A Potential Game Approach for Uplink Resource Allocation in a Multichannel Wireless Access Network

Eitan Altman
INRIA-Sophia Antipolis
Email: eitan.altman@sophia.inria.fr

Anurag Kumar
ECE Dept., Indian Institute of Science
Email: anurag@ece.iisc.ernet.in

Yezekael Hayel
University of Avignon
Email: yezekael.hayel@univ-avignon.fr

***Abstract*—We consider a resource allocation problem in a multichannel wireless access system being shared by several users for uplink transfer of elastic traffic. Each user can allocate its resources (e.g., radios, antennas or power) to one or more of the carriers. In this network scenario we consider a problem of noncooperative allocation of resources by the users, with each user's objective being to maximize its own utility. We apply the theory of potential games to solve this problem by transforming it into an equivalent global optimization one. We obtain structural properties of the equilibrium policies using tools from Schur concave stochastic orders. Finally, we propose a totally distributed algorithm that converges to a Nash Equilibrium of the system.**

I. Introduction

Noncooperative control in wireless communication has been an active academic research field due to its decentralized nature which enables to reduce the burden of resource allocation from a central network manager. Several directions have been studied of which we cite only a few: power control [1], [9], [13], [23], access to a common channel [11], incentive to relay or forward packets [4], [21], and flow or rate control [30]. Today, the noncooperative aspects of decision making in wireless networks is not just an academic issue. It is common practice to delegate decisions to terminals. For example, when one wishes to connect a mobile computer to a wireless LAN, the association decisions (which access point to join) are taken in a decentralized noncooperative way by the users (based on the information they receive concerning the number of available access points and the channel quality to each of them).

Putting intelligence at the terminals rather than in the core network is in line with a whole trend that saw the ATM networks disappearing first from wireline networks and later from wireless (e.g., from the UMTS standard which moved to all IP network). It is also in line with the emerging technology of cognitive radios [6] in which the radios have sufficient intelligence to be able to alter their actions in reaction to changes in their environment. With more intelligent mobiles, it is possible to delegate to each mobile the decisions concerning its performance. The decision making can then be viewed as a noncooperative game. This formalism has been used in [3], [16] in the context of cognitive radios.

In this paper we study an uplink resource allocation problem in a noncooperative setting. The basic model we adopt is based on the one in [5]. Several nonoverlapping channels are available to be shared by some users who wish to transfer elastic traffic over this system. Each user has several radios, and its strategy lies in deploying one or more of its radios on each of the channels. For each such joint strategy, the users each derive some throughput, and suffer the energy cost of operating their radios. The throughputs obtained by each of the users depends on the network resource allocation strategy. We consider two possibilities. **FSU**: Fair Share between Users, in which each channel is equally shared among the users that transmit on that frequency (irrespective of the number of radios each user assigns to each channel), and **FSR**: Fair Share between Radios, introduced in [5], where the capacity of each channel is equally shared among the radios that use it. We recover many of the results of [5] on the structure of the equilibrium and obtain new ones by applying the theory of potential games which allows us to solve the noncooperative frequency selection problem by transforming it into an equivalent global optimization one. We then obtain structural properties of the equilibrium policies using tools from Schur concave stochastic orders.

II. Motivations

Next generation fixed wireless broadband networks are being increasingly deployed as mesh networks in order to provide and extend access to the internet. These networks are characterized by the use of multiple orthogonal channels and nodes with the ability to simultaneously communicate with many neighbors using multiple radios (interfaces) over orthogonal channels [29]. Networks based on the IEEE 802.11a/b/g and 802.16 standards are examples of these systems. Moreover, as the cost of radios plummeted, single radio products evolved to support more radios per mesh node with the additional radios providing specific functions- such as client access, backhaul service or scanning radios for high speed handover in mobility applications. The mesh node design also became more modular - one box could support multiple radio cards - each operating at a different frequency.

There is an important growing area of research on wireless mesh networks with devices equipped with multiple radios. Indeed, the lower and lower cost of RF transceivers allows us to consider two or more radios in the same device which can be also heterogeneous [27]. The use of multiple radios is interesting in a mesh architecture in which this technique permits to increase capacity of the network [29]. It has also been proved in [2] that the capacity of relays with multiple radios is not halved. Using multiple radios offers tradeoffs that can improve robustness, connectivity and performance of the system. In [2], authors study the problem of routing in a context of multiple radios mesh network. Their experimental results on a testbed where each node has two 802.11 multiband radios using different channels, shows good improvements of the throughput for each mobile.

III. System Model and Theoretical Background

We consider the following model. The set of users is denoted by $\mathcal{N}$, with $N := |\mathcal{N}|$. There is a a set $\mathcal{C}$ containing C channels. Each user has a device equipped with $K \leq |\mathcal{C}|$ radio transmitters, each with the same communication capacity. A device can use at the same time any number of its K transmitters. The problem is for each user to distribute some or all of its transmitters over the C channels, so as to optimise certain objectives. There is no limit on the number of radios per channel.

A. System description and notation

User strategies:

The users' strategies comprise the allocation of their radio transmitters to the channels.

