Robust Control in Sparse Mobile Ad-Hoc Networks

Eitan Altman[1], Alireza Aram[3], Tamer Başar[2],
Corinne Touati[4], and Saswati Sarkar[5]

[1] INRIA, BP93, 06902 Sophia Antipolis, France
[2] University of Illinois, 1308 West Main Street, Urbana, IL 61801-2307, USA
[3] Finance Department, The Wharton School, Univ of Pennsylvania, USA
aaram@wharton.upenn.edu
[4] Mescal/LIG project, ENSIMAG, 38330 Montbonnot du St Martiin, France
[5] Dept. of Electrical and Systems Eng., University of Pennsylvania

Abstract. We consider a two-hop routing delay-tolerant network. a When the source encounters a mobile then it transmits, with some probability, a file to that mobile, with the probability itself being a decision variable. The number of mobiles is not fixed, with new mobiles arriving at some constant rate. The file corresponds to some software that is needed for offering some service to some clients, which themselves may be mobile or fixed. We assume that mobiles have finite life time due to limited energy, but that the rate at which they die is unknown. We use an H^{∞} approach which transforms the problem into a worst case analysis, where the objective is to find a policy for the transmitter which guarantees the best performance under worst case conditions of the unknown rate. This problem is formulated as a zero-sum differential game, for which we obtain the value as well as the saddle-point policies for both players.

1 Introduction

We consider in this paper a delay tolerant network, i.e. a sparse network of mobile relay nodes, where connectivity is very low. There is some source that transmits a file to mobiles that are in the communication range. Each mobile is assumed to be in range with the source at some instants that form a Poisson process. A node that receives a copy of the file stores it so that it may transmit it to some potential destinations that may search for a copy of the file.

We assume that it is desirable that the number of mobiles that have a copy of the file be close to some fixed threshold. Distributing the file to a number of mobiles larger than this threshold is not desirable since transmitting and storing the file costs resources (such as memory).

We further assume that the life time of the mobiles is finite due to the finite energy stored in their battery. There is some rate at which the battery empties, which is assumed to be unknown and to change in time in an unpredictable way. We call this the departure rate.

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In order to compensate for the mobiles that become not operational due to battery energy limitation, other mobiles are added to the network (these could be viewed as mobiles that managed to recharge their battery). The new mobiles join at some instants, described by a Poisson process. The rate of this process is assumed to be controlled by the network.

We describe the random evolution of this system and obtain an ODE (ordinary differential equation) for the evolution of the expected number of mobiles that have a copy of the file and of those that do not. We assume that the departure rate od mobiles is unknown and may change in time in an unpredictable way. We then formulate a robust control problem, that of obtaining a policy for the network control that guarantees the best performance under the worst (time varying) departure rate. We formulate the problem as a H^∞ control problem, which we transform into an equivalent zero-sum differential game. We provide an explicit solution to this game using the theory of linear-quadratic zero-sum differential games [5,11]. This problem can thus be seen as one with an additional malicious player that has an opposite performance objective.

Various adversarial approaches have been often used in networking to solve problems in which some parameter may vary in time in an unknown way. A paradigm that has been used frequently in scheduling and routing problems which falls into this category is the competitive online algorithmic paradigm [1]. The H^∞ control theory [5] that we use here is yet another such paradigm that has been used in the context of flow control [3,6,16]. We are not aware of previous applications of adversarial type approachess for the control of epidemic type models.

The two-hop routing protocol considered here was first introduced by Grössglauser and Tse in [9]; the main goal there was to characterize the capacity of mobile ad-hoc networks and the two-hop protocol was meant to overcome severe limitations of static networks capacity obtained in [10]. Two-hop routing, in particular, provides a convenient compromise of energy versus delay compared to epidemic routing; the standard reference work for the analysis of the two-hop relaying protocol is [8]. Fluid approximations and infection spreading models similar to those we use here are described extensively in [13].

