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The wireless multicast coalition game and the non-cooperative association problem

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***Abstract*—We study in this paper the problem of sharing the cost of a multicast service in a wireless network. In a wireless network, multiple users can decode the same signal of the base station provided the received power exceeds a certain minimum threshold. In this work, the cost for broadcasting is taken to be the transmission power. We begin by proposing various schemes to share the cost, and study their properties. We then study the association problem where an user has options of either joining the multicast group or opting for a unicast connection at a given cost. Next, we extend the association problem to the scenarios with partial information - a user knows his own power requirement, but has to make decision without knowledge of the number of other users in the network and their requirements. The unicast alternative that each mobile has, results in limitations on the coverage (area covered by the multicast service) and the capacity (number of users connected to the multicast service). We derive the expected capacity and coverage as a function of the cost sharing mechanism. We finally extend the model to the case where users have the option of joining any one from a given set of multicast service providers. A user's power requirement depends on its association, but its cost share depends on the association profile of all the users. We study the joint problem of the cost allocation and the equilibrium association.**

I. Introduction

A. The Multicast Problem

During the last few years, there has been a growing interest in wireless networks. Many emerging applications such as mobile TV and group oriented mobile commerce aim to deliver the same large volume of data to multiple users in the network. Such applications are naturally amenable to multicast transmission, and largely benefit from the broadcast nature of radio channels. More precisely, in wireless networks a base station (BS) can deliver the multicast data to multiple users only through a single transmission. However, satisfactory reception of applications, measured in term of the quality of service (QoS), is contingent on the BS's transmission power. Each user, depending on its location (with respect to the BS), channel condition and QoS requirement, puts a requirement on the transmission power of the BS. The BS's minimum transmission power depends on the requirements of all the users availing the multicast service. In particular, it corresponds to the largest requirement from among all the users in the group.

The multicast transmission incurs a cost to the service provider that crucially depends on the BS's transmission power. Multicasting data to a large population is likely to incur significant costs that should naturally be shared among all the users. However, a user's cost share should be commensurate to its requirement in terms of the BS's transmission power. We consider the problem of sharing the cost of multicast in a wireless network that consists of selfish users.

Often, users can also use dedicated (unicast) connections for the desired service. While the cost share in the multicast group depends on other users' requirements and decisions (whether they join the multicast group or not) and the cost sharing mechanism, the dedicated option incurs a fixed cost. Clearly, the former cost may not be known a-priori. However, users need to make subscription decisions based on their expected cost shares. We study the users' association problem under each of our proposed cost sharing mechanisms. We consider several setups with different information structures.

Evidently, multicast economizes the usage of the resources (e.g., bandwidth, power) in scenarios where multiple users are interested in the same content. Thus, the size of the multicast group can be viewed as a social performance measure of the cost sharing mechanism. We discuss the notions of *capacity* and *coverage* in this context.

B. Related Work

The problem of sharing the cost of multicast transmission in wired networks is well studied (e.g., see [1], [2], [3]). Penna and Ventre [4] and Bilo et al. [5] study the problem in the context of wireless networks. They consider the possibility that users can misreport their utilities. They obtain strategy-proof cost allocations which are either efficient or budget-balanced. In another paper, Bilo et al. [6] consider multicast in a multihop wireless network and model the selfish nature of users as a noncooperative game. They investigate Nash equilibria (NEs) for several cost allocation methods and also study price of anarchy bounds in a few cases. In all these works, the service provider, and not the users, determine which users will be served.

The cost structure in our problem is identical to that proposed by Littlechild and Owen [7], [8] in the context of *Aircraft landing fees*. Thomson [9] provides a survey on cost allocation for the airport problem.

Myerson [10] developed the theory of large Poisson games, and in particular, proved the existence of equilibria in such games. Under the setup there, utility of a player depends on the aggregate action profile of the whole population, and not on the type-wise action profile. In our setup the cost of a user depends on the type-wise action profile of the population where the type of a user can be identified as its location.

C. Our Contribution

In Section II we briefly describe the concept of cooperative cost games and also discuss a few cost allocations belonging to the core of the game. We present our network and communication model in Section III. We address the problem of splitting the cost of establishing the multicast session among the users. We assume that a user or a set of users that are not satisfied with the way the cost is split, can form an alternative multicast group that would incur less cost to all of these users. Thus we look for the cost sharing rules that make it advantageous for all users to be in one grand multicast group. We formulate the problem as a cooperative cost game and propose a number of cost sharing rules belonging to the core of the game.

We next consider that each user can avail the same service via an alternative unicast connection at a constant cost. Then, depending on the eventual cost share, users may or may not join the multicast session. One may view this problem as a hierarchical non-cooperative association problem - given the service provider's rule for splitting the cost among the users in the multicast group, each user has to decide whether to join the multicast group or opt for the dedicated unicast connection. In Sections IV we study the user association problem under each of the proposed cost sharing mechanisms. We extend our study to scenarios with incomplete or no information in Sections V and VI. We discover a paradoxical behavior in which the multicast user-base improves by providing less information to the users.

In Section VII we study the impact of the cost sharing rule on the number of users in the multicast group (which we call *capacity*), and on the geographical size of the multicast group (which we call *coverage*).[1] Finally, we study the association problem in presence of several multicast service providers. The proofs of a few of the results are omitted for brevity.

II. Cooperative Game Preliminaries

We begin by defining a cooperative cost game [12]. A cooperative cost game is a pair $(\mathcal{M}, c)$ where $\mathcal{M} := \{1, \ldots, M\}$ denotes the set of players and $c : 2^{\mathcal{M}} \to \mathbb{R}$ is the cost function. For any nonempty coalition $\mathcal{S} \subseteq \mathcal{M}$, $c(\mathcal{S})$ is the minimum cost incurred if players in $\mathcal{S}$ work together to serve their purposes; $c(\emptyset) := 0$. The cost function, $c(\cdot)$, is called submodular if

$$c(S_1 \cup S_2) + c(S_1 \cap S_2) \leq c(S_1) + c(S_2),$$

for all $\mathcal{S}_1, \mathcal{S}_2 \subseteq \mathcal{M}$. A cooperative game is called *concave* if the cost function is submodular.

[1] One can formulate this problem as one with multiple coalition structures, as defined in [11, p. 44, section 3.8].

A cost allocation $\mathbf{q} \in \mathbb{R}^{\mathcal{M}}$ charges cost q_i to a player $i \in \mathcal{M}$. An allocation $\mathbf{q}$ is called *efficient* if $\sum_{i \in \mathcal{M}} q_i = c(\mathcal{M})$. An efficient allocation $\mathbf{q}$ is called an *imputation* if $q_i \leq c(\{i\})$ for all $i \in \mathcal{M}$.

