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Stability–throughput tradeoff and routing in multi-hop wireless ad hoc networks

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Abstract

We study the throughput of multi-hop routes and stability of forwarding queues in a wireless ad-hoc network with random access channel. We focus on a wireless network with static nodes, such as community wireless networks. Our main result is characterization of stability condition and the end-to-end throughput using the balance rate. We also investigate the impact of routing on end-to-end throughput and stability of intermediate nodes. We show that (i) as long as the intermediate queues in the network are stable, the end-to-end throughput of a connection does not depend on the load on the intermediate nodes, (ii) we show that if the weight of a link originating from a node is set to the number of neighbors of this node, then shortest-path routing maximizes the minimum probability of end-to-end packet delivery in a network of weighted fair queues. Numerical results are given and support the results of the analysis. Finally, we perform extensive simulation and verify that the analytical results closely match the results obtained from simulations.

Keywords: Ad-hoc networks; Stability; Retransmission; Forwarding; Simulation; Weighted fair queues

1. Introduction

A multi-hop wireless ad-hoc network is a collection of nodes that communicate with each other without any established infrastructure or centralized control. Each of these nodes is a wireless transceiver that transmits and receives at a single frequency band which is common to all the nodes. These nodes can communicate with each other, however, they are limited by their transmitting and receiving capabilities. Therefore, they cannot directly reach all of the nodes in the network as most of the nodes are outside of direct range. In such a scenario, one of the possibilities for the information transmission between two nodes that are not in position to have a direct communication is to use other nodes in the network. To be precise, the source device transmits its information to one of the devices which is within

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transmission range of the source device. The network operates in a multi-hop fashion. Nodes route traffic for each other. Therefore, in a connected ad-hoc network, a packet can travel from any source to its destination either directly, or through some set of intermediate packet forwarding nodes.

Clearly, a judicious choice is required to decide on the set of devices to be used to assist in the communication between any two given pair of devices. This is the standard problem of routing in communication networks. The problem of optimal routing has been extensively studied in the context of wire-line networks where usually a shortest-path routing algorithm is used: Each link in the network has a weight associated with it and the objective of the routing algorithm is to find a path that achieves the minimum weight between two given nodes. Clearly, the outcome of such an algorithm depends on the assignment of the *weights* associated to each link in the network. In the wire-line context, there are many well-studied criteria to select these weights for links, such as delays. In the context of wireless ad-hoc networks, however, not many attempts have been made to (i) identify the characteristics of the quantities that one would like to associate to a *link* as its weight, and in particular (ii) to understand the resulting network performance and resource utilization (in particular, the stability region and the achievable throughput regions). Some simple heuristics have been frequently reported to improve performance of applications in mobile ad-hoc networks (see [12] and reference therein).

To study this problem, we consider in this paper the framework of random access mechanism for the wireless channel where the nodes having packets to transmit in their transmit buffers attempt transmissions by delaying the transmission by a random amount of time. This mechanism acts as a way to avoid collisions of transmissions of nearby nodes in the case where nodes cannot sense the channel while transmitting (hence, are not aware of other ongoing transmissions). We assume that time is slotted into fixed length time frames. In any slot, a node having a packet to be transmitted to one of its neighboring devices decides with some fixed (possibly node dependent) probability in favor of a transmission attempt. If there is no other transmission by the other devices whose transmission can interfere with the node under consideration, the transmission is successful. As examples of this mechanism, we find Aloha-type [14] and IEEE 802.11 CSMA/CA-based mechanism. With these mechanisms, each node determines its transmission times [2,24].

At any instant in time, a device may have two kinds of packets to be transmitted:

1. Packets generated by the device itself. This can be sensed data if we are considering a sensor network.
2. Packets from other neighboring devices that need to be *forwarded*.

Clearly, a device needs to have some scheduling policy to decide on which of these types it wants to transmit, given that it decided to transmit. Having a first come first served scheduling is one simple option. Yet another option is to have two separate queues for these two types and do a weighted fair queueing (WFQ) for these two queues. In this paper we consider the second option.

Working with the above mentioned system model, we study the impact of routing, channel access rates and weights of the weighted fair queueing on throughput, stability and fairness properties of the network.

It is worth mentioning that the above scenario may also be studied in the perspective of game theory in which case the nodes are assumed to be rational and need some incentive to forward data from other nodes. Two queues allow to model the selfish behavior or on the contrary to give higher priority to connections that traverse many hops that could otherwise suffer from large loss rates. Since ad-hoc networks do not have a centralized base-station that coordinates between them, an important question that has been addressed is to know whether we may indeed expect nodes to collaborate in such forwarding. Typically in such scenario, a Nash equilibrium determines the operating point (routing, channel access rates and WFQ weights). Thus, the results of this paper may be helpful in comparing various operating points based on criteria of throughput, stability and fairness in the cases where Nash equilibrium is not unique. Many papers in the literature, have studied the incentive for cooperation in ad-hoc networks, see [7–10]. Almost all previous papers, however, only considered utilities related to successful transmission of a node's packet to its neighbor. In practice, the utility function should depend on the forwarding behavior of all nodes (see [19]).

The main contribution of this paper is to provide approximation expressions of stability. Our main result is concerned with the stability of the forwarding queues at the devices. It states that whether or not the forwarding queues can be stabilized (by appropriate choice of WFQ weights) depends only on the routing and the channel access rates of the devices. Further, the weights of the WFQs play a role in determining the tradeoff between the power allocated for forwarding and the stability of its queue. The end-to-end throughput achieved by the nodes are independent of the choice of the WFQ weight.

1.1. Related literature

Several studies have focused on wireless network capacity and stability. The network capacity problem deals with finding the fundamental limits on achievable communication rates in wireless networks. The closure of the set of achievable rates is the capacity region of the network. In recent year, there has been a considerable effort on trying to compute the capacity region and improve the performance of wireless ad-hoc networks since Gupta and Kumar [6] showed that the capacity of a fixed wireless network decreases as the number of nodes increases. Grossglauser and Tse [5] presented a two-phase packet forwarding technique for mobile ad-hoc networks, utilizing the multiuser diversity, in which a source node transmits a packet to the destination when this destination becomes the closest neighbors of the relay. This scheme was shown to increase the capacity of the MANET, such that it remains constant as the number of users in the MANET increases.

The stability region is the closure of the set of arrivals rates at which the network can be stabilized. Stability depends both on the rate of packet arrivals and the rate of packet departure from the network. The network stability has been studied extensively both for networks with centralized scheduling [15,17] and the Aloha protocol [18,1,20,21]. Among the most studied stability problems are scheduling [15,16] as well as for the Aloha protocol [1,13,23]. Tassiulas and Ephremides [15] obtain a scheduling policy for the nodes that maximizes the stability region. Their approach inherently avoids collisions which allows to maximize the throughput. Radunovic and Le Boudec [3] suggest that considering the total throughput as a performance objective may not be a good objective. Moreover, most of the related studies do not consider the problem of forwarding and each flow is treated similarly (except for [3,11,22]). Our setting is different than the mentioned ones in the following: the number of transmissions is finite, and therefore, in our setting, the output and the input rates need not be the same.

1.2. Main contributions

The main contributions of this paper are (i) Providing a framework for cross-layer study of stability–throughput performance of ad-hoc networks. It has the flexibility for managing at each node forwarded packets and its own packets differently. (ii) Design routing in a way that stabilizes the system. (iii) The context of the stability that we study is new as it takes into account the possibility of a limited number of transmissions of a packet at each node after which it is dropped.

The paper is structured as follows. In Section 2, we present the cross-layer network model. In Section 3, the network stability and the performance are characterized by using a balance equation. The effect of routing and the stability condition of forwarding queues are introduced in Section 4. The validation of analytical results is done with a discrete time simulator in Section 5. Some special cases as linear networks are also studied in Section 6.

2. Network model

In this section, we describe the operation of the network in detail and introduce various quantities that determine the overall performance. We provide also the assumptions underlying this study and introduce appropriate notations.

2.1. Assumptions and definitions

Consider a wireless ad-hoc network consisting of N nodes (we allow $N = \infty$ to study some simple symmetric cases without boundary effects). When N is finite, we number the nodes using integers $1, \ldots, N$.

We assume the following:

- *Simple channel*: nodes use the same frequency for transmitting with an omni-directional antenna. A node j receives successfully a packet from a node i if and only if there is no interference at the node j due to another transmission. A node cannot receive and transmit at the same time.
- *Two types of queues:* two queues are associated with each node. The first one is the forward queue F_i (proper to the node i), which carries

all the packets originated from a given source and destined to a given destination. The second is Q_i which carries the proper packets of the node i (in this case $i \equiv s$ where s designates a source node). We assume that each node has an infinite capacity of storage for the two queues. Packets are served with a first in first served fashion. When F_i has a packet to be sent, the node chooses to send it from F_i with a probability f_i. In other terms, it chooses to send from Q_i with probability $1 - f_i$. When one of these queues is empty then we choose to send a packet from the non empty queue with probability 1. When node i decides to transmit from the queue Q_i, it sends a packet destined for node d, $d \neq i$, with probability $P_{i,d}$.

