A Theoretical Framework for Hierarchical Routing Games

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***Abstract*—Most theoretical research on routing games in telecommunication networks has so far dealt with reciprocal congestion effects between routed entities. Yet in networks that support differentiation between flows, the congestion experienced by a packet depends on its priority level. Another differentiation is made by compressing the packets in the low priority flow while leaving the high priority flow intact. In this paper we study such kind of routing scenarios for the case of non-atomic users and we establish conditions for the existence and uniqueness of equilibrium.**

I. Introduction

The definition of the steady state equilibrium of a traffic network was put forth by J.G. Wardrop in his 1952 treatise [12] which provided two different definitions of traffic assignment concepts. The first is commonly referred to as the Wardrop, or traffic equilibrium, principle. It states that "*The journey time on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route*." In the context of telecommunication networks, the Wardrop equilibrium already advocated by Bertsekas and Gallager in [4], is used most often to model the situation in which the routed entities are packets, and routing decisions are taken at the nodes of the networks (rather than by the users) so as to minimize the (per-packet) delay. In the context of wireless ad-hoc networks, the Wardrop equilibrium has been first introduced by Gupta and Kumar [8] when introducing a routing protocol for Ad-Hoc networks. Wardrop equilibria have also been used in telecommunication networks to model a large number of users that can determine individually their route and in which the routed object is a whole session, see Korilis and Orda [9]. As shown in [3], [13], the setting of wardrop equilibrium turns out to be a potential game. The uniqueness of equilibrium in potential games was established in [7]

None of these mentioned works consider the cases where the congestion effects between the routed entities are not reciprocal. To build a theoretical framework for studying such problems and to study the issues involved is the main objective of this paper. We consider a different kind of *Hierarchical* Routing game in which the routed traffic can be of two types: High priority and Low priority. The high priority traffic is unaffected by the routing profile of the Low priority traffic. On the other hand the low priority traffic faces congestion effects from both high priority and low priority traffic. We consider the non-atomic game in which the individual entities of the traffic (eg. packets) are the decision makers and we seek to minimize the delay per routed entity.

A. Our Contributions

We first introduce the non-atomic game and define the concept of System Wardrop equilibrium which can be seen as an extension of the traditional Wardrop equilibrium concept to our scenario. For the case of parallel links we establish the existence of this equilibrium and give conditions for its uniqueness. Our results are illustrated by an example of multi-channel hierarchical rate allocation under unslotted ALOHA protocol. Natural applications of these models to cognitive radio are also pointed out.

II. Non-atomic routing game over parallel links

Consider a network topology given by $G = (E, \{s, d\})$ where E is a set of $|E| = K$ parallel links connecting source node s to destination node d. The links are assumed to be directed from the source to the destination. We consider a model where the low priority flow is compressed when it is routed while the high priority flow is unaffected. For example if the flow is in the form of a stream of packets, then the packet size is compressed by a factor $\gamma \in (0, 1)$ if it is routed as low priority. Consider a single user who wishes to route a total flow demand of r from the source to the destination. Let rp for $p \in [0, 1]$ be the portion of the total flow that it routes as low priority and $r(1-p)$ be the flow it routes as high priority. Consider a model where the low priority flow is compressed when it is routed while the high priority flow is unaffected. For example if the flow is in the form of a stream of packets, then the packet size is compressed by a factor $\gamma \in (0, 1)$ if it is routed as low priority. After compression, the resulting low priority flow will be γrp while the high priority flow remains unchanged. Thus the total flow being routed is

$$\bar{r}(p) = r(\gamma p + (1 - p)) \tag{1}$$

which is not constant but linearly decreasing in p with $\bar{r}(0) = r$ and $\bar{r}(1) = \gamma r$. Let x_l^H be the high priority flow and x_l^L be the low priority flow shipped over link $l \in E$, $l = 1, \cdots, K$ such that $\sum_{l=1}^{K} x_l^H = r(1 - p)$ and $\sum_{l=1}^{K} x_l^L = \gamma rp$. Let

$x_l = x_l^L + x_l^H$, the total flow on link l. Introduce the following vectors:

$$\bar{x}^H = (x_1^H, \cdots, x_K^H)$$

$$\bar{x}^L = (x_1^L, \cdots, x_K^L)$$

$$\bar{x} = (x_1, \cdots, x_K)$$

Introduce the fixed cost per unit flow C^H and C^L associated with classifying the traffic as High priority and low priority respectively. Introduce for each link l a convex, continuous and monotone increasing routing cost per unit flow function $T_l : [0, \infty) \to [0, \infty)$ which depends only on the total high priority flow x_l^H on that link for the high priority flow but which depends on the total flow x_l on that link for the low priority traffic. Let $T_{min} = \min_{l \in E} T_l(0)$.

