Opportunistic scheduling in cellular systems in the presence of non-cooperative mobiles

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Abstract

A central scheduling problem in wireless communications is that of allocating resources to one of many mobile stations that have a common radio channel. Much attention has been given to the design of efficient and fair scheduling schemes that are centrally controlled by a base station (BS) whose decisions depend on the channel conditions of each mobile. The BS is the only entity taking decisions in this framework based on truthful information from the mobiles on their radio channel. In this paper, we study the scheduling problem from a game-theoretic perspective in which some of the mobiles may be noncooperative. We model this as a signaling game and study its equilibria. We then propose various approaches to enforce truthful signaling of the radio channel conditions: a pricing approach, an approach based on some knowledge of the mobiles' policies, and an approach that replaces this knowledge by a stochastic approximations approach that combines estimation and control. We further identify other equilibria that involve non-truthful signaling. We finally discuss the proportional fair framework under the noncooperative setting.

I. Introduction

Short-term fading arises in a mobile wireless radio communication system in the presence of scatterers, resulting in time-varying channel gains. Various cellular networks have downlink shared data channels that use scheduling mechanisms to exploit the fluctuations of the radio conditions (e.g. 3GPP HSDPA [3] and CDMA/HDR [6] or 1xEV-DO [2]). The scheduler design and the obtained gain are predicated on the mobiles sending information concerning the downlink channel gains in a truthful fashion. In a frequency-division duplex system, the base station (BS) has no direct information on the channel gains, but transmits downlink pilots, and relies on the mobiles' reported values of gains on these pilots for scheduling. A cooperative mobile will truthfully report this information to the BS. A noncooperative mobile will however send a signal that is likely to induce the scheduler to behave in a manner beneficial to the mobile.

Our paper is concerned with game-theoretic analysis of downlink scheduling in the presence of noncooperative mobiles. We consider the BS to be a player as well, and assume that the identity of players that do not cooperate is common knowledge. We model this game as a signaling game which is somewhat uncommon in that there are several players that send signals (the mobiles whose signals correspond to the reported states of the channel) and one follower who reacts to the signal - the BS who decides to whom to allocate the channel. The utility of the BS is assumed to be the social optimum, i.e., the sum of throughputs. The equilibria of this game and distributed ways to achieve them are the focus of our research reported here.

Contribution of the paper: We begin with the case in which the BS does not use any extra intelligence to deal with noncooperative mobiles. We show that the Perfect Bayesian Equilibrium (PBE) of the signaling game, are *babbling* type (i.e., the base station's strategy is to ignore the mobile's signal and use only prior statistical information of the noncooperative mobiles). A somewhat surprising result is that these are the only equilibria.

At the other extreme we have the 'separating' equilibria in which the mobiles signal the true channel value. We present three ways to obtain these as equilibria of the game, each requiring some other type of extra intelligence or involvement of the BS. The first relies on capabilities of the BS to estimate the real downlink channel quality (at a later stage based probably on the rate at which the actual transmission took place), combined with a pricing mechanism that creates incentives for truthful signaling.

The second approach requires that the BS has some knowledge of the policy used by the mobiles. More precisely, it requires the BS to know the probabilities with which various signals are used (in addition to knowing the channel statistics).

The third approach takes advantage of the fact that the game is repeated, and replaces the knowledge required in the previous method by an estimation procedure. The actual procedure that combines estimation and control is based on stochastic approximation algorithm for which truth revelation is an ε-Nash equilibrium. This algorithm and its analysis are inspired by Kushner & Whiting's work on the proportional fair sharing algorithm (PFA) [15].

Within the two last settings we described an equilibrium (or ϵ-equilibrium) similar in nature to mechanism design in which the BS's strategy creates incentives for truth revealing (ITR). We call this a truth revealing equilibrium (TRE) at which the BS uses an ITR policy. However the settings of this paper differ from the mechanism design paradigm; we do not consider explicit pricing and furthermore, the BS is also modeled as a player.

In a game setting, a natural question that arises is whether other equilibria exist. Can a player get more by not signaling truthfully? The answer is a possible yes - when the BS does not use ITR. We show that there indeed exist non-efficient equilibria at which a noncooperative mobile gains more by non-truthful signaling. At such equilibria, the BS gets a utility that may be lesser than that of a TRE but greater than that of a babbling equilibrium.

Finally, we briefly discuss the noncooperation setting within the proportional fairness framework.

Prior work. (1) Proportional fair sharing and other related algorithms were intensely analyzed as applied to the CDMA/HDR system. See [9], [6], [5], [20], [4], [8], [16]. These results are applicable to the 3GPP HSDPA system as well. Kushner & Whiting [15] analyzed the PFA using stochastic approximation techniques and showed that the asymptotic averaged throughput can be driven to optimize a certain system utility function (sum of logarithms of offset-rates). All the above methods assume that the centralized scheduler has complete information of all relevant quantities.

(2) Mobiles in our setting are more informed than the BS; they have access to their individual channel realization which is private information. If mobiles exploit the ignorance of the BS to their advantage, and the BS is aware of this possibility, we have a signaling game as discussed earlier with several lead players or senders (mobiles) and one follower or receiver (BS). See [14] for an illuminating survey on this topic with example applications to marketing, labor services market, insurance deductibles, and evolutionary biology. Our interest will be in a game with “cheap talk”; this is a signaling game where the sender incurs no cost for his signals [14, Sec. 7]. It is well-known that such games admit a babbling equilibrium. There are situations where other equilibria exist. Interestingly, there are situations where cheap talk helps (Farrell [10]), i.e., cheap talk induces a truth-revealing equilibrium while the game with no communication has no equilibrium.

(3) Mechanism design is another area where a less informed social planner induces truth revelation from the more informed players via an appropriate choice of an allocation mechanism and pricing; see [11] for a tutorial survey. Pricing mechanisms have been extensively studied

in the context of networks (see [12], [18]). A related problem of *screening* arises when the receiver is the more informed party and the uninformed senders make the proposals for trade.

(4) Effect of noncooperative mobiles was recently considered in [13], where the special case of maximum rate algorithm was simulated and shown to improve the noncooperative mobile's throughput by 5% but decrease the overall system throughput by 20%. Then, observing that the presence of more mobiles in the system resulted in less aggressive signaling, they proposed that the BS advertise the presence of an inflated number of mobiles in the system; the noncooperative mobiles take this advertisement to be true[1]. Nuggehalli et al [17] considered noncooperation by low-priority latency-tolerant mobiles in an 802.11e LAN setting capable of providing differentiated quality of service. They provide an incentive mechanism in the form of collision free access (CFA) : a guaranteed fraction of allocation to CFA induces truth revelation. Price & Javidi [19] consider an uplink version of a problem where the mobiles are the informed parties on the valuation of the uplinks (queue state information is available only at the mobile). They provide incentives in the form of allocation on the downlink to induce truth revelation.

Paper outline: After providing a brief system and problem description (Section II, Section III), we consider the downlink cheap-talk signaling game comprising of informed noncooperative mobiles (all or a few) and a less-informed BS in Section IV. The signaling game admits only babbling equilibria. We then describe in Sections V, VI the three approaches to obtain a truth revealing equilibrium (TRE) based on ITR policies of BS. Other equilibria are identified in Section VII. Section VIII provides some preliminary results on proportional fair sharing algorithm. We end the paper with a concluding section. Most of the proofs are placed in the Appendix placed at the end of the paper.

II. Problem formulation

We consider the downlink of a wireless network with one base station (BS). There are K mobiles competing for the downlink data channel. Time is divided into small intervals or slots. In each slot one of the K mobiles is allocated the channel. Each mobile m can be in one of the states $h_m \in \mathcal{H}_m$, where $\mathcal{H}_m$ is finite valued. We assume fading characteristics to be independent across the mobiles. Let $\mathbf{h} := [h_1, h_2, \cdots, h_K]^t$ be the vector of channel gains in a particular slot. The channel gains are distributed according to: $p_{\mathbf{H}}(\mathbf{h}) = \prod_{i=1}^{K} p_{H_i}(h_i)$, where

[1] They do not address the possibility of this advertised number being just another signal.

$\{p_{H_m}; m \leq K\}$ represents the statistics of the mobile channels. We assume that the mobiles estimate the channel h_m perfectly using the pilot signals sent by BS. The mobiles send signals $\{s_m\}$ to BS; these are indications of the channel gains. Some mobiles (with indices $1 \leq m \leq K_1$ where $1 \leq K_1 \leq K$) are assumed to be noncooperative and may signal a better channel condition to grab the channel even when their channel is bad. We assume that signals are chosen from the channel space itself, i.e., $s_m \in \mathcal{H}_m$ for all mobiles. BS makes a scheduling decision based on signals $\mathrm{s} := [s_1, s_2, \cdots, s_K]^t$.

Utilities: If the channel is allocated to mobile m, it gets a utility $f(h_m)$ which only depends upon its own channel state. An example utility is $f(h_m) = \log(1+h_m\mathsf{SNR})$ where SNR captures the nominal received signal-to-noise ratio under no channel variation. The utility of the BS is taken to be sum of the utilities of all the mobiles. Optimizing the sum utility at the BS results in an *efficient* solution. It however may be unfair because far-off mobiles may be ignored for they are not likely to contribute to efficiency. In this paper, we mainly study efficient solutions. This is fair in symmetric mobiles case, but not for asymmetric mobiles. Extensions to obtain fairness for asymmetric mobiles are discussed in Section VIII.

Common Knowledge : The channel statistics $\{p_{H_m}; m \leq K\}$ and the information about which mobiles are noncooperative is common knowledge (i.e., known to all the mobiles and the BS). If the BS does not know which mobiles are cooperative, it will treat every mobile as noncooperative and in this case $K_1 = K$. In Sections V-B to VII wherein the BS estimates either the signal statistics or the average utility of the mobile, the BS can detect the noncooperative mobiles.

We conclude this section with a motivating example that illustrates the problems and solution approaches considered in this paper.

A. A motivating example

To illustrate the main concepts, we consider two mobiles. The first one has a constant radio channel that allows it to have a rate of 8 units whenever it receives the channel, i.e.,. $\mathcal{H}_1 = \{h_1^0\}$ and $f(h_1^0) = 8$, and the second has three channel states $\mathcal{H}_2 = \{h_2^1, h_2^2, h_2^3\}$ with probabilities $(p_{H_2}(h_2^1), p_{H_2}(h_2^2), p_{H_2}(h_2^3)) = (1/4, 1/2, 1/4)$ and utilities $(f(h_2^1), f(h_2^2), f(h_2^3)) = (10, 2, 1)$. Mobile 1 (mobile 2) achieves its maximum utility of 8 (of $1/4*10+1/2*2+1/4*1 = 15/4$, respectively) when the channel is always allocated to it. BS achieves its maximum utility (defined as the sum of utilities) of $1/4*10+3/4*8 = 34/4$ if it schedules to mobile 2 when this mobile's channel is 10 and to mobile 1 otherwise. This is thus a game between the BS and the mobile 2 (mobile 1's signal is of no consequence). The deciding entity, the BS, will allocate the channel to the mobile with the maximum utility, thereby maximizing its own (sum) utility. Note that the BS can make this decision based only on the signals from the mobiles. With the above scheduling policy, mobile 2 obtains a utility of only $1/4 * 10 = 10/4$ if it truthfully signals its channel state. However, it can achieve its maximum utility of 15/4 it it always reports a good channel (h_2^1) regardless of its true channel state.

