Broadcasting Forever

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Abstract— **In a pay-per-view scheme, a service provider multicasts the program for which it has the rights on demand only to those who have paid for it. However, once payments are made it is tempting to broadcast the program to all. This occurs both in order to increase advertising revenues (due to a higher rating) and due to the extra costs associated with multicasting in comparison with broadcasting. Of course, this can be done only once as reputation will then be lost. We describe a pricing mechanism which results in broadcasting while still causing those willing to pay, to do so and at the monopoly price.**

I. Pricing in a one shot game

Consider a commodity which is owned and sold by a monopoly. For example, think of a tv channel that bought the broadcasting rights for the final match of the European soccer cup. The channel is not obliged to broadcast it but can deliver it (i.e., multicast) only to those who pay the price it sets (known as a *pay-per-view* scheme). Suppose that the value of the commodity across potential buyers is subjective. Let $F(x)$ be the proportion of those who value it by at most x. We assume that this function is continuous and strictly increasing, and that $F(0) = 0$. Let also $\overline{F}(x) = 1 - F(x)$. If the monopoly is to set a single price, then in order to maximize its profit, it should set the price equal to

$$x_{mo} = \arg\max_x x\overline{F}(x) \quad .$$

Of course, the assumption here is that only those who value the commodity by more than the set price will purchase it.

We consider next two variations of the model and obtain for each of them the gain for broadcasting instead of multicasting. [1]

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[1]We consider each one of the models separately but they can be dealt with in a single model in which both are introduced to the model simultaneously.

a) Variation of the model: income from advertisements: Suppose it is possible to advertise during broadcasting or multicasting. In particular, let $g(p)$ be the provider's gain in the case of a rating to p. Then the optimization problem faced is

$$x_{ad} = \arg\max_x \{x\overline{F}(x) + g(\overline{F}(x))\} \quad .$$

Remark: We assume here that the presence of advertisement does not diminish the will to watch the match. This may be the case when the advertisements are restricted to the break.

Yet, it is tempting here, once payments are collected, to broadcast to all and collect the advertising revenue of $g(1)$ (and not only $g(\overline{F}(x_{ad}))$). If this is the case, then it is even better to charge x_{mo} (rather then x_{ad}) and collect even more revenue due to higher intake in payments.

b) Variation of the model: cost for multicasting: There is another feature which is common in multicasting operations. Multicasting, in comparison with broadcasting, comes with an added cost, borne by the monopoly. This is due to having to ship the commodity to the customers. Assume further that the cost is linear with the number of customers receiving it and denote this cost per customer by c. However, if the commodity is shipped to *all*, then shipping costs nothing. This of course is not in line with standard assumptions in economics. However, it represents the reality where multicasting vs. broadcasting is concerned. In both cases a coded transmission is shipped to all. In case of broadcasting a decoding key is added (at zero added cost). However, multicasting, comes with an extra cost per viewer borne by the operator. This is due to having to keep record of all those who paid (and then mailing the key to all of them (and only them)). They will then be able to decode the otherwise coded broadcast. The optimization problem faced then is

$$x_{mu} = \arg\max_{x \geq c} (x - c)\overline{F}(x) \quad .$$

Yet, here too it is tempting for the operator, once payments are collected, to broadcast the program and save on multicasting costs. As in the case of possible advertisements, had this been possible, the announced price of x_{mo} would lead to an even bigger gain, namely to a gain of $x_{mo}\overline{F}(x_{mo})$.

Of course, the policy of promising pay-per-view and then to broadcast is shortsighted and unsustainable: Customers can be fooled only once. They will soon learn that those who did not pay, watched the match for free so next time they too will try to be free riders. Thus, it seems that the most operators can do is to charge x_{ad} and have a revenue less than $x_{mo}\overline{F}(x_{mo})+g(1)$ (in the first version), or to charge x_{mu} and have a revenue which is less than $x_{mo}\overline{F}(x_{mo})$ (in the second version). In particular, in both cases the end result will be multicasting only to those who pay the price of x_{ad}, in the first version, or x_{mu}, in the second. Yet, we argue below that more can be done with the help of a mechanism that we designed.

II. Broadcasting while still maintaining credibility

In spite of all of the above, we claim next that the operator can still charge x_{mo} per viewer, gain $x_{mo}\overline{F}(x_{mo})$ and broadcast the match to all (payers as well as non-payers). In the case where advertisements are allowed, there will be an extra gain of $g(1)$. This can be achieved as follows. Suppose this scheme is designed. The operator announces a pair of numbers composed of a price x and a fraction q and says (honestly): Those who pay x will be able to view the match (through multicasting or broadcasting). Moreover, if (and only if) at least a fraction of q of the potential subscribers pay this price of x, than the match will be broadcasted to all (including to those who did not pay). What is the optimal choice for x and q for the operator given this scheme?