Algorithms to control forwarding in DTNs have been proposed in the recent literature, e.g, [12], [7]. In [12], the authors describe an epidemic forwarding protocol based on the *susceptible-infected-removed* (SIR) model [13]. They show that it is possible to increase the message delivery probability by tuning the parameters of the underlying SIR model. In [7] a detailed general framework is proposed in order to capture the relative performances of different self-limiting strategies. Finally, under a fluid model approximation, the work in [4] provides a general framework for the optimal control of the broad class of monotone relay strategies, i.e., policies where the number of copies do not decrease over time. It is proved there that optimal forwarding policies are of threshold type.

The present paper is the first one we know of that utilizes tools and framework of robust control and linear-quadratic zero-sum differential games in the context of controlling DTNs, or more generally, controlling propagations of epidemics.

2 The Model

Each mobile of the DTN is assumed to be in the communication range of the source at some time instant governed by a Poisson process with rate η_t. Further:

- x is the **expected number of nodes** that have the file and y is the expected number of nodes that do not, with x_t and y_t denoting their values at time t.
- The source is in contact with each mobile without file at a rate η_t. Thus at a rate of $\eta_t y_t$, mobiles without a file transform into type x_t mobiles.
- There is a stream of new mobiles (without files) that join the system at a rate λ_t
- Mobiles that have the file die at a rate $\nu_t x$, where ν_t is unknown to the source and may change in time.

Introduce the following pair of coupled ordinary differential equations (ODEs), for $t \geq 0$:

$$\begin{cases} \dot{x}_t = \eta_t y_t - \nu_t x_t \,, & x_0 = 0 \,, \\ \dot{y}_t = -\eta_t y_t + \lambda_t \,, & y_0 = 0 \,. \end{cases} \tag{1}$$

These equations can be shown to correspond to the mean field limit of the actual number of mobiles with and without the file, with some appropriate scalings when scaled properly [15,17]. Interestingly, however, these are not just approximations; they in fact describe the precise dynamics of the **expected** numbers of mobiles with and without a copy of the file.

We will now let $u_t := \eta_t y_t$ and $\mu_t := \nu_t x_t$, in view of which (1) is written as:

$$\begin{cases} \dot{x}_t = u_t - \mu_t \,, & x_0 = 0 \,, \\ \dot{y}_t = -u_t + \lambda_t \,, & y_0 = 0 \,, \end{cases} \tag{2}$$

which is a linear, controlled dynamics.

Next we introduce the cost structure. We assume that it is desired for some target number $\overline{x}$ of nodes to have a copy of the file. Due to energy and memory constraints, it is desirable not to exceed this number. The energy for the source to transmit at a rate u_t increases with u_t, so the corresponding cost should be an increasing function of u_t.

The reasoning above as well as practical implementations lead to the following instantaneous cost:

$$c(t) = (x_t - \bar{x})^2 + u_t^2 \tag{3}$$

The controller, u, will be picked to minimize C given by

$$C(u, \mu) = \int_0^{t_f} c(t)dt \,, \tag{4}$$

where $[0, t_f]$ is the horizon of interest.

Remark 1. Since, as it will turn out, u_t is positive and $x_t < \overline{x}$, the instantaneous cost is indeed an increasing function of u_t, and it forces x_t to stay close to $\overline{x}$.

Remark 2. As another motivation or justification for the quadratic cost on the control, we offer the following explanation. Signaling is needed in order for the source to be able to know whether a mobile that does not have the file is within its transmission range. Assume that this signaling is done by each such mobile by periodical sending of beacons. The control variable η_t can then be interpreted as the frequency of beaconing per mobile, and u_t is the total beaconing rate of all mobiles. Thus u_t can be interpreted as the signaling energy expended by all terminals.

Because of the way the control u was introduced, as a control policy we have to restrict it to depend on the current value of y and not on the current value of x, and hence with respect to state x it is an open-loop policy. Note also that since the cost does not depend on y, the value of y (and hence the dynamics that generate y) does not enter into the optimization problem. Hence we can essentially work with a scalar state equation, and once we obtain the optimal u (say u_t^* at time t) the optimal value of the original decision variable η can be obtained by dividing u_t^* by y_t, assuming that the latter is nonzero.