Now, suppose that the BS is aware of this possible noncooperative behavior. If the BS does not use any further information, other than the signals from the mobiles, it can achieve a better utility of 8 by always allocating the channel to mobile 1. When the BS sticks to this policy, mobile 2 can not improve its utility in any way. Thus, the policy of the BS to ignore the signals from mobile 2 and always allocate the channel to mobile 1 and any policy at the mobile 2 is a Nash equilibrium (NE). Such an NE is a babbling equilibrium ([21]).

If the BS is successful in extracting the true channel information from the mobile 2 (mechanisms to do this will be discussed in later sections) then it can achieve its maximum sum utility of $34/4$. This truth revealing NE, if it exists and henceforth called as TRE, is commonly referred to as a separating equilibrium ([21]). It is interesting to note that the above two NE are two extremes in the sense that one is the NE at which the BS's utility is minimum among all possible NE while the other is the NE at which the BS's utility is maximum.

In Sections IV and V we generalize the above example. We then return to this example in Section VII to illustrate further ideas.

III. Model and Background

We now formalize our notation and assumptions : For a set $\mathcal{C}$, let $\mathcal{P}(\mathcal{C})$ be the set of probability measures on $\mathcal{C}$. A policy of mobile m is a function $\{\mu_m(.|h_m)\}$ that maps a state h_m to an element in $\mathcal{P}(\mathcal{H}_m)$. A policy of the BS is a function $\beta = \{\beta(.|\mathbf{s})\}$ that maps each signal $\mathbf{s}$ to an element in $\mathcal{P}(\{1, 2, \cdots, K\})$. In later sections this policy can also be a function of other parameters.

The utility of the mobile m depends only upon the true channel h_m and the allocation A of the BS, where $A \in \{1, \cdots, K\}$, given by: $U_m(s_m, h_m, A) = 1_{\{A=m\}} f(h_m)$, while that of BS is $U_{BS}(\mathbf{s}, \mathbf{h}, A) = \sum_{m=1}^{N} U_m(s_m, h_m, A)$.

Remarks on choice of utility: A noncooperative mobile gets utility $f(h_m)$ when scheduled, regardless of the signal sent by the mobile and without the transmitter knowing the true channel. This is reasonable given the following observations. (1) The reported channel is usually subject to estimation errors and delays, an aspect that we do not consider explicitly in this paper. As a consequence, the current channel may be either higher or lower than the true report. To address this issue, the BS employs a *rateless* code, i.e., starts at an aggressive modulation and coding rate, gets feedback from the mobile after each transmission, and stops as soon as sufficient number of redundant bits are received to meet the decoding requirements. This incremental redundancy technique supported by hybrid ARQ is already implemented in the aforementioned standards (3GPP HSDPA and 1xEV-DO). Then a rate close to the true utility may be achieved. For example, in the first slot the BS transmits a packet at an aggressive rate r_0 bits / second. If the transmission fails, an equal number of additional redundancy bits are transmitted in the next slot, and so on, until the packet is successfully received. The cost of feedback is assumed negligible. If the number of retransmissions were $L \geq 1$, the utility achieved is r_0/L. If each such transmission unit is sufficiently small, r_0/L will be close to $f(h_m)$. The key aspect here is that once allocated, the BS physical layer will persist until the packet is successfully transported or maximum number of retransmissions is reached. (2) Scheduling is done by BS's MAC layer while the coding is done at the physical layer. There may be two different signals sent by the mobile - one to the scheduling entity of the BS, and the other to the physical layer. Suppose that the mobile noncooperatively signals to the BS's scheduler, but is truthful to the physical layer. The latter, knowing h_m, chooses a code rate that matches the true channel state and attains utility $f(h_m)$, but the scheduler decides based on s_m.

Define $\mathbf{h}_{-m} := [h_1, \cdots, h_{m-1}, h_{m+1}, \cdots, h_K]$, $p_{H_{-m}}(\mathbf{h}_{-m}) := \Pi_{j \neq m} p_{H_j}(h_j)$ and $\mu_{-m}(\mathbf{s}_{-m}|\mathbf{h}_{-m})$ $:= \Pi_{j \neq m; j \leq K_1} \mu_j(s_j \mid h_j)\ \Pi_{j \neq m; j > K_1} \delta(h_j = s_j)$ to exclude mobile m. Define $\mu(\mathbf{s}|\mathbf{h}) = \mu_1(s_1 \mid h_1)\mu_{-1}(\mathbf{s}_{-1}|\mathbf{h}_{-1})$.

Hence the instantaneous utility of mobile m, when its channel condition is h_m, when the mobiles use strategies $\mu := \{\mu_m; m \leq K_1\}$, and when the BS uses strategy β, is

$$\begin{aligned} U_m(\mu, h_m, \beta) &= \mathbb{E}_{\mathbf{h}_{-m}} \left[\sum_{\mathbf{s}} U_m(s_m, h_m, m) \beta(m \mid \mathbf{s}) \mu(\mathbf{s} \mid \mathbf{h}) \right] \\ &= f(h_m) \mathbb{E}_{\mathbf{h}_{-m}} \left[\sum_{\mathbf{s}} \beta(m \mid \mathbf{s}) \quad \mu(\mathbf{s} \mid \mathbf{h}) \right]. \end{aligned}$$

Throughout when $\arg\max S$ has more than one element, we write $i = \arg\max S$ to mean $i \in \arg\max S$. By $j := \arg\max S$ we mean that j is a chosen element of $\arg\max S$.

Strategic form games and Nash equilibrium: A strategic form game is described by a triplet consisting of a set of players, a policy/strategy set for each player, and the utility of each player. In our setting, this triplet for the $K_1 + 1$-player game is

$$((1, .., K_1), BS), (\mu_1, .., \mu_{K_1}, \beta), (\mathbb{E}[U_m]; m \leq K_1, \mathbb{E}[U_{BS}]) \,. \tag{1}$$

A Nash Equilibrium (NE) for this game is a strategy-tuple $(\mu_1^* .., \mu_{K_1}^*, \beta^*)$ that satisfies

$$\begin{aligned} \mu_m^* &= \arg\max_{\alpha} \mathbb{E}_{h_m}[U_m(\alpha, h_m, \beta^*)] \text{ for all } m \leq K_1 \\ \beta^* &= \arg\max_{\gamma} \mathbb{E}_{\mathbf{h}}[U_{BS}(\mu^*, \mathbf{h}, \gamma)]. \end{aligned}$$

IV. Signaling Game and Babbling Equilibrium

The signaling game for downlink scheduling is described as follows: K_1 mobiles with the lowest indices are the leaders or senders in the signaling game. The BS is the only follower or receiver. The true channel h_m represents the true *type* of leader m with signal s_m. The policies and utilities of the game are defined in previous paragraphs.

A refinement of NE for such games is a Perfect Bayesian Equilibrium (PBE). This is based on rationale of credible posterior beliefs (Kreps & Sobel [14, Sec. 5], Sobel [21]).

Definition *4.1:* [Posterior beliefs $\pi := \{\pi_m; m \leq K_1\}$] $\pi(h_m \mid s_m)$ is the BS's belief of the posterior probability that the mobile's true channel is h_m given its signal is s_m.

Definition *4.2:* [Perfect Bayesian Equilibrium (PBE)] A PBE is a strategy profile $(\mu_1^*, \cdots, \mu_{K_1}^*; \beta^*)$ and a posterior belief profile π^* such that : Given posterior belief profile π_i^*, for each signal vector $\mathbf{s}$, the BS chooses β^* such that

$$\beta^*(\cdot \mid \mathbf{s}) \;\in\; \arg \max_{\gamma \in \mathcal{P}((1,\cdots,K))} \sum_{j > K_1} \gamma(j) f(s_j) + \sum_{j \leq K_1} \gamma(j) \sum_{h_j} \pi_j^*(h_j \mid s_j) f(h_j). \tag{2}$$

Given β^*, each mobile $i \leq K_1$ chooses μ_i^*, such that,

$$\mu_i^*(\cdot \mid h_i) \;\in\; \arg \max_{\alpha \in \mathcal{P}(\mathcal{H}_i)} \sum_{s_i \in \mathcal{H}_i} \alpha(s_i) \left[\sum_{\mathbf{s}_{-i}} \beta^*(i \mid \mathbf{s}) \sum_{\mathbf{h}_{-i}} p_{H_{-i}}(\mathbf{h}_{-i}) \mu_{-i}^*(\mathbf{s}_{-i} | \mathbf{h}_{-i}) f(h_i) \right] \tag{3}$$

for each $h_i \in \mathcal{H}_i$. For each $i \leq K_1$, $s_i \in \mathcal{H}_i$, the BS updates,

$$\pi_i^*(h_i \mid s_i) = \frac{p_{H_i}(h_i) \mu_i^*(s_i \mid h_i)}{\sum_{h' \in \mathcal{H}_i} p_{H_i}(h') \mu_i^*(s_i \mid h')}, \tag{4}$$

if the denominator in (4) is nonzero, and $\pi_i^*(\cdot \mid s_i)$ is any element in $\mathcal{P}(\mathcal{H}_i)$ otherwise.

In Definition 4.2, (3) ensures that $(\mu_1^*, \cdots, \mu_{K_1}^*)$ is an NE of the subgame of noncooperative mobiles, (2) ensures that β^* is the Bayes-Nash equilibrium of the subgame with the BS, and (4) determines a consistent Bayesian approach to determining posterior beliefs.

In the sequel, we will come across two types of PBE ([21]). The first is the *babbling equilibrium* where the sender's strategy is independent of its type, and the receiver's strategy is independent of signals. The second is the desirable *separating equilibrium* where sender sends signals from disjoint subsets of the set of available signals for each type. Clearly then, the receiver gets complete information about the true types of the leaders. If this equilibrium is achieved, the BS can design a scheduling algorithm as in a fully cooperative environment. Hence a separating PBE is a Truth Revealing Equilibrium (TRE) that we already defined in Section I.

We will now show that without any extra intelligence to combat noncooperation, i.e., if the BS schedules based only upon the signals from the mobiles, there exist only babbling equilibria. The following theorem characterizes all the possible PBE of the signaling game.

Theorem 1: The above $K_1 + 1$ player signaling game has a PBE of the following type (for all $i,\ h_i,\ s_i$): With $i^*_{NC} := \arg\max_{i \le K_1} \mathbb{E}\left[f(h_i)\right]$, $i^*_C(\mathbf{s}) := \arg\max_{i > K_1} f(s_i)$ and $i^*(\mathbf{s}) := \arg\max_{i \in i^*_{NC} \cup i^*_C} \mathbb{E}\left[f\left(h_i\right) \mid s_i\right]$ for all $\mathbf{s}$,

$$\begin{aligned} \mu_i^*(s_i = s_i'|h_i) &\quad \text{equals} \quad \text{any fixed } \mu_i \in \mathcal{P}(\mathcal{H}_i), \\ \pi_i^*(h_i|s_i) &\quad = \quad p_{H_i}(h_i) \text{ and} \\ \beta^*(i|\mathbf{s}) &\quad \text{equals} \quad \text{any fixed } \gamma_{\mathbf{s}} \in \mathcal{P}(i^*(\mathbf{s})). \end{aligned}$$

Further, these are the only type of PBE for this game.

Proof : Please refer to the Appendix. □

In the above equilibrium, the (noncooperative) mobile's strategy is to send a signal independent of the channel value, and the BS's strategy is to ignore the signals from the noncooperative mobiles and use only prior information and the information from the cooperative mobiles. Hence it is a babbling equilibrium. Further, possibility of a 'separating' PBE (TRE) is ruled out.