$k_i^c :=$ the number of radio transmitters that user i allocates to channel c, with $k_i^c \leq K$

$s_i =$ the vector $(k_i^1, ..., k_i^C)$ of radio allocations of user i, with $\sum_{c=1}^{C} k_i^c \leq K$; this is the *strategy* of user i

$\mathcal{S}_i$:= the set of strategy vectors of user i

S:= a strategy vector of all users; i.e., $S = (s_1, \cdots, s_N)$, with s_i being a strategy of user i

S_{-i}:= the strategy vectors of all players except player i in the strategy S

$\mathcal{S}$:= the set of all strategy vectors

Given a strategy S, we define the following.

$k_i :=$ the total number of radio transmitters ($\leq K$) used by user i; thus, $k_i = \sum_{c=1}^{C} k_i^c$

$\mathcal{C}_i$:= the set of channels used by user i

$C_i := |\mathcal{C}_i|$

$\mathcal{K}^c :=$ the set of radio transmitters using channel c

$k^c := |\mathcal{K}^c|$

$\mathcal{N}^c$:= the set of users each having at least one radio that uses channel c

$n^c := |\mathcal{N}^c|$

$[v_i, S_{-i}]$:= the strategy obtained when all users follow the strategy S except user i, which uses strategy $v_i \in \mathcal{S}_i$

Since the previous notation depends on the strategy S under consideration, we sometimes include explicitly the dependence on the strategy in the notation. For example, $k^c(S)$ stands for the number of radio transmitters using channel c under strategy S.

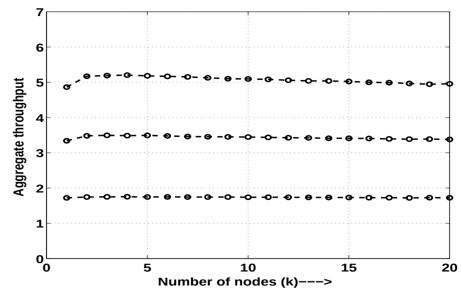

Fig. 1. Total throughput (in Mbps) vs. number of users for radios using CSMA/CA random access, as standardised in IEEE 802.11, when all radios use the same physical transmission rate: 11 Mbps (top), 5.5 Mbps (middle), or 2 Mbps. The queues of the transmitters are saturated. This is an example of the function $\overline{R}^c(k)$ for a network strategy that yields FSR.

User payoffs (or utilities):

When the users employ a joint strategy S, each obtains a payoff that depends on the transmission rate it gets, and the cost (e.g., energy cost) it has to pay in order to use its radios. We model this as follows. When the joint strategy S is employed, then user i obtains the throughput $r_i^c(S)$ on channel c; user i's total throughput is $r_i(S) = \sum_{c=1}^{C} r_i^c(S)$. Then for the strategy S, $U_i(S) = r_i(S) - \lambda_i k_i(S)$ is the utility (net payoff) of user i, where λ_i represents the cost that user i incurs for using each of its radios; this could model energy cost and λ_i may vary from user to user. By changing the values λ_i, one can model various tradeoffs between throughput and energy.

Network policies for allocating bit rates:

The transmission rates that users get on each channel depend on the network's resource allocation policy and the joint strategy of the users. We consider two models for the network's resource allocation strategy.

- **FSR**: Fair Share between Radios [5]: The total bit rate of channel c when k^c radios use channel c is given by $\overline{R}^c(k^c) \geq 0$. Figure 1 (see [8]) shows an example of such a network scenario, where the users employ CSMA/CA in order to access each of the C channels. The rate $\overline{R}^c(k^c)$ is shared equally between the radios allocated to that channel, i.e., for strategy S, each radio using channel c receives a bit rate of (suppressing (S) in the notation)

$$\overline{d}^c(k^c) := \frac{\overline{R}^c(k^c)}{k^c}$$

  Thus, the total transmission throughput for a user i is

$$r_i = \sum_{c \in \mathcal{C}_i} \overline{d}^c(k^c) k_i^c = \sum_{c \in \mathcal{C}_i} \frac{\overline{R}^c(k^c)}{k^c} k_i^c.$$

- **FSU**: Fair Share between Users: The total bit rate of channel c when n^c users use channel c is given by $R^c(n^c) \geq 0$. A TDM mechanism is used to access the channel c in a way that $R^c(n^c)$ is shared equally between the users. Hence, for a strategy S, at channel c, each radio of user i receives a bit rate of (suppressing (S) in the notation)

$$R^c(n^c)/(n^c \times k_i^c),$$

and the total rate that user i receives at channel c is $\frac{R^c(n^c)}{n^c}$, that does not depend on k^c. Define

$$d^c(n^c) = \frac{R^c(n^c)}{n^c}$$

Then the total transmission rate for user i is

$$r_i = \sum_{c \in \mathcal{C}_i} d^c(n^c).$$

B. Potentials, majorization and Shur concavity

The following background will be essential to the later discussions. We provide it here for ready reference.