- *Saturated network*: each node has always packets to be sent from queue Q_i, whereas F_i can be empty. Consequently, the network is considered saturated and depends on the channel access mechanism.

This model allows us to define a neighborhood relation between any two nodes: node i is neighbor of node j if node i can receive transmission from node j. We use the function $A(\cdot,\cdot) : [1,N] \times [1,N] \to \{0,1\}$ to denote the neighborhood relation: $A(i,j) = 1$ if and only if i is neighbor of j. We assume that the (binary) neighborhood relation is symmetric, i.e., $A(i,j) = A(j,i)$. Let $\mathcal{N}(i)$ denote the nodes which are neighbors of node i, i.e., $\mathcal{N}(i) = \{j : A(j,i) = 1\}$.

2.2. Network layer

The network layer handles the two queues Q_i and F_i using the WFQ scheme, as described previously. Each node acts as a router, it permits to relay packets originated from a source s to a destination d. It must carries a routing information which permits sending of packets to a destination via a neighbor. In this paper, we assume that nodes form a static network where routes between any source s and destination d are invariant in the saturated network case. Proactive routing protocols as OLSR [4] (Optimized Link State Routing), construct and maintain a routing table that carries routes to all nodes on the network. These kind of protocols correspond well to our model. We use the notation $R_{s,d}$ to denote the set of nodes between a node s and d (s and d not included). Let $R_{i,s,d}$ be the set of nodes $R_{s,i} \bigcup i$.

2.3. MAC layer

We assume a channel access mechanism only based on a probability to access the network, i.e., when a node i has a packet to transmit from the queue Q_i or F_i, it accesses the channel with a probability P_i. It can be similar to CSMA/CA or any other mechanism to access the channel. For example, in IEEE 802.11 DCF, the transmission probability or attempt rate is given by [24]

$$P = \frac{2(1 - 2P_c)}{(1 - 2P_c)(CW_{\min} + 1) + P_c CW_{\min}(1 - (2P_c)^m)}, \tag{1}$$

where P_c is the conditional collision probability given that a transmission attempt is made, and $m = \log_2(\frac{CW_{\max}}{CW_{\min}})$ is the maximum of backoff stage.

The scheduler of transmission overall the network depends on P_i. We assume that each node is notified about the success or failure of its transmitted packets. A packet is failure only when there is a collision on the intended receiver. We assume that in any slot, a node having a packet to be transmitted to one of its neighboring nodes decides with some fixed (possibly node dependent) probability in favor of a transmission attempt. If there is no other transmission by the other nodes whose transmission can interfere with the node under consideration, the transmission is successful. We have considered previously infinite buffer size, therefore, there is no packet loss due to overflow at the queues. The only source of packet loss is due to collisions. For a reliable communication, we allow a limited number, of successive transmissions of a packet, after that it will be dropped definitively. We denote $K_{i,s,d}$ the maximum number of transmissions allowed for a packet of connection (s,d) at node i.

2.4. Cross-layer representation of the model

The model of Fig. 1 represents our model in this paper. The Network layer and MAC layer are clearly separated. Attempting the channel begins by choosing the queue from which a packet must be selected. And then, this packet is moved from the corresponding queue from the network layer to the MAC layer where it will be transmitted and retransmitted, if needed, until its success or drop. In this manner, when a packet is in the MAC layer, it is itself attempted successively until it is removed from the node.

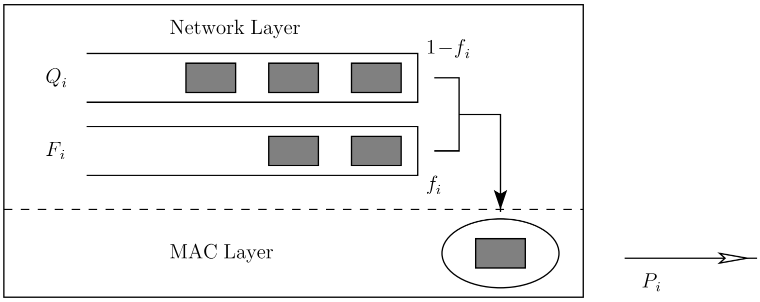


Fig. 1. Network layer and MAC layer of node i.

In this paper, we define the cycle as number of slots needed to transmit a single packet until its success or drop (see Fig. 2). We distinguish two types of cycles: The *forwarding cycles* in which a packet from queue F_i is transmitted during this cycle, and the *source cycles* in which a packet from queue Q_i is transmitted during this cycle. Also, each cycle is affected to a connection. The beginning of each cycle represents the choice of the queue from which we choose a packet and the choice of the connection where to send it. Whereas, the slots that constitute the cycle represents the attempts of the packet itself to the channel, including its retransmissions.

We need to define formally the model, so we will be able to derive some formulas in the next sections. For that, we consider the following counters:

- $C_{t,i}$ is the number of cycles of the node i, till the tth slot (including the tth slot).
- $C^F_{t,i}$ (resp. $C^Q_{t,i}$) is the number of all *forwarding cycles* (resp. *source cycles*) of the node i, till the tth slot (including the tth slot).
- $C^F_{t,i,s,d}$ (resp. $C^Q_{t,i,s,d}$) is the number of *forwarding cycles* (resp. *source cycles*) corresponding to the path $R_{s,d}$ of the node i till the tth slot (including the tth slot).

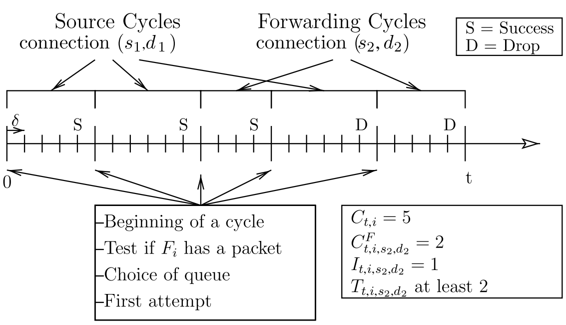


Fig. 2. Illustrative example of node i with cycles approach.

- $T_{t,i,s,d}$ is the number of times we found at the first slot of a cycle and at the first position in the queue F_i a packet for the path $R_{s,d}$ of the node i, till the tth slot (including the tth slot).
- $I_{t,i,s,d}$ is the number of cycles corresponding to the path $R_{s,d}$ of the node i, and where a cycle ends by a success of the transmitted packed, till the tth slot (including the tth slot).
- $A_{t,i,s,d}$ is the number of arrival packets to node i on the path $R_{s,d}$, till the tth slot (including the tth slot).

Fig. 2 shows a simple example with some numerical values of the previous counters for a single node i.

3. Stability properties of the forwarding queues

Our main objective in this section is to derive the rate balance equations from which some properties of the forwarding queues can be deduced. For that we need to write the departure rate from each node i and the end-to-end throughput between a couple of node.

From a practical point of view, each node owns three main parameters P_i, $K_{i,s,d}$ and f_i, that can be managed and set in such a way that each node can maintain stability, or the end-to-end throughput on a path can be optimized. In this paper, by fixing the routing paths (from $R_{s,d}$) and route choice (from $P_{s,d}$), we will obtain forwarding queue stability function of these three main parameters.

For a given routing, let π_i denote the probability that the queue F_i has at least one packet to be forwarded in the beginning of each cycle. $\pi_{i,s,d}$ is the probability that the queue F_i has a packet at the first position ready to be forwarded to the path $R_{s,d}$ in the beginning of each cycle. Let n_i be the number of neighboring nodes of node i.

Given a saturated network case where each node has a packet on its queue Q_i and attempts transmitting all the time to the channel, the forwarding queue F_i of each node will have a π_i load when it tries to forward packets to its neighbors.

3.1. The rate balance equations

The forwarding queue F_i is stable if the departure rate of packets from F_i is equal to the arrival rate into it. This is a simple definition of stability that can be written with a *rate balance equation*. In this paper, we are going to derive this equation for each node i using the cycle approach. In fact, it is judicious to write the rate balance equation of node i for each connection and then do the summation for all others.

For any given nodes i, s and d, let $j_{i,s,d}$ be the entry in the set $R_{s,d}$ just after i. It is possible that there is no such entry, i.e., node i is the last entry in the set $R_{s,d}$. In that case $j_{i,s,d} = d$. Let $P_{i,s,d} = \Pi_{j \in j_{i,s,d} \cup \mathcal{N}(j_{i,s,d}) \setminus i}(1 - P_j)$ be the probability that a transmission from node i on route from node s to node d is successful. Also, let

$$L_{i,s,d} = \sum_{l=1}^{K_{i,s,d}} l(1 - P_{i,s,d})^{l-1} P_{i,s,d} + K_{i,s,d}(1 - P_{i,s,d})^{K_{i,s,d}} = \frac{1 - (1 - P_{i,s,d})^{K_{i,s,d}}}{P_{i,s,d}} \quad (2)$$

be the expected number of attempts till success or consecutive $K_{i,s,d}$ failures of a packet from node i on route $R_{s,d}$.