The scheduling policy of the babbling PBE of Theorem 1 results in the following utility for the BS :

$$U^*_{cop} := \mathbb{E}_{h_m; m > K_1}\left[\max\left\{\max_{i > K_1} f(h_i),\ \max_{m \le K_1} \mathbb{E}[f(h_m)]\right\}\right]. \tag{5}$$

It is the best utility that the BS can get using only information from cooperative mobiles and the statistics of noncooperative mobiles. In Example II-A, U^*_{cop} was 8. Note that the BS uses very little information here to attain U^*_{cop} (only the signals and prior information). To do better, the BS has to use extra intelligence. In the next section we obtain the desirable TRE (Truth Revealing Equilibrium) via two different approaches.

V. SEPARATING EQUILIBRIUM

We realized in the previous section that we have only babbling equilibria in the presence of noncooperative mobiles.

In this section we obtain the desired TRE using two different approaches . The first approach introduces a negative cost into the mobile's payoff for erroneous reporting while in the second approach the BS uses the (predicted) signal statistics along with the signals while scheduling. The next section also obtains a TRE using more practical policies.

A. Penalty for deviant reporting

The BS does not have access to the true channel state of the mobile. But based on the actual throughput seen on the allocated link, the BS may extract the squared error $\{(h_i - s_i)2; i \leq K_1\}$ after the transmission is over. This error can used to punish the mobiles for deviant reporting.

More precisely, let mobile i report s_i when its channel is h_i and suppose it succeeds in getting the channel. For a $\psi \in (0, \infty)$, let us impose a penalty proportional to the squared error if it exceeds c_i, as follows:

$$U_i(h_i, s_i, A) = 1_{\{A=i\}} \left(f(h_i) - \psi 1_{\{(h_i - s_i)2 > c_i\}} (h_i - s_i)2 \right),$$

where $c_i > 0$ is chosen small enough such that for all $h_i \in \mathcal{H}_i$, $\{s_i : (s_i - h_i)2 \leq c_i\} \cap \mathcal{H}_i = \{h_i\}$. If we now choose ψ such that

$$\psi > \max_{\{i \leq K_1,\ (h_i, s_i) \in \mathcal{H}_i \times \mathcal{H}_i : (h_i - s_i)2 > c_1\}} \left\{ \frac{f(h_i)}{(h_i - s_i)\,2} \right\}, \tag{6}$$

then it is clear that $U_i(h_i, s_i, A)$ is negative whenever the action A is i, i.e., any deviant signaling results in negative utility to the mobile. This new utility function is very related to a pricing mechanism which is a powerful tool for achieving a more socially desirable result. Typically, the pricing is used to encourage the mobile to use system resource more efficiency and generate revenue for the system. Usage-based pricing is an approach commonly encountered in the literature. In usage-based pricing, the price a mobile pays for using resource is proportional to the amount of resource consumed by the user. In our case, the price corresponds to the cost a mobile pays for deviant reporting if it exceeds c_i. Through pricing, we obtain a separating PBE for the modified game :

Theorem 2: With ψ satisfying (6), the $K_1 + 1$-noncooperative game with the modified cost has the following separating PBE:

$$\begin{aligned} \mu_i^*(s_i|h_i) &= \delta_{\{h_i=s_i\}}(s_i) \text{ for all } i \leq K_1 \text{ and } h_i \in \mathcal{H}_i, \\ \pi_i^*(h_i|s_i) &= \delta_{\{s_i=h_i\}}(h_i) \text{ for all } i \leq K_1 \text{ and } s_i \in \mathcal{H}_i \text{ and} \\ \beta^*(i|\mathbf{s}) &= 1_{\{i=A^*(\mathbf{s})\}} \text{ for all } \mathbf{s} \\ A^*(\mathbf{s}) &:= i \text{ if } i \in \arg\max_j f(s_j) \end{aligned}$$

Proof : Please refer to the Appendix. □

B. By 'Predicting' the Signal Statistics

In this section, we assume that the BS is able to 'predict' the statistics of the signals generated by all the mobiles, perhaps based on observations of past behavior, before the game is executed. With $p_{S_m}(s) := \sum_h \mu_m(s|h)p_{H_m}(h)$ for all $s \in \mathcal{H}_m$ and for all m, let $\mathcal{P}_s := (p_{S_1}, p_{S_2}, \cdots, p_{S_K})$ represent the tuple of the signal probabilities of all the mobiles. The BS policy is now a probability measure over the set of mobiles for each signal vector s and each $\mathcal{P}_s$ and hence is given by $\beta(.|\mathrm{s}, \mathcal{P}_s)$. This results in a completely different game and we obtain a TRE using this new (more intelligent) policies.

The new game is still an incomplete information game but is no more a signaling game. We now consider a simple $K_1 + 1$ player strategic form game. The rest of the details of the game, i.e., all the utilities and the policies of the mobiles, remain the same as that in the previous sections.

Consider the following 'predictive' policy β_p^* of the BS: *BS allocates the channel to the mobile with maximum signaled rate if its signal statistics are same as its true channel statistics. If not the BS allocates the channel to the cooperative mobile with maximum signaled rate among the cooperative mobiles.*

Policy β_p^* at BS and the truth revealing signals (i.e., $\mu_m^*(s|h) = \delta(s = h)$ at all the mobiles forms a NE, i.e., a TRE, because: If a mobile over signals the channel state such that his signal statistics does not match with the true channel statistics the BS never allocates him the channel. *If a mobile generates a noncooperative signal such that the signal statistics remain the same as the true channel statistics, he will have to lose the channel in one of his good states for a gain of the channel in one of his bad states.*

At a TRE, the BS obtains the following utility :

$$U_{max}^* := \mathbb{E}_{\mathbf{h}} \left[\max_{i \leq K} f(h_i) \right]. \tag{7}$$

It is the utility that the BS can get by using the truthful information from all the mobiles. The utility 8.5 of the motivating example basically represents its U_{max}^*. It is easy to see that this is the maximum utility that the BS can get and hence is also the maximum utility of the BS at equilibrium among all the possible NE.

VI. Stochastic approximation

We saw in the previous section that the desired TRE was obtained by either introducing a negative cost for deviant reporting or by predicting the signal statistics. The negative cost depends on the square of the error between the actual channel and the reported signal. The BS should estimate this error (after successful transmission) using the actual rate obtained by the mobile on the assigned channel. Such a penalty-based mechanism may however be difficult to implement in practice.

The second approach to achieve the TRE needs estimates of signal statistics. Any estimation procedure will have errors and it will be interesting to study the impact of these errors on the desired NE. In this section we directly estimate the average (signaled) throughput of all the mobiles and use these estimates to obtain more realistic (truth revealing) policies at the BS.

A policy based on stochastic approximation is proposed. It operates over several time slots and estimates parameters online. Such an approach is also well-suited to track changes in model parameters. Our policy has a "corrective" feature because the BS continually (i) estimates the average throughput that each mobile gets; (ii) estimates the excess utility that each mobile accumulates beyond its cooperative share (its share in a cooperative setting); (iii) applies a "corrective" term based on the excess utility. The resulting estimates are then used to make scheduling decisions.

We define an appropriate K_1+1-player strategic form game and show that a "corrective" policy at the BS, along with the truth-revealing strategies at mobiles, forms an ϵ-Nash equilibrium, and the policy is thus near-TRE[2].

We begin by first defining the policies and utilities of the players involved in the game (some notations of this section are different from the rest of the paper). The policy of a BS is a time-varying function whose action at time k depends on the signals sent by the mobiles up to and including time k. Throughout this section, we model channel gains as bounded random variables taking values on a continuum and satisfying assumption **A.**3 given below. The policy of a mobile m for $m \leq K_1$ is given by a measurable signal map $s_m : \mathcal{H}_m \mapsto \mathcal{H}_m$ that satisfies assumption **A.**2 given below. For the sake of uniformity we define $s_m(h_m) = h_m$ for $m > K_1$, i.e., the cooperative mobiles report the true channel. Thus, the signaled utility at time k is $f(s_m(h_{m,k})); m \leq K$, while the true utility is $f(h_{m,k}); m \leq K$.

Next we define the utilities of mobiles, BS. Let $\phi_{m,k}$ be the slot-level utility derived by mobile m in slot k. Note that $0 \leq \phi_{m,k} \leq f(h_{m,k})$ and $\phi_{m,k} = 0$ if the channel is not assigned to mobile m in slot k. We then set for all $m \leq K$,

$$U_m = \lim_{k\to\infty} \frac{1}{k} \sum_{l \leq k} \phi_{m,l} \quad \text{and } U_{BS} = \sum_{m=1}^{K} U_m,$$

if all the limits exist. In a TRE, the BS achieves the maximum sum utility U^*_{max} given in (7) while the mth mobile gets

$$U^*_m = \theta^0_m := \mathbb{E}_{\mathbf{h}} \left[f(h_m) 1_{\left\{ f(h_m) \geq f(h_j) \text{ for all } j \neq m \right\}} \right].$$

Note that BS can calculate $\theta^0 := (\theta^0_1, \cdots, \theta^0_K)$ with its available knowledge. We now propose the following iterative "corrective" scheduling algorithm at the BS:

$$\begin{aligned} \theta_{m,k+1} &= \theta_{m,k} + \epsilon_k \left(\tilde{f}_{m,k+1} I_{m,k+1} - \theta_{m,k} \right), \\ \tilde{f}_{m,k+1} &= f(s_m(h_{m,k+1})) - \left(\theta_{m,k} - \theta^0_m \right) \Delta, \qquad (8) \\ I_{m,k+1} &= 1_{\left\{ \tilde{f}_{m,k+1} \geq \tilde{f}_{j,k+1} \text{ for all } j \neq m \right\}}, \qquad (9) \end{aligned}$$

with initial conditions $\theta_{m,0} = \theta^0_m$ for all m. The BS for each mobile i) tracks average reported utility via $\theta_{m,k}$, ii) computes excess utility $\theta_{m,k} - \theta^0_m$, relative to the mobiles cooperative share

[2]This ϵ-Nash equilibrium is in fact slightly stronger equilibrium than NE.

in a TRE, iii) subtracts the excess from the instantaneous signaled utility after magnification by Δ, and uses the updated values to make a current scheduling decision. The choice of Δ depends on ϵ.

If BS schedules mobile m in slot k, the latter gets a utility

$$\bar{f}_{m,k} \quad := \quad \max\left\{0, \min\left\{\tilde{f}_{m,k}, f(h_{m,k})\right\}\right\}. \tag{10}$$

Indeed, if $\tilde{f}_{m,k} < 0$ for the selected mobile, no transmission is made. If $\tilde{f}_{m,k} < f(h_{m,k})$, transmission is made at lesser rate to get a slot-level utility of $\tilde{f}_{m,k}$. If $\tilde{f}_{m,k} \geq f(h_{m,k})$, our remarks on utilities in Section III provide a slot-level utility of $f(h_{m,k})$. Thus the achieved utility in slot k is (10). Consequently, utility for mobile m is $U_m = \lim_{k\to\infty}\frac{1}{k}\sum_{l=1}^{k}\bar{f}_{m,l}I_{m,l}$, which can be rewritten as limit of,

$$\bar{\theta}_{m,k+1} \quad = \quad \bar{\theta}_{m,k} + \epsilon_k\left(\bar{f}_{m,k+1}I_{m,k+1} - \bar{\theta}_{m,k}\right), \tag{11}$$

where $\bar{\theta}_{m,k} = k^{-1}\sum_{l\leq k}\bar{f}_{m,l}I_{m,l}$ and $\epsilon_k = 1/(1+k)$. Thus the mobile and the BS utilities are

$$U_m = \lim_{k\to\infty}\bar{\theta}_{m,k} \text{ for all } m \leq K \text{ and } U_{bs} = \sum_m U_m. \tag{12}$$

Analysis of the policy: Let $S = (s_1, s_2, \cdots s_K)$ represent a strategy profile. (last $K - K_1$ maps are identity maps).