Definition 3.1: [15] A function $G : \mathcal{S} \to \mathbb{R}$ is called a **potential** if for every user i and every strategy $\mathcal{S}$, and for every strategy v_i for user i

$$G([s_i, S_{-i}]) - G([v_i, S_{-i}]) = U_i([s_i, S_{-i}]) - U_i([v_i, S_{-i}]). \tag{1}$$

Remark 3.1: The potential is useful for the following reason. If a game has a potential G, then any policy S that maximises the potential is an equilibrium for the game [15]. Here we have a discrete strategy set, hence, if the potential function satisfies certain types of concavity (e.g., the larger midpoint property (LMP) defined by Ui [26]) the converse is also true: if S is an equilibrium, then it maximizes G. Thus a potential allows us to transform a game into an equivalent optimization problem.

Consider two n-dimensional vectors $\delta(1), \delta(2)$.

Definition 3.2: [12] We say that $\delta(2)$ **majorizes** $\delta(1)$, denoted by $\delta(1) \prec \delta(2)$, if $\delta(2)$ is more "unregular" (less "balanced") in the following sense:

$$\begin{cases} \sum_{i=1}^{k} \delta_{[i]}(1) \leq \sum_{i=1}^{k} \delta_{[i]}(2), \quad k = 1, ..., n-1, \\ \\ \sum_{i=1}^{n} \delta_{[i]}(1) = \sum_{i=1}^{n} \delta_{[i]}(2) \end{cases} \tag{2}$$

where $\delta_{[i]}(j)$ is a permutation of $\delta_i(j)$ satisfying $\delta_{[1]}(j) \geq \delta_{[2]}(j) \geq ... \geq \delta_{[n]}(j)$, $j = 1, 2$.

Note that (2) implies, in particular, that

- the largest element of $\delta(2)$ (i.e. $\delta_{[1]}(2)$) is larger than the largest element of $\delta(1)$ (i.e. $\delta_{[1]}(2)$).
- the smallest element of $\delta(2)$ (i.e. $\delta_{[n]}(2)$) is smaller than the smallest element of $\delta(1)$ (i.e. $\delta_{[n]}(1)$).

Definition 3.3: A function $f : \mathbb{R}^{\mathrm{n}} \to \mathbb{R}$ is **Schur convex** if $\delta(1) \prec \delta(2)$ implies $f(\delta(1)) \leq f(\delta(2))$. f is Schur concave if $\delta(1) \prec \delta(2)$ implies $f(\delta(1)) \geq f(\delta(2))$.

We define strong majorization as in Definition 3.2 but with at least one of the inequalities in (2) being strict. We then define a function to be strictly Schur convex (resp., concave) by requiring in Definition 3.3 that the inequalities as well as the majorization order between $\delta(2)$ and $\delta(1)$ be strict.

The next proposition follows from Proposition C.2 of [12, p. 67] (that states that any function $f : \mathbb{R}^{\mathrm{n}} \to \mathbb{R}$ is Schur-convex if it is symmetric and convex).

Proposition 3.1: (i) ([12, Proposition 2.1]) Let $\psi : \mathbb{R} \to \mathbb{R}$, be a concave (resp., convex) function. Define

$$g(\delta) = \sum_{i=1}^{n} \psi(\delta_i). \tag{3}$$

Then g is Schur concave (resp., convex).

(ii) Let $\psi : \mathbb{R} \to \mathbb{R}$, be a strictly concave (resp., strictly convex) function. Then g is strictly Schur concave (resp., convex).

IV. FSU Scheduling

A. The equilibrium

Theorem 4.1: Consider the FSU network strategy. Let $\lambda_i = \lambda$ for all i; i.e., the energy cost per radio, λ_i, does not depend on i. Then the following hold:
(i) for each user i, and for any fixed policy S_{-i} of users other than i, a necessary condition for s_i to be a best response is that for all c, k_i^c takes either the value 0 or the value 1.
(ii) A Nash equilibrium in pure strategies exists.
(iii) Any equilibrium has the following structure. For each user i and each channel c, k_i^c takes either the value 0 or the value 1.

Proof. Assume that $k_i^c > 1$ for some i and c. Since the rate received by user i at channel c does not depend on the number of radios k_i^c applied there, by reducing k_i^c by 1 the total rate that the user receives in that channel does not decrease, whereas the energy cost strictly decreases. We can therefore reduce the strategy space of the users by restricting k_i^c to take values zero or one only, without loss of optimality. This proves (i).

The game that is thus obtained by restricting the strategy space (as justified above) is a congestion game as defined in [18]. Indeed,

$$U_i(S) = \sum_{c \in \mathcal{C}_i(S)} \left(\frac{R^c(k^c(S))}{k^c(S)} - \lambda \right)$$

where we have used the fact that $k_i^c \in \{0, 1\}$. Thus, the contribution to a user's payoff from channel c is only a function of the number of users, k_i^c, that use channel c; hence, we have a congestion game. As [18] shows, such games always have an equilibrium in pure strategies. This establishes (ii).