For a particular value of p, we characterize the Wardrop equilibrium for the High priority flow, i.e. a vector $\bar{x}^{*H}$ which satisfies the following variational inequalities:

$$x_l^{*H}(T_l(x_l^{*H}) - \alpha(p)) = 0, \, l \in E \tag{2}$$

$$T_l(x_l^{*H}) - \alpha(p) \geq 0, \, l \in E \tag{3}$$

$$\sum_{l=1}^{K} x_l^{*H} = r(1-p) \tag{4}$$

Here $\alpha(p)$ is the cost per unit high priority flow at Wardrop equilibrium for that value of p. The first equation means that the flow on every link l is either zero or its cost per unit flow is equal to the minimum cost on any link which is $\alpha(p)$. The second inequality says that the cost per unit flow on any link is at least as high as this minimum cost. The total cost per unit flow at Wardrop equilibrium is given by $\bar{\alpha}(p) = \alpha(p) + C^H$. Corresponding to the Wardrop equilibrium profile $\bar{x}^{*H}$ of the high priority flow, we then characterize the Wardrop equilibrium for the low priority flow, i.e. a vector $\bar{x}^{*L}$ which satisfies the following variational inequalities:

$$x_l^{*L}(T_l(x_l^{*H} + x_l^{*L}) - \beta(p)) = 0, \, l \in E \tag{5}$$

$$T_l(x_l^{*H} + x_l^{*L}) - \beta(p) \geq 0, \, l \in E \tag{6}$$

$$\sum_{l=1}^{K} x_l^{*L} = \gamma r p \tag{7}$$

where $\beta(p)$ is the cost per unit low priority flow at Wardrop equilibrium for that value of p. The total cost per unit flow at Wardrop equilibrium is given by $\bar{\beta}(p) = \beta(p) + C^L$. It is well known that for a given fraction p, both these Wardrop equilibria for the high priority and the low priority flows exist under the given assumptions on the link cost functions.

We now give the following definition of a system wardrop equilibrium which is the extension of the concept of Wardrop user equilibrium to the case of hierarchical users. It says that at this equilibrium, not only does a member of the population have no incentive to switch his link but he also does not have any incentive to switch his class i.e. High priority or low priority.

Definition 2.1: The flow profiles $\bar{x}^{*H}$, $\bar{x}^{*L}$ and p^* constitute a *System Wardrop equilibrium* if either of the three conditions are satisfied:

- $\bar{x}^{*H}$ and $\bar{x}^{*L}$ satisfy (2)-(4) and (5)-(7) and $\bar{\alpha}(p^*) = \bar{\beta}(p^*)$ such that $p^* \in (0, 1)$
- $p^* = 1$, $\bar{x}^{*L}$ satisfies (5)-(7) and $\bar{\alpha}(1) \geq \bar{\beta}(1)$ where $\bar{\alpha}(1) = T_{min} + C^H$
- $p^* = 0$, $\bar{x}^{*H}$ satisfies (2)-(4) and $\bar{\alpha}(0) \leq \bar{\beta}(0)$ where $\bar{\beta}(0) = \alpha(0) + C^L$

Note that $\alpha(1) = T_{min}$, since if the entire flow is low priority and if a unit flow switches to high priority then it will choose the link with the lowest cost at zero flow. Similarly $\beta(0) = \alpha(0)$ since if entire flow is high priority the congestion cost of a unit flow will not change if it switches to low priority since it will face the same congestion effect.

III. Existence and Uniqueness of the S.W.E

We first have the following lemma:

Lemma 3.1: For the case of parallel links total cost per unit High priority flow at Wardrop equilibrium $\bar{\alpha}(p) = \alpha(p) + C^H$ is convex, continuous and monotone decreasing in $p \in [0, 1]$, the proportion of the low priority flow.