Now, since μ_t drives the state equation (and hence affects the value of the cost function), optimal choice of the control cannot be obtained independently of μ. Further, since the control does not know μ_t, it is reasonable to adopt a worst-case approach, where we see μ as controlled by an adversary. This is the framework of robust control, or H$^\infty$ control [5], where the goal is to minimize the effect of μ on C, by a proper choice of u. We therefore seek to solve the inf sup problem:

$$\inf_u \sup_\mu \frac{C(u,\mu)^{\frac{1}{2}}}{\|\mu\|} =: \gamma^*$$

where $\|\mu\|^2 := \int_0^{t_f} \mu_t^2 dt$, and are interested in the corresponding minimizing control u^*. Since, it is not always possible to achieve γ^*, it would be sufficient to find a control that would achieve a value of γ slightly higher than γ^*, say by an $\epsilon > 0$. Then what we are looking for is a control u^γ achieving[1]

$$\sup_\mu \frac{C(u^\gamma,\mu)^{\frac{1}{2}}}{\|\mu\|} \leq \gamma^* + \epsilon =: \gamma(\epsilon)$$

Now following [5], this is equivalent to finding u^γ that guarantees that

$$L_\gamma(u^\gamma,\mu) := C(u^\gamma,\mu) - \gamma^2\|\mu\|^2 \leq 0$$

for all μ, and doing this for "smallest" possible γ. What we have here is a zero-sum differential game with kernel $L_\gamma(u,\mu)$, parametrized by γ. It turns out [5]

[1] Since the goal is to drive x to $\overline{x}$ and not to *zero* as in standard H$^\infty$ control, the formulation here does not fit the standard H$^\infty$ optimal control formulation, but after the problem is brought into the linear-quadratic differential game format, the standard theory becomes applicable as to be seen shortly.

that for each $\gamma > \gamma^*$ this differential game admits a saddle-point solution, that is (u^γ, μ^γ) such that for every u and μ,

$$L_\gamma(u^\gamma, \mu) \le L_\gamma(u^\gamma, \mu^\gamma) \le L_\gamma(u, \mu^\gamma)\,.$$

Here $L_\gamma(u^\gamma, \mu^\gamma)$ is the *value* of the game with objective function

$$L_\gamma(u, \mu) = \int_0^{t_f} ((x - \overline{x})^2 + u^2 - \gamma^2\mu^2)dt$$

An interpretation of the zero-sum differential game

We may view the above zero-sum differential game as arising in the context of a transmission problem with a malicious player. Instead of assuming that each mobile leaves at an unknown rate of ν_t, we assume that it would stay in the system in the absence of the malicious player. However, the latter transmits to the mobiles some virus at a rate of $\mu_t = \nu_t x_t$, and as a result, these mobiles go out of operation at the rate μ_t. And the malicious player attempts to maximize the quantity the control is trying to minimize, by also respecting some cost on energy

$$D = \int_0^{t_f} \mu_t^2 dt$$

which enters the objective function as a *soft* constraint. Note that, because of the relationship $\mu_t = \nu_t x_t$, we are looking for a maximizing policy that is a function of the state x, that is a closed-loop policy. As well known from theory of H^∞ control, however, whether the maximizing player uses closed-loop or open-loop policy is inconsequential, and the optimum value of γ, γ^*, does not depend on it [5]. Accordingly, in the presentation of the solution in the next section, we will occasionally also use open-loop policies for the maximizer.

3 The Saddle-Point Solution

Our goal is to find a saddle point of L_γ. Introducing the shifted state variable,

$$\tilde{x}_t := x_t - \overline{x}\,,$$

we have:

$$\dot{\tilde{x}} = u_t - \mu_t\,,\ \tilde{x}_0 = -\bar{x}\,, \tag{5}$$

$$L_\gamma(u, \mu) = \int_0^{t_f} (\tilde{x}_t^2 + u_t^2 - \gamma^2\mu_t^2)dt\,. \tag{6}$$

This is a standard linear quadratic zero-sum differential game (the dynamics are linear and the cost is quadratic in both the state and the control variables (of both players)), and recall also that as discussed in the previous section the minimizing player has access to open-loop information (that is u will be only a function of t and $\overline{x}$), whereas the maximizing player has access to closed-loop information but could as well be taken (initially) also to be open loop without any loss (or gain) in performance. The theory of chapter 4 of [5] directly applies, and we have the solution presented in stages in the subsections below.

