Optimal Forwarding in Delay-Tolerant Networks With Multiple Destinations

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***Abstract*—We study the tradeoff between delivery delay and energy consumption in a delay-tolerant network in which a message (or a file) has to be delivered to each of several destinations by epidemic relaying. In addition to the destinations, there are several other nodes in the network that can assist in relaying the message. We first assume that, at every instant, all the nodes know the number of relays carrying the message and the number of destinations that have received the message. We formulate the problem as a controlled continuous-time Markov chain and derive the optimal closed-loop control (i.e., forwarding policy). However, in practice, the intermittent connectivity in the network implies that the nodes may not have the required perfect knowledge of the system state. To address this issue, we obtain an ordinary differential equation (ODE) (i.e., a deterministic fluid) approximation for the optimally controlled Markov chain. This fluid approximation also yields an asymptotically optimal open-loop policy. Finally, we evaluate the performance of the deterministic policy over finite networks. Numerical results show that this policy performs close to the optimal closed-loop policy.**

***Index Terms*—Delay-tolerant networks (DTNs), epidemic relaying, fluid approximation, optimal control.**

I. Introduction

DELAY-TOLERANT networks (DTNs) [1] are sparse wireless ad hoc networks with highly mobile nodes. In these networks, the link between any two nodes is up when these are within each other's transmission range, and is down otherwise. In particular, at any given time, it is unlikely that there is a complete route between a source and its destination.

We consider a DTN in which a short message (also referred to as a *packet*) needs to be delivered to multiple (say M) destinations. There are also N potential relays that do not themselves "want" the message but can assist in relaying it to the nodes that do. At time $t = 0$, N_0 of the relays have copies of the packet. All nodes are assumed to be mobile. In such a network, a common technique to improve packet delivery delay is *epidemic* relaying [2]. We consider a controlled relaying scheme that works as follows. Whenever a node (relay or destination) carrying the packet meets a relay that does not have a copy of the packet, then the former has the option of either copying or not copying. When a node that has the packet meets a destination that does not, the packet can be delivered.

We want to minimize the delay until a significant fraction (say α) of the destinations receive the packet; we refer to this duration as *delivery delay*. Evidently, delivery delay can be reduced if the number of carriers of the packet is increased by copying it to relays. Such copying cannot be done indiscriminately because every act of copying to a relay incurs a transmission cost. Thus, we focus on the problem of the control of packet forwarding.

A. DTNs With Multiple Destinations

DTNs are commonly envisaged for certain applications involving personal mobile devices; such applications have two characteristics: 1) They can work with infrastructure-free direct communication between the devices, and 2) can tolerate moderate delays. Such applications typically involve spreading (or delivering) a content to a set of nodes. The following are three examples of such applications that involve the spread of the same message to multiple destinations, i.e., intended recipients. A widely studied problem is the spread of influence in social networks, e.g., spread of a popular video clip, or diffusion of an alert about a free medical "camp" in a developing country, or an advertisement of a new product. Venkatramanan and Kumar [3] study the joint evolution of content popularity and delivery in mobile peer-to-peer (P2P) networks. The creator of the content aims to maximize its popularity at one level of the model, while at another level the parameters of the spreading process govern the dissemination of the content to those who desire it. Karnik and Dayama [4] study efficient campaigning to yield a desired level of "buzz" at a specified time and also the diffusion of system-wide traffic updates or security alerts through wireless vehicular networks. In another work, Khouzani *et al.* [5] study the dissemination of security patches in mobile wireless networks and seek optimal tradeoffs between security risks and resource consumption.

While each instance has its own resource constraints and performance objectives, we consider one such multiple-destination problem. The introduction of multiple sources and destinations also facilitates study of scaled network dynamics that yields an asymptotically optimal open-loop policy.

B. Related Work

Analysis and control of DTNs with a single source and a single destination has been widely studied. Groenevelt *et al.* [6] modeled epidemic relaying and two-hop relaying using Markov

Manuscript received October 28, 2011; revised June 02, 2012; November 16, 2012; and November 28, 2012; accepted December 01, 2012; approved by IEEE/ACM Transactions on Networking Editor D. Starobinski. This work was supported by the Indo-French Centre for the Promotion of Advanced Research (IFCPAR) under Project 4000-ITA, GANESH (an INRIA Associates program), and the Department of Science and Technology, Government of India, through a J. C. Bose Fellowship. This is an extended version of a paper that appeared in the International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), Princeton, NJ, May 9–13, 2011.

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Digital Object Identifier 10.1109/TNET.2012.2233494

chains. They derived the average delay and the number of copies generated until the time of delivery. Zhang *et al.* [7] developed a unified framework based on ordinary differential equations (ODEs) to study epidemic routing and its variants.

Neglia and Zhang [8] were the first to study the optimal control of relaying in DTNs with a single destination.They assumed that all the nodes have perfect knowledge of the number of nodes carrying the packet. Their optimal closed-loop control is a threshold policy—when a relay that does not have the packet is met, the packet is copied if and only if the number of relays carrying the packet is below a threshold. Due to the assumption of complete knowledge, the reported performance is a lower bound for the cost in a real system.

Spyropoulos *et al.* [9] introduced the *Spray-and-Wait* and *Spray-and-Focus* routing algorithms for intermittently connected mobile networks. Their algorithms disseminate the message to a predetermined number of relays, and then rely on direct delivery from any of these nodes to the destination. In this paper, we propose a finer control of spraying that depends on the instantaneous network state. Consequently, our proposed policy is expected to outperform those in [9].

Altman *et al.* [10] addressed the optimal relaying problem for a class of *monotone relay strategies* that includes epidemic relaying and two-hop relaying. In particular, they derived *static* and *dynamic* relaying policies. Altman *et al.* [11] considered optimal discrete-time two-hop relaying. They also employed stochastic approximation to facilitate online estimation of network parameters. In another paper, Altman *et al.* [12] considered a scenario where active nodes in the network continuously spend energy while *beaconing*. Their paper studied the joint problem of node activation and transmission power control. These works [10]–[12] heuristically obtain fluid approximations for DTNs and study open-loop controls. Li *et al.* [13] considered several families of open-loop controls and obtained optimal controls within each family.

Deterministic fluid models expressed as ordinary differential equations have been used to approximate large Markovian systems. Kurtz [14] obtained sufficient conditions for the convergence of Markov chains to such fluid limits. Darling [15], and subsequently Darling and Norris [16], generalized Kurtz's results. Darling [15] considered the scenario when the Markovian system satisfies the conditions in [14] only over a subset. He showed that the scaled processes converge to a fluid limit until they exit from this subset. Darling and Norris [16] generalized the conditions for convergence, e.g., uniform convergence of the mean drifts of Markov chains and Lipschitz continuity of the limiting drift function, prescribed in [14]. Gast and Gaujal [17] addressed the scenario where the limiting drift functions are not Lipschitz continuous. They proved that under mild conditions, the stochastic system converges to the solution of a differential inclusion. Gast *et al.* [18] studied an optimization problem on a large Markovian system. They showed that solving the limiting deterministic problem yields an asymptotically optimal policy for the original problem.

C. Our Contributions

We formulate the problem as a controlled continuous-time Markov chain (CTMC) [19] and obtain the optimal policy (Section III). The optimal policy relies on complete knowledge of the network state at every node, but availability of such information is constrained by the same connectivity problem that limits packet delivery. In the incomplete information setting, the decisions of the nodes would have to depend upon their beliefs about the network state. The nodes would need to update their beliefs continuously with time, and also after each meeting with another node. Such belief updates would involve maintaining a complex information structure and are often impractical for nodes with limited memory and computation capability. Moreover, designing closed-loop controls based on beliefs is a difficult task [20], even more so in our context with multiple decision-makers and all of them equipped with distinct partial information.

In view of the above difficulties, we adopt the following approach. We show that when the number of nodes is large, the optimally controlled network evolution is well approximated by a deterministic dynamical system (Section IV). The existing differential equation approximation results for Markovian systems [14], [15] do not directly apply as, in the optimally controlled Markov chain that arises in our problem, the mean drift rates are discontinuous and do not converge uniformly. We extend the results to our problem setting in our Theorem 4.1 in Section IV. Note that the differential inclusion-based approach of Gast and Gaujal [17] is not directly applicable in our case, as it needs uniform convergence of the mean drift rates. The limiting deterministic dynamics then suggests a deterministic control that is asymptotically optimal for the finite network problem, i.e., the cost incurred by the deterministic control approaches the optimal cost as the network size grows. We briefly consider the analogous control of two-hop forwarding [21] in Section V. Our numerical results illustrate that the deterministic policy performs close to the complete information-optimal closed-loop policy for a wide range of parameter values (Section VI).

In a nutshell, the ODE approach is quite common in the modeling of such problems. Its validity in situations without control is established by Kurtz [14], Darling and Norris [16], etc. We aim in this paper at rigorously showing the validity of this limit under control in a few DTN problems.

II. System Model

We consider a set of $K := M + N$ mobile nodes. These include M destinations and N relays. At $t = 0$, a packet is generated and immediately copied to N_0 relays (e.g., via a broadcast from an infrastructure network). Alternatively, these N_0 nodes can be thought of as source nodes.

1) Mobility Model: We model the point process of the *meeting instants* between pairs of nodes as independent Poisson point processes, each with rate λ. Groenevelt *et al.* [6] validate this model for a number of common mobility models (random walker, random direction, random waypoint). In particular, they establish its accuracy under the assumptions of small communication range and sufficiently high speed of nodes.

Remarks 2.1: A few studies suggest that traces collected from real-life mobility often demonstrate intercontact times with power-law distributions. However, Karagiannis *et al.* [22] have established that the intercontact times exhibit exponential tails beyond a certain characteristic time. They also validate this finding across a diverse set of mobility traces. The exponential decay beyond the characteristic time is of relevance as available data traces suggest that the mean intercontact time is in many cases of the same order as this characteristic time.

2) Communication Model: Two nodes may communicate only when they come within transmission range of each other, i.e., at *meeting instants*. The transmissions are assumed to be instantaneous. We assume that each transmission of the packet incurs unit energy expenditure at the transmitter.

3) Relaying Model: We assume that a controlled epidemic relay protocol is employed.

Throughout, we use the terminology relating to the spread of infectious diseases. A node with a copy of the packet is said to be *infected*. A node is said to be *susceptible* until it receives a copy of the packet from another infected node. Thus, at $t = 0$, N_0 nodes are infected, while $M + N - N_0$ are susceptible.

A. Forwarding Problem

Our goal is to disseminate the packet to all the M destinations while minimizing the duration until a fraction α ($\alpha < 1$) of the destinations receive the packet.[1]

At each meeting epoch with a susceptible relay, an infected node (relay or destination) has to decide whether to copy the packet to the susceptible relay or not. Copying the packet to a relay incurs a cost, but promotes early delivery of the packet to the destinations. We wish to find the tradeoff between these costs by minimizing

$$\mathbb{E}\{\mathcal{T}_{\rm d} + \gamma \mathcal{E}_{\rm c}\} \tag{1}$$

where $\mathcal{T}_{\rm d}$ is the time until which at least $M_\alpha := \lceil \alpha M \rceil$ destinations receive the packet, $\mathcal{E}_{\rm c}$ is the total number of copies made to relays, and γ is the parameter that relates the number of copies to delay cost. Varying γ helps studying the tradeoff between the delay and the copying costs.

Remarks 2.2: Alternatively, we may interpret γ as a parameter that accounts for the cost of making a copy and also relates this cost to the delivery delay. The cost could include the energy cost of transmission and reception, the cost of storage, the price charged by a receiver for carrying a message, etc. Then, $\gamma\mathcal{E}_{\rm c}$ represents the scaled (for comparison with delay) total cost of copying to relays. In some later discussions, we take this alternative viewpoint.

Remarks 2.3: Copying the packet to the destinations also incurs a cost. However, this cost is fixed irrespective of the forwarding policy and, thus, is not included in our objective function.

III. Optimal Epidemic Forwarding

We derive the optimal forwarding policy under the assumption that, at any instant of time, all the nodes have full information about the number of relays carrying the packet and the number of destinations that have received the packet. This assumption will be relaxed in Section IV.