A. The game among potential subscribers

Before answering the above question we have to agree on how potential subscribers behave once a pair (any pair) of x and q is being announced. The first thing to observe is that the whereabouts of potential subscribers interact, and hence they are facing a non-cooperative game of which they are its players. Also, no dominant strategy exists to any one of them. For example, one who values the match by more than x will pay if all others do not pay, and he will not pay if all others do so. Thus, the solution concept to adopt here is that of Nash equilibrium, namely a strategy profile such that if adopted by all, no individual has an incentive to play other than what is prescribed for him by the profile.

The following observation is quite straightforward and thus a formal proof will not be given.

Observation 2.1: Given a price x and a fraction q, the equilibrium behavior among the customers is unique in the case where $x \geq \overline{F}^{-1}(q)$. In particular, those who value it by x or more, pay, while the rest do not pay. Otherwise, there are many equilibria, among them pay if and only if your value is not smaller than x.[2] However, in any of these equilibria a fraction of $\overline{F}(x)$ pay while the others do not pay.

B. The optimal pricing policy

Assuming the above behaviour among the potential subscribers, the next observation is immediate from the previous one.

Observation 2.2: In the case where the operator announces the pair of x and q, its intake under Nash equilibrium behaviour of the customers (regardless if the latter is unique or not), equals

$$x \min\{\overline{F}(x), q\} \tag{1}$$

Assuming that (1) is indeed the intake of the provider once he announces x and q, the question next is what is the pair of x and q which maximise (1). Specifically, the philosophy of sequential equilibrium and subgame perfection (also, called backwards induction), see, e.g.,[5], pp.268–292 or [7], pp.222–231, says that once a pair of x and q are announced (and the operator cannot renege from it), the other players, in our case the customers, play in accordance to a Nash equilibrium associated with the resulting subgame. This behaviour, in turn, will lead to some payoff to the operator. Things in our model simplify a bit further since there is no ambiguity regarding this payoff (see (1)). This is the case here even if the equilibrium is not unique. Knowing that, the operator will select his move, namely his announced pair of x and q, so as to maximise his gain as given in (1).

The next theorem says what is the optimal pair of x and q.

Theorem 2.1: The optimal pair of (x, q) is $(x_{mo}, \overline{F}(x_{mo}))$. In particular, the operator's intake is as of the monopoly. Moreover, the program is broadcasted to all.

Proof. Given customers' response to any pair of x and q is as described in Observation 2.1, and hence given the corresponding intake by the provider as stated in Observation 2.2, the provider needs to look into the pair of x and q that maximises (1). Clearly, an optimal value q for the provider, given the price x and under such cus-

[2] In summary, to pay if and only if your value is not smaller than $\max\{x, \overline{F}^{-1}(x)\}$ is always an equilibrium.

tomers' behaviour, is $q = \overline{F}(x)$.[3] The provider's profit is hence $x\overline{F}(x)+g(1)$ (in the first version) or $x\overline{F}(x)$ (in the second) as broadcasting is now guaranteed. Optimizing now with respect to x, brings us back to the monopoly's problem! Thus, the optimal pair for x and q is x_{mo} and $\overline{F}(x_{mo})$, respectively. Customers' reaction to that is that $\overline{F}(x_{mo})$ among them pay, making the intake equal to $x_{mo}\overline{F}(x_{mo})$. Finally, as the proportion of customers who pay equals the announced fraction, the program is broadcasted to all. $\diamond$

We like to be a bit more formal regarding the game played. The set of strategies for the provider corresponds to all possible pairs of x and q. The set of strategies for the player is for each pair of x and q, and given the value of the program y, to pay or not to pay. Above we proved that the following is a SPE:

1) The provider: announce x_{mo} and $\overline{F}(x_{mo})$.
2) A potential subscriber whose value is y: for any announced x and q, if $y \geq \max\{x, \overline{F}^{-1}(q)\}$ pay. Otherwise, do not pay.

Any SPE is an equilibrium but not the other way around. We next show, via an example, that multiple equilibria exist here. Suppose, $x_{mo} = 20$ and $\overline{F}(20) = 0.8$. Tag a potential subscriber whose value is 10. Hence, the following is an equilibrium: All do as the SPE prescribes but the tagged customer pays only if the announced x is 25 or above. This is an equilibrium profile. In particular, the tagged subscriber behaves in an optimal way given all behave as stated (due to the fact that $x = 25$ will never be announced). Indeed, it is common in cooperative games that non-optimal behaviour outside the equilibrium path (namely, at those possibilities that will not be reached, given all play according to the equilibrium profile), is sometimes a luxury that some may afford. In any case, it is worth noting that the resulting payoffs do not change and they coincide with those of the SPE. Moreover, the prescription for the provider is the same across all equilibria.

III. Building credibility

The issue of honesty on the provider's side is valid since looking at each match in isolation, multicasting only to those who pay in the event that less than a fraction of q pay for the service, is not part of a subgame perfect equilibrium strategy profile in the case where the provider can renege from his promises. This is the case since broadcasting is always the best response, given any payments were made. Thus, in order to enforce this policy, a credible mediator is needed. In particular, if the service provider wants to gain credibility for future such scenarios, he may hire a respected body who will monitor his behavior and will inform potential clients in cases where the provider does not meet its promises. Of course, in spite of all of the above, the provider needs to keep and maintain its multicasting technology: This is needed in order to make his threat to multicast only to fee payers credible. Note that credibility issues exists in related models. One example is a firm who offers lottery based promotion and hence calls for monitoring. Another, is a firm who offers huge discounts for the first, say, one hundred buyers.