Proof: It follows from the continuity, convexity and monotone increasing nature of the link costs per unit flow $T_l^H(.)$ with respect to the total flow on the links and the fact that the link are parallel and costs per unit flow for all links that are used (i.e. links with positive flows) at Wardrop equilibrium are equal. ■

Lemma 3.2: The routing cost per unit Low priority flow at Wardrop equilibrium $\beta(p)$ is independent of the wardrop equilibrium high priority flow profile and only depends on the total flow at p given by (1). This cost can be obtained from the following variational inequalities which correspond to the Wardrop equilibrium of the total flow profile if the entire flow was low priority:

$$x_l^*(T_l(x_l^*) - \beta(p)) = 0, \, l \in E \tag{8}$$

$$T_l(x_l^*) - \beta(p) \geq 0, \, l \in E \tag{9}$$

$$\sum_{l=1}^{K} x_l^* = \bar{r}(p) = r(\gamma p + (1-p)) \tag{10}$$

where $\bar{x}^*$ is the total flow profile at Wardrop equilibrium. The total cost is then given by $\bar{\beta}(p) = \beta(p) + C^L$.

Proof: If $p = 0$ then the entire flow is high priority with Wardrop equilibrium routing cost per unit flow $\alpha(0)$. Thus the routing cost per unit low profile flow at this equilibrium is the same i.e. $\alpha(0)$ (note that the actual low priority flow is zero). For $p = 0$, the variational inequalities (2)-(4) and (8)-(10) are the same and thus this implies the result.

We now consider $p \in (0, 1]$. Let $\bar{x}^{*H}$ be the Wardrop equilibrium flow profile for the high priority flow at a particular value of p. We first show that for $p > 0$,

$$x_l^* > x_l^{*H}, \, \forall \, l \in E \text{ such that } x_l^* > 0 \tag{11}$$

Suppose this is not true and $x^*_{l'} \leq x^{*H}_{l'}$ for some $l' \in E$ such that $x^*_l > 0$. Since each link cost function $T_l(.)$ is monotone increasing, this means that $T_{l'}(x^*_{l'}) \leq T_{l'}(x^{*H}_{l'})$. Now since both the high priority flow profile and the total flow profile are at Wardrop equilbrium, the link costs for high priority flow $T_l(x^{*H}_l) = \alpha(p)$ for all links such that $x^{*H}_l > 0$ and similarly the link costs for total flow $T_l(x^*_l) = \beta$ for all links such that $x^*_l > 0$. Both these facts imply that $\beta = T_l(x^*_l) \leq T_l(x^{*H}_l) = \alpha(p)$ for each link $l \in E$ such that $x^{H*}_l > 0$. But this implies that if $x^*_l > 0$ then $x^{*H}_l > 0$ for all $l \in E$. To see this, consider a link $l \in E$ where $x^*_l > 0$ but it is unused by the High priority flow. Then $T_l(0) < T_l(x^*_l) = \beta \leq \alpha(p)$ which cannot be true since it is required that the link cost at any unused link should be at least as much as the equilibrium cost. Thus we have that $T_l(x^*_l) \leq T_l(x^{*H}_l)$ for each link $l \in E$ such that $x^*_l > 0$. Again the monotone increasing nature of the cost functions imply that $x^*_l \leq x^{*H}_l$ for each such link. Summing over all links we obtain

$$(1-p)r + \gamma rp \leq \sum_{l \in E; x^*_l > 0} x^{*H}_l \leq (1-p)r$$

which is a contradiction since $p > 0$. Thus equation (11) holds. Next we show the following:

$$\text{If } x^{*H}_l > 0 \text{ then } x^*_l > 0 \, \forall \, l \in E \tag{12}$$

From equation (11) by similar arguments as before, we have $\beta = T_l(x^*_l) > T_l(x^{*H}_l) = \alpha(p)$ for each link $l \in E$ such that $x^*_l > 0$. Suppose there is a link $l \in E$ where $x^{*H}_l > 0$ but $x^*_l = 0$ i.e it is unused in the the total flow profile at equilibrium. Then $T_l(0) < T_l(x^{*H}_l) = \alpha(p) < \beta$ which cannot be true since it is required that the link cost at any unused link should be at least as much as the equilibrium cost. Thus (12) holds. Next define a low priority flow profile $\bar{x}^{*L}$ such that