(iii) is then a direct consequence of (i) and (ii). $\diamond$

We shall call the game with the reduced set of strategies introduced in the proof above the **Reduced Game**. As we saw, for any multistrategy S of the users, there is no loss of optimality to restrict the best response of any player i to S^{-i} to its set of restricted strategies. Thus in the rest of the paper, whenever we consider the network policy to be the FSU, we shall study the restricted game instead of the original one.

We note that when restricting to the reduced game, the FSR policy for the network will yield the same user performance as the FSU policy, since the number of users that share a frequency band is the same as the number of radios that share it. We can then use k^c rather than n^c to describe the network's policy for allocating bit rate.

Remark 4.1: The structural property of the equilibrium given in Theorem 4.1(iii) holds not only for the game problem but also for the team problem, in which all users have a common objctive which is the (possibly weighted) sum of utilities of all players (the weights are some strictly positive constants). The structure thus carries to any Pareto optimal strategy.

B. The potential and its properties

We require that $\lambda_i = \lambda$ for all users $i, 1 \le i \le N$. Then it is straightforward to see (e.g., [15]) that the following function is a potential for this game:

$$G(S) = \sum_{c=1}^{|\mathcal{C}|} h^c(k^c(S)) \quad \text{where} \quad h^c(k) := \sum_{\ell=1}^{k} (d^c(\ell) - \lambda) \tag{4}$$

For example, suppose that user i does not use channel c under strategy v_i. Then (denoting by $e^c \in \mathbb{R}^C$ the unit vector with a 1 in position c) we see that

$$G([v_i + e^c, S_{-i}]) - G([v_i, S_{-i}]) = d^c(k^c([v_i, S_{-i}]) + 1) - \lambda;$$

on the other hand

$$U_i([v_i + e^c, S_{-i}]) - U_i([v_i, S_{-i}]) = d^c(k^c([v_i, S_{-i}]) + 1) - \lambda.$$

Lemma 4.1:
1) If $\frac{R^c(k)}{k}$ is decreasing in k, then $h^c(k)$ is concave in k.
2) If $\frac{R^c(k)}{k}$ is strictly decreasing in k, then $h^c(k)$ is strictly concave in k.

Proof. We see that

$$h^c(k) - h^c(k-1) = d^c(k) - \lambda = \frac{R^c(k)}{k} - \lambda$$

and the result follows from the hypothesis. ◇

Proposition 3.1 then immediately yields the following:

Theorem 4.2: If $h^c(k)$ is concave (resp. strictly concave), then the potential $G(S)$ of the reduced game is Schur concave (resp., strictly Schur concave).

The decreasing property of $\frac{R^c(k)}{k}$ can be obtained under several situations. First, numerical computations on the throughput in Figure 1 shows that this property holds. In addition we can introduce the following assumptions.

A1 (resp., A1s): $R^c(\cdot)$ is concave (resp., strictly concave) and $R^c(0) \ge 0$.

A2 (resp., A2s): $R^c(\cdot)$ is nonincreasing (resp., strictly nonincreasing). This was the case suggested in [5].

Then the following is easily shown to be true.

Lemma 4.2: We have the following results:
1) If either A1 or A2 hold then $\frac{R^c(k)}{k}$ is decreasing in k.
2) If either A1s or A2s hold then $\frac{R^c(k)}{k}$ is strictly decreasing in k.

Proof.
1) Indeed, if A1 holds, then, for $k_2 > k_1 > 0$, using concavity of $R^c(\cdot)$

$$\frac{R^c(k_2) - R^c(k_1)}{k_2 - k_1} \le \frac{R^c(k_1) - R^c(0)}{k_1 - 0}$$

which holds iff

$$\begin{aligned} k_1 R^c(k_2) - k_1 R^c(k_1) \le\ & k_2 R^c(k_1) - k_2 R^c(0) \\ & - k_1 R^c(k_1) + k_1 R^c(0). \end{aligned}$$

Canceling the common term on each side

$$\begin{aligned} k_1 R^c(k_2) &\le k_2 R^c(k_1) - (k_2 - k_1) R^c(0), \\ &\le k_2 R^c(k_1), \end{aligned}$$

where we used the nonnegativity of $R^c(0)$ in the last step. It follows that

$$\frac{R^c(k_2)}{k_2} \le \frac{R^c(k_1)}{k_1}.$$

The conclusion from A2 is evident.
2) This is also immediate.

◇

C. A characterisation of the equilibrium strategies

The Schur concavity of the potential allows us to obtain a regularity property not only for the equilibrium strategies (see Theorem 4.1 (iii)), but also on the **aggregated strategy** k^c (i.e. the total number of radios sent by all user to each channel c) at equilibrium.

Theorem 4.3: Consider the restricted game.

(i) If $h^c(k)$ is concave then there exists an equilibrium multi-strategy S such that for any c,

$$\max_{c,c'} \left(k^c(S) - k^{c'}(S) \right) \le 1. \tag{5}$$

(ii) If $h^c(k)$ is strictly concave then (5) is a necessary condition for S to be an equilibrium.