The probability that a packet is removed from a node i by a successful transmission or a drop (i.e., a successive $K_{i,s,d}$ failure) is the departure rate from F_i. We denote it by d_i. The departure rate concerning only the packets sent on the path $R_{s,d}$ is denoted by $d_{i,s,d}$ which is given by the following lemma:

Lemma 3.1. *For any node i, s and d such that $P_{s,d} > 0$ and $i \in R_{s,d}$, the long term average rate of departure of packets from node i on route from node s to node d is $\frac{\pi_{i,s,d} P_i f_i}{\overline{L}_i}$. The total departure rate is given by*

$$d_i = \sum_{s,d: i \in R_{s,d}} d_{i,s,d} = \pi_i f_i \frac{P_i}{\overline{L}_i}. \quad (3)$$

Proof. For any node i, s and d such that $P_{s,d} > 0$ and $i \in R_{s,d}$, the long term departure rate of packets from node i on the route from s to d is

$$d_{i,s,d} = \lim_{t\to\infty} \frac{C^F_{t,i,s,d}}{t} = \lim_{t\to\infty} \frac{C^F_{t,i,s,d}}{T_{t,i,s,d}} \lim_{t\to\infty} \frac{T_{t,i,s,d}}{C_{t,i}} \cdot \lim_{t\to\infty} \frac{C_{t,i}}{t}, \quad (4)$$

- $\lim_{t\to\infty} \frac{T_{t,i,s,d}}{C_{t,i}}$ is exactly the probability that F_i carried a packet to the path $R_{s,d}$ in the beginning of each cycle. Therefore, $\lim_{t\to\infty} \frac{T_{t,i,s,d}}{C_{t,i}} = \pi_{i,s,d}$.
- $\lim_{t\to\infty} \frac{C^F_{t,i,s,d}}{T_{t,i,s,d}}$ is exactly the probability that we have chosen a packet from F_i to be sent when F_i carried a packet to the path $R_{s,d}$ in the first position and in the beginning of a forwarding cycle. Therefore, $\lim_{t\to\infty} \frac{C^F_{t,i,s,d}}{T_{t,i,s,d}} = f_i$.
- $\lim_{t\to\infty} \frac{t}{C_{t,i}}$ is the average length in slots of a cycle of the node i. A cycle length on the path $R_{s,d}$ is formed by the attempt slots that does not lead to a channel access and the transmission and retransmissions of the same packet until a success or a drop. Thus an average cycle length for a one path $R_{s,d}$ of a node i is given by $\frac{L_{i,s,d}}{P_i}$. When a node transmits to several paths, we need to know the average cycle length. This is given by $\frac{\overline{L}_i}{P_i}$ where $\overline{L}_i$ is the average of $L_{i,s,d}$s of these paths. $\overline{L}_i$ is given by

$$\overline{L}_i = \sum_{s,d: i \in R_{s,d}} \pi_{i,s,d} f_i L_{i,s,d} + \sum_d (1 - \pi_i f_i) P_{i,d} L_{i,i,d}. \quad (5)$$

Therefore, $\lim_{t\to\infty} \frac{C_{t,i}}{t} = \frac{P_i}{\overline{L}_i}$. Consequently, $d_{i,s,d} = \pi_{i,s,d} f_i \frac{P_i}{\overline{L}_i}$.

It is clear that the departure rate $d_{i,s,d}$ on a path $R_{s,d}$ of a node i does not depend on the parameters of only one path but it is also related to the expected number of transmissions to all other paths used by node i. This dependency appears in $\overline{L}_i$. Moreover, it is easy to derive the total departure rate d_i on all paths

$$d_i = \sum_{s,d: i \in R_{s,d}} d_{i,s,d} = \pi_i f_i \frac{P_i}{\overline{L}_i}. \quad \square \quad (6)$$

In the following lemma, we calculate the arrival rate on an intermediate node in ad-hoc networks. The probability that a packet arrives to the queue F_i of the node i, is denoted by a_i. When this rate concerns only packets sent on the path $R_{s,d}$, we denoted it by $a_{i,s,d}$.

Lemma 3.2. *For any fixed choice of nodes i, s and d such that $P_{s,d} > 0$ and $i \in R_{s,d}$, the long term average rate of arrival of packets into F_i for $R_{s,d}$ is*

$$a_{i,s,d} = (1-\pi_s f_s)\cdot P_{s,d}\cdot \frac{P_s}{\overline{L}_s}\cdot \left[(1-(1-P_{s,s,d})^{K_{s,s,d}}) \cdot \prod_{k\in R_{i,s,d}\setminus i}(1-(1-P_{k,s,d})^{K_{k,s,d}})\right]. \quad (7)$$

Proof. For any node i, s and d such that $P_{s,d} > 0$ and $i \in R_{s,d}$, the long term arrival rate of packets into F_i for $R_{s,d}$ is

$$a_{i,s,d} = \lim_{t\to\infty}\frac{A_{t,i,s,d}}{t} \quad (8)$$

$$= \lim_{t\to\infty}\frac{A_{t,i,s,d}}{I_{t,s,s,d}}\cdot \lim_{t\to\infty}\frac{I_{t,s,s,d}}{C^Q_{t,s,s,d}}\cdot \lim_{t\to\infty}\frac{C^Q_{t,s,s,d}}{C^Q_{t,s}} \cdot \lim_{t\to\infty}\frac{C^Q_{t,s}}{C_{t,s}}\cdot \lim_{t\to\infty}\frac{C_{t,s}}{t}. \quad (9)$$

- $\lim_{t\to\infty}\frac{C^Q_{t,s}}{C_{t,s}} = 1-\frac{C^F_{t,s}}{C_{t,s}} = 1-\pi_s f_s$, this is exactly the probability to get a source cycle, i.e., to send a packet from the queue Q_s.
- $\lim_{t\to\infty}\frac{C^Q_{t,s,s,d}}{C^Q_{t,s}}$ is the probability to choose the path $R_{s,d}$ to send a packet from Q_s. Therefore, $\lim_{t\to\infty}\frac{C^Q_{t,s,s,d}}{C^Q_{t,s}} = P_{s,d}$.
- $\lim_{t\to\infty}\frac{C_{t,s}}{t} = \frac{P_s}{\overline{L}_s}$.
- $\lim_{t\to\infty}\frac{I_{t,s,s,d}}{C^Q_{t,s,s,d}}$ is the probability that a source cycle on the path $R_{s,d}$ ends with a success, i.e., the packet sent from Q_s is received on the queue $F_{j_{s,s,d}}$. Therefore, $\lim_{t\to\infty}\frac{I_{t,s,s,d}}{C^Q_{t,s,s,d}} = (1-(1-P_{s,s,d})^{K_{s,s,d}})$.
- $\lim_{t\to\infty}\frac{A_{t,i,s,d}}{I_{t,s,s,d}}$ is the probability that a packet received on the node $j_{s,s,d}$ is also received on the queue F_i of the node i. For that, this packet needs to be received by all the nodes in the set $R_{i,s,d}$. Therefore, $\lim_{t\to\infty}\frac{A_{t,i,s,d}}{I_{t,s,s,d}} = \prod_{k\in R_{i,s,d}\setminus i}(1-(1-P_{k,s,d})^{K_{k,s,d}})$.

Consequently

$$a_{i,s,d} = (1-\pi_s f_s)\cdot P_{s,d}\cdot \frac{P_s}{\overline{L}_s}\cdot \left[(1-(1-P_{s,s,d})^{K_{s,s,d}}) \cdot \prod_{k\in R_{i,s,d}\setminus i}(1-(1-P_{k,s,d})^{K_{k,s,d}})\right]. \quad (10)$$

Remark that when the node i is the destination of a path $R_{s,d}$, then $a_{d,s,d}$ represents the end-to-end average throughput of a connection from s to d. Also, note that the global arrival rate is: $a_i = \sum_{s,d:i\in R_{s,d}} a_{i,s,d}$. □

Finally, in the steady state if all the queues in the network are stable, then for each i, s and d such that $i \in R_{s,d}$ we get $d_{i,s,d} = a_{i,s,d}$, which is the rate balance equation on the path $R_{s,d}$.