A. Markov Decision Process (MDP) Formulation

Let $t_0 := 0$ and $t_k, k = 1, 2, \ldots$ denote the meeting epochs of the infected nodes (relays or destinations) with the susceptible nodes. Define $\delta_k := t_k - t_{k-1}$ for $k \geq 1$. Let $m(t)$ and $n(t)$ be the numbers of infected destinations and relays, respectively, at time t. Thus, $m(0) = 0$ and $n(0) = N_0$, and the forwarding process stops at time t if $m(t) = M$. We use m_k and n_k to mean $m(t_k-)$ and $n(t_k-)$, which are the numbers of infected destinations and relays, respectively, just before the meeting epoch t_k. Let e_k be the type of the susceptible node that an infected node meets at t_k; $e_k \in \mathcal{E} := \{d, r\}$ where d and r stand for destination and relay, respectively. The state of the system at a meeting epoch t_k is given by the tuple

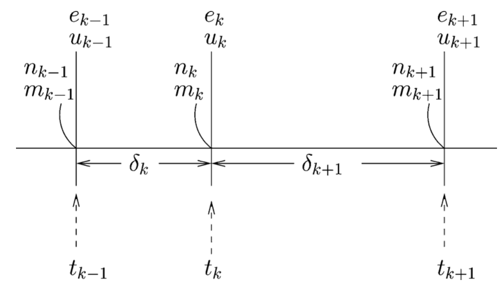

Fig. 1. Evolution of the controlled Markov chain $\{s_k\}$. Note that (m_k, n_k) is embedded at t_k-, i.e., just before the meeting epoch.

$$s_k := (m_k, n_k, e_k).$$

Since the forwarding process stops at time t if $m(t) = M$, the state space is $[M-1] \times [N_0 : N] \times \mathcal{E}$.[2]

Let u_k be the action of the infected node at meeting epoch $t_k, k = 1, 2, \ldots$. The control space is $\mathcal{U} = \{0, 1\}$, where 1 is for *copy* and 0 is for *do not copy*. The embedding convention described above is shown in Fig. 1.

We treat the tuple (δ_{k+1}, e_{k+1}) as the random disturbance at epoch t_k. Note that for $k = 1, 2, \ldots$, the time between successive decision epochs, δ_k, is independent and exponentially distributed with parameter $(m_k + n_k)(M + N - m_k - n_k)\lambda$. Furthermore, with "w.p." standing for "with probability"

$$e_k = \begin{cases} d, & \text{w.p. } p_{m_k,n_k}(d) := \frac{M-m_k}{M+N-m_k-n_k} \\ r, & \text{w.p. } p_{m_k,n_k}(r) := \frac{N-n_k}{M+N-m_k-n_k}. \end{cases}$$

1) Transition Structure: From the description of the system model, the state at time $k+1$ is $s_{k+1} = (m_k + u_k, n_k, e_{k+1})$ if $e_k = d$, and $s_{k+1} = (m_k, n_k + u_k, e_{k+1})$ if $e_k = r$. Recall that e_{k+1} is a component in the random disturbance. Thus, the next state is a function of the current state, the current action, and the current disturbance as required for an MDP.

2) Cost Structure: For a state-action pair (s_k, u_k), the expected single-stage cost is given by

$$g(s_k, u_k) = \mathbb{E}\left\{\delta_{k+1} 1_{\{m_{k+1} < M_\alpha\}}\right\} + \gamma u_k 1_{\{e_k = r\}}$$

where the expectation is taken with respect to the random disturbance (δ_{k+1}, e_{k+1}). It can be observed that

$$g(s_k, u_k) = \begin{cases} \gamma u_k 1_{\{e_k=r\}}, & \text{if } m_k \geq M_\alpha \\ 0, & \text{if } m_k = M_\alpha - 1, e_k = d, \\ & \quad \text{and } u_k = 1 \\ C_d(s_k, u_k) + \gamma u_k 1_{\{e_k=r\}}, & \text{otherwise} \end{cases}$$

[1]Subsequently, we analyze a scaled version of the network in order to obtain a distributed policy. See Footnote 9 for why we restrict to $\alpha < 1$.

[2]We use notation $[a] = \{0, 1, \ldots, a\}$ and $[a : b] = \{a, a+1, \ldots, b\}$ for $b \geq a + 1$ and $a, b \in \mathbb{Z}_+$.

where

$$C_d(s_k,u_k) = \frac{1}{(m_k+n_k+u_k)(M+N-m_k-n_k-u_k)\lambda}$$

is the mean time until the next decision epoch. The quantity γ is expended whenever $u_k=1$, i.e., the action is to copy.

3) Policies: A policy π is a sequence of mappings $\{u_k^\pi, k=0,1,2,\ldots\}$, where $u_k^\pi : [M-1]\times[N_0:N]\times\mathcal{E}\rightarrow\mathcal{U}$. The cost of a policy π for an initial state $s=(m,n,e)$ is

$$J_\pi(s) = \sum_{k=0}^{\infty} \mathbb{E}\Big\{ g(s_k, u_k^\pi(s_k)) \big| s_0 = s \Big\}.$$

Let Π be the set of all policies. Then, the optimal cost function is defined as

$$J(s) = \min_{\pi\in\Pi} J_\pi(s).$$

A policy π is called stationary if u_k^π are identical, say u, for all k. For brevity, we refer to such a policy as the stationary policy u. A stationary policy $u^* \equiv \{u^*, u^*, \ldots\}$ is optimal if $J_{u^*}(s) = J(s)$ for all states s.

4) Total Cost: We now translate the optimal cost-to-go from the first decision instant t_1 into the optimal total cost. Recall that at t_1, the state s_1 is $(0,N_0,r)$ or $(0,N_0,d)$ depending on whether the susceptible node that is met is a relay or a destination. The objective function (1) can then be restated as

$$\begin{aligned}\mathbb{E}_\pi\{\mathcal{T}_d + \gamma\mathcal{E}_c\} = &\frac{1}{\lambda N_0(M+N-N_0)} \\ &+ \left(\frac{N-N_0}{M+N-N_0} J_\pi(0,N_0,r)\right. \\ &\left. + \frac{M}{M+N-N_0} J_\pi(0,N_0,d)\right) \quad (2)\end{aligned}$$

where the subscript π shows dependence on the underlying policy. In the right-hand side, the first term $\frac{1}{\lambda N_0(M+N-N_0)}$ is the average delay until the first decision instant that has to be borne under any policy.

B. Optimal Policy

Since the cost function $g(\cdot)$ is nonnegative, in [19, Ch. 3, Proposition 1.1] implies that the optimal cost function will satisfy the following Bellman equation. For $s=(m,n,e)$

$$J(s) = \min_{u\in\{0,1\}} A(s,u)$$

where

$$A(s,u) = g(s,u) + \mathbb{E}\left(J(s')|s,u\right).$$

Here, s' denotes the next state that depends on s, u and the random disturbance in accordance with the transition structure described above. The expectation is taken with respect to the random disturbance. Since the action space is finite, there exists a stationary policy u^* such that, for all s, $u^*(s)$ attains minimum in the above Bellman equation (see [19, Ch. 3]). Now we characterize this stationary optimal policy.

First, observe that it is optimal to copy whenever a susceptible destination is encountered; it does not incur any cost and increases the number of infected nodes, which in turn also expedites the packet delivery process. Moreover, every destination has to be infected at some time. Thus, $u^*(m,n,d)=1$ for all $(m,n)\in[M-1]\times[N_0:N]$. Next, once M_α destinations have been infected, no further delay cost is incurred, and so further copying to relays does not help. Thus, $u^*(m,n,r)=0$ for all $(m,n)\in[M_\alpha : M-1]\times[N_0:N]$.

Focus now on a reduced state space $[M_\alpha - 1]\times[N_0 : N]\times\{r\}$. Consider the following *one-step lookahead policy* [19, Section 3.4]. At a meeting with a susceptible relay, when the state is (m,n,r), compare the following two action sequences:

1) $0s$: *stop*, i.e., do not copy to this relay or to any susceptible relays met in the future;
2) $1s$: copy to this relay and then *stop*.

The costs to go corresponding to the action sequences $0s$ and $1s$ are respectively

$$J_{0s}(m,n,r) = \sum_{j=m}^{M_\alpha-1} \frac{1}{\lambda(n+j)(M-j)}$$

and

$$J_{1s}(m,n,r) = \gamma + \sum_{j=m}^{M_\alpha-1} \frac{1}{\lambda(n+j+1)(M-j)}.$$

The *stopping set* $\mathcal{S}_\mathcal{S}$ is defined to be

$$\mathcal{S}_\mathcal{S} := \{(m,n,r) : \Phi(m,n)\le 0\} \quad (3)$$

where

$$\begin{aligned}\Phi(m,n) &:= J_{0s}(m,n,r) - J_{1s}(m,n,r) \\ &= \sum_{j=m}^{M_\alpha-1}\frac{1}{\lambda(n+j)(n+j+1)(M-j)} - \gamma \quad (4)\end{aligned}$$

for all $(m,n)\in[M_\alpha-1]\times[N_0:N]$. The one-step lookahead policy is to copy to relay when $(m,n,r)\notin\mathcal{S}_\mathcal{S}$, and to stop copying otherwise.[3]

One-step lookahead policies are shown to be optimal for stopping problems under certain conditions (see [19, Section 3.4] and [23, Section 4.4]). However, our problem is not a stopping problem as an action 0 is not equivalent to *stop*; even if the susceptible relay met now is not copied, the resulting state is not a *terminal state*, and one met in the future may be copied. However, we exploit the cost structure to prove that when an infected node meets a susceptible relay, it can restrict attention to two actions: 1 (i.e., copy now) and *stop* (i.e., do not copy now and never copy again). Subsequently, we also show that the above one-step lookahead policy is optimal.

Theorem 3.1: The optimal policy $u^* : [M-1]\times[N_0 : N]\times\mathcal{E}\rightarrow\mathcal{U}$ satisfies

$$u^*(m,n,e) = \begin{cases} 1, & \text{if } e=d \\ 1, & \text{if } e=r \text{ and } \Phi(m,n)>0 \\ \text{stop} & \text{if } e=r \text{ and } \Phi(m,n)\le 0.\end{cases}$$

[3]Convention: A sum over an empty set is 0. Thus, $\Phi(m,n)=-\gamma$ if $m\ge M_\alpha$. Consequently, for the states $[M_\alpha : M-1]\times[N_0:N]\times\{r\}$, one-step lookahead policy prescribes *stop*. This is consistent with our earlier discussion.

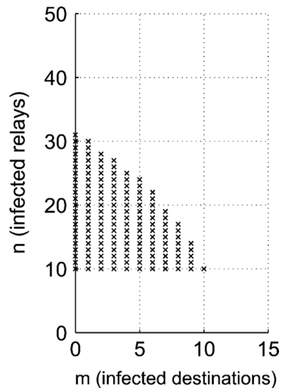

Fig. 2. Illustration of the optimal policy. The symbols "×" mark the states in which the optimal action (at meeting with a relay) is to copy.

Proof: See Appendix A. ■

Remarks 3.1: We can define $\Phi(m,n)$ also for the case $\alpha = 1$, i.e., when we attempt to minimize the delay until all the destinations receive the packet. Theorem 3.1 continues to hold.

We illustrate the optimal policy using an example. Let $M = 15, N = 50, N_0 = 10, \alpha = 0.8, \lambda = 0.001$ and $\gamma = 1$. The "×" symbols in Fig. 2 are the states where the optimal action (at meeting with a relay) is to copy. For example, if only five destinations have the packet, then relays are copied to if and only if there are 24 or less infected relays. If seven destinations already have the packet and there are 19 infected relays, then no further copying to relays is done.

IV. Asymptotically Optimal Epidemic Forwarding

In states $[M_\alpha - 1] \times [N_0 : N] \times \{r\}$, the optimal action, which is governed by the function $\Phi(m,n)$, requires perfect knowledge of the network state (m,n). This may not be available to the decision-maker due to intermittent connectivity. In this section, we derive an asymptotically optimal policy that does not require knowledge of network's state, but depends only on the time elapsed since the generation of the packet. Such a policy is implementable if the packet is time-stamped when generated and the nodes' clocks are synchronized.