Assumptions : We assume the following :

A.1 The function f is bounded, invertible and continuously differentiable. The function f^{-1} is also continuously differentiable;

A.2 The signal maps s_m are such that the density of the random variables $s_m(h_m)$ are bounded;

A.3 The processes $\{h_{m,k}\}_{k\geq 1}$ are stationary Markov chains for each mobile m, and independent across mobiles. The random variable $h_{m,1}$ is a bounded random variable with bounded density for each m.

One of the most common approaches to analyze a stochastic adaptive algorithm is based on the ordinary differential equation (ODE) approximation theory ([15], [7] etc.). In this section we use the ODE approximation theory to analyze the utilities (12) and obtain optimality properties of policy (8). Define $\theta_k := (\theta_{1,k}, \cdots, \theta_{m,k})$, $\bar{\theta}_k := (\bar{\theta}_{1,k}, \cdots, \bar{\theta}_{m,k})$ and $\Theta_k := \left(\theta_k, \bar{\theta}_k\right)$. We will show that the trajectories of $(\theta_k, \bar{\theta}_k)$, in any finite time scale, converge to the solution of the

ODE system

$$\dot{\theta}(t) = H^S(\theta(t)) - \theta(t); \quad \theta(t_0) = \theta_0 \tag{13}$$

$$\dot{\bar{\theta}}(t) = \bar{H}^S(\theta(t)) - \bar{\theta}(t); \quad \bar{\theta}(t_0) = \bar{\theta}_0 \tag{14}$$

$$\begin{aligned} H_m^S(\theta) &:= \mathbb{E}_{\mathbf{h}}\left[\tilde{f}_m^S(h_m,\theta) I_m^S(h,\theta)\right] \\ \bar{H}_m^S(\theta) &:= \mathbb{E}_{\mathbf{h}}\left[\bar{f}_m^S(h_m,\theta_m) I_m^S(h,\theta)\right] \\ \tilde{f}_m^S(h_m,\theta) &:= f(s_m(h_m)) - (\theta_m - \theta_m^0)\Delta \\ I_m^S(h,\theta) &:= 1_{\left\{\tilde{f}_m^S(h_m,\theta) \geq \tilde{f}_j^S(h_j,\theta) \text{ for all } j \neq m\right\}}. \\ \bar{f}_m^S(h_m,\theta_m) &:= \max\left\{0, \min\left\{\tilde{f}_m^S(h_m,\theta_m), f(h_m)\right\}\right\}. \end{aligned}$$

By Lemma 1 of Appendix, the above ODE system has a unique solution for any finite time scale. Define $t(r) := \sum_{k=0}^{r} \epsilon_k$, $m(n,T) := \arg\max_{r \geq n}\{t(r) - t(n) \leq T\}$. Let $\Theta_k := \left(\theta_k, \bar{\theta}_k\right)$. Let $\Theta(t, t_0, (\theta_0, \bar{\theta}_0))$ represent the solution pair of the ODEs (13) and (14) with the initial condition $\Theta(t_0) = (\theta_0, \bar{\theta}_0)$.

Theorem 3: Assume **A**.1–**A**.3 and that the step sizes $\{\epsilon_k\}$ satisfy assumption **B.**0 of Appendix. For any $T > 0$, for all $\delta > 0$, for any $\mathbf{h}$, and when $(\mathbf{h}_n, \Theta_n)$ is initialized with $(\mathbf{h}, (\theta, \bar{\theta}))$,

$$P_{n:\mathbf{h},\theta,\Theta_n}\left\{\sup_{\{n \leq r \leq m(n,T)\}} \left|\Theta_r - \Theta\left(t(r),\ t(n),\ \Theta_n\right)\right| \geq \delta\right\} \to 0$$

as $n \to \infty$ uniformly for all $(\theta, \bar{\theta}) \in Q_1$, where Q_1 is a subset of the bounded set of solutions of the ODEs (13), (14) until time T. In the above, $P_{n:\mathbf{h},\Theta_n}$ denotes distribution of $(\mathbf{h}_{n+k}, \Theta_{n+k})$ with $\mathbf{h}_n = h, \Theta_n = (\theta, \bar{\theta})$.

Proof : Please refer to the Appendix. □

By the above theorem the trajectory $\{\bar{\theta}_{i,k}; i \leq K\}$ is approximated by the solution of the ODE (14). Thus, one can study its limit via the attractors of the ODE (14). We hence analyze the utilities (12) by replacing the limits of the trajectoies with the the attractors of the above ODE.

Any attractor θ^* of the ODE (13) satisfies

$$\theta_m^* - \theta_m^0 = \frac{\mathbb{E}_{\mathbf{h}}\left[f(s_m(h_m)) I_m^S(h,\theta^*)\right] - \theta_m^0}{1 + \Delta E\left[I_m^S(h,\theta^*)\right]}.$$

If $E\left[I_m^S(h,\theta^*)\right] = 0$ (mostly occurs when $\theta_m^0 = 0$) then clearly $\theta_m^* = 0$. If not, for large enough values of Δ, $\theta_m^* \approx \theta_m^0$, i.e., either $\theta_m^* = 0$ or $\theta_m^* \leq \theta_m^0 + o(1/\Delta)$. Further, any attractor of ODE

(14) satisfies $\bar{\theta}^* = \bar{H}^S(\theta^*)$ leading to $\bar{\theta}^* \leq \theta^*$. Thus,

$$U_m \approx \bar{\theta}^* \leq \theta_m^* \leq \theta_m^0 + o(1/\Delta). \tag{15}$$

Let $S = I$ be the truth revealing strategy profile ($s_m(h_m) = h_m$ for all h_m, m). Clearly θ^0 is a zero of RHS of ODEs (13), (14). One can easily show that it will indeed be an attractor by showing that the derivative of $H^S(\theta) - \theta$ is negative definite near θ^0. Hence $U_i = \theta_i^0$ when $S = I$.

From (15) none of the mobiles, no matter what strategy they use or no matter what strategy the other mobiles use, can gain more than θ_i^0. This along with para above implies that *the 'corrective' policy (with appropriately large Δ) of BS together with the truth-revealing signals at all the mobiles forms an ϵ-Nash equilibrium.*

Some further Remarks : Note for large values of Δ,

$$\begin{aligned}(\theta_m^* - \theta_m^0)\Delta &= \frac{\Delta(\mathbb{E}_{\mathbf{h}}\left[f(s_m(h_m))I_m^S(h,\theta^*)\right] - \theta_m^0)}{1 + \Delta E\left[I_m^S(h,\theta^*)\right]} \\ &\approx \frac{\mathbb{E}_{\mathbf{h}}\left[f(s_m(h_m))I_m^S(h,\theta^*)\right] - \theta_m^0}{E\left[I_m^S(h,\theta^*)\right]}\end{aligned}$$

which can be significant but is bounded (independently of Δ) because of the boundedness of h. Now, if any mobile reports much more than its true value, i.e., if $f(h_m) \ll f(s_m(h_m))$ for significant values of h_m, and if in fact it is large enough such that $f(h_m) \ll f(s_m(h_m)) - (\theta_m^* - \theta_m^0)\Delta$ then,

$$U_m \ll \mathbb{E}_{\mathbf{h}}[(f(s_m(h_m)) - (\theta_m^* - \theta_m^0)\Delta)I_m^S(h,\theta^*)] = \theta_m^*.$$

Hence $U_m \ll \theta_m^0$, i.e., that particular mobile's output is much lesser than θ_m^0, its own cooperative share. *Hence the mobile which deviates the most from true values gets the least share.*

We present an example to illustrate the above with 3 mobiles. The mobiles are truncated Gaussian distributed, i.e., $h_m \sim \min\{0, \max\{\mathcal{N}(3, \sigma_m^2), 6\}\}$. We set $\Delta = 100$. Our $s_m(h_m) := \max\{h_m + \delta_m, 6\}$. We see from the table that all the (asymptotic) reported rates, θ^* are close

$\sigma_1^2, \sigma_2^2, \sigma_3^2$	$\delta_1, \delta_2, \delta_3$	$\theta_1^0, \theta_2^0, \theta_3^0$	$\theta_1^*, \theta_1^*, \theta_3^*$	U_1, U_2, U_3
1.5, .9, 1.5	4, 4, .5	.58, .44, .58	.587, .442, .584	.543, .38, .574
1.5, 1.5, 1.5	0, 4, 0	.54, .54, .54	.545, .548, .543	.542, .48, .535
1.5, 1.5, 1.5	0, 0, 0	.54, .54, .54	.546, .543, .543	.542, .543, .543

to the cooperative shares θ^0 in all cases. But the actual rates gained by the mobiles, U_m, are close to θ_m^0 only when the particular mobile is cooperative. Further, the more the value of δ (i.e., more a mobile deviates from its true type), the more it loses.

We consider another numerical example to illustrate the accuracy of ODE approximation as well as the robustification of the proposed stochastic approximation algorithm against non-cooperation. We consider 2 mobiles in a Ricean fading environment. The rates of mobile m are given by $f(h_m) = log\left(1 + P\sqrt{h_m(i)^2 + h_m(r)^2}\right)$ with $h_m(i), h_m(r) \sim \mathcal{N}(3, 1/2)$. The signaled rates are : $f(s_m(h_m)) = \max\{f(h_m) + \delta, 2.2\}$. We plot the trajectories of the first mobile, $(\theta_{1,k}, \bar{\theta}_{1,k})$ against iteration number k for two values of δ equal to 0 and 4. We set $\Delta = 5000$, $\epsilon_k = 0.0001k^{-.1}$. We also plot the first mobile's cooperation share, θ_1^0 in the same figure. The trajectories converge close to the attractors of the ODE as is shown in the theory.

From figure 1 we can see that the signaled rates $\theta_{1,k}$ for both values of δ (actually both the lines almost merge with each other in the figure) converge close to θ_1^0 while the actual rates $\bar{\theta}_{1,k}$ converges close to θ_1^0 only for $\delta = 0$ i.e. when mobile 1 is cooperative. With $\delta = 4$ even though the signaled rate converges close to θ_1^0 the true rate experienced by the mobile is much below θ_1^0 which illustrates the robustification of the 'corrective' SA policy against noncooperation. Thus the SA policy has properties as suggested by the theory.

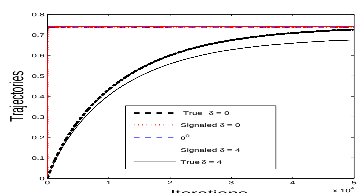

Fig. 1. Robust SA algorithm in the presence of noncooperation

VII. Existence of other Nash Equilibria

We obtained two types of NE till now. Under the first equilibrium (babbling equilibrium of Theorem 1) BS schedules only using the signals from the cooperative mobiles and the channel statistics of the noncooperative mobiles. The BS utility is the minimum among all the possible equilibrium utilities and equals U_{cop}^* given in (5).