The core: The core, $\mathcal{C}$, of the game is defined as follows

$$\mathcal{C} = \{\mathbf{q} \in \mathbb{R}^{\mathcal{M}} : \sum_{i \in \mathcal{M}} q_i = c(\mathcal{M}), \\ \sum_{i \in \mathcal{S}} q_i \leq c(\mathcal{S}),\ \forall\ \mathcal{S} \subset \mathcal{M}\} \quad (1)$$

The core of a concave cooperative game is nonempty [13].

Next we state a number of appealing rules for cost allocation. Each of these results in a cost vector that lies in the core.

- *Shapley value:* For any i, and $\mathcal{S} \subseteq \mathcal{M}$ such that $i \notin \mathcal{S}$, let $\Delta_i(\mathcal{S}) = c(\mathcal{S} \cup \{i\}) - c(\mathcal{S})$. The Shapley value is the cost allocation $\mathbf{q}$ for which

$$q_i = \frac{1}{M!} \sum_{U \in \mathcal{U}} \Delta_i(\mathcal{S}_i(U)), \quad (2)$$

where $\mathcal{U}$ is the set of all orderings of $\mathcal{M}$, and $\mathcal{S}_i(U)$ is the set of players preceding i in ordering U. The Shapley value of a concave cooperative game lies in the core [14, Chapter 14].
- *Nucleolus:* The *excess* of a coalition $\mathcal{S}$ under an imputation $\mathbf{q}$ is $e_S(\mathbf{q}) = \sum_{i \in \mathcal{S}} q_i - c(\mathcal{S})$; this is a measure of dissatisfaction of $\mathbf{S}$ under $\mathbf{q}$. Let $E(\mathbf{q}) = (e_S(\mathbf{q}), S \in 2^{\mathcal{M}})$ be the vector of excesses arranged in monotonically increasing order. The *nucleolus* is the set of imputations $\mathbf{q}$ for which the vector $E(\mathbf{q})$ is lexicographically minimal. The nucleolus is a singleton and belongs to the *core* whenever the latter is nonempty.
- *Egalitarian Allocation:* The egalitarian allocation for cooperative games was introduced by Dutta and Ray [15]. They showed that the egalitarian allocation is unique whenever it exists. The following characterization of the egalitarian allocation for concave cost games is due to Jain and Vazirani [16]. It is based on the notion of *Lorentz ordering*.

Definition 2.1: Let $\mathbf{q^1}, \mathbf{q^1} \in \mathbb{R}^{\mathcal{M}}$ be such that $q_1^1 \leq \cdots \leq q_M^1$, $q_1^2 \leq \cdots \leq q_M^2$ and $\sum_{i \in \mathcal{M}} q_i^1 = \sum_{i \in \mathcal{M}} q_i^2$. We say that $\mathbf{q^1}$ Lorentz dominates $\mathbf{q^2}$ if for all $1 \leq k \leq M$,

$$\sum_{i=1}^{k} q_i^1 = \sum_{i=1}^{k} q_i^2,$$

and the inequality is strict for at least one k.

Now, for $\mathbf{q} \in \mathbb{R}^{\mathcal{M}}$ define $I(\mathbf{q})$ to be the vector obtained by arranging the components of $\mathbf{q}$ in increasing order. Then, $\mathbf{q} \in \mathcal{C}$ is the egalitarian allocation if $I(\mathbf{q})$ Lorentz dominates $I(\mathbf{r})$ for all other cost allocations $\mathbf{r} \in \mathcal{C}$. For concave cost games, the egalitarian allocation always exists and lies in the core [15].

Remark 2.1: Max-min (or, min-max) fairness has been used in the networking community in the context of bandwidth allocation and routing. Jain and Vazirani [16] show that for concave cost games, the unique egalitarian allocation is also max-min fair and min-max fair allocation in $\mathcal{C}$.

III. System Model

A. Network and Communication Model

We consider a wireless network with a multicast transmission source (say, a BS) and a random number (say, N) of users. We assume that N is a Poisson random variable with mean λ. Users' locations are also random, and both the channel conditions as well as the required QoS levels vary from one user to another. User i requires transmission at power p_i in order to meet its QoS needs. The power requirements, p_is, are independently identically distributed (i.i.d.) random variables with distribution $G(p)$. For simplicity of exposition, we assume that this distribution has an associated well defined density function $g(\cdot)$ ($g(p) := G'(p)$) that has no mass at isolated points. [2]

Any subset $\mathcal{M}$ of users can subscribe for a multicast session. The BS then broadcasts information with the minimum power p that guarantees that all users in $\mathcal{M}$ receive the satisfactory levels of QoS. Evidently $p = \max\{p_i : i \in \mathcal{M}\}$. The BS incurs a cost $f(p)$ per unit of time when it transmits at power p; $f(\cdot)$ is a continuous increasing function of power. The cost $f(p)$ has to be shared by all the users in the multicast group.

Every user has an alternative option of using a dedicated connection based on some other technology. We assume that the alternative option incurs an identical cost V to every user.

Our analysis can be easily extended to more general power requirement distributions, or to the scenario where different users pay different costs for using the alternative option.

B. Cost Sharing Mechanisms

The cost sharing mechanisms discussed in this section apply to any given realization of the network. Thus, we assume a known set $\mathcal{N} = \{1, \ldots, N\}$ of users with power requirements $\mathbf{p} := (p_1, \ldots, p_N)$. We also use notation $\mathbf{p}_{\mathcal{M}} := (p_i, i \in \mathcal{M})$ for all $\mathcal{M} \subseteq \mathcal{N}$.