The implementation of these policies during a long series of such scenarios, each of which with its own demand function $F(x)$, will lead to situations where a fraction of the customers always pay (a fraction that depends on the actual $F(x)$), while the provider always broadcasts. One can claim that a few instances of multicasting is needed in order to gain some credibility so announcing q which is larger than $\overline{F}(x_{mo})$ periodically might be needed. Note also that in practice (and after credibility is achieved), the firm should charge somewhat less than x_{mo}: If announcing x_{mo} and a less than the fraction of $\overline{F}(x_{mo})$ of potential customers pay, the firm will suffer a quantum cost of $g(1) - g(\overline{F}(x_{mo}))$ (in the first version) or $c\overline{F}(x_{mo})$ (in the second) due to multicasting. Charging a bit less will only lead to an infinitesimal loss in revenue.[4]

IV. Discussion

c) Fairness issues: Some may find this price mechanism as unjust or discriminatory due to the large number of free riders. This claim can be refuted by observing that many pricing schemes which are used today were unheard of just a few years ago. For example, nobody raises objections when airline passengers pay different prices for the same flight. Another example are the lower prices for services or products obtained by efficient users who purchase via the Internet. This occurs not only because effort invested in a web search is rewarding, but because vendors deliberately design two (or more) webpages offering the same item with a different price stated in each page. They may charge more if the site is linked through a paid ad in the portal of a search engine and charge less for the same item in a site where one

[3] Note that if $q > \overline{F}^{-1}(x)$ then the net profit is $x\overline{F}(x) + g(\overline{F}(x))$ in the first version or is $(x - c)\overline{F}(x)$ in the second.

[4] As stated in Observation 2.1, charging a bit less than x_{mo} will lead to multiple equilibria. Yet, the equilibria differ only by the identities of the free riders: Their fraction among the entire population is fixed and hence the profits are invariant with the executed equilibrium.

needs to invest more time and/or effort in searching in order to make a 'hit'.

d) Related work: A related paper to this is [9] which considers the above version in which multicasting comes with an extra cost. They deal with a similar situation in which many tv programs exist, each of which with its own demand function. In their model, a pair of (x, q) is announced: The price x is charged and a lottery decides whether to broadcast or multicast, the former with a probability q (and the latter with a probability $1-q$). The values for x and q are uniform and they do not vary with the offered program. In case of multicasting, only those who pay can watch the program. They show how to derive the optimal q given (a common) x for a given set of distributions over the demand functions.

In the framework of our above model, it is possible to see that if a pair of a price x and a probability q are announced (and broadcasting takes place with probability q regardless of customers' payment behavior), then only those who value the program by more than $x/(1-q)$ pay.[5] Thus, the optimal pair of x and q in this scheme with lottery for the model with income from advertisements is

$$\arg\max_{x,q} x\overline{F}(x/(1-q)) + qg(\overline{F}(1)) + (1-q)g(\overline{F}(x/(1-q))).$$

A similar optimization model holds for the case where multicasting is costly.

e) Economic context: The free riding phenomenon has been identified and considered by economists for a long time, see e.g. [4], [11]. With the appearance of P2P networks, this phenomenon has started to attract attention of many computer scientists [1], [3] as well, and credit based mechanisms have been proposed and deployed in order to diminish this phenomenon.

Another relevant issue in economic theory is that of *public good*. Public good can be the construction of a bridge, treating public waste, laying communication infrastructure, or, back to our model, broadcasting. The society at large has the option of going for it or not. In the former case, an added question is who is to foot the bill. It makes sense that those who gain from the common good, will pay for it. Moreover, the larger the gain, the larger should be the payment. Hence, many questions arise. Among them is how is it possible to tell among participants who is who, or how one can make those who gain more, reveal their true identity (and pay). After all, they too would prefer to be free riders. A good introduction to this issue appears in [5], Chapter 11. With public goods, contributors' benefits are public or available to all, while provision costs impact only the contributors [8], [6], [10]. When the operator, as in our problem, decides to broadcast the program to all users, the program can be considered as a public good. Here, as in a public good, if not enough resources are allocated by all, the good is not made available to all. What is unique in our model is that the good is always made available to those who contribute, regardless of how much is collected in total.

Assurance mechanisms that induce or strengthen cooperation, necessary for gathering an amount of individuals that is sufficient for the construction of a public good, are the issue of [10]. Our problem differs from the setting in [10] in the following. In our problem, (i) users are not symmetric and value differently the public good, (ii) the operator that broadcasts or multicasts a program is part of the game where as in [10] the assurance mechanism is only a tool for coordination and is not related to another player's utility.

[5] This is the value which makes one indifferent between paying or not. Namely, it solves for y, $y - x = qy$.

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