components. ◇

In Figure 3 we compare the global optimal solution with the best solution among the symmetric policies. The upper subfigure is obtained with the same parameters as used for the equilibrium in Figure 1: $g = 1, n = 100, \lambda = 10, \tau = 1$. The vertical axis is the utility of the symmetrical and it is presented as a function of the policy u given by the ratio $u = k/n$; k that varies between 1 to 10 appears in the horizontal axis of the figure. The second subfigure repeats the same experiment but with $\lambda = 1$ and with $n = 100$.

In both subfigures we see that there is indeed a difference between the global optimal solution and the one obtained with the best symmetric policy. The latter is indeed seen to provide a smaller optimum.

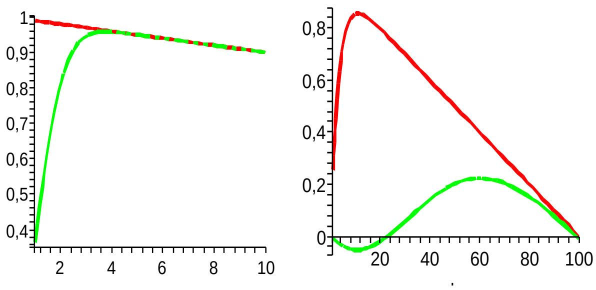

Fig. 3. Team case: Utility as a function of k

Remark 1. The fact that an optimal policy exists among U_d means that there is an optimal number of mobiles that should participate. It can also be viewed as an optimal coalition size. We plan to study in the future the question of optimal coalition size in the case that there is competition between a given number N of coalitions.

5 The dynamic DTN game

In the last section we assumed a static game: a mobile took one decision, at time 0, on whether to participate or not. We now consider a dynamic game in which a mobile can switch on or off at any time. A policy for a mobile consists of the choice of time periods during which it is activated.

We introduce next threshold policies. A time threshold policy R is a policy that keeps a mobile active till time R and then deactivates it. We shall identify threshold equilibria for our problem. More precisely, assume that all mobiles use threshold policies with a common threshold R. We shall consider deviation of a single mobile to another threshold policy s and look for R such that $s = R$ is an optimal response of the deviating mobile.

The probability that the deviating mobile is the first to deliver the packet to the destination is

$$P_{succ}(s,R) =$$
$$\begin{cases} \dfrac{1-(Q_s)^n}{n} & \text{if } s \le R, \\ \dfrac{1-(Q_{R})^n}{n} + Q_{R}^{n-1}\Big(q_{s-R}(1-Q_{s-R}) \\ \qquad + \lambda R q_{R}(1-q_{s-R})\Big) & \text{otherwize} \end{cases}$$

The first term corresponds to the event that the first successful delivery occurs before time R, and the second term is related to its occurrence between time R and s. More precisely, the second term corresponds to the event that no one of the other $n-1$ mobiles met the source till time t, where as the tagged mobile either (i) did not meet the source before time R and then, during time interval $(R, s]$ met the source and then the destination, or (ii) it met the source at least once before R but did not meet the destination before R, and then it met the destination during $(R, s]$.

The utility for a player is given by

$$U(s, R) = P_{succ}(s, R) - gs$$

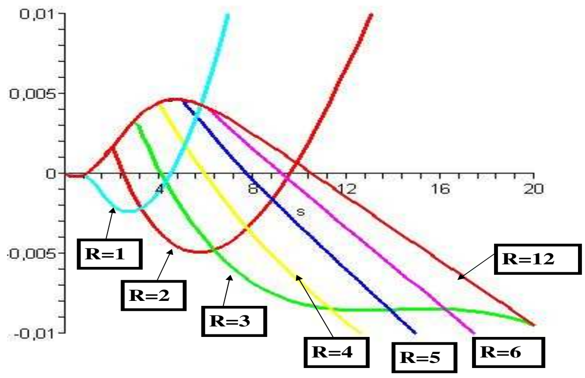

Fig. 4. Equilibria in threshold policies: The utility $U(s, R)$ as a function of s for various values of R.

Figure 4 shows the utility $U(s, R)$ for s varying between 0 and 20 and for $R = 1, 2, 3, 4, 5, 6, 12$. The parameters area $g = .001, lambda = .049, m = 2, n = 95$. We observe the following.

– There are many equilibria. For R between 3 and 5, the best response is $s = R$ and hence any value of R in the interval [3,5] is a symmetric threshold equilibrium (for any horizon τ that is greater than R).
– $1 \leq R < 3$ are also equilibria but only for some value of τ. For example, for $R = 2$, $s = 2$ is the best response as long as we restrict s to be smaller than 10.5; thus if $\tau \leq 10.5$ then all R's between 2 and 5 are symmetric threshold equilibria. If we restrict to $\tau \leq 4$ then the values of R in the whole range [1,5] provide symmetric equilibria.

An alternative way to see the multiple equilibria phenomenon is by plotting $U(s, R)$ as a function of R for various values of s. We do so in Figure 5. This time we take $s = 1, 2, 3, 4, 5, 6, 12$. The intersection of the curves corresponding to the different values of s with the vertical axis are increasing with s. We indeed see that the best response to $R = 3$ is $s = 3$ but at the same time, $s = 4$ is the best response to $R = 4$.