$$x^{*L}_l = x^*_l - x^{*H}_l, \, l \in E \tag{13}$$

Then (11) and (12) together imply that $x^{*L}_l \geq 0 \forall l \in E$ and that $x^{*L}_l > 0$ if and only if $x^*_l > 0$. So for a total flow profile which satisfies the variational inequalities (8)-(10) and for a Wardrop equilibrium High priority profile at any $p \in (0, 1]$ we have a Low priority flow profile given by (13) which satisfies the following equivalent set of variational inequalities:

$$x^{*L}_l(T_l(x^{*L}_l + x^{*H}_l) - \beta) = 0, \, l \in E \tag{14}$$

$$T_l(x^{*L}_l + x^{*H}_l) - \beta \geq 0, \, l \in E \tag{15}$$

$$\sum_{l=1}^{K} x^{*L}_l = \gamma pr \tag{16}$$

But these are the variational inequalities at Wardrop equilibrium for the low priority flow. Hence the Low priority flow profile at Wardrop equilibrium for any $p \in (0, 1]$ can be obtained from the total flow profile at Wardrop equilibrium and the High priority flow profile at wardrop equilibrium by the construction (13) ■

Corollary 3.1: If $\gamma = 1$, the routing cost per unit Low priority flow at Wardrop equilibrium $\beta(p)$ is constant and does not vary with p, the proportion of Low priority flow. This cost β is obtained from the following variational inequalities which correspond to the Wardrop equilibrium of the total flow profile if the entire flow was low priority:

$$x^*_l(T_l(x^*_l) - \beta) = 0, \, l \in E \tag{17}$$

$$T_l(x^*_l) - \beta \geq 0, \, l \in E \tag{18}$$

$$\sum_{l=1}^{K} x^*_l = r \tag{19}$$

where $\bar{x}^*$ is the total flow profile at Wardrop equilibrium. The total cost is then given by $\bar{\beta} = \beta + C^L$.

Lemma 3.3: The total cost per unit High priority flow and Low priority flow at Wardrop equilibrium i.e. $\bar{\alpha}(p) = \alpha(p) + C^L$ and $\bar{\beta}(p) = \beta(p) + C^L$ respectively are convex, continuous and monotone decreasing in p, the proportion of the low priority flow. Further $\beta(p) = \alpha(p(1-\gamma)))$ for $p \in [0, 1]$.

Proof: The result for the high priority flow follows from lemma (3.1). For the low priority flow, we see that the total flow decreases continuously and linearly in p and thus the result follows from lemma (3.1) and the fact that the link cost functions are continuous, convex and monotone increasing in the link flow. For the second part, see that the routing cost at wardrop equilibrium for the low priority flow at any p depends on the total flow being routed which is $r(\gamma p + (1-p))$ and for the High priority flow it depends on the total high priority flow which is $r(1-p)$. These costs will be equal if these flows are equal i.e. when

$$r(\gamma p + (1-p)) = r(1-p') \text{ for any } p, p' \in [0, 1]$$

which leads to

$$p' = p(1-\gamma) \tag{20}$$

and hence the result follows. ■

Theorem 3.1: Assume the link costs T_l for each link $l \in E$ are convex, continuous and monotone increasing in the flow on that link. Then,

- If $C^H > C^L$ and $\beta(1) + C^L > C^H + T_{min}$, then there exists a System Wardrop equilibrium, i.e. there exists a $p^* \in [0, 1]$ such that $\bar{\beta}(p^*) = \bar{\alpha}(p^*)$. Further assume that $\alpha(p)$ and $\beta(p)$ are continuously differentiable. Then if $\gamma > \max_{p \in [0,1]}(1 - \frac{\alpha'(p)}{\alpha'(p(1-\gamma))})$ then this equilibrium is unique.
- If $C^H \leq C^L$ then the unique System Wardrop equilibrium occurs at $p = 0$, i.e. the entire flow is routed as high priority.
- If $C^H > C^L$ and $\beta(1) + C^L \leq C^H + T_{min}$ then the unique System Wardrop equilibrium occurs at $p = 1$, i.e. the entire flow is routed as low priority.