A cost sharing mechanism, $\mathbf{h} = (\mathbf{h}(\mathcal{M}), \mathcal{M} \subseteq \mathcal{N})$, specifies the cost shares of users in any set $\mathcal{M}$ that constitutes the multicast group. More precisely, $\mathbf{h}(\mathcal{M}, \cdot) : \mathbb{R}^{\mathcal{M}} \to \mathbb{R}^{\mathcal{M}}$, maps the vector of power requirements to the vector of cost shares of users in $\mathcal{M}$, i.e., $h_j(\mathcal{M}, \mathbf{p}_{\mathcal{M}})$ gives the cost share of user $j \in \mathcal{M}$. We study the cost sharing mechanisms that satisfy the following economical constraints.

budget-balancedness: A cost sharing mechanism $\mathbf{h}$ is called *budget-balanced* if users pay exactly the total cost of the service, i.e.,

$$\sum_{j \in \mathcal{M}} h_j(\mathcal{M}, \mathbf{p}_{\mathcal{M}}) = \max\{f(p_i) : i \in \mathcal{M}\}$$

for all $\mathcal{M} \subseteq \mathcal{N}$.

cross-monotonicity: A cost sharing mechanism is called *cross-monotonic* if each user's cost decreases as the service set expands. To be precise, consider a cost sharing rule $\mathbf{h}$. If a subset $\mathcal{M} \subset \mathcal{N}$ of users avail the multicast service, the resulting cost share vector is $\mathbf{h}(\mathcal{M}, \mathbf{p}_{\mathcal{M}}) \in \mathbb{R}^{\mathcal{M}}$. However, if another user $k \in \mathcal{N} \backslash \mathcal{M}$ joins the multicast group, the resulting cost vector becomes $\mathbf{h}(\mathcal{M} \cup \{k\}, \mathbf{p}_{\mathcal{M} \cup \{k\}}) \in \mathbb{R}^{\mathcal{M} \cup \{k\}}$. The rule $\mathbf{h}$ is cross-monotonic if $h_i(\mathcal{M} \cup \{k\}, \mathbf{p}_{\mathcal{M} \cup \{k\}}) \leq h_i(\mathcal{M}, \mathbf{p}_{\mathcal{M}})$ for all $i \in \mathcal{M}$, $k \in \mathcal{N} \backslash \mathcal{M}$ and $\mathcal{M} \subset \mathcal{N}$.

Remark 3.1: A cost sharing mechanism is called *strategy-proof* if revealing true utilities is a dominant strategy for each user. For any cost allocation scheme *strategy-proofness* is a desirable feature (e.g., see [5]). However, in the cost sharing problem studied here, an user's utility can be assumed to be equal to the dedicated connection's cost which is known.

Now, suppose that a subset $\mathcal{M} = \{1, \ldots, M\}$ of users join the multicast session. Without loss of generality, we assume that users are indexed such that $p_1 < \cdots < p_M$. Thus, the BS transmits with power p_M and incurs a cost $f(p_M)$. The cost sharing problem can be formulated as a cooperative cost game $(\mathcal{M}, c)$. Here, $c : 2^{\mathcal{M}} \to \mathbb{R}$, for a coalition $\mathcal{S} \subseteq \mathcal{M}$, gives the cost to support communication to all the users in $\mathcal{S}$, i.e., $c(\mathcal{S}) = \max\{f(p_i) : i \in \mathcal{S}\}$.

Now consider two coalitions $S_1, S_2 \subseteq \mathcal{M}$. Observe that

$$\begin{aligned} c(S_1 \cup S_2) &= \max\{c(S_1), c(S_2)\} \\ \text{and } c(S_1 \cap S_2) &\leq \min\{c(S_1), c(S_2)\}. \end{aligned}$$

Hence,

$$c(S_1 \cup S_2) + c(S_1 \cap S_2) \leq c(S_1) + c(S_2), \tag{3}$$

i.e., the cost function is *submodular*. This implies the following results.

Theorem 3.1: (i) The core of the cost allocation game is nonempty.
(ii) The Shapley value lies in the core.
(iii) The egalitarian allocation lies in the core and is min-max (also max-min) fair.

The core of the multicast game can be expressed as

$$\Big\{\mathbf{q} \in \mathbb{R}^{\mathcal{M}} : \sum_{i \in \mathcal{M}} q_i = f(p_M), \sum_{i \in \mathcal{S}} q_i \leq f(p_{\bar{S}}), \mathcal{S} \subset \mathcal{M}\Big\}$$

where $\bar{S} := \max\{i : i \in \mathcal{S}\}$. We make the following observations.

1) All the cost allocations in the core are nonnegative; if $q_i < 0$, $\mathbf{q}$ can not satisfy the constraint corresponding to the subset $\mathcal{N} \backslash \{i\}$.
2) The constraint $\sum_{i=1}^{j} q_i \leq f(p_j)$ makes the constraints corresponding to the subsets $S \subset \{1, \ldots, j\}$ redundant.

In view of these, the core can be rewritten as

$$\Big\{\mathbf{q} \in \mathbb{R}_+^{\mathcal{M}} : \sum_{i=1}^{M} q_i = f(p_M), \sum_{i=1}^{j} q_i \leq f(p_j), 1 \leq j < M\Big\}.$$

A budget-balanced cost sharing mechanism is cross-monotonic only if it belongs to the core of the associated cooperative cost game. Hence we focus on cost allocations from the core. Following criteria can be used.

- *Highest cost allocation (HCA):* The user requiring the highest power (in our case, user M) pays the whole cost. Of course $(0, \ldots, 0, f(p_M))$ is in the core.

[2] Consequently, in almost all the network realizations, the power requirements are different for different users, i.e., $p_i \neq p_j$ if $i \neq j$. We consider only such realizations in Sections III-B and IV.

• *Incremental cost allocation (ICA):* A more fair cost allocation is where user 1 pays $f(p_1)$ and for $i \geq 2$, user i pays $f(p_i) - f(p_{i-1})$. We call it incremental cost allocation.

• *Shapley value (SV):* Following [7], the Shapley value is given as follows

$$h_i(\mathcal{M}, \mathbf{p}_{\mathcal{M}}) = \sum_{j=1}^{i} \frac{f(p_j) - f(p_{j-1})}{M+1-j}, 1 \leq i \leq M.$$

• *Nucleolus (NS):* The following algorithm for calculating the nucleolus was given by Littlechild [8] in the context of the *airport cost game*.

Define $i_0 = r_0 = 0$. For $k \geq 1$, iteratively define

$$r_k = \min_{i \in \{i_{k-1}+1, \dots, M-1\}} \frac{f(p_i) - f(p_{i_{k-1}}) + r_{k-1}}{i - i_{k-1} + 1},$$

and i_k as the largest value of i for which the minimum is attained in the above expression. Continue this until $k = k'$ where $i_{k'} = M - 1$. The nucleolus of the multicast game is given as

$$\begin{aligned} h_i(\mathcal{M}, \mathbf{p}_{\mathcal{M}}) &= r_k, \ i_{k-1} < i \leq i_k, \ k = 1, \dots, k' \\ h_M(\mathcal{M}, \mathbf{p}_{\mathcal{M}}) &= f(p_M) - f(p_{M-1}) + r_{k'} \end{aligned}$$

• *Egalitarian allocation (EA):* The egalitarian allocation for the multicast cost game can be computed by applying the following algorithm [17].