The other type of NE (equilibria of Sections V, VI), are the truth revealing equilibria (TRE). BS achieved these equilibria by using ITR (incentives for truth revealing) policies. When in a

TRE, the BS schedules using the true channel information of all the mobiles. BS now achieves the maximum possible equilibrium utility U^*_{max} given in (7).

Clearly $U^*_{cop} \leq U^*_{max}$. This raises a natural question about the existence of other NE with BS's equilibrium utility taking any value in the interval $[U^*_{cop}, U^*_{max}]$. In this section we continue with the 'predictive' policies, $\beta(.|\mathrm{s}, \mathcal{P}_s)$, of Section V-B and investigate the existence of other NE in Theorem 4.

We will first return to the motivating example of the Section II to describe the main ideas of this section.

A. Motivating Example continued

optimal policy for mobile 2 Here we assume that the mobile uses the following policy $\bar{\mu}_2$:

- It declares that it is in state 10 when it is in that state, i.e., $\bar{\mu}_2(h^1_2 \mid h^1_2) = 1$ and in addition
- it declares with probability ρ that it is in 10 whenever it is in state 2, i.e., $\bar{\mu}_2(h^1_2|h^2_2) = \rho = 1 - \bar{\mu}_2(h^2_2|h^2_2)$.
- Finally let $\bar{\mu}_2(h^3_2|h^3_2) = 1$.

Let's choose ρ such that the best response of the BS to this policy is to allocate to mobile 2 whenever state 10 is declared. For this to hold, ρ should be such that the utility of the BS is at least 8, since the BS can always secure the $U^*_{cop} = 8$ by always allocating to the cooperative mobile 1. For such ρ, the utility of BS and mobile 2 are given by

$$\begin{aligned} U_{BS} &= \frac{1}{4} \cdot 10 + \rho \cdot \frac{1}{2} \cdot 2 + (1-\rho) \cdot \frac{1}{2} \cdot 8 + \frac{1}{4} 8 = 8.5 - 3\rho \\ U_2 &= 2\frac{1}{2}\rho + 10\frac{1}{4}. \end{aligned} \tag{16}$$

Since ρ should satisfy the constraint $8.5 - 3\rho \geq 8$, ρ that maximizes the utility of mobile 2 is $\rho = 1/6$. The probability that mobile 2 declares that its channel to be in state 10 is

$$p_{S_2}(h^1_2) = \frac{1}{4} + \frac{1}{2}\frac{1}{6} = \frac{7}{24}.$$

Thus with the above policy $\bar{\mu}_2$ of the mobile, the best response policy of the BS among the simple policies is to select mobile 2 whenever it declares a 10. Denote it by $\bar{\beta}$.

The couple $(\bar{\mu}_2, \bar{\beta})$ is not an equilibrium since the best response of mobile 2 against $\bar{\beta}$ is simply to declare always that the channel is 10. Hence the BS should allocate channel to mobile 2 whenever it declares 10, only if it is guaranteed a utility that is at least equal to 8. This can be done in a way similar to that in Section V-B by allocating the channel to mobile 2 after further verifying that the mobile 2 declares to be in 10 for not more than $\frac{7}{24}$ of time.

More precisely, the BS choses the following signal 'predictive' policy (this policy knows apriori the signal probabilities of the mobile 2 and uses it for decision making) policy $\hat{\beta}$: whenever mobile 2 declares 10 allocate channel to mobile 2 with probability $q(p_{S_2})$ where

$$q(p_{S_2}) := \min \left\{ 1, \frac{p_{S_2}(h_2^1)}{7/24} \right\}$$

Hence $(\bar{\mu}_2, \widehat{\beta})$ is an equilibrium that guarantees a rate of $8/3$ to mobile 2, a rate of $16/3$ to mobile 1 and a total rate of 8 to the BS.

Infinitely many equilibria in feedback policies. In the sequel, we show that there is an infinity of Nash equilibria in which the BS gets a utility greeter than $U^*_{cop} = 8$. We use the same type of policy $\bar{\mu}_2$ for mobile 2, but we choose $\rho < 1/6$. Then the probability that mobile 2 declares that it is in state 10 is $\bar{p}_{S_2}(h_2^1) = 1/4 + \rho/2$. Consider now that the BS uses policy $\tilde{\beta}$: BS selects mobile 2 with probability q whenever the mobile declares that it in state 10 where

$$q = \min \left(1, \frac{p_{S_2}(h_2^1)}{1/4 + \rho/2} \right)$$

Thus the utility of BS and mobile 2 are $8 + 1/2q(1 - 6\rho)$ and $10/4.q + q\rho$, respectively. It is easy to show that the couple $(\bar{\mu}_2, \tilde{\beta})$ is a Nash equilibrium for each $\rho \in [0, 1/6]$. In the figure 2, we plot the utility of BS, mobile 1 and mobile 2 at equilibrium as function of ρ.

B. Main result : Generalization of the above example

In this section we generalize the example of the previous section to arbitrary number of players and states.

In this section, we assume that signal statistics of all the mobiles $\mathcal{P}_s$ is known to the BS. Hence the BS's policy is given by $\beta(.|\mathbf{s}, \mathcal{P}_s)$ as in Section V-B

Let $E^{\mu_m}[f(h_m)|s_m]$ represent the conditional expectation of the mobile's utility conditioned on the signal s_m when mobile m uses strategy μ^m, i.e., for every $s_m \in \mathcal{H}_m$ define

$$E^{\mu_m}[f(h_m) \mid s_m] := \sum_{h_m \in \mathcal{H}_m} \frac{p_{H_m}(h_m)\mu_m(s_m \mid h_m)}{\sum_{\tilde{h}_m \in \mathcal{H}_m} p_{H_m}(\tilde{h}_m)\mu_m(s_m|\tilde{h}_m)} f(h_m).$$

With this, the payoff for mobile m is,

$$U_m(\mu_m, \beta) = \mathbb{E}_{\mathbf{s},\mathbf{h}}\left[\beta(m|\mathbf{s}, \mathcal{P}_s) f(h_m)\right] = \mathbb{E}_{\mathbf{s}}\left[\beta(m|\mathbf{s}, \mathcal{P}_s) E^{\mu_m}\left[f(h_m)|s_m\right]\right]. \quad (17)$$

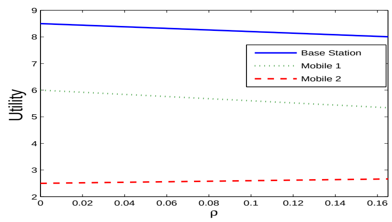

Fig. 2. The utility of Base station, mobile 1 and mobile 2 at equilibrium as function of ρ

In the above, $\mathbb{E}_{\mathbf{s}}$ represents the expectation w.r.t. the probability measure $\mathcal{P}_s$.

Given a signal probability K-tuple $\mathcal{P}_s$, let μ^* (or more appropriately $\mu^*(\mathcal{P}_s)$) represent 'best' mobile strategy that gives for every $h^i, h^j \in \mathcal{H}_m$ and for all $m \leq K$

$$f(h^i) \geq f(h^j) \implies E^{\mu_m^*}[f(h_m)|s_m = h^i] \geq E^{\mu_m^*}[f(h_m)|s_m = h^j].$$

Construction of μ^* : Consider mobile 1 without loss of generality. Let $\mathcal{H}_1 = \{h^1, h^2, \cdots, h^{N_1}\}$ and assume $f(h^1) \geq f(h^2) \geq \cdots \geq f(h^{N_1})$. In the following few lines we leave subscript 1 to improve readability i.e., the random variables h_1, s_1 etc. are represented by h, s etc. Strategy μ_1^* is defined in a iterative way.

We will first define $\{\mu_1^*(s = h^1|h); h \in \mathcal{H}_1\}$, i.e., the conditional probabilities of declaring to be in its best state h^1 by mobile 1 when it is actually in any arbitrary state $h \in \mathcal{H}_1$. Find the minimum index j_1^* such that probability of the channel to be in one of the top j_1^* states is greater than or equal to $p_{S_1}(h^1)$, i.e., let, $j_1^* := \arg\min_j \left\{ \sum_{i=1}^{j} p_{H_1}(h^i) \geq p_{S_1}(h^1) \right\}$. Declare state h^1 whenever the true channel is one among the top $j_1^* - 1$ states, i.e., for all h with $f(h) > f(h^{j_1^*})$, set $\mu_1^*(s = h^1|h) := 1$. When $h = h^{j_1^*}$ signal to be in best state h^1, for a fraction of time, where the fraction is chosen such that the overall probability of signal $s = h^1$ will be equal to $p_{S_1}(h^1)$, i.e., $\mu_1^*(s = h^1|h = h^{j_1^*}) = \frac{p_{S_1}(h^1) - \sum_{l<j_1^*} p_{H_1}(h^l)}{p_{H_1}(h^{j_1^*})}$. Set $\mu_1^*(s = h^1|h^i) = 0$ whenever $i > j_1^*$.

Now we define $\{\mu_1^*(s = h^2|h); h \in \mathcal{H}_1\}$, i.e., the conditional probabilities of declaring to be in the second best state h^2 by mobile 1, when in any arbitrary state $h \in \mathcal{H}_1$. Let $j_2^* := \arg\min_j \left\{\sum_{l=1}^{j} p_{H_1}(h^l) - p_{S_1}(h^1) \geq p_{S_1}(h^2)\right\}$. Now define (the below definitions are for $j_2^* > j_1^*$. If not one can appropriately modify the definitions.),

$$\begin{aligned}
\mu_1^*(s = h^2 \mid h^{j_1^*}) &= 1 - \mu_1^*(s = h^1|h^{j_1^*}), \\
\mu_1^*(s = h^2 \mid h^i) &= 1 \text{ whenever } j_1^* < i < j_2^*, \\
\mu_1^*(s = h^2 \mid h^{j_2^*}) &= \frac{p_{S_1}(h^1) + p_{S_1}(h^2) - \sum_{l<j_2^*} p_{H_1}(h^l)}{p_{H_1}(h^{j_2^*})} \\
\mu_1^*(s = h^2 \mid h) &= 0 \text{ for the remaining } h.
\end{aligned}$$

Note that with the above definitions

$$E^{\mu_1^*}[f(h_1)|s = h^1] \;\; \geq \;\; f(h^{j_1^*}) \geq E^{\mu_1^*}[f(h_1)|s = h^2].$$

Continue in the same way to obtain

$$E^{\mu_1^*}[f(h_1)|s = h^1] \geq E^{\mu_1^*}[f(h_1)|s = h^2] \geq \cdots \geq E^{\mu_1^*}[f(h_1)|s = h^{N_1}]. \tag{18}$$

By Result 1 given below, if mobile 1 uses any other strategy μ_1 resulting again in the same signal probability p_{S_1} of $\mathcal{P}_S$ while all other mobiles use their 'best' strategy and the BS uses policy $\beta(m|\mathbf{s}, \mathcal{P}_s) := 1_{\{m = \arg max_m \mathbb{E}[f(h_m)|s_m]\}}$, then $U_1(\mu_1^*, \beta) \geq U_1(\mu_1, \beta)$. These strategies are called 'best' because with these, the mobile m gets the best payoff for masquerading a signal probability p_{S_m}.