Proof: We first prove the first statement. Define a function $f : [0, 1] \to \mathbb{R}$ such that $f(p) = \bar{\alpha}(p) - \bar{\beta}(p) = \bar{\alpha}(p) - \bar{\alpha}(p(1-\gamma))$ where the last equality follows from Lemma (3.3). It again follows from the two lemmas that f is continuous. Also for the first part, $f(0) = C^H - C^L > 0$ and $f(1) = C^H + T_{min} - (\beta(1) + C^L) < 0$. Both these facts imply that there exists a $p^* \in (0,1)$ not necessarily unique such that $f(p^*) = 0$. This proves the result. Further if $\alpha(p)$ and $\beta(p)$ are continuously differentiable, then

$$f'(p) = \alpha'(p) - \alpha'(p(1-\gamma))(1-\gamma)$$

Hence if $\gamma > \max_{p\in[0,1]}(1 - \frac{\alpha'(p)}{\alpha'(p(1-\gamma))})$ then $f(p)$ is monotone decreasing in $p \in [0,1]$ and there exists a unique $p^* \in (0,1)$ such that $f(p^*) = 0$. The second and third statements follow from the definition of System Wardrop equilibrium. ■

Remark 3.1: Even in the absence of differentiability of $\alpha(p)$ and $\beta(p)$ we can have uniqueness of the System Wardrop equilibrium for high enough values of γ. Indeed if $\gamma = 1$ then corollary (3.1) says that $\beta(p)$ is constant and hence $f(p)$ is monotone decreasing. Hence we have the following corollary.

Corollary 3.2: If $\gamma = 1$ then under the assumptions on the link costs, the system wardrop equilibrium is unique.

For any given p, $\bar{\alpha}(p)$ and $\bar{\beta}(p)$ can be found out by computing general Wardrop equilibrium for the high and low priority users at this proportion for which there exist several methods. Particularly since the users belong to a single class, for the kind of cost functions that we have considered here, a potential function exists for these games and the equilibrium can be computed as a global convex optimization problem. Also certain population dynamics converge to the equilibrium. For these methods, see [13].

Now if $f = \bar{\alpha}(p) - \bar{\beta}(p)$ is monotone decreasing, if for example $\gamma = 1$, then we can find its zero by using the simple bisection method and thus the system Wardrop equilibrium can be found.

IV. Hierarchical rate control over multiple ALOHA channels

A hierarchical routing game arises naturally in some context of practical interests. For example, this hierarchy is naturally present in contexts where there are primary (licensed) users and secondary (unlicensed) users who can sense their environment because there are equipped with a cognitive radio [10]. But in cognitive radio networks it requires that secondary users dynamically and opportunistically utilize unused licensed spectrum on a non-interfering with primary users. In the same spectrum band, users in secondary group (unlicensed) are seeking to exploit possible transmission opportunities whenever it would not cause any interference to primary group. Since the primary user has a license to operate in the spectrum band, its access should not be affected by the operations of any other unlicensed users. In order not to disturb the primary users spectrum usage, priority to access the spectrum is given to primary users. We assume that the unlicensed users equipped with cognitive radios are capable to detect the primary users activities or to transmit with lower power in order to keep the interference for primary user limited.

Consider a population of communicating terminals using an ALOHA protocol over several channels to send packets to a single destination, with two possible levels of power High and Low. In this example we consider the Non-atomic game where each terminal in the population takes the decision of choosing a single channel amongst the available channels.

A. The non-atomic game

As mentioned before consider an infinite population of mobile terminals. We use the model of unslotted aloha where the global arrival of new packets from all the mobiles follows a poisson process with rate λ. All mobiles are identical in the sense that the contribution to the global rate λ of the arrival rate from each mobile, though negligible, is the same for all mobiles. Let η be the rate contribution of a single mobile where η is very small compared to λ. Consider M channels over which the packets can be routed under the ALOHA protocol. Also two levels of power can be chosen by each mobile: High and Low. The time required to send the packet is $\frac{1}{2}$ time unit. So if there is a transmission in the period $[t - \frac{1}{2}, t + \frac{1}{2}]$, then there is a collision. If a High power packet collides with another High power packet then both packets are lost, similarly if a low power packet collides with a low power packet. If a high power packet collides with a low priority packet, then the high power packet transmission is successful while the low power packet is lost. This is the capture phenomenon. Let λ_l^H and λ_l^L denote the High power and Low power rate respectively routed through channel l such that:

$$\sum_l \lambda_l^H + \lambda_l^L = \lambda$$

In spite of the losses, the input rates are fixed at these values. The probability of successful transmission for a packet choosing high power on a link l is