3.1 Computing the Value γ^*

Before solving for the saddle point, we first need to determine the values of $\gamma > 0$ for which such a saddle point exists, or in other words the upper value of the game is bounded. The answer lies in the solution of the following Riccati differential equation

$$\dot{s} + 1 + \frac{1}{\gamma^2}s^2 = 0\,,\ s(t_f) = 0\,, \tag{7}$$

which has the following general solution:

$$s = \gamma\frac{a\cos\frac{t}{\gamma} - b\sin\frac{t}{\gamma}}{a\sin\frac{t}{\gamma} + b\cos\frac{t}{\gamma}} \tag{8}$$

Using the terminal condition $s(t_f) = 0$, we get

$$\tan\frac{t_f}{\gamma} = \frac{a}{b} \tag{9}$$

A feasible γ is one for which s is finite in the interval $[0, t_f]$, that is there is no finite escape. S goes to infinity only if the term in the denominator goes to zero, which will happen for all times t that satisfy $\tan(t/\gamma) = -b/a = -1/\tan(t_f/\gamma)$. Hence γ is feasible if the t that satisfies this condition falls outside the interval $[0, t_f]$, which happens if $t_f/\gamma < \pi$, or in other words, $\gamma > t_f/\pi$. Therefore, the open-loop zero sum differential game has a unique open-loop saddle point for all $\gamma > \frac{t_f}{\pi}$, and the γ^* introduced earlier is equal to this value, that is $\gamma^* = \frac{t_f}{\pi}$. For $\gamma < \gamma^*$, the upper value of the game is unbounded when the minimizing player is restricted to open-loop policies.

3.2 The Solution

We now proceed to solve for the saddle point. The relevant Riccati differential equation in this case is (see, [5], p. 135):

$$\dot{z} = -1 + qz^2;\ z(t_f) = 0, \quad q := (1 - \frac{1}{\gamma^2}) \tag{10}$$

The general solution for z is $-t + k$ if $\gamma = 1$. Else, $z = -\dot{w}/qw$, where w solves

$$\ddot{w} - qw = 0\,, \tag{11}$$

which admits the general solution

$$w = \begin{cases} a\exp{+\sqrt{q}t} + b\exp{-\sqrt{q}t}\,,\ q > 0 \\ a\sin\sqrt{-q}t + b\cos\sqrt{-q}t\,, \quad q < 0 \end{cases} \tag{12}$$

where a and b are parameters to be determined from the terminal condition (actually, the only relevant quantity is a/b, and thus we henceforth let $b = 1$).

Note also that we are interested in this solution to the extent that w remains bounded away from *zero*, because at $w(t) = 0$, z will be unbounded. As we will see next, this will place a restriction on the range of values q and thus γ can take. What we know (without doing any computation), however, is that the the Riccati differential equation (10) will admit a unique solution whenever (7) does, and hence if $(\overline{\gamma}, \infty)$ is the range of feasible values of γ for (10), then $\overline{\gamma} < \gamma^*$, and further that $\overline{\gamma} < 1$; see [5].

Therefore, we have (as the solution to the original Riccati differential equation (10)):

$$z = \begin{cases} \frac{-1}{\sqrt{q}} \frac{a \exp \sqrt{q}t - \exp(-\sqrt{q}t)}{a \exp \sqrt{q}t + \exp(-\sqrt{q}t)} & \gamma > 1 \\ -t + k & \gamma = 1 \\ \frac{1}{\sqrt{-q}} \frac{a \cos \sqrt{-q}t - \sin \sqrt{-q}t}{a \sin \sqrt{-q}t + \cos \sqrt{-q}t} & \overline{\gamma} < \gamma < 1 \end{cases} \tag{13}$$