For $m > K_1$, we set $\mu^*(h_m \mid s_m) = 1_{\{h_m = s_m\}}$. With the help of 'best' strategies we obtain the existence of other NE:

Theorem 4: For every signal probability tuple $\bar{\mathcal{P}}_s$ with the 'best' strategies $\bar{\mu}^*$ such that,

$$U_{cop}^* \;\; \leq \;\; \mathbb{E}_{\mathbf{h},\mathbf{s}}\left[f\left(h_{m^*}\right)\right] \text{ with } m^* := \arg\max_{1\leq m\leq K} E^{\bar{\mu}_m^*}[f(h_m)|s_m],$$

the ordered pair $\left(\bar{\mu}^*, \bar{\beta}^*\right)$ is a Nash equilibrium where the feedback policy $\bar{\beta}^*$ of the BS is given by following:

Let $\mu = (\mu_1, \mu_2, \cdots \mu_{K_1})$ be any arbitrary signaling policy of the mobiles and let $\mathcal{P}_s = \{p_{S_m}; m \leq K_1\}$ be the signaling probabilities resulting from these actions of the noncooperative mobiles. Define,

$$q_m(\mathcal{P}_s, s_m) := \min\left\{1, \frac{\bar{p}_{S_m}(s_m)}{p_{S_m}(s_m)}\right\} \text{ for all } s_m \in \mathcal{H}_m,$$

and for all $m \leq K_1$. For $m > K_1$ define $q_m(\cdot,\cdot) = 1$. For any given signal vector s define, $m_1^*(\mathbf{s}) := \arg\max_{m \leq K} \quad E^{\bar{\mu}_m^*}[f(h_m)|s_m]$ (the best among all the mobiles) and $m_2^*(\mathbf{s}) := \arg\max_{m > K_1} f(s_m)$ (best among cooperative mobiles). Then define,

$$\begin{aligned} \bar{\beta}^*(m|\mathbf{s}, \mathcal{P}_s) &= 0 \quad \text{for all} \quad m \neq m_1^*, m_2^*, \\ \bar{\beta}^*(m_1^*|\mathbf{s}, \mathcal{P}_s) &= q_{m_1^*}\left(\mathcal{P}_s, s_{m_1^*}\right), \\ \bar{\beta}^*(m_2^*|\mathbf{s}, \mathcal{P}_s) &= \left(1 - q_{m_1^*}\left(\mathcal{P}_s, s_{m_1^*}\right)\right). \end{aligned} \tag{19}$$

Proof : If all the noncooperative mobiles are fixed with signaling policy $\bar{\mu}^*$ then the signaling probabilities will be given by $\bar{\mathcal{P}}_s$ and we have, $q_m(\bar{\mathcal{P}}_s, s_m) = 1$ for all $s_m \in \mathcal{H}_m$ and for all $m \leq K$. Hence $\bar{\beta}^*(m|\mathbf{s}, \bar{\mathcal{P}}_s) = 1_{\{m = m_1^*(\mathbf{s})\}}$.

From (17), the total payoff of the BS with signal probabilities fixed at $\bar{\mathcal{P}}_s$, when it uses some arbitrary channel allocation say $\beta(.|\mathbf{s})$, is given by,

$$U_{BS} = \mathbb{E}_{\mathbf{s}}\left[\sum_{m=1}^{K} \beta(m|\mathbf{s}) \mathbb{E}^{\bar{\mu}_m^*}[f(h_m)|s_m]\right].$$

Clearly, the BS achieves the maximum with $\bar{\beta}^*$.

Say BS uses the policy $\bar{\beta}^*$. Without loss of generality assume mobile 1 unilaterally deviates from strategy $\bar{\mu}_1^*$ and signals instead using μ_1 such that the signal probabilities remain the same. Then by the Result 1 mobile 1 gets lesser than before. If now μ_1 is such that even the signal probabilities are different from $\bar{p}_{S_1}$ then the payoff of the mobile 1 is further reduced as is seen from (19), as now it is possible that $q_1(\mu_1, s_1) < 1$ for some values (note that $(1 - q_1(\mu_1, s_1))$ fraction of the time channel is allocated to a cooperative mobile) and the rest steps are as in the proof of Result 1 stated next. □

We would like to emphasize here that the BS calculates m_1^*, m_2^* of the above theorem using $\bar{\mu}^*$ irrespective of the strategies actually used at the mobiles.

Result 1: Say all mobiles other than 1 use their 'best' strategies, i.e., mobiles m with $m > 1$ use strategy $\bar{\mu}_m^*$. Also assume that BS uses the policy in(19). The payoff of mobile 1 is maximized with its 'best' strategy $\bar{\mu}_1^*$, when its signal probabilities are restricted to $\bar{p}_{S_1}$.

Proof : Under the given hypothesis, the BS's policy from (19) would be $1_{\{m=m_1^*(\mathbf{s})\}}$. Hence the payoff of the mobile if it uses some arbitrary policy μ_1 from (17) will be :

$$U_1(\mu_1, \bar{\beta}^*) = \mathbb{E}_{\mathbf{s}}\left[1_{\{m=m_1^*(\mathbf{s})\}} E^{\mu_1}\left[f(h_1) \mid s_1\right]\right]$$

Define the probability measure:

$$\tilde{p}(u) \ := \ \Pr\left(\mathbf{s}_{-1} : u = \max_{1<m\leq K} E^{\bar{\mu}^*_m}[f(h_m) \mid s_m]\right)$$

for all possible u resulting from $\bar{\mathcal{P}}_s$ and $\{\bar{\mu}^*_m; m > 1\}$.

By independence, we are interested in the following constrained optimization problem :

$$\max_{\{\mu_1\}} \sum_{\tilde{u}} \sum_{s\in\mathcal{H}_1} \tilde{p}(\tilde{u}) 1_{\{\bar{E}(s)>\tilde{u}\}} \sum_{h\in\mathcal{H}_1} p_{H_1}(h)\mu_1(s|h)f(h)$$

$$\text{where } \bar{E}(s) \ := \ E^{\bar{\mu}^*_1}[f(h_1)|s_1 = s] = \sum_{\tilde{h}\in\mathcal{H}_1} \frac{\bar{\mu}^*_1(s|\tilde{h})p_{H_1}(\tilde{h})}{\bar{p}_{S_1}(s)} f(\tilde{h}) \text{ subject to}$$

$$\sum_{s\in\mathcal{H}_1} \mu_1(s|h) \ = \ 1 \text{ for all } h \in \mathcal{H}_1 \text{ and } \sum_{h\in\mathcal{H}_1} p_{H_1}(h)\mu_1(s|h) = \bar{p}_{S_1}(s) \text{ for all } s \in \mathcal{H}_1.$$

If say as before, $\mathcal{H}_1 = \{h^1, h^2, \cdots, h^{N_1}\}$ is arranged in the decreasing order of their utilities (i.e. such that $f(h^1) \geq f(h^2) \cdots \geq f(h^{N_1})$), then clearly by definition of 'best' strategies $\bar{\mu}^*$,

$$\bar{E}(h^1) \geq \bar{E}(h^2) \geq \cdots \geq \bar{E}(h^{N_1}). \quad (20)$$

One can rewrite the above objective function after defining $r(h, s) := p_{H_1}(h)\mu_1(s|h)$, as,

$$\sum_s \sum_h r(h, s)f(h) \left(\sum_{\tilde{u}} 1_{\{\bar{E}(s)>\tilde{u}\}} \tilde{p}(\tilde{u})\right) \quad (21)$$

From (20) it is easy to see that,

$$\left(\sum_{\tilde{u}} 1_{\{\bar{E}(h^1)>\tilde{u}\}} \tilde{p}(\tilde{u})\right) \geq \left(\sum_{\tilde{u}} 1_{\{\bar{E}(h^2)>\tilde{u}\}} \tilde{p}(\tilde{u})\right) \geq \cdots \left(\sum_{\tilde{u}} 1_{\{\bar{E}(h^{N_1})>\tilde{u}\}} \tilde{p}(\tilde{u})\right).$$

Hence, the maximum in (21) is achieved under the above mentioned constraints, by first maximizing the term (the constraints on this alone will be less strict)

$$\sum_h r(h, h^1)f(h) \text{ subject to}$$
$$\sum_h r(h, h^1) \ = \ \bar{p}_{S_1}(h^1), \text{ and } r(h, h^1) \leq p_{H_1}(h) \text{ for all } h.$$

to obtain $\{r^*(h, h^1), \mu^*_1\}$ and then maximizing the term (while ensuring that these variables and the optimal variables from the previous step jointly satisfy the required constraints)

$$\sum_h r(h, h^2)f(h) \text{ subject to}$$
$$\sum_h r(h, h^2) \ = \ \bar{p}_{S_1}(h^2) \text{ and } r(h, h^2) \leq (1 - \mu^*_1(h, h^1))p_{H_1}(h) \text{ for all } h.$$

and so on. Easy to see that (assuming the obvious $\bar{p}_{S_1}(h^1) > p_{H_1}(h^1)$,

$$\begin{aligned}
r^*(h^1, h^1) &= p_{H_1}(h^1) \\
r^*(h^2, h^1) &= \min\{(p_S(h^1) - p_{H_1}(h^1)), \quad p_{H_1}(h^2)\} \\
&\vdots \\
r^*(h^k, h^1) &= \max\left\{0, \min\left\{\left(p_S(h^1) - \sum_{l<k} p_{H_1}(h^l)\right), \quad p_{H_1}(h^k)\right\}\right\} \\
&\vdots \\
\text{Further, } r^*(h^1, h^2) &= 0 \\
r^*(h^2, h^2) &= \min\{p_S(h^2), \quad (1 - p^*_{h,s}(h^2, h^1))p_{H_1}(h^2)\} \\
r^*(h^3, h^2) &= \min\left\{p_S(h^2) - r^*(h^2, h^2), \ (1 - p^*_{h,s}(h^3, h^1))p_{H_1}(h^3)\right\} \\
&\vdots \\
r^*(h^k, h^2) &= \min\left\{p_S(h^2) - \sum_{l<k} r^*(h^l, h^2), \ (1 - p^*_{h,s}(h^k, h^1))p_{H_1}(h^k)\right\}.
\end{aligned}$$

It is easy to see that $r^*(h, s) = \bar{\mu}^*_1(s \mid h)p_{H_1}(h)$. $\square$

VIII. Proportional Fairness

All the policies mentioned till this point were aiming at achieving maximum throughput at the BS, i.e. they maximize $\sum_{i=1}^K U_m$, where U_m is the utility of the mobile m. But it is well known that these policies are only efficient and are not fair (mobiles at a far away point are starved of the signal).