A. Asymptotic Deterministic Dynamics

Our analysis closely follows Darling [15]. It is straightforward to show that the equations that follow are the conditional expected drift rates of the optimally controlled CTMC. For $(m(t), n(t)) \in [M-1] \times [N_0 : N]$, using the optimal policy in Theorem 3.1, we get

$$\frac{\mathrm{d}\mathbb{E}(m(t)|(m(t),n(t)))}{\mathrm{d}t} = \lambda(m(t)+n(t))(M-m(t)) \quad (5a)$$

$$\frac{\mathrm{d}\mathbb{E}(n(t)|(m(t),n(t)))}{\mathrm{d}t} = \lambda(m(t)+n(t))(N-n(t)) \times 1_{\{\Phi(m(t),n(t))>0\}}. \quad (5b)$$

Recalling that $K = M + N$, the total number of nodes, we study large K asymptotics. More precisely, we consider a sequence of problems with increasing M, N, N_0 (and thus also $K := M + N$) such that the ratios $\frac{M}{K}$, $\frac{N}{K}$, and $\frac{N_0}{K}$ remain constant. The problems are indexed by K. The parameters of the Kth problem are denoted using the superscript K. Normalized versions of these parameters and normalized versions of the system state are denoted as follows:

$$\left.\begin{array}{r} X = \frac{M^K}{K}, \quad Y = \frac{N^K}{K} \\ X_\alpha = \frac{\alpha M^K}{K}, \quad Y_0 = \frac{N_0^K}{K} \\ \lambda^K = \frac{\Lambda}{K}, \quad \gamma^K = \frac{\Gamma}{K} \\ x^K(t) = \frac{m^K(t)}{K} \quad \text{and} \quad y^K(t) = \frac{n^K(t)}{K} \end{array}\right\}. \quad (6)$$

Remarks 4.1: The pairwise meeting rate and the copying cost must both scale down as K increases. Otherwise, the delivery delay will be negligible and the total transmission cost will be enormous for any policy, and no meaningful analysis is possible.

For each K, we define a scaled two-dimensional integer lattice

$$\Delta^K = \left\{\left(\frac{i}{K}, \frac{j}{K}\right) : (i,j) \in [M^K - 1] \times [N_0^K : N^K]\right\}.$$

Clearly, $(x^K(t), y^K(t)) \in \Delta^K$. Now, using the notation in (6), the drift rates in (5a) and (5b) can be rewritten as follows:

$$\frac{\mathrm{d}\mathbb{E}(x^K(t)|(x^K(t),y^K(t)))}{\mathrm{d}t} = f_1^K(x^K(t),y^K(t)) := \Lambda(x^K(t)+y^K(t))(X-x^K(t)) \quad (7a)$$

$$\frac{\mathrm{d}\mathbb{E}(y^K(t)|(x^K(t),y^K(t)))}{\mathrm{d}t} = f_2^K(x^K(t),y^K(t)) := \Lambda(x^K(t)+y^K(t))(Y-y^K(t))1_{\{\phi^K(x^K(t),y^K(t))>0\}} \quad (7b)$$

where, for $(x,y) \in \Delta^K$

$$\phi^K(x,y) := \sum_{j=Kx}^{\lceil KX_\alpha \rceil - 1} \frac{1}{K\Lambda(y+\frac{j}{K})(y+\frac{j+1}{K})(X-\frac{j}{K})} - \Gamma. \quad (8)$$

We also define $(x(t), y(t)) \in [0, X] \times [Y_0, Y]$ as functions satisfying the following ODEs: $x(0) = 0, y(0) = Y_0$, and for $t \geq 0$

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = f_1(x(t),y(t)) := \Lambda(x(t)+y(t))(X-x(t)) \quad (9a)$$

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = f_2(x(t),y(t)) := \Lambda(x(t)+y(t))(Y-y(t)) \times 1_{\{\phi(x(t),y(t))>0\}} \quad (9b)$$

where[4]

$$\phi(x,y) = \int_{z=x}^{X_\alpha} \frac{\mathrm{d}z}{\Lambda(y+z)^2(X-z)} - \Gamma. \quad (10)$$

Finally, we redefine the delivery delay $\mathcal{T}_d$ [see (1)] to be

$$\tau^K = \inf\{t \geq 0 : x^K(t) \geq X_\alpha\} \quad (11)$$

and

$$\tau = \inf\{t \geq 0 : x(t) \geq X_\alpha\}. \quad (12)$$

Note that τ^K is a stopping time for the random process $(x^K(t), y^K(t))$, whereas τ is a deterministic time instant. Since $f_1^K(x,y)$ is bounded away from zero, $\tau^K < \infty$ with

[4]Convention: An integral assumes the value 0 if its lower limit exceeds the upper limit. Therefore, $\phi(x,y) = -\Gamma$ if $x \geq X_\alpha$.

probability 1. Similarly, on account of $f_1(x, y)$ being bounded away from zero, $\tau < \infty$.

Kurtz [14] and Darling [15] studied convergence of CTMCs to the solutions of ODEs. The following are the hypotheses for the version of the limit theorem that appears in Darling [15].

1) $\lim_{K \to \infty} \mathbb{P}\left(\|(x^K(0), y^K(0) - (x(0), y(0))\| > \epsilon\right) = 0$.
2) In the scaled process $(x^K(t), y^K(t)$, the jump rates are $O(K)$ and drifts are $O(K^{-1})$.
3) $(f_1^K(x, y), f_2^K(x, y))$ converges to $(f_1(x, y), f_2(x, y))$ uniformly in (x, y).
4) $(f_1(x, y), f_2(x, y))$ is Lipschitz continuous.

Observe that, in our case, only the first two hypotheses are satisfied. In particular, $f_2^K(x, y)$ does not converge uniformly to $f_2(x, y)$, and $f_2(x, y)$ is not Lipschitz over $[0, X_\alpha] \times [Y_0, Y]$. Hence, the convergence results do not directly apply in our context. Thankfully, there is some regularity we can exploit that we now summarize as easily verifiable facts.

a) $\phi^K(x, y)$ converges uniformly to $\phi(x, y)$.
b) The drift rates $f_1(x, y)$ and $f_2(x, y)$ are bounded from below and above.
c) $f_1(x, y)$ is Lipschitz, and $f_2(x, y)$ is locally Lipschitz.
d) For all small enough $\nu \in \mathbb{R}$, and all (x, y) on the graph of "$\phi(x, y) = \nu$," the direction in which the ODE progresses, $(f_1(x, y), f_2(x, y))$, is not tangent to the graph.

We then prove the following result, which is identical to [15, Theorem 2.8].

Theorem 4.1: Assume that $\alpha < 1$ and $Y_0 > 0$. Then, for every $\epsilon, \delta > 0$, for the optimally controlled epidemic forwarding process

$$\lim_{K \to \infty} \mathbb{P}\left(\sup_{0 \le t \le \tau} \|(x^K(t), y^K(t)) - (x(t), y(t))\| > \epsilon\right) = 0$$
$$\lim_{K \to \infty} \mathbb{P}\left(|\tau^K - \tau| > \delta\right) = 0.$$

Proof: See Appendix B. ■

We illustrate Theorem 4.1 using an example. Let $X = 0.2, Y = 0.8, \alpha = 0.8, Y_0 = 0.2, \Lambda = 0.05$, and $\Gamma = 50$. In Fig. 3, we plot $(x(t), y(t))$ and sample trajectories of $(x^K(t), y^K(t))$ for $K = 100, 200$, and 500. We indicate the states at which the optimal policy stops copying to relays, i.e., $\Phi^K(x^K(t), y^K(t))$ goes below 0 (see Theorem 3.1), and the states at which the fraction of infected destinations crosses X_α. We also show the corresponding states in the fluid model. The plots show that for large K, the fluid model captures the random dynamics of the network very well.

B. Asymptotically Optimal Policy

Observe that $\phi(x, y)$ is decreasing in x and y, both of which are nondecreasing with t. Consequently $\phi(x(t), y(t))$ decreases with t. We define

$$\tau^* := \inf\{t \ge 0 : \phi(x(t), y(t)) \le 0\}. \quad (13)$$

The limiting deterministic dynamics suggests the following policy u^∞ for the original forwarding problem.[5]

$$u^\infty(m, n, e) = \begin{cases} 1, & \text{if } e = d \\ 1, & \text{if } e = r \text{ and } t \le \tau^* \\ 0, & \text{if } e = r \text{ and } t > \tau^*. \end{cases}$$

[5]Observe that the policy u^∞ does not require knowledge of m and n. The infected node readily knows the type of the susceptible node (d or r) at the decision epoch.

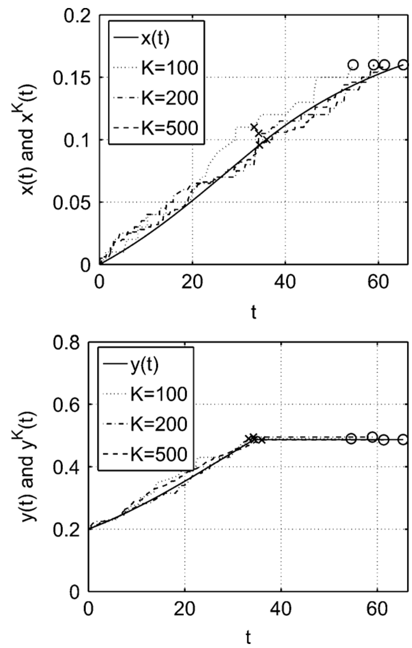

Fig. 3. Simulation results: fractions of infected *(top)* destinations and *(bottom)* relays as a function of time. $(x^K(t), y^K(t))$ are obtained from a simulation of the controlled CTMC, and $(x(t), y(t))$ from the ODEs. The marker "×" indicates the states at which copying to relays is stopped, whereas "o" indicates the states at which a fraction α of destinations have the packet.

We show that the policy u^∞ is asymptotically optimal in the sense that its expected cost approaches the expected cost of the optimal policy u^* as the network grows. Let us restate (2) as

$$\mathbb{E}_\pi^K\{\mathcal{T}_d + \gamma\mathcal{E}_c\} = \frac{1}{K\Lambda Y_0(1 - Y_0)} + \left(\frac{Y - Y_0}{1 - Y_0} J_\pi^K(0, Y_0, r) + \frac{X}{1 - Y_0} J_\pi^K(0, Y_0, d)\right).$$

We have used superscript K to show the dependence of cost on the network size. We then establish the following asymptotic optimality result.

Theorem 4.2: Assume that $\alpha < 1$ and $Y_0 > 0$. Then

$$\lim_{K \to \infty} \mathbb{E}_{u^*}^K\{\mathcal{T}_d + \gamma\mathcal{E}_c\} = \lim_{K \to \infty} \mathbb{E}_{u^\infty}^K\{\mathcal{T}_d + \gamma\mathcal{E}_c\} = \tau + \Gamma y(\tau^*).$$

Proof: See Appendix C. ■

Remarks 4.2: Observe that we do not compare the limiting value of the optimal cost with the optimal cost on the (limiting) deterministic system. In general, these two may differ.[6] However, the deterministic policy u^∞ can be applied on the finite K-node system. The above theorem asserts that given any $\epsilon > 0$, cost of the policy u^∞ is within ϵ of the optimal cost *on the K-node system* for all sufficiently large K.

Distributed Implementation: The asymptotically optimal policy can be implemented in a distributed fashion. We assume that the system parameters $M, N, \alpha, N_0, \lambda$, and γ are known at

[6]In our case, these two indeed match. See [24, Appendix D] for a proof.

the source, and also that all the nodes are time-synchronized.[7] Suppose that the packet is generated at the source at time t_0 (we assumed $t_0 = 0$ for the purpose of analysis). Given the system parameters, the source first extracts $X, Y, X_\alpha, Y_0, \Lambda$, and Γ as in (6). Then, it calculates τ^* [see (13)] and stores $t_0 + \tau^*$ as a header in the packet.

The packet is immediately copied to N_0 relays, perhaps by means of a broadcast from an infrastructure "base station." When an infected node meets a susceptible relay, it compares $t_0 + \tau^*$ with the current time. The susceptible relay is not copied to if the current time exceeds $t_0 + \tau^*$; the nodes do not need to know the transient numbers of infected relays and infected destinations. However, all the infected nodes continue to carry the packet and to copy to susceptible destinations as and when they meet.

Remarks 4.3: Consider a scenario, where the interest is in copying packet to only a fraction α of the destinations. Observe from Theorem 4.1 that for every $\epsilon > 0$

$$\lim_{K \longrightarrow \infty} \mathbb{P}\left(\left|\frac{m^K(\tau)}{M} - \alpha\right| > \epsilon\right) = 0.$$

Thus, in large networks, copying to destinations can also be stopped at time τ [see (12)] while ensuring that with large probability the fraction of infected destinations is close to α. Consequently, all the relays can delete the packet and free their memory at τ. This helps when packets are large and relay (cache) memory is limited.

V. Optimal Two-Hop Forwarding

Instead of epidemic relaying, one can consider two-hop relaying [21]. Here, the N_0 source nodes can copy the packet to any of the $N - N_0$ relays or M destinations. The infected destinations can also copy the packet to any of the susceptible relays or destinations. However, the relays are allowed to transmit the packet only to the destinations. Here also, a similar optimization problem as in Section II-A arises.