Define $i_0 = r_0 = 0$. For $k \geq 1$, iteratively define

$$r_k = \min_{i \in \{i_{k-1}+1, \dots, M\}} \frac{f(p_i) - f(p_{i_{k-1}})}{i - i_{k-1}},$$

and i_k as the largest value of i for which the minimum is attained in the above expression. Continue this until $k = k'$ where $i_{k'} = M$. The egalitarian allocation is given as

$$h_i(\mathcal{M}, \mathbf{p}_{\mathcal{M}}) = r_k, \ i_{k-1} < i \leq i_k, \ k = 1, \dots, k'$$

All the proposed cost sharing mechanisms are budget-balanced by their definitions.

Cross-monotonicity of the proposed cost allocations: Evidently HCA and ICA are cross-monotonic. The Shapley value is well known to be cross-monotonic (see Moulin [18]). Dutta [19] showed that the egalitarian allocation is also cross-monotonic in concave games. Sonmez [20] showed that the nucleolus of a generic concave cost game need not be cross-monotonic. However, he also proved the cross-monotonicity of nucleolus for the airport game which has identical formulation as ours.

In the rest of this section, we present a few monotonicity properties of the proposed cost sharing mechanisms.

Lemma 3.1: Under cost sharing mechanisms HCA, SV, NS and EA, for any two users i, j such that $p_j > p_i$, $q_j \geq q_i$.

Next, we investigate how, under the proposed cost sharing mechanisms, an user's cost share varies with its power requirement if all other users' requirements are kept fixed. Towards this, let us consider a tagged user i. Assuming other users' power requirements constant, we define a function $\hat{q}_i : p_i \mapsto q_i$ as follows

$$\hat{q}_i(p_i) = h_i(\mathcal{M}, \mathbf{p}_{\mathcal{M}}).$$

Lemma 3.2: Under cost sharing mechanisms HCA, SV, NS and EA, $\hat{q}_i$ is a monotone increasing function.

Proof:

• *HCA:* Clearly the claim is true for HCA.

• *SV:* Let us consider user i with required power p_i. Assume that its power requirement is increased to p_i'. If $p_i' \leq p_{i+1}$ then the cost share increases by

$$\hat{q}(p_i') - \hat{h}(p_i) = \frac{f(p_i') - f(p_i)}{M+1-i}.$$

Let us consider the case when $p_{i+1} < p_i' < p_{i+2}$. Other cases can be analyzed with a repeated application of this procedure. Now, the new cost share of player i is

$$\begin{aligned} \hat{q}(p_i') &= \sum_{j=1}^{i-1} \frac{f(p_j) - f(p_{j-1})}{M+1-j} \\ &\quad + \frac{f(p_{i+1}) - f(p_{i-1})}{M+1-i} + \frac{f(p_i') - f(p_{i+1})}{M-i} \\ &\geq \sum_{j=1}^{i} \frac{f(p_j) - f(p_{j-1})}{M+1-j} \\ &= \hat{q}(p_i) \end{aligned}$$

• *EA:* Let us revisit the algorithm used to obtain EA and assume $i = i_k$ for some k. Suppose user i increases its power requirement to p_i'. As before, first consider the case when $p_i' \leq p_{i+1}$. Clearly, $\hat{q}(p_i') \geq \hat{q}(p_i)$ in this case. The same holds true if $p_{i+1} < p_i' < p_{i+2}$. Similar arguments can be made in the case when $i \neq i_k$ for any k.

• *NS:* The proof is similar to that for EA.

■

The expression for the nucleolus has the similar form as that for the egalitarian allocation. Hence, in the following we analyze the egalitarian allocation but do not discuss the nucleolus.

IV. Non-cooperative Subscription Game: Complete Information

Each user independently decides whether to join the multicast group or not. Recall that a player bears a cost V if it does not use the multicast service and chooses the alternative dedicated option. We formulate the decision problem as a noncooperative game with users as players.

In this section, we assume that each user knows the total number of users in the network, N, and their power requirements, $\mathbf{p} = (p_1, \dots, p_N)$. An equilibrium is a multicast group $\mathcal{M} \subseteq \mathcal{N}$ of users such that the cost share of each user in $\mathcal{M}$ is less than or equal to V, and the cost share of each user in $\mathcal{N} \backslash \mathcal{M}$ would be larger than V if it joined the multicast group $\mathcal{M}$. Following is the precise definition.

Definition 4.1: An equilibrium is a multicast group $\mathcal{M} \subseteq \mathcal{N}$ of users such that $h_i(\mathcal{M}, p_{\mathcal{M}}) \leq V$ for all $i \in \mathcal{M}$, and $h_i(\mathcal{M} \cup \{i\}, p_{\mathcal{M} \cup \{i\}}) > V$ for all $i \in \mathcal{N} \backslash \mathcal{M}$.

We next provide the characterization of equilibria (NEs) corresponding to the proposed cost sharing policies. These are in fact *strong equilibria* - these are robust not only to unilateral deviations but also to deviations by subsets of users.

- *HCA:* The unique NE is $\mathcal{M} = \{i : f(p_i) \leq V\}$.
- *ICA:* The unique NE is

$$\mathcal{M} = \begin{cases} \emptyset & \text{if } p_1 > V, \\ \{1, \ldots, j-1\} & \text{otherwise,} \end{cases} \tag{4}$$

where $j = \min\{i : f(p_i) - f(p_{i-1}) > V\}$.

- *SV:* Let us define

$$b_j = \sum_{i=1}^{j} \frac{f(p_i) - f(p_{i-1})}{j+1-i},$$

Then the unique NE is $\mathcal{M} = \{1, \ldots, j\}$ where j is the largest index such that $b_j \leq V$.

- *EA:* Define $i_0 = r_0 = 0$. For $k \geq 1$, iteratively define

$$r_k = \min_{i \in \{i_{k-1}+1, \ldots, N\}} \frac{f(p_i) - f(p_{i_{k-1}})}{i - i_{k-1}},$$

and i_k as the largest value of i for which the minimum is attained in the above expression. Continue this until $r_k > V$. The unique NE is $\mathcal{M} = \{1, \ldots, i_{k-1}\}$.