where the parameters a and k are determined by the condition $z(t_f) = 0$, and $\overline{\gamma}$ is the value of γ which makes the denominator of the third expression in (13) *zero*. Carrying this out, we obtain: $k = t_f$,

$$a = \begin{cases} \exp(-2\sqrt{q}t_f)\,, & \gamma > 1 \\ \tan \sqrt{-q}t_f\,, & \overline{\gamma} < \gamma < 1 \end{cases} \tag{14}$$

and $\overline{\gamma} = 2t_f/\sqrt{\pi^2 + 4t_f^2}$. In view of these, (13) becomes (parameterized by γ)

$$z_\gamma = \begin{cases} \frac{1}{\sqrt{q}} \tanh(\sqrt{q}(t_f - t)) & \gamma > 1 \\ t_f - t & \gamma = 1 \\ \frac{1}{\sqrt{-q}} \tan(\sqrt{-q}(t_f - t)) & \overline{\gamma} < \gamma < 1 \end{cases} \tag{15}$$

The open-loop saddle-point solution is given by

$$u_t^* = -z_\gamma(t)\tilde{x}_t^*\,, \quad \mu_t^* = -\gamma^{-2} z_\gamma(t)\tilde{x}_t^*\,,$$

where γ has to satisfy $\gamma > \gamma^* = t_f/\pi$, and $\tilde{x}_t^*,\ t \geq 0$, is the corresponding trajectory of the shifted state, obtained from

$$\dot{\tilde{x}}_t^* = -(1 - \gamma^{-2}) z_\gamma(t)\tilde{x}_t^*, \ \ \tilde{x}_0^* = -\bar{x}\,. \tag{16}$$

The corresponding trajectory for y can be obtained by substituting u back into the second ODE of (1):

$$y_t^* = \int_0^t [z_\gamma(s)\tilde{x}_s^* + \lambda_s]\, dt\,.$$

To ensure that y^* stays positive, the input rate should be chosen to satisfy

$$\lambda_t > -z_\gamma(t)\tilde{x}_t^*$$

A few remarks are in order here:

First, for the saddle-point solution obtained above to relate to the original problem posed, we have to have x_t^* positive for all t, which means that $\tilde{x}_t^*$ should be nondecreasing in t. This will be achieved only if $\gamma > 1$, and hence we need (in view of the earlier open-loop condition)

$$\gamma > \max(1, t_f/\pi)\,.$$

Second, to obtain the corresponding contact rate (η_t), we have to divide u_t^* by y_t^*, leading to

$$\eta_t^* = -z_\gamma(t)\tilde{x}_t^*/y_t^*\,,$$

which is positive. This, however, is well defined as long as we start y^* not at 0 at $t = 0$, but at some positive value (that is, the expected value of mobiles without the file initially, or at the time optimization kicks in, should be positive); otherwise the rate will be infinite.

Third, to obtain the corresponding dying rate, we have to divide μ_t^* by x_t^*, leading to

$$\nu_t^* = -\gamma^{-2} z_\gamma(t)\tilde{x}_t^*/(\tilde{x}_t^* + \overline{x})\,,$$

which is positive. Again, the denominator of this expression is 0 at $t = 0$, and hence an adjustment has to be made so that $x_0^* > 0$ and not 0, which would be a small perturbation on the initial state.

Note that the condition on the positivity of the initial conditions is not consistent with the assumption that we had made in eq (1) (that the initial conditions are zero). However, since the solutions are continuous in the initial conditions, the departure from optimality will be tolerable if we take them to be positive but close to zero.