Theorem 3.1. *In the steady state, if all the queues in the network are stable, then for each i, s and d such that* $i \in R_{s,d}$

$$\frac{\pi_{i,s,d}P_i f_i}{\overline{L}_i} = (1-\pi_s f_s)\cdot P_{s,d}\cdot \frac{P_s}{\overline{L}_s}\cdot \left[(1-(1-P_{s,s,d})^{K_{s,s,d}}) \cdot \prod_{k\in R_{i,s,d}\setminus i}(1-(1-P_{k,s,d})^{K_{k,s,d}})\right]. \quad (11)$$

Let $y_i = 1-\pi_i f_i$ and $z_{i,s,d} = \pi_{i,s,d} f_i$. Thus $y_i = 1-\sum_{s,d:i\in R_{s,d}} z_{i,s,d}$. Then rate balance equation becomes

$$\sum_{d:i\in R_{s,d}} z_{i,s,d} = \frac{y_s\left(\sum_{s',d'} z_{i,s',d'}L_{i,s',d'} + \sum_{d''} y_i P_{i,d''} L_{i,i,d''}\right) w_{s,i}}{\left(\sum_{s',d'} z_{s,s',d'}L_{s,s',d'} + \sum_{d''} y_s P_{s,d''} L_{s,s,d''}\right)}, \quad (12)$$

where

$$w_{s,i} = \sum_{d:i\in R_{s,d}} \frac{P_{s,d}P_s}{P_i} \prod_{k\in R_{i,s,d}\cup s\setminus i}(1-(1-P_{k,s,d})^{K_{k,s,d}}). \quad (13)$$

3.2. Interpretation and applications

The relation of Eq. (12) has many interesting interpretation and applications. Some of these are:

- *The effect of f_i*: At the heart of all the following points is the observation that the quantities $z_{i,s,d}$ and y_i are *independent* of the choice of f_j, $1 \leqslant j \leqslant N$. It only depends on the routing and the value of P_j.
- *Stability*: Since the values of y_i are independent of the values of f_j, $j = 1,\ldots,N$, and since we need $\pi_i < 1$ for the forwarding queue of node i to be stable, we see that for any value of $f_i \in (1-y_i, 1)$, the forwarding queue of node i will be stable. Thus we obtain a lower bound on the weights given to the forwarding queues at each node in order to guarantee stability of these queues. To ensure that these lower bounds are all feasible, i.e., are less than 1, we need that $0 < y_i \leqslant 1$; $y_i = 0$ corresponds to the case where F_i is unstable. Hence, if the routing, $P_{s,d}$ and

P_js are such that all the y_i are in the interval $(0,1]$, then all the forwarding queues in the network *can be made stable by appropriate choice of* f_is. Now, since y_i is determined only by routing and the probabilities P_js and $P_{s,d}$, we can then *choose* f_i (thereby also fixing π_i, hence the forwarding delay) to satisfy some further optimization criteria so that this extra degree of freedom can be exploited effectively.

- *Throughput*: We see that the long term rate at which node s can serve its own data meant for destination d is $P_{s,d}P_s(1-\pi_s f_s) = P_{s,d}P_s y_s$ which is *independent of* f_s. The throughput, i.e., the rate at which data from node s reaches their destination d which is given by $a_{d,s,d}$, turns out to be independent of the choice of $f_j, 1 \leqslant j \leqslant N$. Similarly, the long term rate at which the packets from the forwarding queue at any node i are attempted transmission is $P_i\pi_i f_i = P_i(1-y_i)$, which is also independent of the choice of $f_j, 1 \leqslant j \leqslant N$.
- *Choice of* f_i: Assume that we restrict ourselves to the case where $f_i = f$ for all the nodes. Then, for stability of all the nodes we need that

  $$f > 1 - \min_i y_i.$$

  Since the length of the interval that f_i is allowed to take is equal to y_i, we will also refer to y_i as stability region.
- *Throughput–delay tradeoff*: For a given set of P_js, $P_{s,d}$ and routing, the throughput obtained by any route $R_{l,m}$ is fixed, independent of the forwarding probabilities f_i. Hence there is no *throughput–delay* tradeoff that can be obtained by changing the forwarding probabilities. A real tradeoff is caused by the maximum number of attempts: the throughput is ameliorated when reattempting many times on a path, while the service rate on a forwarding queue is slowed down causing low stability region and delay will be increased.
- *Energy consumption of forwarding packets*: Note that the value $\pi_i f_i E_r$ represents the energy consumption used by node i to forward the packets of other connections where E_r is the energy spent for transmission of one packet. This quantity turns out to be independent of the choice of f_i. Hence, the node can use f_i to improve the expected delay without affecting the energy consumption.
- *Per-route behavior*: Note that the above observations are based on the global rate balance equation for forwarding queue F_i of node i. Similar observations can be made when considering the detailed balance equation for queue F_i for some fixed source destination pair s,d such that $i \in R_{s,d}$.

3.3. Special cases

In this part, we discuss some special case when the system of equation (12) is linear. It can be obtained when for each node i $(0 \leqslant i \leqslant N)$, we have that $\overline{L}_i$ is independent from the unknowns y_i and $z_{i,s,d}$. In other terms, we need $\overline{L}_i = L_{i,s,d}$ for all $s,d : i \in R_{s,d}$ and for all $0 \leqslant i \leqslant N$. A symmetric network with $n_i = n$, $P_i = P$ and $K_{i,s,d} = K$ is an example of this case. In asymmetric network, this condition is satisfied when each node in the network, uses the same neighbor as a next hop to forward all its packets or $K_{i,s,d} = 1$ for all $0 \leqslant i \leqslant N$. Consequently, the system from Eq. (12) can be written as

$$1 - y_i = \sum_s y_s \bar{w}_{s,i}, \tag{14}$$

where

$$\bar{w}_{s,i} = \sum_{d:i\in R_{s,d}} \frac{P_s P_{s,d} P_{s,s,d} L_{i,s,d}}{P_i} \prod_{k\in R_{i,s,d}\setminus i} (1-(1-P_{k,s,d})^{K_{k,s,d}}). \tag{15}$$

Therefore, the system of equations (14) can be written in a matrix form as following and resolved easily:

$$\underline{y}(I + \overline{W}) = \underline{1}, \tag{16}$$

where $\overline{W}$ is an $N \times N$ matrix whose (s,i)th entry is $\bar{w}_{s,i}$ (independent on y_i) and $\underline{y}$ is an N-dimensional row vector.

3.4. Balance equations under unlimited attempts: $K_{i,s,d} \equiv \infty$

In this subsection, we consider an extreme case in which a node attempts forwarding of a packet until the transmission is successful. This case provides some further important observations while keeping the expressions simple. The detailed balance equation for queue F_i on route from node s to node d is

$$\pi_{i,s,d} f_i P_i / \overline{L}_i = P_{s,d} P_s (1 - \pi_s f_s) / \overline{L}_s. \tag{17}$$

By assuming that all nodes have same channel access rate $P_i = P, \forall i$, we have

$$\pi_i f_i = \sum_{s,d:i\in R_{s,d}} \frac{P_{s,d}(1-\pi_s f_s)\overline{L}_i}{\overline{L}_s}. \tag{18}$$

Observe that if a source has at most one destination, i.e, $P_{s,d} \in \{0,1\}$, and if the number of neighbor is same for all the nodes so that $L_{i,s,d} = L_{s,s,d}$, then the rate balance equations become

$$y_i + \sum_{s,d:i \in R_{s,d}} y_s = 1. \tag{19}$$

Thus

$$-\pi_i f_i + \sum_{s,d:i \in R_{s,d}} (1 - \pi_s f_s) = 0. \tag{20}$$

The above relation has many interesting interpretations/implications. Some of these are

Stability: if a node s' which is also a source for some destination d' does not forward packets of any other connection, i.e., if $\pi_{s'} = 0$ then for any $i \in R_{s',d'}$, the rate balance equation (20) becomes

$$\pi_i f_i = \sum_{s,d:i \in R_{s,d}, s \neq s'} (1 - \pi_s f_s) + 1 \tag{21}$$

implying that the forwarding queues of all the nodes in $R_{s',d'}$ are unstable since the above requirement requires $\pi_i \geqslant 1$ as f_i is bounded by 1. This implies that a *necessary condition for the forwarding queues in the network to be stable is that all the sources must also forward data*. This can have serious implications in case of ad-hoc networks. There is also an advantage of the above result as it reduces the allowed set of routes and thus makes the search for the optimal route easier. From the above rate balance equation it follows that, for a given P and f, the stability of the forwarding queue of node i depends in *an inverse manner* on the stability of the forwarding queues of the source nodes of the routes that pass through node i. Precisely, observe that the value of π_i increases with a decrease in value of π_s. This implies that if the routing is such that node i carries traffic of a source s which does not forward any route's packet, i.e., $\pi_s = 0$, then the value of π_i is more as compared to the case where, keeping everything else fixed, now node s forwards traffic from some route.

4. Stability of forwarding queues and routing for some special cases

In the following we will restrict ourselves to symmetric networks, i.e., we will assume that $P_i = P, \forall i$ and $f_i = f, \forall i$. Hence the balance rate becomes linear and the solution y_i for all i is given by

$$1 - y_i = \sum_s y_s \bar{w}_{s,i}, \tag{22}$$

where

$$\bar{w}_{s,i} = \sum_{d:i \in R_{s,d}} \frac{P_s P_{s,d} P_{s,s,d} L_{i,s,d}}{P_i} \prod_{k \in R_{i,s,d} \setminus i} (1 - (1 - P_{k,s,d})^{K_{k,s,d}}). \tag{23}$$

However, we allow for general source–destination pair combinations and general routing. We will also assume that the number of neighbors of all the nodes are same, i.e., $n_i = n, \forall i$. Also, we will be assuming that $K_{i,s,d} \equiv 1$. Note that assuming a symmetric network need not imply that the number of nodes is infinite. *We mention that the restriction to symmetric case is only to simplify the presentation and all the following development will work for a general network as well.*

We give some necessary and some sufficient conditions for stability of the forwarding queues. These stability conditions can be grouped into two categories: (i) stability conditions specific to a particular routing, and (ii) stability conditions independent of the routing.