Efficiency of the proposed cost allocations: An *efficient* cost sharing mechanism is one that maximizes the net social utility, i.e., minimizes the social disutility. We define the social disutility to be the aggregate cost paid by all the users in the network. Assume that, under a cost sharing mechanism, users $\mathcal{M} = \{1, \ldots, M\}$ opt for the multicast service (out of the total N users). Then, the total cost paid by the users in the multicast group is $f(p_M)$, and the total cost of all the users is $f(p_M) + (N - M)V$. The most efficient rule is the one that minimizes this quantity. Also recall that a user joins the multicast group only if its cost share is less than or equal to V. Thus, in our model, an efficient cost sharing mechanism is the one that maximizes the size of the multicast group. It is an straight-forward observation that HCA is the least efficient among all the proposed rules.

V. Non-cooperative Subscription Game: Incomplete Information

Here, we assume that any user does not know the number of other users in the network and their power requirements. In other words, all the users know only their own requirements. However, they know the distribution of the number of users in the network (Poisson(λ)) and also the distribution of users' power requirements ($G(p)$). Under this setup, we investigate the equilibrium strategies of the users.

We restrict ourselves to symmetric action profiles - players' actions depend on their identities only through their power requirements.[3] Since the number of players is Poisson distributed, we have an *environmental equivalence* property [10] - from the perspective of any player also, the number of other players in the game is a Poisson random variable with the same mean λ.

The above arguments imply that a pure strategy equilibrium is characterized by a set such that users with power requirements in that set join the multicast group and others do not. Let $P \subset \mathbb{R}_+$ be such that users with power requirements in P have joined the multicast group. Consider user i with power requirement p_i. Let ω_{-i} denote a realization of user i's environment which consists of other users in the networks and their power requirements. Adding i and its power requirement to ω_{-i} yields a realization of the whole network; we call it ω. We express the cost share of user i (assuming it joins the multicast group) as a function of its power requirement, P (the strategy of other users) and the realization of its environment. More precisely, we define

$$\bar{q}_i(p_i, P, \omega_{-i}) = h_i\left(\mathcal{M}_P(\omega) \cup \{i\}, p_{\mathcal{M}_P \cup \{i\}}(\omega)\right)$$

where $\mathcal{M}_P(\omega) = \{i \in \mathcal{N}(\omega) : p_i(\omega) \in P\}$. Then, the expected cost share of user i is $\mathbb{E}\bar{q}_i(p_i, P)$ where the expectation is taken over all possible ω_{-i}s.[4]

Definition 5.1: In the multicast game with population uncertainty, an NE is characterized by a set $P \subset \mathbb{R}_+$ such that any user i with power requirement $p_i \in P$ joins the multicast group and others do not . More precisely, $\mathbb{E}\bar{q}_i(p_i, P) \leq V$ if $p_i \in P$, and $\mathbb{E}\bar{q}_i(p_i, P) > V$ if $p_i \notin P$.

Following Lemma 3.2, $\bar{q}_i(p_i, P, \omega_{-i})$ is increasing in p_i for fixed P and ω_{-i}. Averaging over all possible ω_{-i}, we find that $\mathbb{E}\bar{q}_i(p_i, P)$ is increasing in p_i for any fixed P whence we obtain the following corollary.

Corollary 5.1: Under cost sharing mechanisms HCA, SV and EA, for any symmetric strategy P, there exists a threshold p^* such that $\mathbb{E}\bar{q}_i(p_i, P) \leq V$ if and only if $p_i \leq p^*$.

Evidently, any best response strategy is of the form $[0, p^*]$ or $[0, \infty)$. Thus, we also conclude the following.

Corollary 5.2: Under cost sharing mechanisms HCA, SV and EA, the only candidates for NEs are the sets of the form $[0, p^*]$ or $[0, \infty)$.

The following proposition asserts that under the cost sharing mechanism ICA also, NEs exhibit identical property.

Proposition 5.1: Under the cost sharing mechanism ICA, the only candidates for NEs are the sets of the form $[0, p^*]$ or $[0, \infty)$.

Proof: We prove the claim via contradiction. Assume that $P = \cup_{k=1}^{K} [a_k, b_k]$ is an NE where $b_{k-1} < a_k$ for all k ($b_0 := 0$). For $0 \leq p_i \leq a_1$, user i's expected cost $f(p_i)$ will be increasing in p_i. For $b_1 \leq p_i \leq a_2$, user i's expected cost $\mathbb{E}\bar{q}_i(p_i, P)$ is

$$f(p_i) - \int_{a_1}^{b_1} \lambda f(p) g(p) \exp\Big(-\lambda \int_p^{b_1} g(s)\ ds\Big)\ dp.$$

In writing the above expression we have used the *decomposition property* of Poisson distribution - the number of users with power requirements in the range $[p, b_1]$ is a Poisson random variable with rate $\lambda \int_p^{b_1} g(s)\ ds$. Finally, it is seen that the

[3]In the multicast game with population uncertainty, players with same requirements have no commonly known attributes by which others can distinguish them. Also, all players with the same requirement must have the same predicted behavior. See Myerson [10] for a detailed discussion.

[4]For brevity, we omit the third argument ω_{-i} of $\bar{q}_i(p_i, P, \omega_{-i})$.

above expression is increasing in p_i. Similarly, it can be shown that $\mathbb{E}\bar{q}_i(p_i, P), b_{k-1} \le p_i \le a_k$, is increasing in p_i for all $1 \le k \le K$. Hence if all other users are using the strategy P, user i's best response can not be P. Thus, the only candidates for symmetric NEs are the threshold strategies $[0, p^*]$.

On the other hand, if $\mathbb{E}\bar{q}_i(p_i, [0, \infty)) \le V$ for all p_i then $[0, \infty)$ is also an NE. ■

A. Expressions for the NEs

We have shown that under each of the proposed cost sharing mechanisms the NEs are characterized by certain thresholds. In this section we derive expressions for the thresholds.

Theorem 5.1: Under cost sharing mechanisms HCA, SV and EA, a symmetric multi-strategy $[0, p^*]$ is an NE if and only if $\mathbb{E}\bar{q}_i(p^*, [0, p^*]) = V$. If $\mathbb{E}\bar{q}_i(p_i, [0, \infty)) \le V$ for all p_i, $[0, \infty)$ is also an NE.

Proof: Suppose $\mathbb{E}\bar{q}_i(p^*, [0, p^*]) = V$. Then, from Corollary 5.2, $\mathbb{E}\bar{q}_i(p_i, [0, p^*]) \le V$ for all $p_i \le p^*$. Also, $\mathbb{E}\bar{q}_i(p_i, [0, p^*]) > V$ for all $p_i > p^*$. Thus $[0, p^*]$ is indeed an NE.