Now, the decision epochs $t_k, k = 1, 2, \ldots$ are the meeting epochs of the infected nodes (sources, relays, or destinations) with the susceptible destinations and the meeting epochs of the sources or infected destinations with the susceptible relays. We can formulate an MDP with state

$$s_k := (m_k, n_k, e_k)$$

at instant t_k where m_k, n_k, and e_k are as defined in Section III-A. The state space is $[M_\alpha - 1] \times [N_0 : N] \times \mathcal{E}$. The control space is $\mathcal{U} = \{0, 1\}$, where 1 is for *copy* and 0 is for *do not copy*. We also get a transition structure identical to that in Section III-A.

For a state action pair (s_k, u_k), the expected single-stage cost is given by

$$\begin{aligned} g(s_k, u_k) &= \mathbb{E}\left\{\delta_{k+1} 1_{\{m_{k+1} < M_\alpha\}}\right\} + \gamma u_k 1_{\{e_k = r\}} \\ &= \begin{cases} \gamma u_k 1_{\{e_k=r\}}, & \text{if } m_k \geq M_\alpha \\ 0, & \text{if } m_k = M_\alpha - 1, e_k = d, \\ & \quad \text{and } u_k = 1 \\ C_d(s_k, u_k) + \gamma u_k 1_{\{e_k=r\}}, & \text{otherwise} \end{cases} \end{aligned}$$

where

$$\begin{aligned} C_d(s_k, u_k) &= \Big((m_k + n_k + u_k)(M - m_k - u_k 1_{\{s_k=d\}})\lambda \\ &\quad + (m_k + u_k 1_{\{s_k=d\}} + N_0)(N - n_k - u_k 1_{\{s_k=r\}})\lambda\Big)^{-1} \end{aligned}$$

is the mean time until the next decision epoch. As before, the quantity γu_k accounts for the transmission energy.

Let $u^* : [M_\alpha - 1] \times [N_0 : N] \times \mathcal{E} \longrightarrow \mathcal{U}$ be a stationary optimal policy. As in Section III-B, the optimal policy satisfies $u^*(m, n, d) = 1$ for all $(m, n) \in [M - 1] \times [N_0 : N]$ and $u^*(m, n, r) = 0$ for all $(m, n) \in [M_\alpha : M - 1] \times [N_0 : N]$. Thus, we focus on a reduced state space $[M_\alpha - 1] \times [N_0 : N] \times \{r\}$. As before, we look for the one-step lookahead policy, which turns out to be the same as that for epidemic relaying. Finally, Theorem 3.1 holds for two-hop relaying as well (see the proof in Appendix A).

Next, we turn to the asymptotically optimal control for two-hop relaying. The following are the conditional expected drift rates. For $(m(t), n(t)) \in [M_\alpha - 1] \times [N_0 : N]$

$$\begin{aligned} \frac{\mathrm{d}\mathbb{E}(m(t)|(m(t), n(t)))}{\mathrm{d}t} &= \lambda(m(t) + n(t))(M - m(t)) \\ \frac{\mathrm{d}\mathbb{E}(n(t)|(m(t), n(t)))}{\mathrm{d}t} &= \lambda(m(t) + N_0)(N - n(t)) \\ &\quad \times 1_{\{\Phi(m(t), n(t)) > 0\}}. \end{aligned}$$

We employ the same scaling and notations as in (6). The drift rates in terms of $(x^K(t), y^K(t)) \in [0, X_\alpha] \times [Y_0, Y]$ are

$$\begin{aligned} &\frac{\mathrm{d}\mathbb{E}(x^K(t)|(x^K(t), y^K(t)))}{\mathrm{d}t} \\ &\quad = f_1^K(x^K(t), y^K(t)) \\ &\quad := \Lambda(x^K(t) + y^K(t))(X - x^K(t)) \\ &\frac{\mathrm{d}\mathbb{E}(y^K(t)|(x^K(t), y^K(t)))}{\mathrm{d}t} \\ &\quad = f_2^K(x^K(t), y^K(t)) \\ &\quad := \Lambda(x^K(t) + Y_0)(Y - y^K(t)) 1_{\{\phi^K(x^K(t), y^K(t)) > 0\}}. \end{aligned}$$

Now, $x(t), y(t)$ are defined as functions satisfying $x(0) = 0, y(0) = Y_0$ and for $t \geq 0$

$$\begin{aligned} \frac{\mathrm{d}x(t)}{\mathrm{d}t} &= f_1(x(t), y(t)) := \Lambda(x(t) + y(t))(X - x(t)) \\ \frac{\mathrm{d}y(t)}{\mathrm{d}t} &= f_2(x(t), y(t)) := \Lambda(x(t) + Y_0)(Y - y(t)) \\ &\quad \times 1_{\{\phi(x(t), y(t)) > 0\}}. \end{aligned}$$

The analysis in Section IV applies to two-hop relaying as well. In particular, Theorems 4.1 and 4.2 hold. However, for the identical system parameters (M, N, α, λ and γ) and initial state (N_0), the value of the time-threshold τ^* will be larger due to the slower rates of infection of relays and destinations.

[7] In practice, due to variations in the clock frequency, the clocks at different nodes will drift from each other. However, the time differences are negligible compared to the delays caused by intermittent connectivity in the network. Moreover, when an infected node meets a susceptible node, clock synchronization can be performed before the packet is copied. Distributed algorithms for time synchronization are also available (e.g., see [25]).

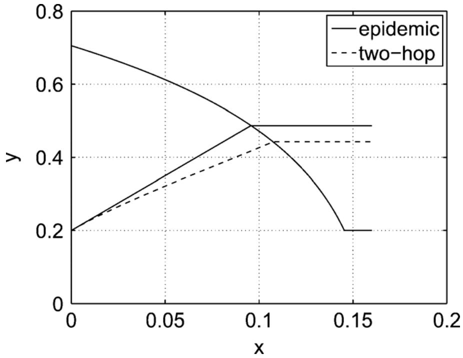

Fig. 4. Illustration of the epidemic and two-hop trajectories. The plots also show the graph of "$\phi(x, y) = 0$."

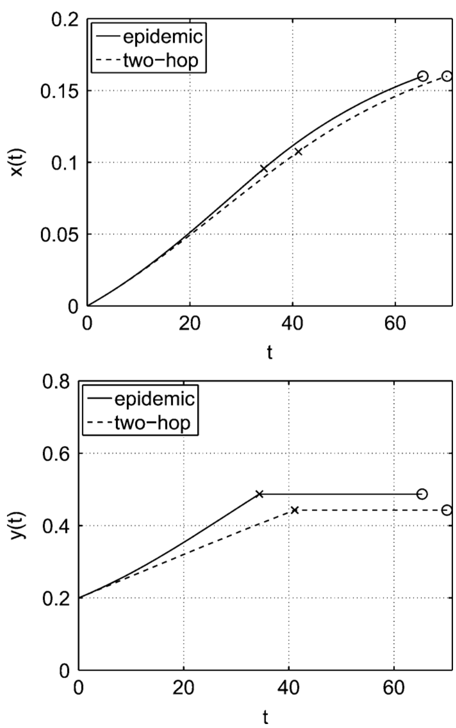

Fig. 5. Fractions of infected *(top)* destinations and *(bottom)* relays as a function of time. The marker "×" indicates the states at which copying to relays is stopped (i.e., the states at τ^*), and "o" indicates the states at which α fraction of destinations have been copied (i.e., the states at τ).

We illustrate the comparison between epidemic and two-hop relaying via an example. Let $X = 0.2, Y = 0.8, \alpha = 0.8, Y_0 = 0.2, \Lambda = 0.05$, and $\Gamma = 50$. In Fig. 4, we plot the graph of "$\phi(x, y) = 0$" and also the "y versus x" trajectories corresponding to epidemic and two-hop relayings. In Fig. 5, we plot the trajectories of $x(t)$ and $y(t)$ corresponding to epidemic and two-hop relayings. As anticipated, the value of the time-threshold τ^* is larger for two-hop relaying than epidemic relaying. Moreover, the number of transmissions is less, while the delivery delay is more under the controlled two-hop relaying.

VI. Simulation and Numerical Results

We start with simulations that validate the independent Poisson point process model for the meeting instants in the presence of control. We then show a few numerical results to demonstrate the good performance of the deterministic control and also to compare the performance of optimal epidemic and two-hop relayings.

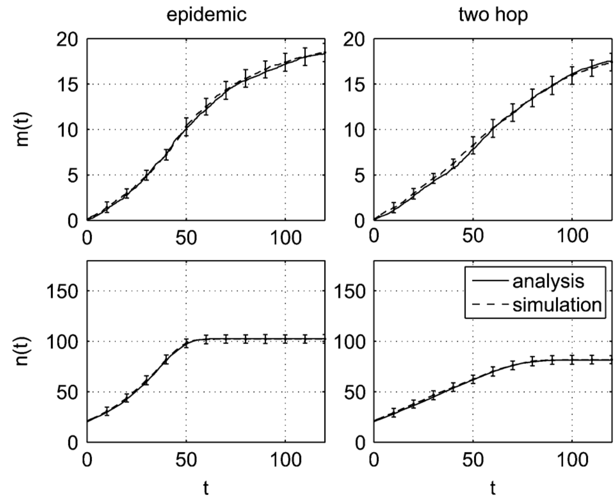

Fig. 6. Numbers of infected *(top)* destinations and *(bottom)* relays as a function of time. The simulation plots are based on traces generated according to random waypoint mobility, while the analysis plots are based on the Poisson point process model for the meeting instants. *(left)* Optimally controlled epidemic relaying. *(right)* Optimally controlled two-hop relaying. We also show 95% confidence intervals for our simulation.

For simulation, we use the mobility traces generated using OMNeT++ according to the *random waypoint* mobility model [6]. In our setup, $K = 200$ nodes move at a speed 10 m/s in a square of size 2×2 km^2 and have a communication radius 28.7115 m. These values are chosen to yield a pairwise meeting rate $\lambda = 0.00025/\text{s}$. We take $M = 20, N = 180, \alpha = 0.8$, and $N_0 = 20$ and assume quarter a unit cost per copy $(\gamma = 0.25)$. We perform 1000 runs of the simulation and average $m(t)$ and $n(t)$; averaging leads to fractional values of these otherwise integer variables. We choose destinations and initially infected relays uniformly at random from among all the nodes for each iteration. We consider both epidemic and two-hop relayings. As Fig. 6 depicts, in both the cases, the Poisson point process model for the meeting instants quite accurately predicts the dynamics of the fraction of infected destinations and relays under optimal control.

Now, we use the Poisson point process model for the meeting instants to illustrate the performance of optimal and deterministic open-loop controls. We set $M = 20, N = 80, \alpha = 0.8, N_0 = 10$, and $\lambda = 0.0005$ and vary γ from 0.01 to 10. In Fig. 7, we plot the total number of copies to relays, delivery delays, and total costs. The figures on the left are for epidemic relaying, while those on the right are for two-hop relaying. Each figure contains four plots: the optimal policy, the deterministic open-loop policy, the spray-and-wait policy [9], and uncontrolled forwarding policy (copying to all the relays).[8] The authors in [9] propose *spray-and-wait* and *spray-and-focus* routings for single destination networks and select the number of copies to meet certain expected delay target. We have adapted these for a multidestination network, and we choose the number of copies to minimize the weighted sum of delivery delay and copying cost as in (1). In the *spray* phase, we use *binary spraying* in the left-hand-side plots and *source spraying* in the right-hand-side

[8]In our symmetric Poisson meeting model, *spray-and-wait* and *spray-and-focus* routings are identical; no relays are copied in the *focus* phase.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

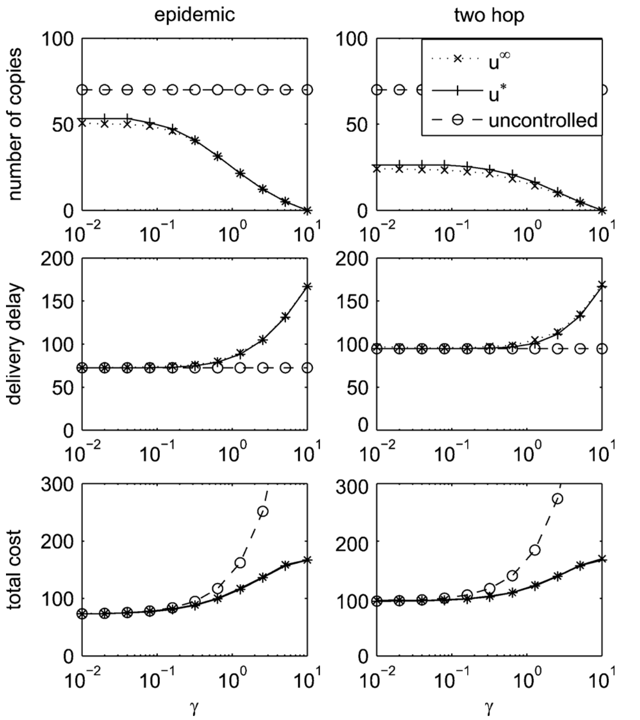

Fig. 7. *(top)* Expected total number of copies to relays, *(middle)* expected delivery delays, and *(bottom)* expected total costs corresponding to the optimal, the deterministic, and *spray-and-wait* policies, and also corresponding to uncontrolled forwarding. *(left)* Epidemic relaying. *(right)* Two-hop relaying.