Clearly, the stability conditions which account for routing will give tighter conditions. However, obtaining stability conditions that do not depend on the routing is in itself significant simplification in tuning the network parameters. For example, suppose that we are deploying a grid (or, mesh) network for which $n_i = 4$. In this case, if we can find a pair of values P and f such that *all the forwarding queues are guaranteed to be stable*, then one can decouple the problem of finding an optimal route and that of stability. We will use this decoupling later in the paper.

Let $r \triangleq (1 - P)^n$. Note that $P_{i,s,d} = r$. Also, for a given routing, let $d(i,s,d)$ be the number of elements in the set $R_{i,s,d} \setminus i$.

4.1. Stability conditions

Proposition 4.1. (1) *A necessary condition for stability of F_i for a given routing is that*

$$Pf \geqslant \sum_{s,d:i \in R_{s,d}} (1 - f) P_{s,d} P (1 - r)^{d(i,s,d)+1}. \tag{24}$$

(2) *A sufficient condition for stability of F_i, irrespective of routing is that*

$$Pf \geqslant (1 - P)^n.$$

Proof. (1) For a given routing, the input rate into the forwarding queue F_i is

$$\sum_{s,d:i\in R_{s,d}} y_s PP_{s,d}(1-r)^{d(i,s,d)+1}. \quad (25)$$

Now, $y_s = 1 - \pi_s f \geqslant 1 - f$. Hence, the minimum rate at which packets can arrive to F_i is

$$\sum_{s,d:i\in R_{s,d}} (1-f)PP_{s,d}(1-r)^{d(i,s,d)+1}. \quad (26)$$

The maximum rate at which F_i can be served is clearly Pf. The proof is complete for 1.

(2) The maximum arrival rate of packets into the queue F_i is $(1-P)^n = r$, because in any slot F_i can receive packet only if the node i and $(n-1)$ of its neighbors are not transmitting. Similarly, the maximum rate at which the queue F_i is served is Pf. For stability we need the service rate to be at least the arrival rate. The proof is complete. □

4.2. Effect of routing

Assume a symmetric network and assume that the condition of Proposition 4.1 is satisfied so that all the forwarding queues are always stable, irrespective of the routing of packets.

Under the present situation where stability is guaranteed irrespective of the routing used, we can change routing to obtain better throughput for the various routes while maintaining stability of the forwarding queues.

The probability that a packet on route $R_{s,d}$ reaches its destination is $r^{d(d,s,d)}$. Here, the quantity $d(d,s,d)$ depends on the routing used. We then have the following easy result:

Lemma 4.1. *Shortest-path routing maximizes the probability of success of a packet between a source–destination pair.*

Proof. From the expression of probability of success of a packet on a route, we need minimum value of $d(d,s,d)$ to maximize the probability. □

The above result was fairly straightforward to obtain and is also intuitive. It is similarly easily shown that

Corollary 4.1. *If the number of neighbors is not the same for all the nodes then a route with shortest number of interfering nodes achieves maximum probability of success of packet.*

Even though we are able to ensure that the forwarding queues are stable independent of the routing used, it is clear that maximizing the probability of success of a packet on any route does not necessarily maximize the *throughput* on that route. This is because the throughput on a route $R_{s,d}$ is $y_s PrP_{s,d}r^{d(d,s,d)}$, so that it is possible that the probability of success on a route increases but the forwarding queue of the source itself is loaded so much that the throughput that the source decreases.

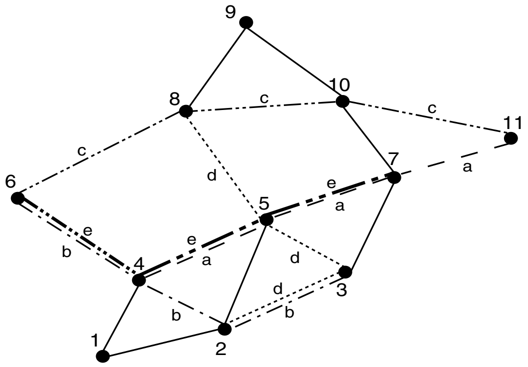
However, we know that the minimum rate at which queue Q_s is served is $P_s(1-f_s) = P(1-f)$, independent of the load on queue F_s. Hence, by maximizing the probability of success for each source–destination pair by using shortest-path routing maximizes the minimum guaranteed throughput for the source–destination pair. This in itself is important consequence of Lemma 4.1.

Remark 4.1. The results of this section deal with the effect of routing on the minimum guaranteed throughput. We assumed that the system is always stable, independent of the routing used (we also gave a sufficient condition for this to happen). However, we have not answered the question of maximizing the throughput itself. This is a hard problem in general as can be seen by the complex dependence of y_s on the routing. Moreover, assuming a shortest-path routing does not always uniquely determine the routing in a network. This is because in a network there may be many paths between a given source–destination pair which qualify to be shortest path. A simple example is a Grid network. In our ongoing research work we are looking at the problem where we restrict ourselves to the space of shortest-path routing and then aim at maximizing the throughput obtained by the routes. This amounts to maximizing y_s for each value of s. This also amounts to minimizing the value of π_s for each s. Clearly, this need not always be possible since two vectors need not always be component-wise comparable.

5. Numerical results and simulations

In this section, we present some numerical results and validate the expressions found in the previous sections with a discrete time network simulator. We have implemented this simulator according to the model of Section 2. Hence, it appears to be a valuable tool of measurement. We deploy an asymmetric static wireless network with 11 nodes as shown in Fig. 3.

Five connections are established a, b, c, d and e as indicated in the same figure (a dashed or complete line between two nodes in this figure means that there is a neighboring relation). These connections choose the shortest-path in terms of hops to route their packets. We choose the parameters $K_{i,s,d} \equiv K$, $f_i \equiv f$ and P_i in a manner of enabling stability, for all i, s and d. We fix $f = 0.8$ except contraindication. Let $P_2 = 0.3, P_3 = 0.3, P_4 = 0.4, P_5 = 0.5, P_7 = 0.3, P_8 = 0.3, P_{10} = 0.4$ be the fixed transmission probabilities for nodes 2, 3, 4, 5, 7, 8 and 10 while $P_i \equiv P$ for all other i. Many nodes need to have fix transmission probabilities so as to get a stable queues for all nodes.



connection a : 4 - 5 - 7 - 11
connection b : 3 - 2 - 4 - 6
connection c : 11 - 10 - 8 - 6
connection d : 8 - 5 - 3 - 2
connection e : 6 - 4 - 5 - 7

Fig. 3. Wireless network.

First, we validate the analysis results via simulation. A simulated ad-hoc network scenario is configured to study our analytical model. In Figs. 4 and 6 (resp. 5 and 7), we plot the throughput computed by analytical model (resp. simulation) on various routes and the quantities π_i versus the channel access probability for $K = 4$. We observe that the analytical results match closely the simulator result.

In Figs. 8 and 9, we plot the throughput on various routes and the probability π_i versus the transmission probability. The existence of an optimal channel access rate (or, the transmission probability) is evident from the figures. Moreover, as expected, the optimal transmission probability increases with K. The Figs. 4–9 show that increasing the parameter K significantly improves the throughput but the region of stability decreases. It is, therefore, clear, there is a throughput–stability tradeoff which can be obtained by changing the maximum number of transmission (K).

From Figs. 11 and 12, we vary the load of the forwarding queues by changing the forwarding probability of each node. The parameter of the network are as follow: $K = 4$, $P_2 = 0.5, P_3 = 0.3, P_4 = 0.45, P_5 = 0.5, P_7 = 0.3, P_8 = 0.3, P_{10} = 0.4$ and the rest of the nodes have $P = 0.2$. We observe that when f is small the system is not stable, more precisely nodes 2, 4, 5, 8 and 10 are suffering from a congestion as shown in Fig. 10. They need to deliver

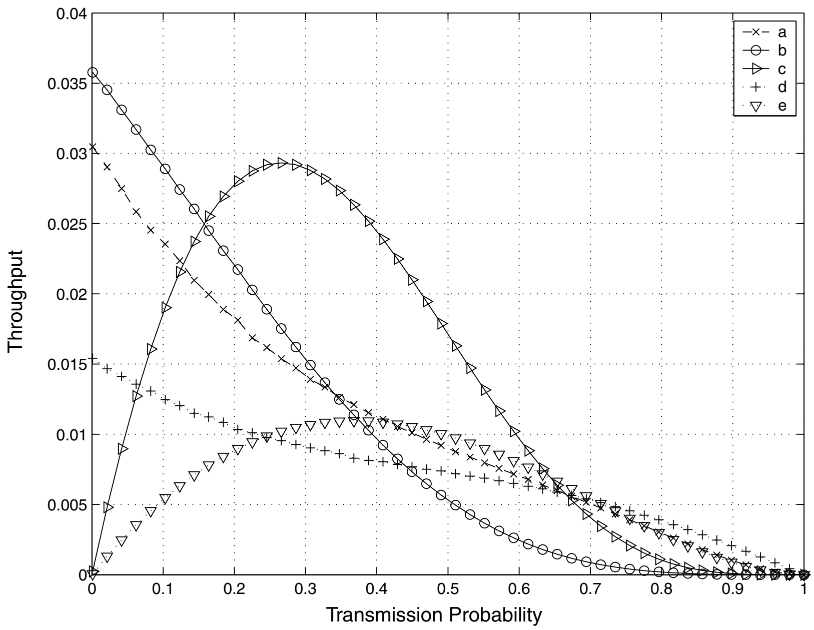


Fig. 4. Throughput from analytical model for $K = 4$ and $f = 0.8$.