Now, assume the $\mathbb{E}\bar{q}_i(p^*, [0, p^*]) > V$. Consider user i with $p_i = p^*$. Then $[0, p^*]$ can not be an equilibrium strategy of user i. Finally, assume that $\mathbb{E}\bar{q}_i(p^*, [0, p^*]) < V$ and denote $\epsilon := V - \mathbb{E}\bar{q}_i(p^*, [0, p^*])$. Since $\mathbb{E}\bar{q}_i(p_i, [0, p^*])$ is continuous and increasing in p_i for $p_i \ge p^*$, there exists a $\delta > 0$ such that $\mathbb{E}\bar{q}_i(p_i, [0, p^*]) \le V$ for $p_i = p^* + \delta$. Thus $[0, p^*]$ can not be an equilibrium strategy of user i.

If $\mathbb{E}\bar{q}_i(p_i, [0, \infty)) \le V$, user i's best response is to join the multicast group given that all others have joined. Hence $[0, \infty)$ is also an NE. ■

Corollary 5.3: Under the cost sharing mechanism HCA $[0, f^{-1}(V)]$ is an NE. $[0, \infty)$ is also an NE provided $f(p_i) \le V$ for all p_i.

Proof: Under HCA $\mathbb{E}\bar{q}_i(p_i, [0, p_i]) = f(p_i)$. Since $f(\cdot)$ is strictly increasing, the unique solution to $\mathbb{E}\bar{q}_i(p^*, [0, p^*]) = V$ is $p_i = f^{-1}(V)$. The claim follows from Theorem 5.1. ■
We next discuss the cost sharing mechanism ICA. Consider user i with power requirement p_i. Let us assume that all other users join the multicast group. Again using the *decomposition property* of Poisson distribution, the expected cost of user i, $\mathbb{E}\bar{q}_i(p_i, [0, \infty))$, is

$$f(p_i) - \int_0^{p_i} \lambda f(p) g(p) \exp\Big(-\lambda \int_p^{p_i} g(s) ds\Big) dp.$$

Lemma 5.1: Under cost sharing mechanism ICA, $[0, p^*]$ is an NE if and only if $\mathbb{E}\bar{q}_i(p_i, [0, \infty)) \le V$ for all $p_i \le p^*$ and $\mathbb{E}\bar{q}_i(p^*, [0, \infty)) = V$. If $\mathbb{E}\bar{q}_i(p_i, [0, \infty)) \le V$ for all p_i, $[0, \infty)$ is also an NE.

Proof: if part: Recall that, under ICA, user i's cost share depends on only those users that have power requirements less than p_i. Hence $\mathbb{E}\bar{q}_i(p_i, [0, p])$ are same for all $p \ge p_i$. Now, from the conditions on p^*, $\mathbb{E}\bar{q}_i(p_i, [0, p^*]) \le V$ for all $0 \le p_i \le p^*$. Also, following the proof of Proposition 5.1, $\mathbb{E}\bar{q}_i(p_i, [0, p^*]) > V$ for all $p_i > p^*$. Hence $[0, p^*]$ is indeed an NE.

only if part: Consider a symmetric strategy $[0, p']$, and assume that there exists a $0 < p \le p'$ such that $\mathbb{E}\bar{q}_i(p, [0, p']) > V$. Clearly $[0, p']$ can not be an equilibrium strategy of user i. Finally consider the case when $[0, p']$ is a symmetric strategy of all the users while $\mathbb{E}\bar{q}_i(p', [0, \infty)) < V$. Denote $\epsilon := V - \mathbb{E}\bar{q}_i(p', [0, \infty))$. Since $f(p)$ is continuous and increasing, there exists a $\delta > 0$ such that $f(p' + \delta) - f(p') \le \epsilon$ implying $\mathbb{E}\bar{q}_i(p_i, [0, \infty)) \le V$ for $p_i = p' + \delta$. Thus $[0, p']$ can not be an equilibrium strategy of user i. ■

Numerical examples: To illustrate our findings, we consider a linear network topology. The BS is a origin, 0. The number of users is a Poisson random variable with mean 50. Users are uniformly and independently deployed on the line segment $[0, 100]$. Let x_i be the location of user i. We assume that the transmission power requirement of a user is equal to the square of its distance from the BS, i.e., $p_i = x_i^2$ for all i. Furthermore, we assume $f(p) := p$ for all $p \in \mathbb{R}_+$.

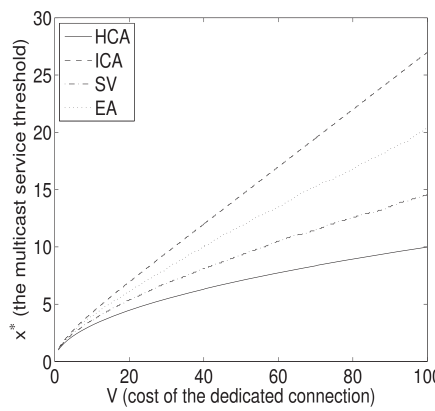

Fig. 1. Thresholds determining the multicast group as a function of the cost of the dedicated connection for various cost allocation schemes

In the figure, x^* is the threshold location for the multicast group: users in $[0, x^*]$ join the multicast service while the users out side this set opt for dedicated connections. We plot x^* as a function of V, the cost of the dedicated connection. The four curves are for the four different cost allocation schemes: HCA, ICA, SV and EA. As expected, thresholds increase as the cost of the dedicated connection increases. Moreover, the most number of users join the multicast group if ICA is in place, while the least number of users join if HCA is the underlying cost allocation scheme.

VI. Non-cooperative Subscription Game: No Information

As in Section V, we assume that any user does not know the number of other users in the network and their power requirements. To start with, we also assume that a user does not know even its own power requirement. To motivate the setup, consider mobile users that need to decide in advance whether to join the multicast session or not. Users do not know their exact future locations and channel conditions. However, all the users have the probability distributions of the number

N of users (Poisson(λ)). They also know the distribution of users' distances (from the BS) that are assumed to be i.i.d. and yield distribution $G(p)$ on the users' power requirements. As before, we formulate the multicast group subscription problem as a noncooperative game among users, and seek to obtain users' equilibrium strategies.