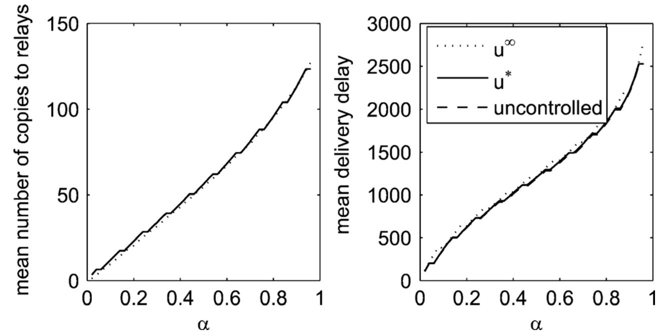

Fig. 8. *(left)* Expected total number of copies to relays and *(right)* expected delivery delays corresponding to both the optimal and the deterministic policies under controlled epidemic relaying. The mean delivery delay under uncontrolled epidemic relaying is also shown.

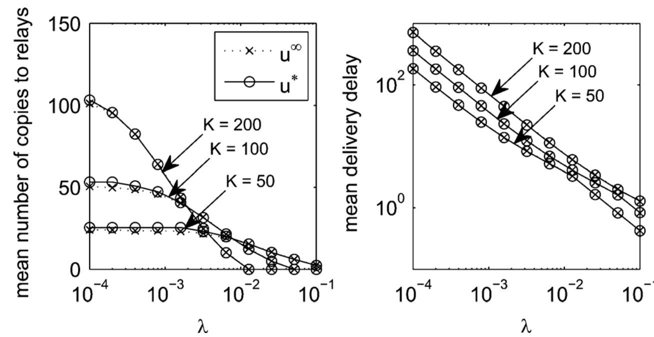

Fig. 9. *(left)* Expected total number of copies to relays and *(right)* expected delivery delays corresponding to both the optimal and the deterministic policies under controlled epidemic relaying.

plots. Clearly, the deterministic policy performs close to the optimal policy for all the considered parameter sets. Both these policies outperform *spray-and-wait* routing as well as uncontrolled forwarding. Performance improvements with respect to *spray-and-wait* routing are substantial for small values of γ. On the other hand, performance improvements with respect to the uncontrolled forwarding are meager for small values of γ; for small copying costs, the controlled protocols also make a large number of copies (because the difference in the number of copies affects the total cost only marginally), and hence the controlled protocols incur approximately similar delivery delays as the uncontrolled one. However, for higher values of γ, optimal control brings in considerable performance benefits.

Often, static (probabilistic) controls have been considered in the literature (e.g., see [10]). In our context, probabilistic controls end up copying to all the relays, but incur higher delivery delay than uncontrolled forwarding. In particular, such controls incur higher total cost than uncontrolled forwarding and are clearly suboptimal.

Our results also characterize a few typical features of epidemic and two-hop relayings. As expected, for each set of parameters, the optimal number of copies is less and the optimal delivery delay is more under two-hop relaying when compared to epidemic relaying. We also observe that the optimal epidemic relaying always preforms better than the optimal two-hop relaying. This also is expected because in epidemic relaying we optimally control forwarding without the constraint that relays cannot copy to other susceptible relays. However, for large values of γ, controlled epidemic as well as controlled two-hop relayings provision few copies to relays. Thus, in this regime, both these schemes incur almost same delivery delays, and so incur almost same total costs.

Now, we investigate the effect of α on the number of copies made and the delivery delay. Toward this, we set $M = 40$, $N = 160$, $N_0 = 20$, $\lambda = 0.00001$, and $\gamma = 2$ and vary α from 0.02 to 0.97 such that M_α assumes all the integral values from 1 to 39. We focus only on epidemic relaying. Expectedly, both the number of copies and the delivery delay increase with α (see Fig. 8). In particular, as α approaches one, even the optimal policy copies to almost all the susceptible relays (in this example, 140), and all the policies (including uncontrolled forwarding) incur huge delivery delays.[9]

Finally, we study the effect of varying the network size K and the pairwise meeting rate λ. Let $X = 0.2$, $Y = 0.8$, $\alpha = 0.8$, $Y_0 = 0.1$, and $\gamma = 0.1$. We vary λ from 0.0001 to 0.1 and use $K = 50, 100$, and 200. In Fig. 9, we plot the number of copies to relays and the delivery delays corresponding to both the optimal and the asymptotically optimal deterministic policies in the case of epidemic relaying. We observe that, for a fixed K, both the mean delivery delay and the mean number of copies decrease as λ increases. We also observe that, for a fixed λ, the mean delivery delay decreases as the network size grows. Finally, for smaller values of λ, the mean number of copies to relays increases with the network size, and for larger values of λ, the opposite happens.

[9]As α approaches one, $\phi(x, y)$ approaches infinity [see (10)], and consequently τ^* also approaches infinity [see (13)]. Thus, the deterministic policy prescribes copying to all the relays in an attempt to mitigate the enormous delivery delay. This illustrates why we restrict to $\alpha < 1$ to get a meaningful distributed policy.

VII. Conclusion

We studied the epidemic forwarding in DTNs, formulated the problem as a controlled continuous-time Markov chain, and obtained the optimal policy (Theorem 3.1). We then developed an ordinary differential equation approximation for the optimally controlled Markov chain, under a natural scaling, as the population of nodes increases to ∞ (Theorem 4.1). This ODE approximation yielded a forwarding policy that does not require global state information (and, hence, is implementable) and is asymptotically optimal (Theorem 4.2).

The optimal forwarding problem can also be addressed following the result of Gast *et al.* [18]. They study a general discrete-time MDP [19]. However, they do not solve the finite problem citing the difficulties associated with obtaining the asymptotics of the optimally controlled process (see [18, Section 3.3]). Instead, they consider the fluid limit of the MDP and analyze optimal control over the deterministic limiting problem. They then show that the optimal reward of the MDP converges to the optimal reward of its mean field approximation, given by the solution to a Hamilton–Jacobi–Bellman (HJB) equation [23, Section 3.2]. On the other hand, our approach is more direct. We have a continuous-time controlled Markov chain at our disposal We explicitly characterize the optimal policy for the finite (complete information) problem and prove convergence of the optimally controlled Markov chain to a fluid limit. An asymptotically optimal deterministic control is then suggested by the limiting deterministic dynamics and does not require solving HJB equations. Our notion of asymptotic optimality is also stronger in the sense that we apply both the optimal policy and the deterministic policy to the finite problem and show that the corresponding costs converge.

There are several directions in which this work can be extended. In the same DTN framework, there could be a deadline on the delivery time of the packet (or message); the goal of the optimal control could be to maximize the fraction of destinations that receive the packet before the deadline subject to an energy constraint. Our work in this paper assumes that network parameters such as $M,\ N,\ \lambda$, etc., are known; it will be important to address the adaptive control problem when these parameters are unknown.

Appendix A
Proof of Theorem 3.1

We first prove that for the optimal policy it is sufficient to consider two actions: 1 (i.e., copy now) and *stop* (i.e., do not copy now and never copy again). More precisely, under the optimal policy, if a susceptible relay that is met is not copied, then no susceptible relay is copied in the future as well. Let us fix an $N_0 \le n \le N-1$. Let m_n^* be the maximum j such that $u^*(j,n,r)=1$.[10] We show that $u^*(j,n,r)=1$ for all $0 \le j < m_n^*$; see Fig. 2 for an illustration of this fact. The proof is via induction.

Proposition A.1: If $u^*(j,n,r)=1$ for all $m+1 \le j \le m_n^*$, then $u^*(m,n,r)=1$.

Proof: Define

$$\psi(m,n) := J_{0s}(m,n,r) - J(m,n,r)$$

$$\theta_0(m,n) := J_{0s}(m,n,r) - A((m,n,r),0)$$

and

$$\theta_1(m,n) := J_{1s}(m,n,r) - A((m,n,r),1).$$

The action sequences that give rise to $J_{0s}(m,n,r)$ and $A((m,n,r),0)$ do not copy to the susceptible relay that was just met.

More formally[11]

$$J_{0s}(m,n,r) = C_d((m,n,r),0) + \sum_{j=m}^{M_\alpha-1}\left(\prod_{l=m}^{j-1} p_{l,n}(d)\right)p_{j,n}(r) \times \left(\sum_{l=m}^{j-1} C_d((l,n,d),1) + J_{0s}(j,n,r)\right)$$

and

$$A((m,n,r),0) = C_d((m,n,r),0) + \sum_{j=m}^{M_\alpha-1}\left(\prod_{l=m}^{j-1} p_{l,n}(d)\right)p_{j,n}(r) \times \left(\sum_{l=m}^{j-1} C_d((l,n,d),1) + J(j,n,r)\right).$$

Thus, subtracting the latter from the former

$$\theta_0(m,n) = \sum_{j=m}^{M_\alpha-1}\left(\prod_{l=m}^{j-1} p_{l,n}(d)\right)p_{j,n}(r)\psi(j,n). \quad (14)$$

Since $A((m,n,r),0) \ge J(m,n,r)$, it follows that $\psi(m,n) \ge \theta_0(m,n)$, and so

$$\begin{aligned}\psi(m,n) &\ge \sum_{j=m}^{M_\alpha-1}\left(\prod_{l=m}^{j-1} p_{l,n}(d)\right)p_{j,n}(r)\psi(j,n)\\ &= p_{m,n}(r)\psi(m,n)\\ &\quad + p_{m,n}(d)\sum_{j=m+1}^{M_\alpha-1}\left(\prod_{l=m+1}^{j-1} p_{l,n}(d)\right)p_{j,n}(r)\psi(j,n)\end{aligned}$$

which implies upon rearrangement

$$\psi(m,n) \ge \sum_{j=m+1}^{M_\alpha-1}\left(\prod_{l=m+1}^{j-1} p_{l,n}(d)\right)p_{j,n}(r)\psi(j,n). \quad (15)$$

Next, we establish the following lemma.

Lemma A.1: $\theta_1(m,n) \ge \theta_1(m+1,n)$.

Proof: Note that both the action sequences that lead to the two cost terms in the definition of $\theta_1(m,n)$ copy at state (m,n,r). Subsequently, both incur equal costs until a decision epoch when an infected node meets a susceptible relay. Also, at any such state $(j,n+1,r),\ j \ge m$, the costs to go differ by $\psi(j,n+1)$. Hence

$$\begin{aligned}\theta_1(m,n) &= \sum_{j=m}^{M_\alpha-1}\left(\prod_{l=m}^{j-1} p_{l,n+1}(d)\right)p_{j,n+1}(r)\psi(j,n+1)\\ &= p_{m,n+1}(r)\psi(m,n+1) + p_{m,n+1}(d)\theta_1(m+1,n)\end{aligned}$$

[10] Note that, for a given n, m_n^* could be 0; in that case we do not copy to any more relays.

[11] Convention: A sum over an empty index set is 0, and a product over an empty index set is 1, which happens when $j=m$.