Proof. Recall that the potential G as defined in (4) is Schur concave (Theorem 4.2). Now by Eqn. 4, and Lemma 4.1, we see that G is a separable concave function, and, thus, satisfies the larger midpoint property (LMP) [26]. It follows that if S is a Nash equilibrium multistrategy, then it maximizes the potential; see also Remark 3.1. By the definition of Schur concavity, it is majorized by any other multistrategy S'. This then implies that S can be chosen such that for any c, (5) holds. Indeed, assume that under any S that maximizes the potential, there are some c and c', such that $k^c(S) - k^{c'}(S) > 1$. Now consider the policy S' obtained from policy S by transfering one radio transmission from c to c'. Then S majorizes S'. Since G is Schur concave, it follows by definition that $G(S') \ge G(S)$. Since S maximizes G this is only possible when in fact $G(S') = G(S)$. By repeating this procedure we obtain a policy S that maximizes G and satisfies (5). The second part follows since in this case G is strictly Shur concave. ◇

D. Convergence of the single user improving policy

The **Single User Improving Policy** (SUIP) is the following. Consider any strictly increasing time sequence T_n where at each T_n one user changes its strategy so as to strictly improve its own utility.

We restrict without loss of optimality to the reduced game defined in the previous subsection. It is a potential game, where the potential is given by (4). Hence at each improvement step of the SUIP policy by a user, the potential strictly increases (by the same amount as the improvement of the payoff of the user). Since there are finitely many strategies in $\mathcal{S}$, after a finite number of improvements the potential will reach a maximum. The strategies of the users at this maximum are in Nash equilibrium; a deviation of a single user that would improve that user's utility is not possible anymore (since such a deviation would mean that the potential can further increase).

Note that while the potential increases at each improvement step by some user, say user i, the utilities of the other users need not improve as a result of the deviation of user i.

V. Additional Results

As we saw in the last section, if the network uses the FSU policy, the users can restrict without loss of optimality to the restricted game. It is evident that the same results apply to the FSR policy if the strategies are restricted so that each user deploys 1 or 0 radio on each channel.

We assume throught this section that the problem is symmetric. In particular, d^c are the same functions for all channels c, and the players have $\lambda_i = \lambda$.

A. Global optimization

Theorem 5.1: Consider either the FSR policy with the restricted strategies or the FSU policy,
(i) Assume that Assumption A1 holds. Then $U(S)$ is Schur concave, and there exists an optimal strategy S satisfying (5).
(ii) If Assumption A1s holds then U is strictly Schur concave, and a necessary condition for a strategy S to be optimal is that (5) holds.

Proof of Theorem 5.1.
Define $g(x) = R^c(x) - x\lambda$. Then g is also concave, and the global utility U has the form (in the FSR case with restricted strategies and the FSU case)

$$U(S) = \sum_{c=1}^{|\mathcal{C}|} g(k^c(S)). \tag{6}$$

It then follows Proposition 3.1 that $U(S)$ is Schur concave. The rest follows from the same steps as those in the proof of Theorem 4.3. ◇

Note that the Schur concavity of U allows us to obtain not only the structure of optimal policies, but also to compare the performance of two non-optimal policies, whenever one of the policies majorises the other.

B. The game problem for FSR

We consider the FSR model without the restricted strategies.

Assume that a strategy S is such that for every c, $|k^c|$ radios use channel c. Then we can show by induction over the number of players that $G(S)$ as given in (7) can be expressed as (4).

Note that for any user i and any strategy S_{-i} of other players, $U_i([0, S_{-i}]) = 0$, where 0 stands for the strategy of player i that does not use any radio. Hence, using repeatedly (1), and setting $G(0) = 0$, we obtain (by induction) the following expression that a potential G has to satisfy: Let i be an integer not greater than N, and define the policy $s^i = (s_1, ..., s_i, 0, ..., 0)$. Then

$$\begin{aligned} G(s^j) &= \sum_{i=1}^{j}[G(s^i) - G(s^{i-1})] = \sum_{i=1}^{j}[U_i(s^i) - U_i(s^{i-1})], \\ &= \sum_{i=1}^{j} U_i(s^i). \end{aligned} \tag{7}$$

In particular, for any policy S we have

$$G(S) = \sum_{i=1}^{N} U_i(s^i). \tag{8}$$

But (8) should further be independent of the order: for any permutation $\pi : \mathcal{N} \to \mathcal{N}$ we require

$$G(S) = \sum_{i=1}^{N} U_i(s^{\pi(i)}). \tag{9}$$

But this is not always the case. This is illustrated in the next example.

Example 5.1: Consider a single channel with 2 users. The total number of radios is $x + y$, because user 1 has x radios and user 2 has y radios. Then the utility obtained, using the FSR model, by each user is:

$$U_1(x, y) = xd(x + y), \quad \text{and} \quad U_2(x, y) = yd(x + y),$$

because the total bit rate is shared among the total number of radios. Given relation 9, in order to be a potential, G should satisfy the two permutations:

$$G(S) = U_1(s^1) + U_2(s^2) = U_1(s^2) + U_2(s^1),$$

with $s^1 = (x, 0)$ and $s^2 = (x, y)$. But

$$U_1(x, 0) + U_2(x, y) \quad = \quad xd(x) + yd(x + y),$$

and

$$U_1(x, y) + U_2(x, 0) \quad = \quad xd(x + y).$$

We observe that both expressions are different in the FSR model where $d(x) = \frac{\overline{R}^c(x)}{x}$.