One can obtain explicit expressions for the corresponding contact rate and the dying rate by first noting that

$$\frac{d\tilde{x}}{dt} = -qz_\gamma(t)\tilde{x}$$

Hence

$$\begin{aligned}
ln(-\tilde{x}(T)) - ln(-\tilde{x}(0)) &= -q\int_0^T \frac{1}{\sqrt{q}}\tanh(\sqrt{q}(t_f - t))dt \\
&= -\sqrt{q}\int_0^T \tanh(\sqrt{q}(t_f - t))dt \\
&= -\sqrt{q}\int_0^T \frac{exp(\sqrt{q}(t_f - t)) - exp(-\sqrt{q}(t_f - t))}{exp(\sqrt{q}(t_f - t)) + exp(-\sqrt{q}(t_f - t))}dt \\
&= \int_{exp(\sqrt{q}t_f)+exp(-\sqrt{q}t_f)}^{exp(\sqrt{q}(t_f-T))+exp(-\sqrt{q}(t_f-T))} \frac{1}{u}du \quad \text{(with the change of variable } u = exp(\sqrt{q}(t_f - t)) + exp(-\sqrt{q}(t_f - t))) \\
&= ln\left(\frac{cosh(\sqrt{q}(t_f - T))}{cosh(\sqrt{q}t_f)}\right)
\end{aligned}$$

Hence, for any $\gamma > 1$, we have:

$$\tilde{x}_t = -\overline{x}\left(\frac{cosh(\sqrt{q}(t_f - t))}{cosh(\sqrt{q}t_f)}\right)$$

We obtain finally the following expression for the control η_t, which gives the the arrival rate of mobiles with the file:

$$\begin{aligned}\frac{u_t}{y_t} &= \frac{-z_\gamma(t)\tilde{x}_t^*}{\int_0^t [z_\gamma(s)\tilde{x}_s^* + \lambda_s]\, dt} = \frac{\frac{\overline{x}}{\sqrt{q}}\frac{sinh(\sqrt{q}(t_f-t))}{cosh(\sqrt{q}t_f)}}{\frac{-\overline{x}}{\sqrt{q}}\int_0^t \frac{sinh(\sqrt{q}(t_f-t))}{cosh(\sqrt{q}t_f)}dt + \int_0^t \lambda_s dt + y_0} \\ &= \frac{\frac{\overline{x}}{\sqrt{q}}\frac{sinh(\sqrt{q}(t_f-t))}{cosh(\sqrt{q}t_f)}}{\frac{\overline{x}}{q}\frac{cosh(\sqrt{q}(t_f-t))-cosh(\sqrt{q}t_f)}{cosh(\sqrt{q}t_f)} + \int_0^t \lambda_s dt + y_0} \\ &= \frac{\overline{x}\sqrt{q}sinh(\sqrt{q}(t_f - t))}{\overline{x}cosh(\sqrt{q}(t_f - t)) - \overline{x}cosh(\sqrt{q}t_f) + qcosh(\sqrt{q}t_f)\left(\int_0^t \lambda_s dt + y_0\right)} \end{aligned} \quad (17)$$

For ν we get the following expression:

$$\frac{\frac{1-q}{\sqrt{q}}sinh(\sqrt{q}(t_f - t))}{cosh(\sqrt{q}t_f) - cos(\sqrt{q}(t_f - t))}$$

Numerical examples. Below we present the evolution of the states and controllers over the time interval $\leq 0 \leq 10$. We use the parameters $\lambda = 3$ (constant in time), $y_0 = 4, \gamma = 5, \overline{x} = 40$. The figures display the mean number of infected nodes, the control actions of both controllers and the instantaneous cost; all these are given as function of time.

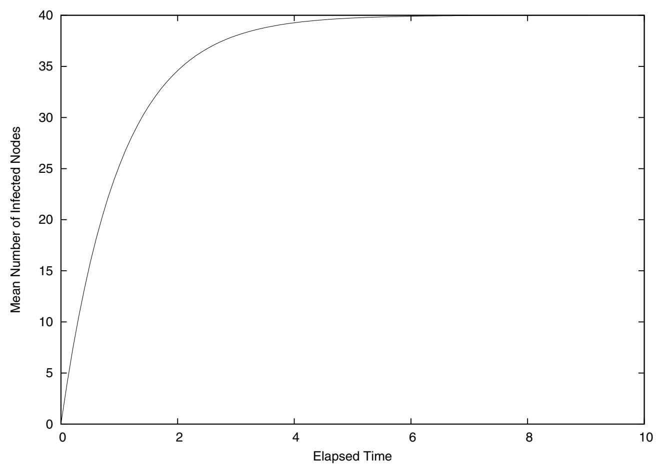

Fig. 1. Mean number of infected nodes (vertical axis) as a function of time (the horizontal axis)