Let us focus on any one of the proposed cost sharing mechanisms (say, EA). Consider a tagged user, say i. If n other users join the multicast session, the expected cost share of user i is

$$q_i'(n) = \mathbb{E}\Big[h_i(\mathcal{M}(\omega), p_{\mathcal{M}}(\omega))\Big||\mathcal{M}(\omega)| = n+1\Big].$$

Here, we have averaged over all the network realizations, ω, that consist of $n+1$ users including i. Using the cross-monotonicity of the cost sharing mechanism and a coupling argument, it can be shown that $q_i'(n)$ is decreasing in n. Now, consider a symmetric multi-strategy where each user joins the multicast session with probability s. From user i's perspective, the number of other users in the multicast group will be Poisson distributed with mean $s\lambda$. Hence, unconditioning over the size of the multicast group, the expected cost share of user i will be

$$\tilde{q}_i(s) = \sum_{n=0}^{\infty} \frac{(s\lambda)^n \exp(-s\lambda) q_i'(n)}{n!}$$

Since the family of Poisson distributions, Poisson($s\lambda$), parametrized by $s \in [0,1]$ is stochastically increasing, $\tilde{q}_i(s)$ will be decreasing in s.

Lemma 6.1: 1. If $\tilde{q}_i(0) > V$ then 0 is an NE, a pure strategy equilibrium where none of the users joins the multicast group.
2. If $\tilde{q}_i(1) \leq V$ then 1 is an NE, a pure strategy equilibrium where all the users join the multicast group.
3. If $\tilde{q}_i(0) \geq V > \tilde{q}_i(1)$ the the symmetric multi-strategy s^* such that $\tilde{q}_i(s^*) = V$ is the unique mixed strategy NE.

In the rest of this section we restrict to HCA and discuss few more information structures.

A. Information on the Number of Users

Assume that the multicast service provider broadcasts, N, the number of users in the network. Consider a tagged user, say i. Let U be the random variable for i's cost, and $F(\cdot)$ be the distribution of the random cost. Since i's power requirement has distribution $G(\cdot)$, and the cost associated with power p is $f(p)$,

$$F(u) = G(f^{-1}(u)).$$

If n other users join the multicast session, the expected cost share of user i is

$$q_i'(n) = \frac{\int_0^\infty (1 - F^{n+1}(u))du}{n+1}.$$

Consider a symmetric multi-strategy where each user joins the multicast session with probability s. Then, the unconditional expected cost share of user i will be

$$\tilde{q}_i(s) = \sum_{n=0}^{N-1} \binom{N-1}{n} s^n (1-s)^{N-1-n} q_i'(n).$$

As in Lemma 6.1, the equilibrium strategy s is characterized by the solution of $\tilde{q}_i(s) = V$.

B. Some More Information on Power Requirements

We assume a little more information - the multicast source tells each user whether its required power is below or above V. It further broadcasts, N, the number of users having power requirements above $f^{-1}(V)$. Note that for the users with requirements below $f^{-1}(V)$ joining the multicast group is the dominant strategy. They also do not affect the costs of other N users. Hence we consider a noncooperative game with N users only. Consider one of these users, say i. The conditional distribution i's cost is

$$\tilde{F}(u) = \frac{F(u) - F(V)}{1 - F(V)}.$$

If n other users join the multicast session, the expected cost share of user i is

$$q_i'(n) = \frac{\int_V^\infty (1 - \tilde{F}^{n+1}(u))du}{n+1}.$$

Also consider a symmetric multi-strategy where each user joins with probability s. As before, the unconditional expected cost share of user i will be

$$\tilde{q}_i(s) = \sum_{n=0}^{N-1} \binom{N-1}{n} s^n (1-s)^{N-1-n} q_i'(n).$$

Again, the equilibrium policy s is characterized by the solution of $\tilde{q}_i(s) = V$. In particular, a symmetric multi-strategy s^* such that $\tilde{q}_i(s^*) = V$ is an NE.

Remark 6.1: 1. As expected $s^* = 0$ is an NE.
2. Observe that $\tilde{q}_i(1) = q_i'(N-1)$. Thus, if $q_i'(N-1) \leq V$, $s^* = 1$ is also an NE. Therefore, providing less information may potentially improve the multicast user base.

VII. Expected Capacity and Coverage

We consider the system model as described in Section III-A. We focus on the scenario in which all the users have complete information about the network. We also assume that all the users have identical QoS requirements, and thus their power requirements are entirely governed by their locations (more precisely, their distances from the BS). Recall that, every user has an alternative option of using a dedicated connection that incurs a fixed cost V. Let us fix the cost sharing mechanism. Let $\mathcal{S} \subseteq \mathcal{N}$ be such that assuming all the users in $\mathcal{S}$ join the multicast group none of them can benefit from leaving the multicast group,.i.e., the cost share for every user in $\mathcal{S}$ is less than or equal to V. We call such a set a "V-stable set". Assume that $\mathcal{S}$ is a maximal V-stable set, i.e., if we add to $\mathcal{S}$ any other user $i \in \mathcal{N} \backslash \mathcal{S}$, the new set $\mathcal{S} \cup \{i\}$ is not V-stable anymore[5].

We define the *capacity* associated with a cost sharing mechanism to be the size of the maximal V-stable set.

We define the *coverage* of a cost sharing mechanism to be the set of locations where if we place another user and add

[5]It can be easily shown that there will be a unique maximal V-stable set

this user to the maximal V-stable set then the new set will still be V-stable.

In the following, we compute the capacity and coverage of the cost sharing mechanisms HCA and ICA. We restrict ourselves to a linear network topology. More precisely, we assume that the BS is located at the origin and the users are located along a straight line according to a stationary Poisson point process with intensity λ. We also assume linear transmission cost for the BS, i.e., $f(p) = p$.

A. HCA: Capacity and Coverage

We enumerate the users according to the increasing distance from the BS. Define $x_0 := 0$ and let x_i be the location of the ith user. Now, consider a user at location x. If the BS transmits at power p_t then the received signal power at the tagged user will be $\beta p_t x^{-\alpha}$. If the user needs a minimum received power p in order to meet the desired OoS, the BS's transmission power needs to be $p_t = \frac{px^\alpha}{\beta}$ which is also the cost incurred by the BS. Now, recall the equilibrium analysis in Section IV. Evidently, the tagged user joins the multicast group if

$$\frac{px^\alpha}{\beta} \leq V$$
$$\text{or, } x \leq C_0^{1/\alpha}$$

where $C_0 := \frac{\beta V}{p}$. Thus, the capacity is a Poisson random variable with parameter $2\lambda C_0^{1/\alpha}$. In particular, the expected capacity is $2\lambda C_0^{1/\alpha}$. The coverage is the interval $[-C_0^{1/\alpha}, C_0^{1/\alpha}]$.