Then we conclude that in the setting with the FSR model, we do not have a potential game.

VI. Backward Learning Algorithm

In [5], the authors propose three algorithms to enable the selfish users to converge to a Nash Equilibrium. Their algorithms use different set of available information. The first one is a centralized where each user has perfect information about the number of radios on each channels. The second is distributed and each user has also perfect information about the overall system. The last one considers imperfect information where each user knows the total number of radios only on those channels on which he operates. Due to the insufficiency of the local information, the system can be stabilized at a false Nash equilibrium, i.e., a stable point in which local information might be insufficient for the players to determine if it is a Nash equilibrium. In order to solve this problem, they allow each user to change a channel even if it is not necessary. This mechanism induces convergence to a Nash equilibrium with high probability but it does not stay in a Nash equilibrium solution. One can observe in their simulation that even if a Nash equilibrium is reached, the algorithm runs and can leave this stable point. For the last two algorithms, they consider only the restricted game.

We propose here an algorithm which always converges to a Nash equilibrium for the game with the FSU model and in the restricted game with the FSR model. This algorithm is specifically related to cognitive radio technology [7] because each user's decision is taken considering past perceived utility as **learning process**. This algorithm, provided in [24], is distributed and needs no information. Learning is one important specificity of cognitive radio technology [14], [17] and is often ignored in the study of such wireless networks using normal form game model such as in [25]. This algorithm has been used in [28], for the study of a power control allocation problem.

It is also important to notice that the algorithm does not need a backoff mechanism as proposed in [5], that delays the convergence. Also in their algorithms, each user has to know lots of information, like the total number of radios on each link it operates, that is very inconvenient for the deployment of such distributed algorithm.

A. Distributed Algorithm

The action of each player $i \in \{1, \ldots, N\}$ is a vector s_i composed of K elements whose elements are the channel allocation of each radio, i.e., for each $j \in \{1, \ldots, K\}$, $s_{i,j}$ is the channel chosen by user i for his j^{th} radio (for all i, j, $s_{i,j} \in \{1, \ldots, C\}$).

Note that the iterations could pass through strategies that are not restricted. Then this algorithm can be used for both models, FSR and FSU.

The strategy of any user is a probability matrix $P_i(t)$ of size $C \times K$ where $(P_i(t))_{c,k}$ represents the probability for user i of using channel c for the transmitter k at time slot t. We denote by $s_{i,k}(t)$ the channel used by user i for his transmitter k at time slot t. The parameter $0 < b < 1$ is the step size. We denote by P the vector composed of the N matrices P_i. Then, the expected utility of player i depending on mixed strategies of all other users and his own is:

$$g^i(P) = \sum_S U_i(S) \prod_{i=1}^{N} \prod_{k=1}^{K} (P^i)_{s_{i,k},k},$$

where $S = (s_{1,1}, s_{1,2}, \ldots, s_{N,K})$ is the vector all decisions of all users, of length $N \times K$. The expected utility of user i, if he chooses action $s_i = (s_{i,1}, s_{i,2}, \ldots, s_{i,K})$ is defined as:

$$H_{is_i}(P) = \sum_{S \setminus s_i} U_i(S) \prod_{j=1, j \neq i}^{N} \prod_{k=1}^{K} (P^j)_{s_{i,k},k},$$

with $S \setminus s_i = s_{1,1}, \ldots, s_{i-1,K}, s_{i+1,1}, \ldots, s_{N,K}$ describes the actions of all users but not user i. We have also the expected utility of user i choosing the channel c for his radio k:

$$(H_i)_{c,k}(P) = \sum_{S \setminus s_{i,k}} U_i(S)|_{s_{i,k}=c} \prod_{j=1, j \neq i}^{N} \prod_{k=1}^{K} (P^j)_{s_{i,k},k} \prod_{l=1, l \neq k}^{K} (P^i)_{s_{i,l},l},$$

with $S \setminus s_{i,k} = (s_{1,1}, \ldots, s_{i,k-1}, s_{i,k+1}, \ldots, s_{N,K})$ represents the actions of all users but not the channel of the radio k of user i. Note that we have the following relation:

$$\begin{aligned} g^i(P) &= \sum_{s_{i,1}, \ldots, s_{i,K}} H_{is_i}(P) \prod_{k=1}^{K} (P^i)_{s_{i,k},k}, \\ &= \sum_{c=1}^{C} (H_i)_{c,k}(P)(P^i)_{c,k}. \end{aligned}$$

We describe the totally decentralized algorithm defined in [24] adapted to our multi-decision game where each player strategy is a matrix.