$$P^H(\lambda_l^H) = e^{-\lambda_l^H}$$

and that for a packet choosing low power is

$$P^L(\lambda_l^H + \lambda_l^L) = e^{-(\lambda_l^H + \lambda_l^L)}$$

The payoff to a mobile terminal per unit rate is an increasing function of the transmission probability. We choose the logarithmic function, and then the cost to the mobile (negative of the payoff) is rendered linear in the rates on that channel:

$$c_l^H = -\eta k_l \ln(e^{-\lambda_l^H}) = \eta k_l(\lambda_l^H)$$

and

$$c_l^L = -\eta k_l \ln(e^{-(\lambda_l^H + \lambda_l^L)}) = \eta k_l(\lambda_l^H + \lambda_l^L)$$

where c_l^H is the channel cost to a mobile choosing channel l and high power, c_l^L is the channel cost to the mobile choosing channel l and low power and k_l is a constant for each channel l. Let the cost for sending a unit rate with high power be given by C^H and that with lower power be given by C^L. Then the cost per mobile is given by ηC^H and ηC^L respectively. The total channel costs are then given by :

$$\bar{c}_l^H = \eta k_l(\lambda_l^H) + \eta C^H$$

and

$$\bar{c}_l^L = \eta k_l(\lambda_l^H + \lambda_l^L) + \eta C^L$$

We look for a system wardrop equilibrium such that no mobile has any incentive to change its channel or its power level at equilibrium. By using the fact that the number of High or low power mobiles on a link is given by $\frac{\lambda_l^H}{\eta}$ and $\frac{\lambda_l^L}{\eta}$, we can pose it as a problem where the total rate is to be divided amongst the M channels rather than the population of mobiles.The total channel costs for a unit rate at High power and Low power are respectively given by:

$$\hat{c}_l^H = k_l(\lambda_l^H) + C^H$$

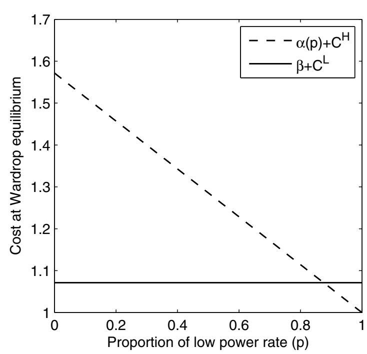
and

$$\hat{c}_l^L = k_l(\lambda_l^H + \lambda_l^L) + C^L$$

1) Example: For the non-atomic game we consider $\lambda = 1$, there are three channels, $1, 2$ and 3 and $k_1 = 1$, $k_2 = 2$ and $k_3 = 4$. $C^H = 1$ and $C^L = \frac{1}{2}$. If p is the proportion of the rate which is being transmitted with low power, then the wardrop equilibrium cost per unit high power rate is given by: $\bar{\alpha}(p) = \alpha(p) + C^H = \frac{4}{7}(1-p) + 1$ and the cost at wardrop equilibrium per unit low power rate is constant from corollary 3.1 and is given by: $\bar{\beta}(p) = \beta + C^L = \frac{4}{7} + \frac{1}{2}$

Thus the system wardrop equilibrium occurs at $p^* = \frac{7}{8}$ (Refer to the figure below)

The the high and low power channel rates at the System wardrop equilibrium are given by: $\bar{\lambda}^H = (\bar{\lambda}_1^H, \bar{\lambda}_2^H, \bar{\lambda}_3^H) = (\frac{1}{14}, \frac{1}{28}, \frac{1}{56})$ and $\bar{\lambda}^L = (\bar{\lambda}_1^L, \bar{\lambda}_2^L, \bar{\lambda}_3^L) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{8})$



V. Conclusions

We introduced a new class of routing games which allows for a hierarchy amongst the routed entities with non-reciprocal congestion effects, applications of which arise frequently in practical scenarios especially with the advent of cognitive radio technology. Through our results we were able to highlight the issues that arise in modeling routing scenarios in the absence of reciprocal congestion effects between classes of users. We introduced the concept of a System Wardrop equilibrium for the non-atomic game and established its existence and uniqueness under certain conditions. Our results were illustrated by an application: multichannel hierarchical rate allocation under unslotted ALOHA for which we explicitly computed the System Wardrop equilibrium.

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