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where

$$\theta_1(m+1,n) = \sum_{j=m+1}^{M_\alpha-1}\left(\prod_{l=m+1}^{j-1} p_{l,n+1}(d)\right)p_{j,n+1}(r)\psi(j,n+1).$$

Thus, it suffices to show that

$$\psi(m,n+1) \geq \theta_1(m+1,n)$$

which is same as (15) with n replaced by $n+1$. ■

Next, observe that for all $m \leq j \leq m_n^*$

$$\begin{aligned}\psi(j,n) &= J_{0s}(j,n,r) - \min\{A((j,n,r),0), A((j,n,r),1)\}\\ &= \max\{\theta_0(j,n), \Phi(j,n)+\theta_1(j,n)\}. \qquad (16)\end{aligned}$$

Moreover, from the induction hypothesis, the optimal policy copies at states (j,n,r) for all $m+1 \leq j \leq m_n^*$. Hence, for $m+1 \leq j \leq m_n^*$

$$\psi(j,n) = \Phi(j,n) + \theta_1(j,n).$$

Finally, $\psi(j,n) = 0$ for all $m_n^* < j \leq M_\alpha - 1$ as the optimal policy does not copy in these states. Hence, from (14)

$$\begin{aligned}&\theta_0(m,n)\\ &= p_{m,n}(r)\max\{\theta_0(m,n), \Phi(m,n)+\theta_1(m,n)\}+p_{m,n}(d)\\ &\quad\times\sum_{j=m+1}^{m_n^*}\left(\prod_{l=m+1}^{j-1} p_{l,n}(d)\right)p_{j,n}(r)\big(\Phi(j,n)+\theta_1(j,n)\big)\\ &< p_{m,n}(r)\max\{\theta_0(m,n), \Phi(m,n)+\theta_1(m,n)\}+p_{m,n}(d)\\ &\quad\times\big(\Phi(m,n)+\theta_1(m,n)\big)\sum_{j=m+1}^{m_n^*}\left(\prod_{l=m+1}^{j-1} p_{l,n}(d)\right)p_{j,n}(r)\\ &\leq p_{m,n}(r)\max\{\theta_0(m,n), \Phi(m,n)+\theta_1(m,n)\}\\ &\quad + p_{m,n}(d)\big(\Phi(m,n)+\theta_1(m,n)\big)\\ &= \max\big\{p_{m,n}(r)\theta_0(m,n)+p_{m,n}(d)\big(\Phi(m,n)+\theta_1(m,n)\big),\\ &\qquad \Phi(m,n)+\theta_1(m,n)\big\} \qquad (17)\end{aligned}$$

where the first (strict) inequality holds because $\Phi(m,n)$ is strictly decreasing [see (4)] and $\theta_1(m,n)$ is decreasing (see Lemma A.1) in m for fixed n. The second inequality follows because the summation term is a probability that is less than 1. Now suppose that $\theta_0(m,n) \geq \Phi(m,n)+\theta_1(m,n)$. Then

$$\begin{aligned}&\max\big\{p_{m,n}(r)\theta_0(m,n) + p_{m,n}(d)\big(\Phi(m,n)+\theta_1(m,n)\big),\\ &\qquad \Phi(m,n)+\theta_1(m,n)\big\}\\ &\quad = p_{m,n}(r)\theta_0(m,n) + p_{m,n}(d)\big(\Phi(m,n)+\theta_1(m,n)\big)\\ &\quad \leq \theta_0(m,n)\end{aligned}$$

which contradicts (17). Thus, we conclude that

$$\theta_0(m,n) < \Phi(m,n) + \theta_1(m,n).$$

This further implies that $\psi(m,n) = \Phi(m,n) + \theta_1(m,n)$ [see (16)], and so that $u^*(m,n,r) = 1$. ■

We now return to the proof of Theorem 3.1. We show that the one-step lookahead policy is optimal for the resulting stopping problem. To see this, observe that $\Phi(m,n)$ is decreasing in m for a given n and also decreasing in n for a given m. Thus, if $(m,n,r) \in \mathcal{S}_\mathcal{S}$, i.e., $\Phi(m,n) \leq 0$ [see (3)], and the susceptible relay that is met is copied, the next state $(m,n+1,r)$ also belongs to the stopping set $\mathcal{S}_\mathcal{S}$. In other words, $\mathcal{S}_\mathcal{S}$ is also an absorbing set [19, Section 3.4]. Consequently, the one-step lookahead policy is optimal.

Appendix B
Proof of Theorem 4.1

We start with a preliminary result and a few definitions.

Proposition B.1: Let $\alpha < 1$ and $Y_0 > 0$. Let ϕ^K and ϕ be as given in (8) and (10), respectively. Then, the functions $\phi^K(\cdot)$ converge to $\phi(\cdot)$ uniformly, i.e., for every $\nu > 0$, there exists a K_ν such that

$$\sup_{(x,y)\in\Delta^K} |\phi^K(x,y) - \phi(x,y)| < \nu$$

for all $K \geq K_\nu$.

Proof: See [24, Appendix B]. ■

In the following, to facilitate a parsimonious description, we use the notation $z^K(t) = (x^K(t), y^K(t))$, $z(t) = (x(t), y(t))$ and $\mathcal{Z} = [0, X_\alpha] \times [Y_0, Y]$. Let us define, for a $\nu \in \mathbb{R}$

$$\begin{aligned}\mathcal{S}_\nu &= \{z \in \mathcal{Z} : \phi(z) > \nu\}\\ \tau_\nu &= \inf\{t \geq 0 : z(t) \notin \mathcal{S}_\nu\}\end{aligned}$$

and a stopping time

$$\tau_\nu^K = \inf\{t \geq 0 : z^K(t) \notin \mathcal{S}_\nu\}$$

the time when $z^K(t)$ exits the limiting set $\mathcal{S}_\nu$. Observe that

$$\frac{\partial\phi}{\partial x} = -\frac{1}{\Lambda(x+y)^2(X-x)} \leq -\frac{1}{\Lambda(X_\alpha+Y)^2X} \qquad (18)$$

and $f_1^K(x,y)$ [see (7a)] is positive and bounded away from zero. These imply $\tau_\nu^K < \infty$ with probability 1. Similarly, $\tau_\nu < \infty$. The following assertion is a corollary of Proposition B.1.

Corollary B.1: Let K_ν be as in Proposition B.1. For $K \geq K_\nu$

$$\phi^K(z) > 0 \qquad \text{for all } z \in \mathcal{S}_\nu$$

and

$$\phi^K(z) \leq 0 \qquad \text{for all } z \notin \mathcal{S}_{-\nu}.$$

We define the uncontrolled dynamics (i.e., the one in which the susceptible relays are always copied) as a Markov process $\bar{z}^K(t) = (\bar{x}^K(t), \bar{y}^K(t))$, $t \geq 0$ for which $\bar{z}^K(0) = z^K(0)$. Let $\bar{z}(t) = (\bar{x}(t), \bar{y}(t))$, $t \geq 0$ be the corresponding limiting deterministic dynamics. Formally, $\bar{z}(0) = z(0)$, and for $t \geq 0$

$$\begin{aligned}\frac{\mathrm{d}\bar{x}(t)}{\mathrm{d}t} &= \Lambda(\bar{x}(t)+\bar{y}(t))(X-\bar{x}(t))\\ \frac{\mathrm{d}\bar{y}(t)}{\mathrm{d}t} &= \Lambda(\bar{x}(t)+\bar{y}(t))(Y-\bar{y}(t)).\end{aligned}$$

The quantities on the right-hand side of the above equations are at most Λ, and so $\left\|\frac{\mathrm{d}\bar{z}}{\mathrm{d}t}\right\| \leq \sqrt{2}\Lambda$. Also observe that the processes $\bar{z}^K(t)$ and $\bar{z}(t)$ satisfy the hypotheses of Darling [15] (see Section IV-A), and thus convergence of $\bar{z}^K(t)$ to $\bar{z}(t)$ follows.

We also define a Markov process $\tilde{z}^K(t) = (\tilde{x}^K(t), \tilde{y}^K(t))$, $t \geq \tau_\nu$ for which $\tilde{z}^K(\tau_\nu) = z^K(\tau_\nu)$ and

$$\begin{aligned}\frac{\mathrm{d}\mathbb{E}(\tilde{x}^K(t)|(\tilde{x}^K(t), \tilde{y}^K(t))}{\mathrm{d}t} &= \Lambda(\tilde{x}^K(t)+\tilde{y}^K(t))(X-\bar{x}^K(t))\\ \frac{\mathrm{d}\mathbb{E}(y^K(t)|(x^K(t), y^K(t))}{\mathrm{d}t} &= 0.\end{aligned}$$

TABLE I
VARIABLES AND THEIR DESCRIPTION

variables	description
$z^K(t)$	controlled dynamics with discontinuity at τ^K
$z(t)$	$z^K(t)$'s fluid limit with discontinuity at τ^*
τ_ν^K	instant when $z^K(t)$ exits $\mathcal{S}_\nu$
τ_ν	instant when $z(t)$ exits $\mathcal{S}_\nu$
$\bar{z}^K(t)$	uncontrolled dynamics with no discontinuity
$\bar{z}(t)$	$\bar{z}^K(t)$'s fluid limit with no discontinuity
$\tilde{z}^K(t)$	identical to $z^K(t)$ until τ_ν at which copying to relays is stopped
$\tilde{z}(t)$	$\tilde{z}^K(t)$'s fluid limit with discontinuity at τ_ν
$\tilde{\tau}_{-\nu}^K$	instant when $\tilde{z}^K(t)$ exits $\mathcal{S}_{-\nu}$
$\tilde{\tau}_{-\nu}$	instant when $\tilde{z}(t)$ exits $\mathcal{S}_{-\nu}$

In other words, $\tilde{z}^K(t)$ is the process in which relays are not copied after τ_ν. Similarly, we define $\tilde{z}(t) = (\tilde{x}(t), \tilde{y}(t)), t \geq \tau_\nu$, as the solution of the corresponding differential equations. In other words, $\tilde{z}(\tau_\nu) = z(\tau_\nu)$, and for $t \geq \tau_\nu$

$$\frac{\mathrm{d}\tilde{x}(t)}{\mathrm{d}t} = f_1(\tilde{x}(t), \tilde{y}(t)) := \Lambda(\tilde{x}(t) + \tilde{y}(t))(X - \tilde{x}(t))$$
$$\frac{\mathrm{d}\tilde{y}(t)}{\mathrm{d}t} = f_2(\tilde{x}(t), \tilde{y}(t)) := 0.$$

We define

$$\tilde{\tau}_{-\nu}^K = \inf\{t \geq \tau_\nu : \tilde{z}^K(t) \notin \mathcal{S}_{-\nu}\}$$
$$\tilde{\tau}_{-\nu} = \inf\{t \geq \tau_\nu : \tilde{z}(t) \notin \mathcal{S}_{-\nu}\}.$$

Since

$$\Lambda Y_0(X - X_\alpha) \leq \frac{\mathrm{d}\tilde{x}}{\mathrm{d}t} \leq \Lambda$$

the lower bound implies that there is a strictly positive increase in $\tilde{x}$ after time τ_ν. Since $\Phi(x, y)$ decreases with increasing x at a rate bounded away from 0 [see (18)], $\tilde{z}(t)$ must exit $S_{-\nu}$ within a short additional duration. Thus, we have that $\tilde{\tau}_{-\nu} - \tau_\nu \leq b\nu$ for a suitably chosen $b < \infty$.

We summarize the variables in Table I. We also illustrate sample trajectories of a controlled CTMC and the corresponding ODE via an example (Fig. 10). We choose $M = 40$, $N = 160$, $\alpha = 0.8$, $N_0 = 40$, $\lambda = 0.00025$, and $\gamma = 0.25$. We plot the graphs of "$\phi(x, y) = \nu$" and "$\phi(x, y) = -\nu$" for $\nu = 0.2$. We also show the trajectories "y^K versus x^K," "y versus x," "$\tilde{y}$ versus $\tilde{x}$," and the epochs τ_ν, $\tau_{-\nu}$, and $\tilde{\tau}_{-\nu}$.

We also need the following lemmas.

Lemma B.1: For every $\epsilon > 0$, there exists a $\bar{\tau}_\epsilon$ such that for all $t \geq 0$, $0 \leq u \leq \bar{\tau}_\epsilon$

$$\mathbb{P}\left(\|\bar{z}^K(t + u) - \bar{z}^K(t)\| > \epsilon\right) = O(K^{-1}).$$

Proof: See [24, Appendix B]. ∎

Lemma B.2: Suppose u is a fixed time and u^K is a random time that satisfies $\mathbb{P}\left(|u - u^K| > \delta\right) = O(K^{-1})$ for every $\delta > 0$. Then, for every $\epsilon > 0$

$$\mathbb{P}\left(\|\bar{z}^K(u) - \bar{z}^K(u \wedge u^K)\| > \epsilon\right) = O(K^{-1}).$$

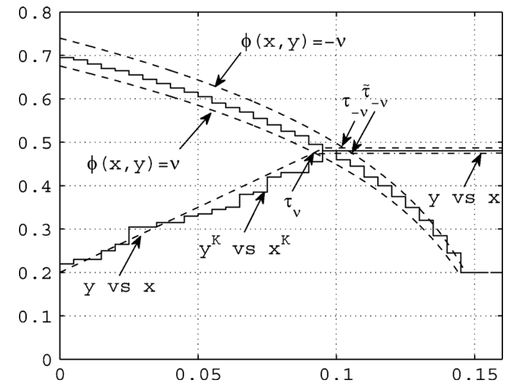

Fig. 10. Illustration of the trajectories of the controlled CTMC and the corresponding ODE and the associated variables.