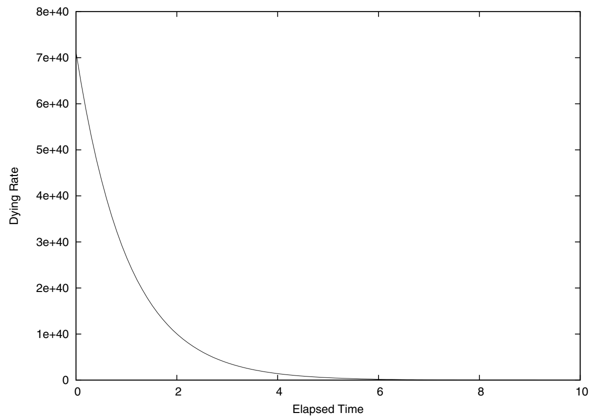

Fig. 2. Dying (or departure) rate (vertical axis) as a function of time (horizontal axis)

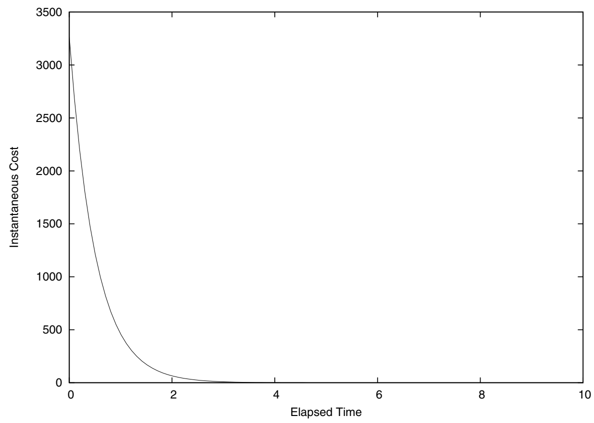

Fig. 3. Instantaneous cost (vertical axis) as a function of time

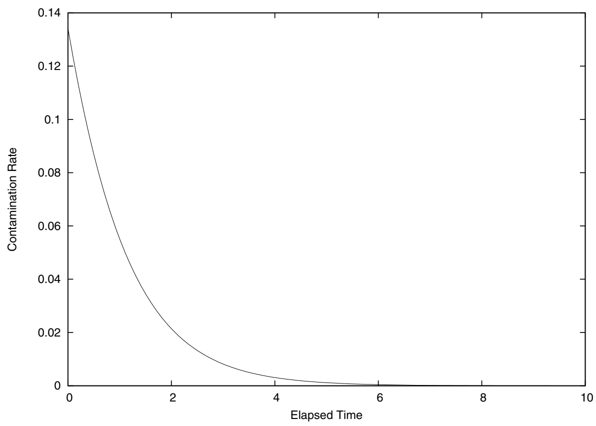

Fig. 4. Contamination rate (controlled by the original controller) at the vertical axis, as a function of time (horizontal axis)

4 Conclusions

DTNs exhibit many special features that make them hard to control: the population of mobiles may be large, relevant information may be unavailablle or may take time to arrive. The complexity of these systems often pushes us to come with simplified models (including Poisson contact processes) which allows us to describe some features of the model through exact closed form formulas. The linear quadratic control is one central framework that enables derivation of optimal control through the use of closed form formulae.

In deriving simplified control models, it is desirable to have not only simplicity but also robustness. In particular, the model should not to be too sensitive to the simplifying assumptions that lead to it. The H^∞ control is an appropriate framework that allows one on the one hand to include the simplicity of the linear quadratic framework, and, on the other, to account for some complexities and imprecisions To our knowledge, this is the first attempt to use the H^∞ paradigm in the context DTNs. The problem presented here is not directly one of security, but the solution approach is well adapted to many potential security problems.

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