1) Set an initial mixed strategy probability matrix $P_i(0)$, for each user $i = 1, \ldots, N$.
2) At every time step t, each user chooses a channel $s_{i,k}(t)$ for each transmitter k according to its probability matrix $P_i(t)$.
3) Each player obtains his payoff $U_i(t)$ based on the global channel allocation. The utility is normalized using:
$$\tilde{u}_i(t) = \frac{U_i(t)}{M_i},$$
where M_i is the maximum utility that can be perceived by user i. One user gets the maximum utility if he is the only one to transmit on each channel, then his maximum utility is $K \times \max_x R_c(x)$.
4) Each user updates its action matrix probability according to the following rule
   - $(P_i(t+1))_{c,k} = (P_i(t))_{c,k} - b\tilde{u}_i(t)(P_i(t))_{c,k}$ if $c \neq s_{i,k}(t)$,
   - $(P_i(t+1))_{c,k} = (P_i(t))_{c,k} + b\tilde{u}_i(t)(1 - P_i(t))_{c,k}$.
5) A stopping criterion met; else go to step 2.

The normalization process in step 3 ensures that the updated probabilities described in step 4 lie in the interval]0,1[. Note that depending on the system, the value of M_i, the maximum of the utility of user i, can be not known in advance and has to be estimated [28].

In [24], the authors prove that as $b \to 0$, the sequence of probabilities converges weakly to solution of the following ordinary differential equation $\forall i, c, k$:

$$\frac{d(P_i)_{c,k}}{dt} = (P_i)_{c,k} \sum_{s \neq c} (P_i)_{s,k}[(H_i)_{c,k}(P) - (H_i)_{s,k}(P)],$$

where $P(t)$ denotes the the channel allocation strategy at time t (a matrix of size $C \times K \times N$) and $(H_i)_{c,k}(P)$, the expected utility of user i for choosing action channel c for the transmitter k. Then, they show that (a) all stationary points that are not Nash equilibria are unstable and (b) all pure strategies that are strict Nash equilibria are asymptotically stable. By contradiction, we state the following result.

Proposition 6.1 ([24]): The algorithm does not converge to a stable channel allocation which is not a Nash Equilibrium.

It is equivalent to say that a stable stationary point must be a Nash Equilibrium. The problem is that the algorithm can exhibit limit cycle behavior. Hence, in [24] (Theorem 3.3) the authors propose a necessary condition for convergence, and they propose an example when users have a common payoff, which is the case here considering the FSU scheduling policy with the same unit energy cost.

Proposition 6.2: The learning algorithm, for any initial condition $P_i(0)$ for each user $i = 1, \dots, N$, always converges to a Nash equilibrium in the case of FSU or the restricted game with FSR.

Proof of Proposition 6.2. We use the potential function G defined in (4) in order to construct the following function:

$$F(P) = \sum_{s_{1,1}, s_{1,2}, \dots, s_{N,K}} G(S) \prod_{i=1}^{N} \prod_{k=1}^{K} (P_i)_{s_{i,k},k}.$$

This function has the following property:

$$\frac{\partial F}{\partial (P_i)_{c,k}}(P) = \lim_{t \to 0} \frac{F(P)|_{(P_i)_{c,k}+t} - F(P)}{t} = \lim_{t \to 0} \frac{\Delta_t}{t}.$$

We have:

$$\begin{aligned}
\Delta_t &= \sum_{s \backslash s_{i,k}} G_i(S)|_{s_{i,k}=c} \prod_{\substack{j=1 \\ j \neq i}}^{N} \prod_{k=1}^{K} (P^j)_{s_{j,k},k} \prod_{\substack{l=1 \\ l \neq k}}^{K} (P^i)_{s_{i,l},l} (P^i_{c,k} + t) \\
&\quad - \sum_{s \backslash s_{i,k}} G_i(S)|_{s_{i,k}=c} \prod_{\substack{j=1 \\ j \neq i}}^{N} \prod_{k=1}^{K} (P^j)_{s_{j,k},k} \prod_{\substack{l=1 \\ l \neq k}}^{K} (P^i)_{s_{i,l},l} P^i_{c,k}, \\
&= t \sum_{s \backslash s_{i,k}} G_i(S)|_{s_{i,k}=c} \prod_{\substack{j=1 \\ j \neq i}}^{N} \prod_{k=1}^{K} (P^j)_{s_{j,k},k} \prod_{\substack{l=1 \\ l \neq k}}^{K} (P^i)_{s_{i,l},l}, \\
&= t (H_i)_{c,k}(P).
\end{aligned}$$

Then we obtain:

$$\frac{\partial F}{\partial (P_i)_{c,k}}(P) = (H_i)_{c,k}(P).$$

Combining this result with the ode (10), we have:

$$\begin{aligned}
\frac{dF}{dt} &= \sum_{i,c,k} \frac{\partial F}{\partial (P_i)_{c,k}} \frac{d(P_i)_{c,k}}{dt}, \\
&= \sum_{i,c,k} \frac{\partial F}{\partial (P_i)_{c,k}} (P_i)_{c,k} \sum_{s \neq c} (P_i)_{s,k} [(H_i)_{c,k}(P) - (H_i)_{s,k}(P)], \\
&= \sum_{i,c,k} \sum_{s \neq c} (P_i)_{c,k} (P_i)_{s,k} [(H_i)_{c,k}(P)^2 - (H_i)_{c,k}(P)(H_i)_{s,k}(P)], \\
&= \sum_{i,c,k} \sum_{s > c} (P_i)_{c,k} (P_i)_{s,k} [(H_i)_{c,k}(P) - (H_i)_{s,k}(P)]^2 \geq 0.
\end{aligned}$$

We have proved that the function F is nondecreasing along the trajectories of the ode. Then all solutions of the ode will be in the set of probabilities such that $\frac{\partial F}{dt}(P) = 0$. Thus all solutions converge to some stable stationary points and, as from theorem 6.1, all stable stationary points are Nash equilibria, the result follows.