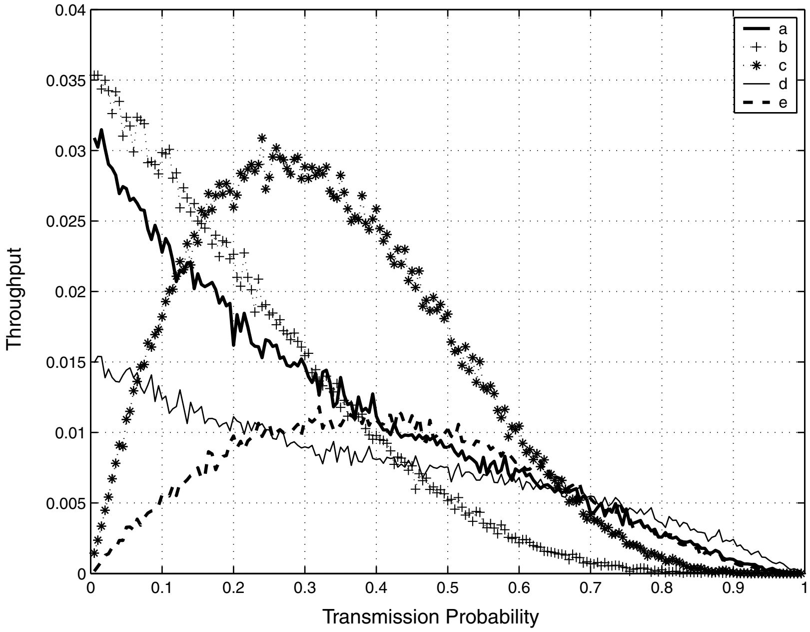


Fig. 5. Throughput from simulation for $K = 4$ and $f = 0.8$.

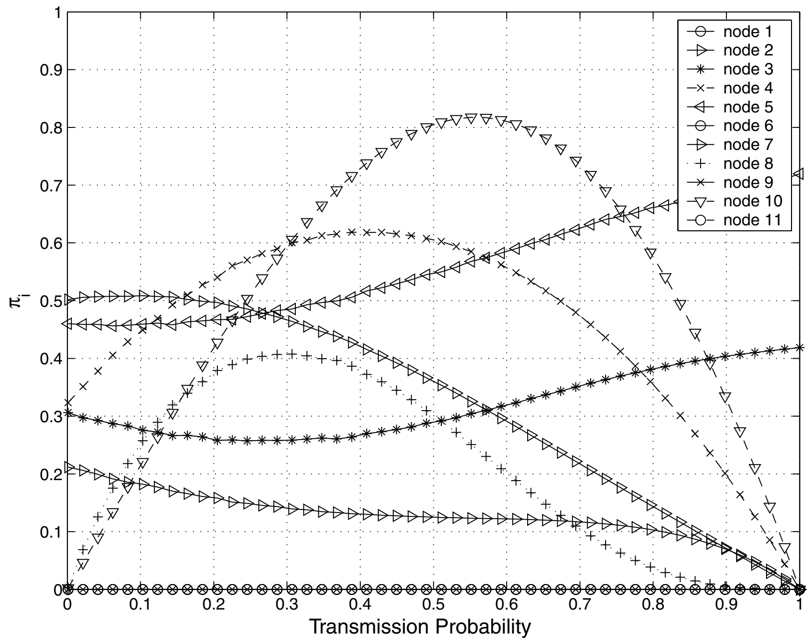


Fig. 6. π_i from analytical model for $K = 4$ and $f = 0.8$.

more packets from the forwarding queue in a faster manner. In this unstable case, the throughput of all connections is sensitive with the f variation. As we see in Fig. 11, it increases with f until the system becomes stable around $f = 0.4$. The y_i as shown in Fig. 12 remains independent of f in the stability region. Consequently, the throughput that does depend on y, is also independent of f, thus it is inde-

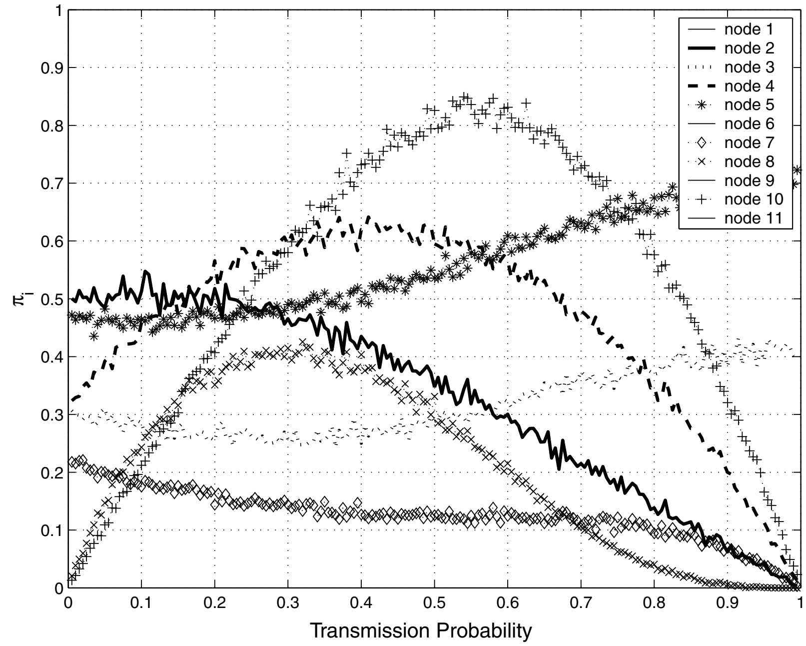


Fig. 7. π_i from simulation for $K = 4$ and $f = 0.8$.

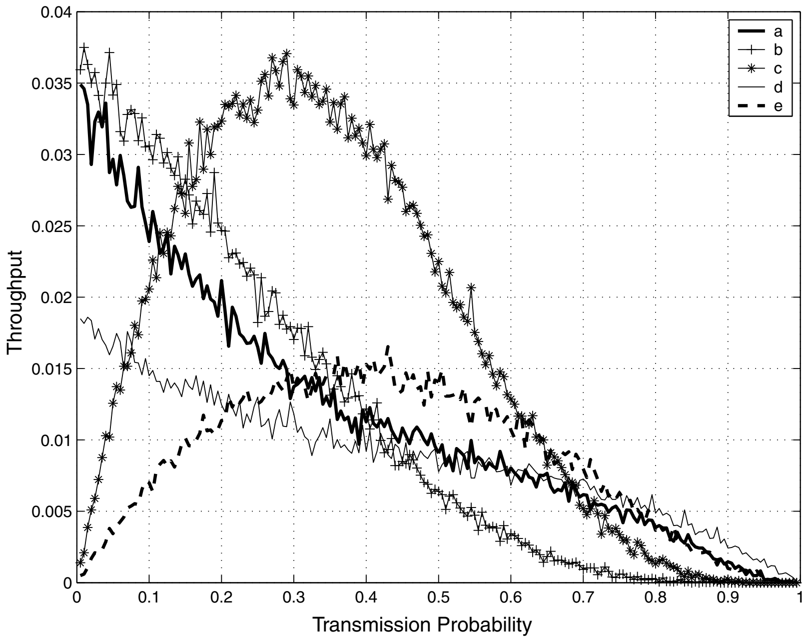


Fig. 8. Throughput from simulation for $K = 5$ and $f = 0.8$.

pendent of the load π_i of the nodes engaged in a connection.