Proof: See [24, Appendix B]. ∎

We now prove the assertion in Theorem 4.1 in three steps: a) over $[0, \tau_\nu]$; b) over $[\tau_\nu, \tilde{\tau}_{-\nu}]$; and c) over $[\tilde{\tau}_{-\nu}, \tau]$.

a) First, we prove the convergence of $z^K(t)$ to $z(t)$ over $[0, \tau_\nu]$. Fix a $\nu > 0$. Then, Corollary B.1 implies that $z^K(t)$ converges to $z(t)$ in the region $\mathcal{S}_\nu$. Following [15, Theorem 2.8] we have, for all $\epsilon,\ \delta > 0$

$$\mathbb{P}\left(\sup_{0 \leq t \leq \tau_\nu} \|z^K(t \wedge \tau_\nu^K) - z(t)\| > \epsilon\right) = O(K^{-1}) \quad (19a)$$

and

$$\mathbb{P}(|\tau_\nu^K - \tau_\nu| > \delta) = O(K^{-1}). \quad (19b)$$

Since for all $t \geq 0$

$$\|z^K(t) - z(t)\| \leq \|z^K(t \wedge \tau_\nu^K) - z(t)\| + \|z^K(t) - z^K(t \wedge \tau_\nu^K)\|$$

we obtain

$$\sup_{0 \leq t \leq \tau_\nu} \|z^K(t) - z(t)\| \leq \sup_{0 \leq t \leq \tau_\nu} \|z^K(t \wedge \tau_\nu^K) - z(t)\| + \sup_{0 \leq t \leq \tau_\nu} \|z^K(t) - z^K(t \wedge \tau_\nu^K)\|.$$

If the left side is larger than ϵ, at least one of the two terms on the right side is larger than $\epsilon/2$, and so by the union bound, we get

$$\begin{aligned}\mathbb{P}&\left(\sup_{0 \leq t \leq \tau_\nu} \|z^K(t) - z(t)\| > \epsilon\right)\\ &\leq \mathbb{P}\left(\sup_{0 \leq t \leq \tau_\nu} \|z^K(t \wedge \tau_\nu^K) - z(t)\| > \frac{\epsilon}{2}\right)\\ &\quad + \mathbb{P}\left(\sup_{0 \leq t \leq \tau_\nu} \|z^K(t) - z^K(t \wedge \tau_\nu^K)\| > \frac{\epsilon}{2}\right)\\ &\leq O(K^{-1}) + \mathbb{P}\left(\|z^K(\tau_\nu) - z^K(\tau_\nu \wedge \tau_\nu^K)\| > \frac{\epsilon}{2}\right) \quad (20)\end{aligned}$$

where the first term in the last inequality follows from (19a). Also, from corollary B.1, for $K \geq K_\nu$, $\phi^K(z^K(\tau_\nu^K)-) > 0$, i.e., the process $z^K(t)$ follows uncontrolled dynamics until τ_ν^K. Thus, for $K \geq K_\nu$, $z^K(\tau_\nu^K) = \bar{z}^K(\tau_\nu^K)$ and

$$\|z^K(\tau_\nu) - z^K(\tau_\nu \wedge \tau_\nu^K)\| \leq \|\bar{z}^K(\tau_\nu) - \bar{z}^K(\tau_\nu \wedge \tau_\nu^K)\|$$

sample path wise. The inequality is an equality if $\tau_\nu \le \tau_\nu^K$; both sides equal 0 in this case. Otherwise, it is an inequality because the possible change in dynamics of $z^K(t)$ after τ_ν^K makes it increase (in both its components) at a slower pace than the uncontrolled $\bar{z}^K(t)$. Thus

$$\mathbb{P}\left(\|z^K(\tau_\nu) - z^K(\tau_\nu \wedge \tau_\nu^K)\| > \frac{\epsilon}{2}\right) \le \mathbb{P}\left(\|\bar{z}^K(\tau_\nu) - \bar{z}^K(\tau_\nu \wedge \tau_\nu^K)\| > \frac{\epsilon}{2}\right) \le O(K^{-1})$$

where the last inequality follows from (19b) and Lemma B.2. Using this in (20), we get

$$\mathbb{P}\left(\sup_{0\le t\le\tau_\nu} \|z^K(t) - z(t)\| > \epsilon\right) \le O(K^{-1}) + O(K^{-1}) = O(K^{-1}).$$

b) Now we prove the convergence of $z^K(t)$ to $z(t)$ over $[\tau_\nu, \tilde{\tau}_{-\nu}]$.
Observe that, for $t \in [\tau_\nu, \tilde{\tau}_{-\nu}]$

$$\|z^K(t) - z(t)\| \le \|z^K(\tau_\nu) - z(\tau_\nu)\| + \|z^K(t) - z^K(\tau_\nu)\| + \|z(t) - z(\tau_\nu)\|.$$

Hence

$$\begin{aligned}\sup_{\tau_\nu\le t\le\tilde{\tau}_{-\nu}} \|z^K(t) - z(t)\| &\le \|z^K(\tau_\nu) - z(\tau_\nu)\| + \sup_{\tau_\nu\le t\le\tilde{\tau}_{-\nu}} \|z^K(t) - z^K(\tau_\nu)\| + \sup_{\tau_\nu\le t\le\tilde{\tau}_{-\nu}} \|z(t) - z(\tau_\nu)\| \\ &= \|z^K(\tau_\nu) - z(\tau_\nu)\| + \|z^K(\tilde{\tau}_{-\nu}) - z^K(\tau_\nu)\| + \|z(\tilde{\tau}_{-\nu}) - z(\tau_\nu)\| \\ &\le \|z^K(\tau_\nu) - z(\tau_\nu)\| + \|z^K(\tilde{\tau}_{-\nu}) - z^K(\tau_\nu)\| + \sqrt{2}\Lambda b\nu\end{aligned}$$

where the equality follows because the $z(t)$ and $z(t)$ are nondecreasing. The last inequality holds because $\|\mathrm{d}z/\mathrm{d}t\| \le \|\mathrm{d}\bar{z}/\mathrm{d}t\| \le \sqrt{2}\Lambda$ and $\tilde{\tau}_{-\nu} - \tau_\nu \le b\nu$. Moreover

$$\begin{aligned}\mathbb{P}\left(\sup_{\tau_\nu\le t\le\tilde{\tau}_{-\nu}} \|z^K(t) - z(t)\| > \sqrt{2}\Lambda b\nu + \frac{\epsilon}{2}\right) &\le \mathbb{P}\left(\|z^K(\tau_\nu) - z(\tau_\nu)\| > \frac{\epsilon}{4}\right) + \mathbb{P}\left(\|z^K(\tilde{\tau}_{-\nu}) - z^K(\tau_\nu)\| > \frac{\epsilon}{4}\right) \\ &= O(K^{-1}) + \mathbb{P}\left(\|z^K(\tilde{\tau}_{-\nu}) - z^K(\tau_\nu)\| > \frac{\epsilon}{4}\right)\end{aligned}$$

where the equality follows from the result of part (a). We now redefine the Markov process $\bar{z}^K(t) = (\bar{x}^K(t), \bar{y}^K(t))$ for $t \ge \tau_\nu$, to be the uncontrolled dynamics with initial condition $\bar{z}^K(\tau_\nu) = z^K(\tau_\nu)$. Again, it can be easily observed that

$$\|z^K(\tilde{\tau}_{-\nu}) - z^K(\tau_\nu)\| \le \|\bar{z}^K(\tilde{\tau}_{-\nu}) - \bar{z}^K(\tau_\nu)\|.$$

Thus

$$\begin{aligned}\mathbb{P}\left(\sup_{\tau_\nu\le t\le\tilde{\tau}_{-\nu}} \|z^K(t) - z(t)\| > \sqrt{2}\Lambda b\nu + \frac{\epsilon}{2}\right) &\le O(K^{-1}) + \mathbb{P}\left(\|\bar{z}^K(\tilde{\tau}_{-\nu}) - \bar{z}^K(\tau_\nu)\| > \frac{\epsilon}{4}\right) \\ &\le O(K^{-1}) + \mathbb{P}\left(\|\bar{z}^K(\tau_\nu + b\nu) - \bar{z}^K(\tau_\nu)\| > \frac{\epsilon}{4}\right).\end{aligned}$$

Set $\nu = \min\{\frac{\epsilon}{2\sqrt{2}\Lambda b}, \bar{\tau}_{\frac{\epsilon}{4b}}\}$, and apply Lemma B.1 to get

$$\begin{aligned}\mathbb{P}\left(\sup_{\tau_\nu\le t\le\tilde{\tau}_{-\nu}} \|z^K(t) - z(t)\| > \epsilon\right) &\le \mathbb{P}\left(\sup_{\tau_\nu\le t\le\tilde{\tau}_{-\nu}} \|z^K(t) - z(t)\| > \sqrt{2}\Lambda b\nu + \frac{\epsilon}{2}\right) \\ &\le O(K^{-1}) + O(K^{-1}) = O(K^{-1}).\end{aligned}$$

c) Finally, we prove the convergence of $z^K(t)$ to $z(t)$ over $[\tilde{\tau}_{-\nu}, \tau]$. Reconsider the process $\tilde{z}^K(t),\ t \ge \tau_\nu$ and the associated function $\tilde{z}(t)$. Recall that for any $\nu > 0$, $\tilde{z}^K(t)$ and $\tilde{z}(t)$ exit $\mathcal{S}_{-\nu}$ at $\tilde{\tau}_{-\nu}^K$ and $\tilde{\tau}_{-\nu}$, respectively. Clearly, $\tilde{\tau}_{-\nu/2} < \tilde{\tau}_{-\nu}$; say $\tilde{\tau}_{-\nu} - \tilde{\tau}_{-\nu/2} = \delta_\nu$. Also, using [15, Theorem 2.8]

$$\mathbb{P}\left(\tilde{\tau}_{-\nu/2}^K - \tilde{\tau}_{-\nu/2} > \delta_\nu\right) = O(K^{-1})$$

i.e.,

$$\mathbb{P}\left(\tilde{\tau}_{-\nu/2}^K > \tilde{\tau}_{-\nu}\right) = O(K^{-1}).$$

Furthermore, $\tau_{-\nu/2}^K \le \tilde{\tau}_{-\nu/2}^K$ sample pathwise. The inequality holds because $z^K(t)$ may continue to increase (in both its components) at a higher pace than $\tilde{z}^K(t)$ even after τ_ν. Thus

$$\mathbb{P}\left(\tau_{-\nu/2}^K > \tilde{\tau}_{-\nu}\right) = O(K^{-1})$$

implying that the probability that $z^K(t)$ has changed its dynamics by $\tilde{\tau}_{-\nu}$ approaches 1 as K approaches ∞. In these realizations, the dynamics of $z^K(t)$ and $z(t)$ match for $t \ge \tilde{\tau}_{-\nu}$. We restrict ourselves to only these realizations. We also have from part (b) that for every $\epsilon > 0$

$$\mathbb{P}\left(\|z^K(\tilde{\tau}_{-\nu}) - z(\tilde{\tau}_{-\nu})\| > \epsilon\right) = O(K^{-1}).$$

Once more using [15, Theorem 2.8], for any $\epsilon,\ \delta > 0$

$$\mathbb{P}\left(\sup_{\tilde{\tau}_{-\nu}\le t\le\tau} \|z^K(t) - z(t)\| > \epsilon\right) = O(K^{-1})$$

and

$$\mathbb{P}\left(|\tau^K - \tau| > \delta\right) = O(K^{-1}).$$

Appendix C
Proof of Theorem 4.2

Observe that $\mathcal{T}_d = \tau^K$ by definition [see (11)] and that all the destinations are copied under any policy. Hence, the total expected cost under the optimal policy u^* is

$$\mathbb{E}^K_{u^*}\{\mathcal{T}_d + \gamma\mathcal{E}_c\} = \mathbb{E}^K_{u^*}\{\tau^K + \Gamma y^K(\tau^K)\}.$$