$\diamond$

This kind of result is obtained in [22] in the context of population game with the continuous version of the potential.

B. Numerical applications

In this section we consider the following parameters: number of users $N = 3$, number of channels $C = 4$, number of radios per user $K = 3$. The stopping criterion for our distributed algorithm is that

$$\max_{c,k,i} (|(P_i)_{c,k}(t) - (P_i)_{c,k}(t-1)|) < 10^{-9},$$

and the updating parameter is $b = 0.1$. We compare different rate functions obtained from different network management or wireless technology.

1) FSU and constant rate function: We consider as specified in [5] the constant rate function $R_c(x) = 1Mbps$. We take the cost per radio is $\lambda = 0.1$.

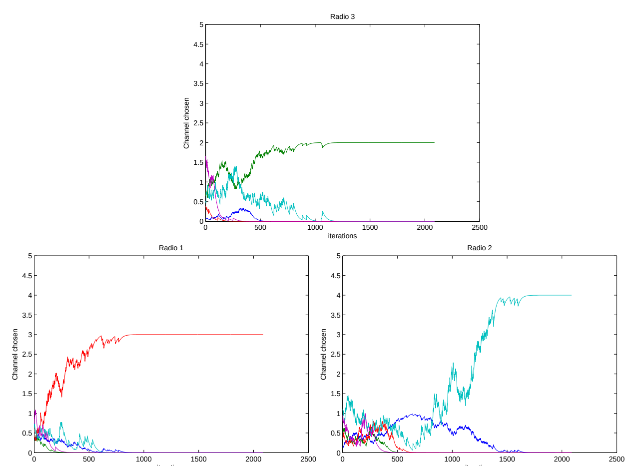

Fig. 2. Radios position of user 1 radios at the NE when $\lambda = 0.1$.

We observe on figures2 that at equilibrium, the user 1 puts his radios on channels 3, 4 and 2. The nash equilibrium is:

- for user 1, one radio on channels 3, 4 and 2,
- for user 2, one radio on channels 3, 2 and 1,
- for user 2, one radio on channels 1, 4 and 3.

We observe that all users have decided to use all their radios. We now increase the cost by $\lambda = 0.3$. We observe now that the algorithm converges to the following nash equilibrium:

- for user 1, one radio on channels 2, 1 and 3,
- for user 2, one radio on channels 2 and 4, and his last radio is not used,
- for user 2, one radio on channels 3, 4 and 1.

Then the total number of radios on each channel is 2 and one user uses only 2 of his 3 radios.

2) FSR and IEEE 802.11 DCF: Using the FSR model we don't have the proof of the convergence of the distributed algorithm because we cannot use a potential function (it does not exist). We still observe the convergence of the distributed algorithm in this numerical example.

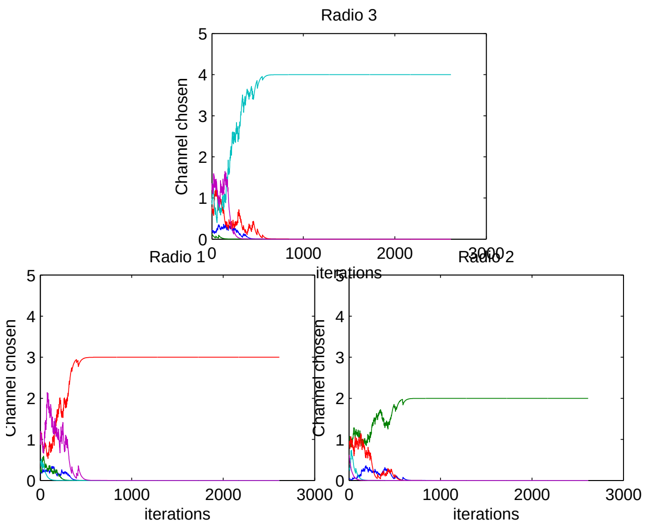

Fig. 3. Radios position of user 1 radios at the NE when $\lambda = 2$.

VII. Conclusions

This paper considers a resource allocation problem where several users, equipped with multiple radios, share in a multichannel wireless access. We study two model of network allocation strategy and we prove the existence of a well know structure of congestion game in one model. Therefore we find properties of the equilibrium strategy and finally we propose a totally distributed algorithm that converge to a Nash equilibrium of this resource allocation game.

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