In [15], Kushner and Whiting studied a stochastic approximation based algorithm that achieves proportional fairness, i.e., the algorithm asymptotically maximizes, $\sum_{m=1}^K \log(U_m + d_m)$, where d_m are small positive constants. The PFA algorithm is given by

$$\begin{aligned}
\theta^F_{m,k+1} &= \theta^F_{m,k} + \epsilon_k \left[I^F_{m,k+1} f(h_{m,k+1}) - \theta^F_{m,k}\right] \\
I^F_{m,k+1} &= 1_{\left\{m = \arg\max_j \frac{f(h_{j,k+1})}{d_j + \theta^F_{j,k}}\right\}}
\end{aligned}$$

The SA algorithm proposed in the previous section is robust against noncooperation and achieves the efficient solution (maximizes the sum utility) for BS. We now illustrate via an example that the PFA may be robust to noncooperative behavior, i.e., it may be an ITR policy. The PFA algorithm in the presence of noncooperative mobiles will be:

$$\theta^F_{m,k+1} \;=\; \theta^F_{m,k} + \epsilon_k \left[I^F_{m,k+1} f\left(s_m\left(h_{m,k+1}\right)\right) - \theta^F_{m,k} \right]$$

with $I^F_{m,k+1} = 1_{\left\{ m = \arg\max_j \frac{f(s_j(h_{j,k+1}))}{d_j + \theta^F_{j,k}} \right\}}$. By this, true rates that mobile m actually achieves, is (in a way similar to our SA algorithm) updated by,

$$\bar\theta^F_{m,k+1} = \bar\theta^F_{m,k} + \epsilon_k \left[I^F_{m,k+1} f\left(h_{m,k+1}\right) - \bar\theta^F_{m,k} \right].$$

In the examples given below we consider two mobiles one of which experiences Ricean fading while the second experiences Rayleigh fading. The rates of mobile m are given by $f(h_m) = \log\left(1 + P\sqrt{h_m(i)^2 + h_m(r)^2}\right)$ where the real and complex channel gains are Gaussian distributed, $h_m(i), h_m(r) \sim \mathcal{N}(\mu_m, 1/2)$ with $\mu_1 = 3/\sqrt{2}$, $\mu_2 = 0$. The mobile m when noncooperative increases its rate directly using the rule $f(s_m(h_m)) = \max\{f(h_m)+10, r_{max,m}\}$, where the maximum rates $r_{max,1}, r_{max,2}$ are set equal to $2.1, 1.6$ respectively. The maximum rates are chosen such that $f(h_m) \le r_{max,m}$ with probability very close to 1.

Algo	Signaled rates $\lim_{k\to\infty}\theta_{i,k}$	True Rates U_i	$U_{BS} = U_1 + U_2$
Eff :All Coop	1.387, 0.0058	1.378, 0.0059	1.384
PFA:All Coop	0.79, .364	0.79, 0.364	1.154
Eff:Mob 1 Non	1.387, 0.0057	1.316, 0.0052	1.321
PFA:Mob 1 Non	1.168, .363	0.782, 0.363	1.144
Eff:Mob 2 Non	1.387, 0.0061	1.380, 0.0042	1.384
PFA:Mob 2 Non	0.737, 0.837	0.737, 0.315	1.052

TABLE I

COMPARISON OF PFA VERSUS SA ALGORITHM

Remarks : The asymptotic true rate of any mobile is lesser when it is noncooperative and when the other mobile remains cooperative. This is true in both PFA and our SA algorithm.

It was shown in the previous section that the truth revealing strategy at all the mobiles and SA algorithm is an (ϵ) Nash equilibrium for the game when the BS tries to optimize sum utility. In a similar way simulations indicate that the truth revealing strategy at all mobiles and PFA at BS is a Nash equilibrium, for similar game as in Section VI with only the BS's utility changed to sum of log utilities.

But this is just a preliminary simulation result. The PFA may be robust against the kind of noncooperation we considered in this example but may fail in general. In fact, in the example given below, we will see that PFA fails.

However, the following modification of 'corrective' SA algorithm can be proved to be robust against noncooperation while providing a fair solution. This can be proved using ODE theory, exactly following the steps as in Section VI, with θ_i^{F0} being cooperative fair share

$$\theta^F_{m,k+1} = \theta^F_{m,k} + \epsilon_k \left(\tilde{f}^F_{m,k+1} I^F_{m,k+1} - \theta^F_{m,k} \right),$$

$$\tilde{f}^F_{m,k+1} = f^F(s_m(h_{m,k+1})) - \left(\theta^F_{m,k} - \theta^{F0}_m \right) \Delta, \tag{22}$$

$$I^F_{m,k+1} = 1_{\left\{ m = \arg\max_j \frac{f(s_j(h_{j,k+1}))}{d_j + \theta^F_{j,k}} \right\}}. \tag{23}$$

We will conclude this section with an example where Kushner et al's PFA fails against non-cooperation but our proposed Robust PFA algorithm (23) is indeed robust against non-cooperation. In this example one of the users has a fixed channel set at 4, while the other users utility is directly given by $4 + \max\{z^2, 400\}$ where $z \sim \mathcal{N}(0, 4)$ is Gaussian distributed. One

Algo	Signaled rates $\lim_{k\to\infty} \theta_{i,k}$	True Rates U_i	$U_{BS} = U_1 + U_2$
Robust PFA :All Coop	2.75, 13.98	2.72, 13.77	16.5
PFA:All Coop	2.75, 13.95	2.75, 13.95	16.7
Efficient :All Coop	0.0023, 19.98	0.006, 19.6	19.56
Robust PFA :Mob 2 Non	2.75, 14.1	2.72, 13.80	16.54
PFA:Mob 2 Non	2.18, 61.64,	2.18,16.28	18.47

TABLE II

COMPARISON OF PFA VERSUS SA ALGORITHM

can see from the last two rows of the table that Mobile 2 was successful in fooling the BS, when the later uses Kushner et al's PFA algorithm but not when it uses Robust PFA. In fact, the non-cooperative mobile 2 increases it's utility by more than 10%. We can see that because of this non-cooperation the fairness is lost to a good extent (the total utility is in fact closer to the efficient sum utility).

CONCLUDING REMARKS

We studied centralized downlink transmissions in a cellular network in the presence of noncooperative mobiles. We modeled this as a signaling game with several players controlling signals and where the BS serves as follower. In absence of extra intelligence, only babbling equilibrium is obtained, at which both the BS and the noncooperative players make no use of the signaling opportunities. We then proposed three approaches to obtain an efficient equilibrium (TRE), both of which required extra intelligence from the BS but resulted in the mobiles signaling truthfully. We further showed the existence of other non efficient equilibria at which a noncooperative mobile achieves a better utility while the BS achieves better utility than that at a babbling equilibrium but a lower one than that at a TRE.

We see several avenues open for further research on scheduling under noncooperation. We recall that we assumed that a player is either cooperative or not. What if the player can choose? Preliminary research show that there is no clear answer: it depends on the channel statistics of the player as well as that of others. Another related question: what if the BS does not know whether a mobile cooperates or not?

Finally, it should be clear that our approach is applicable not just to wireless networks, but is equally applicable to other resource allocation situations as for example in wireline networks.

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APPENDIX

Proof of Theorem 1 : By definition at any PBE, for any $i \leq K_1$ and for any $h_i \in \mathcal{H}_i$,

$$\mu_i^*(\cdot \mid h_i) \quad \in \quad \arg\max_{\alpha \in \mathcal{P}(\mathcal{H}_i)} \sum_{s_i \in \mathcal{H}_i} \alpha(s_i) \left[\sum_{\mathbf{s}_{-i}} \beta^*(i \mid \mathbf{s}) \sum_{\mathbf{h}_{-i}} p_{H_{-i}}(\mathbf{h}_{-i}) \mu_{-i}^*(\mathbf{s}_{-i} | \mathbf{h}_{-i}) f(h_i) \right].$$

Since $f(h_i)$ is independent of $s_i, \mathbf{h}_{-i}, \mathbf{s}_{-i}$ (true for every i),

$$\mu_i^*(\cdot \mid h_i) \in \arg\max_{\alpha \in \mathcal{P}(\mathcal{H}_i)} \sum_{s_i \in \mathcal{H}_i} \alpha(s_i) \left[\sum_{\mathbf{s}_{-i}} \beta^*(i \mid \mathbf{s}) \sum_{\mathbf{h}_{-i}} p_{H_{-i}}(\mathbf{h}_{-i}) \mu_{-i}^*(\mathbf{s}_{-i} | \mathbf{h}_{-i}) \right]. \quad (24)$$

This shows that $\mu_i^*(\cdot \mid h_i) = \mu_i^*(.)$ for some probability distribution μ_i^* on $\mathcal{H}_i$, for all h_i and for all $i \leq K_1$, i.e., the optimal signaling policy does not depend upon the channel value h_i. Note that $\mu_i(\cdot)$ can depend on i since p_{H_i} need not be identical.

With the above for any $1 \leq i \leq K_1$ and for any $s_i \in \mathcal{H}_i$ with $\mu_i^*(s_i) \neq 0$ from definition 4.2,

$$\pi_i^*(h_i \mid s_i) = \frac{p_{H_i}(h_i)\mu_i^*(s_i)}{\sum_{h'} p_{H_i}(h')\mu_i^*(s_i)} = p_{H_i}(h_i).$$

When $\mu_i^*(s_i) = 0$, the denominator is zero, but we may set $\pi_i^*(h_i \mid s_i) = p_{H_i}(h_i)$. This implies that in equilibrium the posterior beliefs can not be improved.

For any $\mathbf{s} = (s_1, \cdots, s_K)$, the first optimization in the definition of PBE can be written as (with γ any arbitrary probability distribution on $\{1, \cdots, K\}$),

$$\beta^*(\cdot \mid \mathbf{s}) \in \arg\max_{\gamma} \left[\sum_{j=1}^{K_1} \sum_{h_j} p_{H_j}(h_j) f(h_j) \gamma(j) + \sum_{j > K_1} f(s_j)\gamma(j) \right].$$

The above optimization is independent of $\{s_i; \;\; i \leq K_1\}$ and hence the optimization reduces to maximizing,

$$\sum_{j=1}^{K_1} \gamma(j) \mathbb{E}\left[f(h_j)\right] + \sum_{j > K_1} \gamma(j) f(s_j).$$

This justifies the definition of $\beta^*(. \mid .)$ in the statement of the theorem.

Since $\beta^*(. \mid \mathbf{s}) = \beta^*(.|s_{K_1+1}, \cdots, s_K)$ for all $\mathbf{s}$, the optimization in (24) can be rewritten as,

$$\mu_i^*(\cdot \mid h_i) \in \arg\max_{\alpha \in \mathcal{P}(\mathcal{H}_i)} \sum_{s_i \in \mathcal{H}_i} \alpha(s_i) \left[\sum_{s_{K_1+1}, \cdots, s_K} \beta^*(i \mid s_{K_1+1}, \cdots, s_K) \Pi_{l > K_1} p_{H_l}(s_l) \right].$$

Thus the Theorem follows. $\square$

Proof of Theorem 2: Assume π_i^*, for every i, is as in hypothesis. Then,

$$\beta^*(.|\mathbf{s}) \in \arg\max_{\gamma} \left[\sum_{k=1}^{K} f(s_k)\gamma(k) \right] \text{ for all } \mathbf{s}.$$

Hence, $\beta^*(i|\mathbf{s}) = 1_{\{i = A^*(\mathbf{s})\}}$, i.e. the unit mass at $A^*(\mathbf{s})$ defined in (7).