Under the deterministic policy u^∞, copying to relays is stopped at the deterministic time instant τ^*. Thus, it incurs the total expected cost

$$\mathbb{E}^K_{u^\infty}\{\mathcal{T}_d + \gamma\mathcal{E}_c\} = \mathbb{E}^K_{u^\infty}\{\tau^K + \Gamma y^K(\tau^*)\}.$$

Also, under u^∞, the fluid limits of $(x^K(t), y^K(t))$ are the same deterministic dynamics $(x(t), y(t))$ defined in Section IV-A [i.e., solutions of (9a)–(9b)]. Indeed, $(x^K(t), y^K(t))$ and $(x(t), y(t))$ satisfy the hypotheses assumed in Darling [15] over the intervals $[0, \tau^*]$ and $[\tau^*, \infty)$. Thus [15, Theorem 2.8] applies, and we obtain[12]

$$\lim_{K\longrightarrow\infty} \mathbb{P}^K_{u^\infty}\left(\sup_{0\le t\le\tau} \|(x^K(t), y^K(t)) - (x(t), y(t))\| > \epsilon\right) = 0$$
$$\lim_{K\longrightarrow\infty} \mathbb{P}^K_{u^\infty}\left(|\tau^K - \tau| > \delta\right) = 0.$$

We first show that, under the control u^*, $y^K(\tau^K)$ converge to $y(\tau^*)$ in probability. Recall that $\phi(x(t), y(t))$ is decreasing in t, $\phi(x(\tau^*), y(\tau^*)) = 0$ [see (13)] and $\phi(x(\tau), y(\tau)) = -\Gamma$ [see (10) and(12)]. Thus, $\tau^* < \tau$, and from (9b), $y(\tau) = y(\tau^*)$. Hence, it suffices to show that $y^K(\tau^K)$ converge to $y(\tau)$ in probability. To see this, observe that

$$|y^K(\tau^K) - y(\tau)| \le |y^K(\tau^K) - y^K(\tau)| + |y^K(\tau) - y(\tau)|. \quad (21)$$

From Theorem 4.1, $y^K(\tau)$ and τ^K converge to $y(\tau)$ and τ, respectively, in probability. The latter result and arguments similar to those in the proof of Lemma B.2 imply that

$$\mathbb{P}\left(|y^K(\tau^K) - y^K(\tau)| > \epsilon\right) = O(K^{-1})$$

for every $\epsilon > 0$. Using these facts in (21), we conclude that

$$\mathbb{P}\left(|y^K(\tau^K) - y(\tau)| > \epsilon\right) = O(K^{-1})$$

for every $\epsilon > 0$, which is the desired claim.

Furthermore, $y^K(\tau^K)$ are bounded uniformly over all K. Thus, following [26, Remark 9.5.1], $y^K(\tau^K)$ are uniformly integrable under u^*. Similar arguments imply that, under u^∞ also, $y^K(\tau^*)$ converge in probability to $y(\tau^*)$ and are uniformly integrable. Then, the convergence in probability along with [26, Theorem 9.5.1] implies that

$$\lim_{K\longrightarrow\infty} \mathbb{E}^K_{u^*} y^K(\tau^K) = \lim_{K\longrightarrow\infty} \mathbb{E}^K_{u^\infty} y^K(\tau^*) = y(\tau^*). \quad (22)$$

Next, it is shown that under both the controls u^* and u^∞, the delivery delays τ^K have second moments that are bounded uniformly over all K. To see this, consider a policy u^0 that never copies to relays. Clearly

$$\mathbb{E}^K_{u^*}(\tau^K)^2 < \mathbb{E}^K_{u^0}(\tau^K)^2$$
$$\mathbb{E}^K_{u^\infty}(\tau^K)^2 < \mathbb{E}^K_{u^0}(\tau^K)^2$$

for each K. Thus, it suffices to show that

$$\sup_K \mathbb{E}^K_{u^0}(\tau^K)^2 < \infty. \quad (23)$$

Note that

$$\tau^K = \sum_{m=0}^{M^K_\alpha - 1} \bar{\delta}_m$$

where $\bar{\delta}_m$ is the time duration for which $m(t) = m$. Under the policy u^0, $\bar{\delta}_m$ is exponentially distributed with mean $\frac{1}{\lambda^K(m+N^K_0)(M^K-m)}$. Thus

$$\begin{aligned}\mathbb{E}^K_{u^0}\tau^K &= \sum_{m=0}^{M^K_\alpha - 1} \frac{1}{\lambda^K(m + N^K_0)(M^K - m)} \\ &\le \sum_{m=0}^{M^K_\alpha - 1} \frac{1}{\lambda^K N^K_0(M^K - M^K_\alpha)} \\ &= \frac{M^K_\alpha}{\lambda^K N^K_0(M^K - M^K_\alpha)} \\ &= \frac{X_\alpha}{\Lambda Y_0(X - X_\alpha)} < \infty.\end{aligned}$$

Also, $\bar{\delta}_m,\ m = 0, 1, \ldots$ are independent. Thus

$$\begin{aligned}\mathrm{Var}^K_{u^0}\tau^K &= \sum_{m=0}^{M^K_\alpha - 1} \frac{1}{\left(\lambda^K(m + N^K_0)(M^K - m)\right)^2} \\ &\le \frac{M^K_\alpha}{\left(\lambda^K N^K_0(M^K - M^K_\alpha)\right)^2} \\ &= \frac{X_\alpha}{K\Lambda^2 Y_0^2(X - X_\alpha)^2} \longrightarrow 0\end{aligned}$$

as $K \longrightarrow \infty$. These results together imply (23).

Again, from [26, Remark 9.5.1], τ^K are uniformly integrable under both u^* and u^∞. Recall from Theorem 4.1 that τ^K converge to τ in probability. Once more using [26, Theorem 9.5.1]

$$\lim_{K\longrightarrow\infty} \mathbb{E}^K_{u^*}\tau^K = \lim_{K\longrightarrow\infty} \mathbb{E}^K_{u^\infty}\tau^K = \tau. \quad (24)$$

Finally, combining (22) and (24), we conclude

$$\lim_{K\longrightarrow\infty} \mathbb{E}^K_{u^*}\{\mathcal{T}_d + \gamma\mathcal{E}_c\} = \lim_{K\longrightarrow\infty} \mathbb{E}^K_{u^\infty}\{\mathcal{T}_d + \gamma\mathcal{E}_c\} = \tau + \Gamma y(\tau^*).$$

[12] Applying [15, Theorem 2.8] over $[0, \tau^*]$ yields $\lim_{K\longrightarrow\infty} \mathbb{P}\left(\|(x^K(\tau^*), y^K(\tau^*) - (x(\tau^*), y(\tau^*))\| > \epsilon\right) = 0$, which is a necessary condition to apply [15, Theorem 2.8] over $[\tau^*, \infty)$.

References

[1] K. Fall, "A delay-tolerant network architecture for challenged internets," in *Proc. SIGCOMM*, Karlsruhe, Germany, 2003, pp. 27–34.

[2] A. Vahdat and D. Becker, "Epidemic routing for partially-connected ad hoc networks," Duke University, Durham, NC, Tech. Rep. CS-2000-06, 2000.

[3] S. Venkatramanan and A. Kumar, "Co-evolution of content popularity and delivery in mobile P2P networks," in *Proc. IEEE INFOCOM*, Orlando, FL, Mar. 2012, pp. 3006–3010.

[4] A. Karnik and P. Dayama, "Optimal control of information epidemics," in *Proc. COMSNETS*, Bangalore, India, Jan. 2012, pp. 1–7.

[5] M. Khouzani, S. Sarkar, and E. Altman, "Dispatch then stop: Optimal dissemination of security patches in mobile wireless networks," in *Proc. IEEE CDC*, Atlanta, GA, Dec. 2010, pp. 2354–2359.

[6] R. Groenevelt, P. Nain, and G. Koole, "The message delay in mobile ad hoc networks," *Perform. Eval.*, vol. 62, no. 1–4, pp. 210–228, Oct. 2005.

[7] X. Zhang, G. Neglia, J. Kurose, and D. Towsley, "Performance modeling of epidemic routing," *Comput. Netw.*, vol. 51, no. 10, pp. 2867–2891, Jul. 2007.

[8] G. Neglia and X. Zhang, "Optimal delay-power tradeoff in sparse delay tolerant networks: A preliminary study," in *Proc. SIGCOMM Workshop Challenged Netw.*, Pisa, Italy, Sep. 2006, pp. 237–244.

[9] T. Spyropoulos, K. Psounis, and C. S. Raghavendra, "Efficient routing in intermittently connected mobile networks: The multiple-copy case," *IEEE/ACM Trans. Netw.*, vol. 16, no. 1, pp. 77–90, Feb. 2008.

[10] E. Altman, T. Basar, and F. D. Pellegrini, "Optimal monotone forwarding policies in delay tolerant mobile ad-hoc networks," *Perform. Eval.*, vol. 67, no. 4, pp. 299–317, Apr. 2010.

[11] E. Altman, G. Neglia, F. D. Pellegrini, and D. Miorandi, "Decentralized stochastic control of delay tolerant networks," in *Proc. IEEE INFOCOM*, Rio de Janeiro, Brazil, Apr. 2009, pp. 1134–1142.

[12] E. Altman, A. P. Azad, F. D. Pellegrini, and T. Basar, "Optimal activation and transmission control in delay tolerant networks," in *Proc. IEEE INFOCOM*, San Diego, CA, Mar. 2010, pp. 1–5.

[13] Y. Li, Y. Jiang, D. Jin, L. Su, L. Zeng, and D. Wu, "Energy-efficient optimal opportunistic forwarding for delay-tolerant networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 9, pp. 4500–4512, Nov. 2010.

[14] T. G. Kurtz, "Solutions of ordinary differential equations as limits of pure jump Markov processes," *J. Appl. Probab.*, vol. 7, pp. 49–58, 1970.

[15] R. W. R. Darling, "Fluid limits of pure jump Markov processes: A practical guide," arXiv:math/0210109v3, 2002.

[16] R. W. R. Darling and J. R. Norris, "Differential equation approximations for Markov chains," *Probab. Surveys*, vol. 5, pp. 37–79, 2008.

[17] N. Gast and B. Gaujal, "Mean field limit of non-smooth systems and differential inclusions," INRIA, 2010.

[18] N. Gast, B. Gaujal, and J.-Y. L. Boudec, "Mean field for Markov decision processes: From discrete to continuous optimization," *IEEE Trans. Autom. Control*, vol. 57, no. 9, pp. 2266–2280, Sep. 2012.

[19] D. P. Bertsekas, *Dynamic Programming and Optimal Control*, 3rd ed. Belmont, MA: Athena Scientific, 2007, vol. 2.

[20] C. Singh, A. Kumar, and R. Sundaresan, "Delay and energy optimal two-hop relaying in delay tolerant networks," in *Proc. 8th WiOpt*, Avignon, France, May–Jun. 2010, pp. 256–265.

[21] M. Grossglauser and D. N. C. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *IEEE/ACM Trans. Netw.*, vol. 10, no. 4, pp. 477–486, Aug. 2002.

[22] T. Karagiannis, J.-Y. L. Boudec, and M. Vojnovic, "Power law and exponential decay of intercontact times between mobile devices," *IEEE Trans. Mobile Comput.*, vol. 9, no. 10, pp. 1377–1390, Oct. 2010.

[23] D. P. Bertsekas, *Dynamic Programming and Optimal Control*, 3rd ed. Belmont, MA: Athena Scientific, 2005, vol. 1.

[24] C. Singh, E. Altman, A. Kumar, and R. Sundaresan, "Optimal forwarding in delay tolerant networks with multiple destinations," Indian Institute of Science, Bangalore, India, 2012 [Online]. Available: http://arxiv.org/abs/1110.6127

[25] R. Solis, V. S. Borkar, and P. R. Kumar, "A new distributed time synchronization protocol for multihop wireless networks," in *Proc. IEEE CDC*, San Diego, CA, Dec. 2006, pp. 2734–2739.

[26] K. B. Athreya and S. N. Lahiri, *Measure Theory and Probability Theory*. New York: Springer, 2006.

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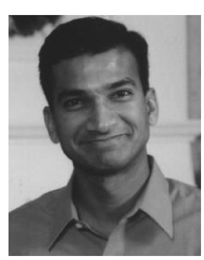

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