All the above analysis and simulations were done in the saturated case where each node has always a packet to transmit from its queue Q. We were wondering if the forwarding probability has an impact in the unsaturated case. For that, we consider that packets generated in each node arrive to the queue Q following a Bernoulli process with mean λ. We show in Fig. 13 the connection a

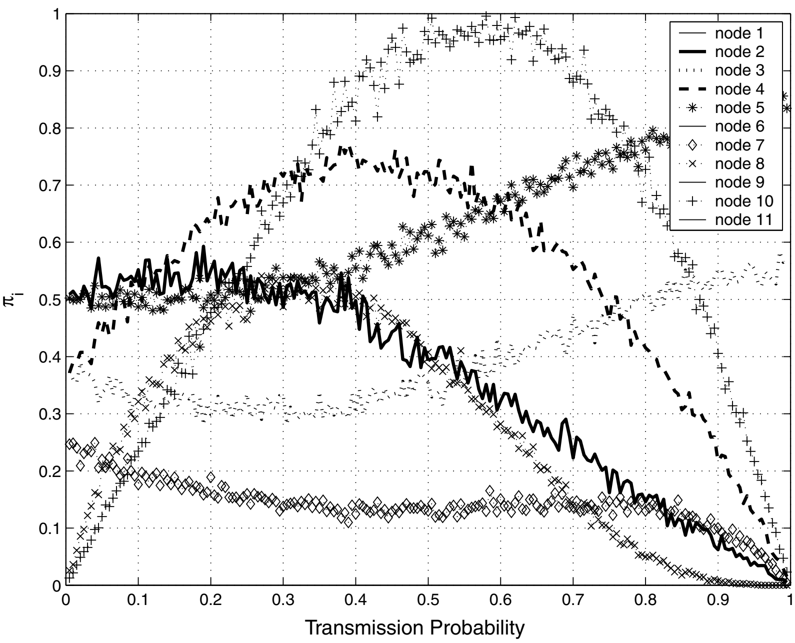


Fig. 9. π_i from simulation for $K = 5$ and $f = 0.8$.

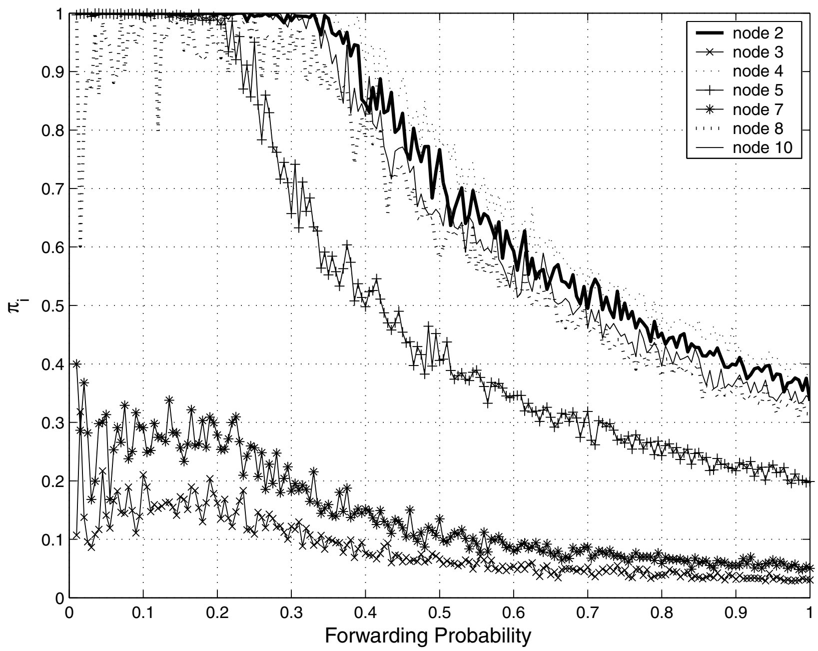


Fig. 10. π_i from simulation versus the forwarding probability for $K = 4$.

throughput as function of the forwarding probability for different λ. It appears that the throughput reaction is similar to the saturated case. It is independent of the weight given to the forwarding queue when the stability is reached for a given f. Moreover, it exists an optimal λ in the interval $[0.01, 0.1]$ as shown in the same figure that maximizes the throughput. For a very small λ the success probability of packets is very high and for high λ there are many collisions.

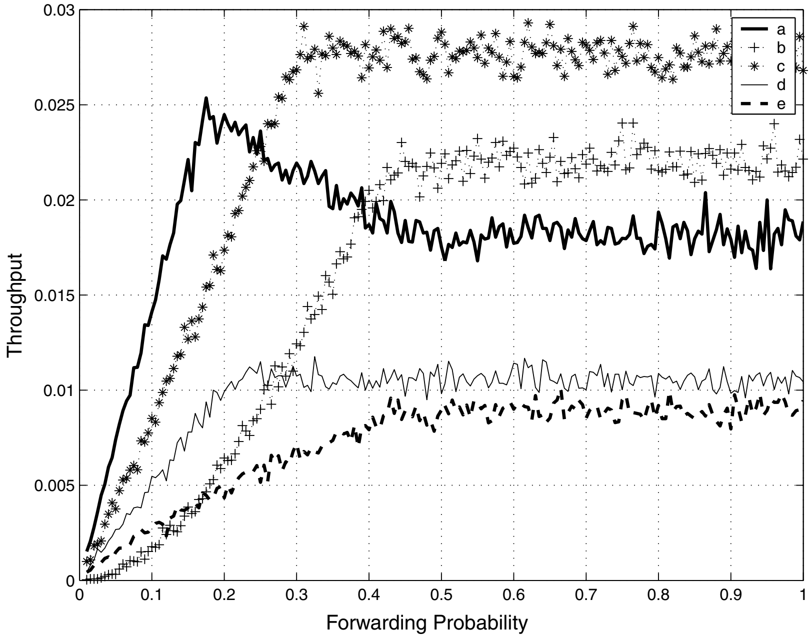


Fig. 11. Throughput from simulation for $K = 4$ as function of the forwarding probability.

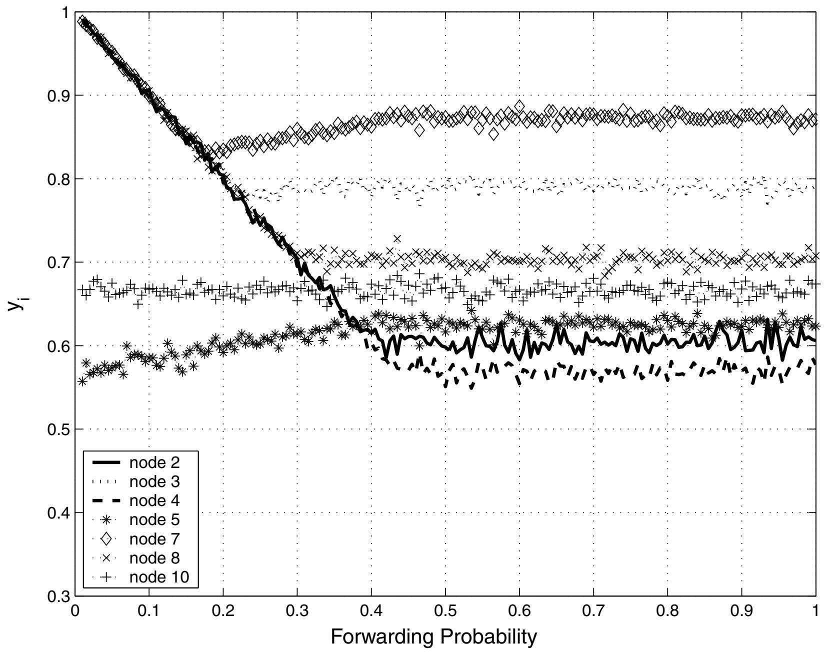


Fig. 12. The quantity y_i from simulation as function of the forwarding probability for $K = 4$.

Now, we use the shortest-path routing (based on the number of interferers on a path as defined in Section 4) under the present situation where the stability of all the forwarding queues in the network is guaranteed. The routes for all connections under this shortest-path routing are $R_{1,11} = \{2, 3, 7\}$, $R_{9,3} = \{10, 7\}$ and $R_{6,7} = \{8, 10\}$.

In Figs. 14a and b, we compare the throughput of all connections under the old and new routings. We observe that the throughput of all connections

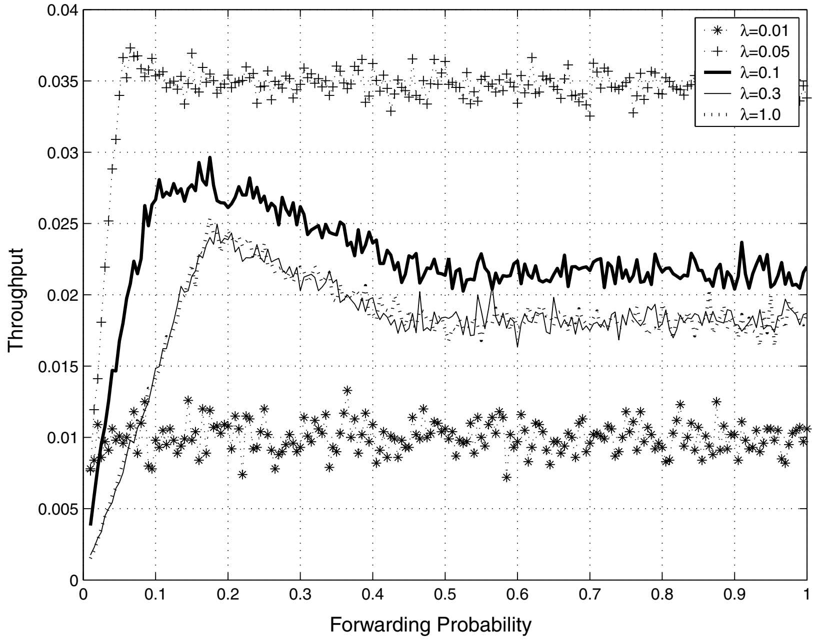


Fig. 13. Throughput of connection a from simulation as function of the forwarding probability for different values of λ in the unsaturated case and $K = 4$.