B. ICA: Capacity and Coverage

Recall the NE characterization with complete information under ICA (see (4)). Assume that there is a user at location x, and it participates in the multicast session. Now, consider another users at location $y > x$. The latter user will participate in the multicast session if

$$\begin{aligned} & \frac{py^\alpha}{\beta} - \frac{px^\alpha}{\beta} \leq V \\ \text{or,} \quad & \frac{py^\alpha}{\beta} \leq \frac{px^\alpha}{\beta} + V \\ \text{or,} \quad & y \leq (C_0 + x^\alpha)^{1/\alpha} \end{aligned} \tag{5}$$

where $C_0 := \frac{\beta V}{p}$. Evidently, the capacity for ICA is given by

$$M = \sup\{j : {x_i}^\alpha - x_{i-1}^\alpha \leq C_0 \text{ for all } i = 1, \ldots, j\}$$

where $\sup \emptyset := 0$. The coverage is given by

$$C = (C_0 + x_M^\alpha)^{1/\alpha}$$

Now, we derive the expected capacity and the expected coverage. Define $M(0) := M$ and

$$M(k) := \sup\{j : x_i^\alpha - x_{i-1}^\alpha \leq C_0 \text{ for all } i = k+1, \ldots, j\}.$$

Observe that $M(k)$ is the capacity conditioned on the coverage satisfying $C > x_k$. Define $C(k)$ as

$$C(k) := (C_0 + x_{M(k)}^\alpha)^{1/\alpha}$$

1) *The linear case:* $\alpha = 1$:

Lemma 7.1: If $\alpha = 1$ then
(i) $M(k) - k$ are identically distributed, $k = 0, 1, 2, ...$
(ii) $C(k) - x_k$ are identically distributed, $k = 0, 1, 2, ...$

The expected coverage C can be computed as follows. The location x_1 of the nearest user (to the BS) is exponentially distributed with parameter λ. With probability $\exp(-\lambda C_0)$ we have $x_1 > C_0$ and then $C = C_0$. With the complementary probability, $x_1 < C_0$ so that $C > x_1$. In that case, $C = C(1)$ where $C(1) - x_1$ has the same distribution as C (due to Lemma 7.1). We conclude that

$$\begin{aligned} \mathbb{E}[C] &= \mathbb{E}[C1_{\{x_1 > C_0\}}] + \mathbb{E}[(x_1 + C)1_{\{x_1 \leq C_0\}}] \\ &= C_0 \exp(-\lambda C_0) + \mathbb{E}[x_1 1_{\{X \leq C_0\}}] \\ &\quad + \mathbb{E}[C]\mathbb{E}[1_{\{x_1 \leq C_0\}}] \\ &= \frac{1 - \exp(-\lambda C_0)}{\lambda} + (1 - \exp(-\lambda C_0))\mathbb{E}[C] \end{aligned}$$

Thus the expected coverage is given by

$$\mathbb{E}[C] = \frac{1 - \exp(-\lambda C_0)}{\lambda \exp(-\lambda C_0)}$$

Similarly, the expected capacity can be seen to be

$$\mathbb{E}[N] = \frac{1 - \exp(-\lambda C_0)}{\exp(-\lambda C_0)}$$

2) $\alpha > 1$:

We observe that, for $\alpha_2 \geq \alpha_1 \geq 1$,

$$\Big(C_0 + a^{\alpha_2}\Big)^{1/\alpha_2} \leq \Big(C_0 + a^{\alpha_1}\Big)^{1/\alpha_1}$$

for all $a \geq 0$. A similar analysis as the one for $\alpha = 1$, shows that the expected capacity and the expected coverage reduce as α is increased. Thus, the linear case gives an upper bound on both.

VIII. Non-cooperative Subscription problem: Multiple Resources

A. Model

Finally, we consider a network with N users and M multicast sources (say, BSs) all of which provide the same multicast service. An user's power requirement depends upon which provider it associates with (say, owing to its different distances to different BSs). Assume that if user i joins the multicast session of provider j, then it requires a transmission with power p_{ij}. When provider j transmits with a power p, it incurs a cost $f_j(p)$. We assume that all the providers apply the same cost sharing mechanism which is HCA.

Users have choice to associate with any one of the providers. We define an *assignment* to be a partition $I = \{I_1, ..., I_M\}$ of the set of all users where I_j is set of users that join provider j. We view the association problem as a non-cooperative game and characterize an equilibrium assignment.

B. Min-max equilibrium

Let us consider an assignment I. We define $v_j(I)$ to be the power with which BS j needs to transmits; $v_j(I) = \max_{i \in I_j} p_{ij}, j = 1, \ldots, M$ ($v_j(I) := 0$ if $I_j = \emptyset$). Let $v^*(I)$ be the vector of transmission powers of BSs arranged in a decreasing order.

Definition 8.1: We call an assignment I a min-max assignment if $v^*(I)$ is lexicographically smaller than $v^*(I')$ for any other assignment I'.

Let $j(i)$ be the base station that is closest to user i. Then, player i can guarantee to pay a cost no more than $f_{j(i)}(p_{i,j(i)})$. Also, define $i(S)$ to be the user whose power requirement achieves the max-min cost among a set of users S. In other words, $i(S)$ is defined to be the user i' whose distance to the closest BS $j(i')$ is the largest among all the users in S:

$$i(S) = \arg\max_{i \in S} \min_j p_{ij}.$$

Consider the following algorithm A1:

1) Set $k = 1$ and let $\mathcal{N}_k$ be the set of all users.
2) Define $i_k = i(\mathcal{N}_k)$
3) Let I_k be the set of all users in $\mathcal{N}_k$ that are closer to BS $j(i_k)$ than i_k.
4) If the set is nonempty then increase k by one, define $\mathcal{N}_{k+1} = \mathcal{N}_k \setminus I_k$ and go to step 2.
5) Let I_j be the set set of users that connect to the multicast session j.

Theorem 8.1: Algorithm A1 determines a max-min equilibrium assignment I.

IX. Conclusion

We have addressed the problem of sharing the cost for a multicast session among the subscribers in a wireless network. We proposed various methods that had in common an stability aspect with the meaning that no sub-coalition can emerge availing the same service to the users at better price. With these basic building blocks at hand we then considered the association problem in which users decide, based on the cost sharing mechanism of the service provider, whether to join the multicast session or opt for a unicast alternative. We formulated the association problem as a non-cooperative game among users. Further, we extended the analysis to the cases where a user has incomplete or no information about the network topology, e.g., its own requirement, number of other users in the network and their requirements etc. Finally, we considered an association problem in presence of several multicast service providers; users take association decisions based on their eventual cost shares in different multicast groups. We give an algorithm that leads to a max-min equilibrium assignment.

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