Fix i, h_i. With β^*, μ_{-i}^* as in theorem hypothesis,

$$\mu_i^*(\cdot \mid h_i) \in \arg\max_{\alpha \in \mathcal{P}(\mathcal{H}_i)} \sum_{s_i \in \mathcal{H}_i} \alpha(s_i) \left[\sum_{\mathbf{s}_{-i}} p_{H_{-i}}(\mathbf{s}_{-i}) U_i(h_i, s_i, A^*(\mathbf{s})) \right] \quad (25)$$

By the choice of $\{c_i\}$ and ψ as in (6), for any α,

$$\begin{aligned}
\sum_{s_i} \alpha(s_i) \left[\sum_{\mathbf{s}_{-i}} p_{H_{-i}}(\mathbf{s}_{-i}) U_i(h_i, s_i, A^*(\mathbf{s})) \right] &= \sum_{\{s_i:(s_i-h_i)2>c_i\}} \alpha(s_i) \left[\sum_{\mathbf{s}_{-i}} p_{H_{-i}}(\mathbf{s}_{-i}) U_i(h_i, s_i, A^*(\mathbf{s})) \right] \\
&\quad + \sum_{(s_i-h_i)2 \le c_i} \alpha(s_i) \left[\sum_{\mathbf{s}_{-i}} p_{H_{-i}}(\mathbf{s}_{-i}) U_i(h_i, s_i, A^*(\mathbf{s})) \right] \\
&\le \alpha(h_i) \left[\sum_{\mathbf{s}_{-i}} p_{H_{-i}}(\mathbf{s}_{-i}) U_i(h_i, h_i, A^*(h_i, \mathbf{s}_{-i})) \right] \\
&\le \left[\sum_{\mathbf{s}_{-i}} p_{H_{-i}}(\mathbf{s}_{-i}) U_i(h_i, h_i, A^*(h_i, \mathbf{s}_{-i})) \right].
\end{aligned}$$

Hence maximum in (25) is achieved by the choice of μ_i^* given in theorem hypothesis. $\square$

Proof of Theorem 3 : We will be using the stochastic approximation results of Benvensite et al ([7]) to obtain this proof. These results are reproduced in at the end of this Appendix for easier readability.

Let $G_k := \mathbf{h}_k = (h_{1,k}, h_{2,k}, \cdots, h_{K,k})$. The equations (8), (11) can be rewritten in the format of Benveniste et al as,

$$\begin{aligned}
\theta_{k+1} &= \theta_k + \epsilon_k W(G_k, \theta_k), \\
\bar{\theta}_{k+1} &= \bar{\theta}_k + \bar{\epsilon}_k \bar{W}(G_k, \theta_k, \bar{\theta}_k) \\
W_i(G_k, \theta_k) &= \left(\tilde{f}_{i,k+1} I_{i,k+1} - \theta_{i,k} \right), \\
\bar{W}_i(G_k, \theta_k, \bar{\theta}_k) &= \left(\bar{f}_{i,k+1} I_{i,k+1} - \bar{\theta}_{i,k} \right).
\end{aligned}$$

Define

$$\begin{aligned}
w(\theta) &:= \mathbb{E}_{\mathbf{h}}[W(G, \theta)] = H^S(\theta) - \theta, \\
\bar{w}(\theta, \bar{\theta}) &:= \mathbb{E}_{\mathbf{h}}[\bar{W}(G, \theta, \bar{\theta})] = \bar{H}^S(\theta) - \bar{\theta}.
\end{aligned}$$

One can easily show that the above algorithm satisfies the assumptions **B.**1-4 with $\Theta = (\theta, \bar{\theta})$, $h = (w, \bar{w})$ and

$$\begin{aligned}
\nu_\theta(G) &= W(G, \theta) - w(\theta), \\
\bar{\nu}_{\theta,\bar{\theta}}(G) &= \bar{W}(G, \theta, \bar{\theta}) - \bar{w}(\theta, \bar{\theta}) \text{ because of the following,}
\end{aligned}$$

- The ODE (13) is independent of ODE (14). We use this fact in the proofs.

- By assumption **A.**3, $P_\theta(g,.) = P(g,.)$ for all θ.
- By Lemma 1 w, $\bar{w}$ are locally Lipschitz.
- The boundedness of the ODE solution (for any finite time) satisfies assumption (28) of Appendix B.

Thus the theorem follows by Theorem 5 □

Lemma 1: The function $H^S(\theta)$ is continuously differentiable and the function $\bar{H}^S(\theta)$ is locally Lipschitz, both w.r.t. θ.

Proof : By independence of h_i across the mobiles,

$$\begin{aligned} H_i^S(\theta) &= \mathbb{E}_{\mathbf{h}}\left[\tilde{f}_i^S(h_i,\theta_i) I_i^S(\mathbf{h},\theta)\right] \\ &= \mathbb{E}_{h_i}\left[\tilde{f}_i^S(h_i,\theta_i)\Pi_{j\neq i}\Pr(A_j(h_i,\theta))\right] \end{aligned}$$

$$\text{where } A_j(h_i,\theta) := \left\{h_j : f(s_i(h_i)) - f(s_j(h_j)) \geq \Delta(\theta_i - \theta_j - \theta_i^0 + \theta_j^0)\right\}.$$

The first part of the lemma is proved by BCT if we show that the functions $\{\Pr(A_j(h_i,\theta))\}_{j\neq i}$ and $\tilde{f}_i^S(h_i,\theta_i)$ are continuously differentiable (w.r.t. θ) with uniformly bounded derivatives for almost all h_i. This is immediately evident for $\tilde{f}_i^S$. The same holds for $\{\Pr(A_j(h_i,\theta))\}_{j\neq i}$ by assumptions **A**.1, **A**.2 as,

$$\begin{aligned} \frac{\partial \Pr(A_j(h_i,\theta))}{\partial\theta_l} &= (-1)^{\delta(l=i)} g_{s_j}\left(f^{-1}\left(f(s_i(h_i)) - \Delta(\theta_i-\theta_j-\theta_i^0+\theta_j^0)\right)\right) \\ & \quad f^{-1'}\left(f(s_i(h_i)) - \Delta\left(\theta_i-\theta_j-\theta_i^0+\theta_j^0\right)\right)\Delta \end{aligned} \tag{26}$$

for $l = i, j$, where g_{s_j} is the (bounded) density of signal $s_j(h_j)$. Note in the above that the continuous derivative $f^{-1'}$ will also be uniformly bounded for all θ coming from a compact set, because of boundedness of f.

Easy to see that $\bar{f}_i^S(h_i,\theta_i) - \bar{f}_i^S(h_i,\theta_i') \leq \Delta\,|\theta-\theta'|$. Hence, with C_f representing the upper bound on function f,

$$\begin{aligned} \bar{H}^S(\theta) - \bar{H}^S(\theta') &= \mathbb{E}_{\mathbf{h}}\left[\left(\bar{f}_i^S(h_i,\theta_i) - \bar{f}_i^S(h_i,\theta_i')\right) I_i^S(\mathbf{h},\theta)\right] \\ &\quad + \mathbb{E}_{\mathbf{h}}\left[\bar{f}_i^S(h_i,\theta_i')\left(I_i^S(\mathbf{h},\theta) - I_i^S(\mathbf{h},\theta')\right)\right] \\ &\leq \Delta|\theta_i-\theta_i'|\mathbb{E}_{\mathbf{h}}\left[I_i^S(\mathbf{h},\theta)\right] + C_f\mathbb{E}_{\mathbf{h}}\left|I_i^S(\mathbf{h},\theta) - I_i^S(\mathbf{h},\theta')\right|. \end{aligned}$$

Let $\mathcal{D}$ be any compact set. Let C_{g_S} represent the common upper bound on density g_{s_i} for all i, $C_{f^{-1}}(\mathcal{D})$ represent the upper bound (which is independent of $\mathbf{h}$) on $f^{-1'}$ for all $\theta \in \mathcal{D}$. Then, from (26), for appropriate constant $C(K)$,

$$\mathbb{E}_{\mathbf{h}} \left| I_i^S(\mathbf{h}, \theta) - I_i^S(\mathbf{h}, \theta') \right| \;\; \leq \;\; C(K) C_{g_S} C_{f^{-1}}(\mathcal{D}) \Delta \left| \theta - \theta' \right|$$

and hence $\bar{H}^S(\theta)$ is locally Lipschitz. □

Benveniste et al's ODE approximation result

In [7] (Theorem 9, p.232), Benveniste et al obtained the ODE approximation for the system,

$$\Theta_{k+1} \;\; = \;\; \Theta_k + \mu_k H(\Theta_k, G_{k+1}). \tag{27}$$

We reproduce the result here in a form which is suitable for this paper.

Let Θ take values in an open subset D of $\mathcal{R}^m$. We make the following assumptions:

B.0 $\{\mu_k\}$ is a decreasing sequence with $\sum_k \mu_k = \infty$ and $\sum_k \mu_k^{1+\delta} < \infty$ for some $\delta > 0$.

B.1 There exists a family $\{P_\theta\}$ of transition probabilities $P_\theta(g, \mathbf{A})$ such that, for any Borel set $\mathbf{A}$,

$$P[G_{n+1} \in \mathbf{A} | \mathcal{F}_n] = P_{\Theta_n}(G_n, \mathbf{A}) \text{ where } \mathcal{F}_k \stackrel{\triangle}{=} \sigma(\Theta_0, G_0, G_1, \cdots, G_k).$$

Thus $\{(G_k, \Theta_k\}$ forms a Markov chain.

B.2 For any compact $Q \subset D$, there exist C_1, q_1 such that $|H(\theta, g)| \leq C_1(1 + |g|^{q_1})$ for all $\theta \in D$.

B.3 There exists a function h on D, and for each $\theta \in D$ a function $\nu_\theta(.)$ such that

a) h is locally Lipschitz on D.

b) $(I - P_\theta)\nu_\theta(g) = H(\theta, g) - h(\theta)$ where $P_\theta \nu_\theta(g) = E[\nu_\theta(G_1) | G_0 = g, \Theta_0 = \theta_0]$.

c) For all compact subsets Q of D, there exist constants C_3, C_4, q_3, q_4 and $\lambda \in [0.5, 1]$, such that for all $\theta, \theta' \in Q$

i) $|\nu_\theta(g)| \leq C_3(1 + |g|^{q_3})$,

ii) $|P_\theta \nu_\theta(g) - P_{\theta'} \nu_{\theta'}(g)| \leq C_4 \left(1 + |g|^{q_4}\right) \left|(\theta) - (\theta')\right|^\lambda$.

B.4 For any compact set Q in D and for any $q > 0$, there exists a $\mu_q(Q) < \infty$, such that for all $n, g, (\theta) \in \mathcal{R}^d$ (with $E_{g,\theta}$ representing the expectation taken with $G_0, \Theta_0 = g, \theta$),

$$E_{g,\theta} \left\{ I((\Theta_k) \in Q, k \leq n) \left(1 + |G_{n+1}|^q\right) \right\} \;\; \leq \;\; \mu_q(Q) \left(1 + |g|^q\right).$$

Define $t(r) := \sum_{k=0}^r \mu_k$, $m(n, T) := max_{r \geq n}\{t(r) - t(n) \leq T\}$. Let $\Theta(t, t_0, \theta)$ represent a solution of

$$\dot{\Theta}\,(t) = h(\Theta(t)),$$

with initial condition $\Theta(t_0) = \theta$. Let Q_1 and Q_2 be any two compact subsets, such that $Q_1 \subset Q_2$ and we can choose a $T > 0$ such that there exists an $\delta_0 > 0$ satisfying

$$d\left(\Theta(t, 0, \theta), Q_2^c\right) \geq \delta_0, \tag{28}$$

for all $\theta \in Q_1$ and all t, $0 \leq t \leq T$.

Theorem 5: Assume **B**.0–**B**.4. Furthermore, pick Q_1, Q_2, T and δ_0 satisfying (28). Then for all $\delta \leq \delta_0$, for any (θ, g) and when (G_n, Θ_n) is initialized with (g, θ),

$$P_{n:g,\theta}\left\{\sup_{\{n \leq r \leq m(n,T)\}} \left|\Theta_r - \Theta\left(t(r),\ t(n),\ \theta\right)\right| \geq \delta\right\} \to 0$$

as $n \to \infty$ uniformly for all $\theta \in Q_1$. In the above, $P_{n:g,\theta}$ denotes distribution of $(G_n + k, \Theta_{n+k})$ with $G_n = g, \Theta_n = \theta$.