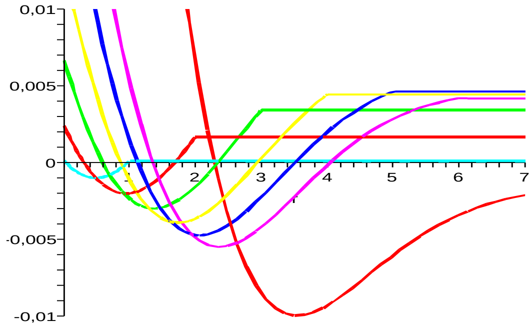

Fig. 5. Equilibria in threshold policies: The utility $U(s, R)$ as a function of R for various values of s.

6 The dynamic team problem

We consider the optimization problem restricted to symmetric policies, i.e. where all mobiles use threshold policies and the threshold value R is the same for all mobiles. Using the theory of Markov Decision Processes it can be shown that there is no loss of optimality in doing so.

The global utility is then $U(R) = 1 - (Q_R)^n - ng \times R$. We have

$$\frac{\partial U(R)}{\partial R} = \frac{(Q_R)^n n \lambda^2 R}{1 + \lambda R} - ng$$

$$\frac{\partial^2 U(R)}{\partial R^2} = \frac{n(Q_R)^n \lambda^2 (n\lambda^2 R^2 - 1)}{(1 + \lambda R)^2}$$

From the first derivative we see that $U(R)$ is monotone. For all R sufficiently large, it is negative if $g < 1$ and is positive if $g > 1$. From the second derivative we see that $U(R)$ is convex for $R < 1/(\lambda\sqrt{n})$ and is concave for $R > 1/(\lambda\sqrt{n})$.

An example is given in Figure 6 The experiment was done with $n = 95$. Each curve corresponds to another value of the parameter g: $g = 0.0001$ (top curve), $g = 0.0033$ (next to top), $g = 0.001$ and $g = 0.002$. We see that there may be one or two optimal values to the threshold R: it is either an extreme point ($R = 0$ or $R = \infty$) or it is an interior point.

Theorem 1. *There exists a unique optimal threshold policy. A policy is optimal if and only if it is of a threshold type.*

Proof We have

$$1 - F_D(n) = E\left[\exp\left(-\lambda \int_0^\tau X_t dt\right)\right] = \exp \int_0^\tau \log(X_t^*(\lambda))dt$$

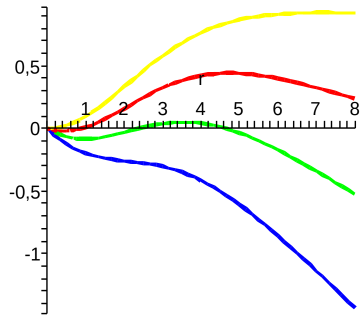

Fig. 6. The dynamic team case: utility as a function of R.

where the last equality follows from the Lévy Khinchine formula. Hence

$$P(no\ success) = \exp\left(-\lambda \int_0^\tau \log E\left[\exp\left(-\lambda X_t\right)\right] dt\right)$$

$$= \exp\Big(-\lambda n \int_0^\tau \log\left[e^{-\lambda} - (1-e^{-\lambda})\exp\left(-\lambda \int_0^t u_s ds\right)\right] dt\Big)$$

Assume that u is not a threshold. Let v be the threshold policy that transmits till time $s^* := \int_0^\tau u_t dt$ and then stops transmitting. Then for every t,

$$\int_0^t u_s ds \geq \int_0^t v_s ds$$

This implies that the first integral $\int_0^\tau$ is smaller under u and hence also the success probability. $\diamond$

A similar characterization of the optimal policy has been derived in [3] for the case of discrete time.

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Appendix: Marjotization and Schur concavity

Definition 1. *(Majorization and Schur-Concave Function [5])*
Consider two n-dimensional vectors $d(1), d(2)$. $d(2)$ majorizes $d(1)$, which we denote by $d(1) \prec d(2)$, if

$$\sum_{i=1}^{k} d_{[i]}(1) \leq \sum_{i=1}^{k} d_{[i]}(2), \quad k = 1, ..., n-1, \quad \textit{and}$$

$$\sum_{i=1}^{n} d_{[i]}(1) = \sum_{i=1}^{n} d_{[i]}(2),$$

where $d_{[i]}(m)$ is a permutation of $d_i(m)$ satisfying $d_{[1]}(m) \geq d_{[2]}(m) \geq ... \geq d_{[n]}(m)$, $m = 1, 2$.

A function $f : R^n \to R$ is Schur concave if $d(1) \prec d(2)$ implies $f(d(1)) \geq f(d(2))$. It is strictly Schur concave if strict inequality holds whenever $d(1)$ is not a permutation of $d(2)$.

Lemma 4. [5, Chapter 3] *Assume that a function $g : R^n \to R$ can be written as the sum $g(d) = \sum_{i=1}^{n} \psi(d_i)$ where ψ is a concave (resp. strictly concave) function from R to R. Then g is Schur (resp. strictly) concave.*