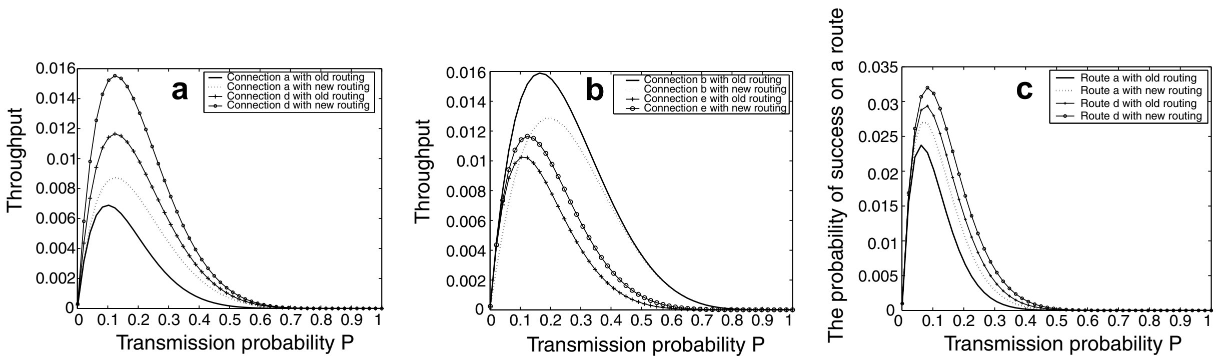


Fig. 14. (a) and (b) show the throughput of connections a, c, b and e as function of the transmission probability P for $K = 1$ and (c) shows the probability of success on a route a and d as function of the transmission probability P for $K = 1$.

(except that of connection b), is better with new routing than those obtained under the old routing. The reason of decreasing the throughput of connection b is the change in quantity y_3. In old routing, $y_3 = 1$ (node 3 with old routing, does not forward packets of any connections). With the new routing, node 3 forwards the packets of connection a. However, the value of y_3 decreases with new routing, explaining the decrease of throughput of connection b (because now the source node of connection b, i.e., node 3 gives some of its resources to forwarding of packets on route a). In conclusion, the question of maximizing the throughput *uniformly for all nodes* is a hard problem. The complexity of this problem comes from the dependence of throughput and the quantity y. In Fig. 14c, we plot the probability of success of a packet on all connections versus the transmission probability P. We observe that, as predicted already in Section 4, the new routing improves the probability of success of *all* connections.

Remark 5.1. Studying an asymmetric network numerically requires one to consider all possible combinations of the network parameters. Since the degree of freedom (the parameters to choose) are usually very large in asymmetric networks, such a numerical study is not carried out generally. In a symmetric network we have $n_j = n$ for all nodes; some examples are a grid network, a circular network or a linear network. Moreover, for the symmetric networks, we can simplify the expressions in the detailed balance equation (Proposition 3.1) while getting important insights into the working of the network.

6. Linear networks

We now study the observations made in Section 3 in detail for some symmetric networks. In a symmetric network we have $n_j = n$ for all nodes; some examples are a grid network, a circular network or a linear network. Moreover, for the symmetric networks, we can simplify the expressions in the detailed balance equation (Proposition 3.1) while getting important insights into the working of the network.

In this section we will restrict ourselves to the study of linear networks only. We have chosen linear network because in these networks there is only one route between any two given nodes, thus avoiding the issue of routing.

Hence the numerical results for the networks where there is possibility of multiple routes between two nodes will be presented in Section 4. Moreover, studying the linear network is like studying a general network with a pre-specified routing.

Consider a *symmetric* linear network with infinitely many nodes so that $n = 2$ with $f_i = f$ and $P_i = P$. We number the nodes from $-\infty$ to $+\infty$ and use the number of a node to refer to that node. The probability that a node s sends a new packet to a given destination d, depends only on the distance between nodes s and d, i.e., $P_{s,d} = P(|s-d|)$.

6.1. Unlimited attempts $K_{i,s,d} \equiv \infty$

Here, we assume that each node attempts a forwarding of a packet until the transmission is successful. Because of symmetry, and since there is only one possible routing, it is seen that $y_i = y$ for all the nodes. Hence the global rate balance equations becomes

$$1 - y = 2\sum_{j=1}^{\infty} y \sum_{h=j+1}^{\infty} P(h), \tag{27}$$

so that

$$y = \frac{1}{1 + 2\sum_{j=1}^{\infty}\sum_{h=j+1}^{\infty} P(h)} = \frac{1}{1 + E[H]}, \tag{28}$$

where $E[H]$ is the expected hop length. Thus

$$\pi = \frac{E[H]}{f(1 + E[H])} \tag{29}$$

This indicates that as $E[H] \to \infty$, $\pi_i \to 1$ and $y \to 0$, i.e., the forwarding queues become unstable and the source throughput, $P(1-P)^2 y \to 0$. For this reason, we focus on the case when the number of hops between a source and a destination is bounded. Each node is a source of packets that have destination which are h hops away (left or right) with the probability $P(h)/2$, $h \leqslant B < \infty$. Here, we consider two forms of the probability distribution $P(h), 1 \leqslant h \leqslant B$:

1. The destinations are uniformly distributed, i.e., $P(h) = \frac{1}{2B}$. This choice is referred to as Fixed probability (FP).
2. The values $P(h)$ increase with h. We refer to this case as higher probability for long hops (HPLH).
3. The values $P(h)$ decrease with h. This choice is referred to as Lower probability for long hops (LPLH).

$P(h)$	y	π	Throughput
FP: $P(h) = \frac{1}{2B}$	$\frac{2}{B+1}$	$\frac{B-1}{f(B+1)}$	$\frac{2P(1-P)^2}{B+1}$
HPLH: $P(h) = \frac{h}{2\sum_{k=1}^{x} k}$	$\frac{3}{1+2B}$	$\frac{2(B-1)}{f(1+2B)}$	$\frac{3P(1-P)^2}{1+2B}$
LPLH: $P(h) = \frac{B-h+1}{B(B+1)}$	$\frac{3}{2+B}$	$\frac{B-1}{f(B+2)}$	$\frac{3P(1-P)^2}{B+2}$

By comparing these cases, we observe that the LPLH has larger stability region (lower bound on the values of f is smaller) as compared to other scenarii. Also, for a given value of P, LPLH achieves the maximum throughput. Moreover, the optimal throughput for all cases is obtained when $P = 1/3$.

Fig. 15 shows the values of the lower bound $f^{\min}$ on the forwarding probability f for the different choices of the distribution $P(\cdot)$ studied above. The parameter that is varied is the bound B on the lengths of routes. $f^{\min}$ is independent of the choice of the channel access probabilities P_s. The first observation from this figure is that, irrespective of the choice of $P(\cdot)$, the forwarding queues tend to instability as B increases. Also, for a fixed choice

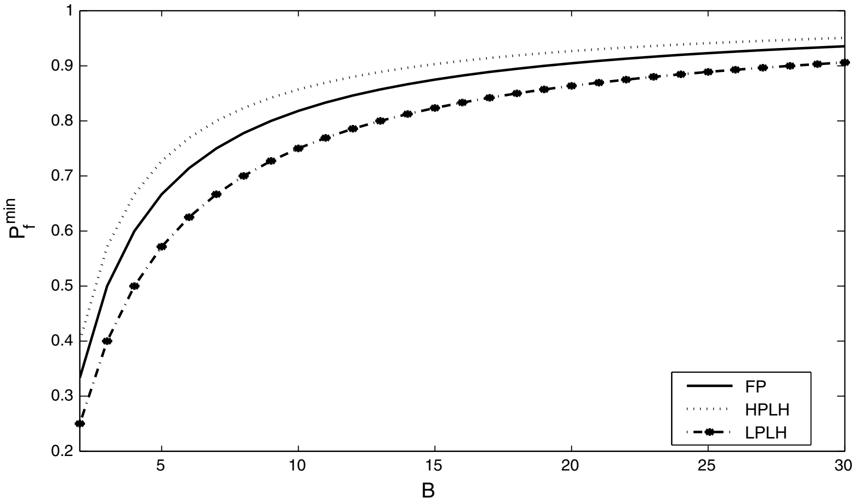


Fig. 15. The minimum weight given to the forwarding queues for stability versus B.

of B, the LPHL policy requires less value forwarding probability to ensure the stability of forwarding queues. In Fig. 16 we plot the throughput against the channel access probabilities for the stable system (so that the value of f does not matter). The existence of an optimal choice of the channel access probability is evident from the figure. As already observed, the optimal channel access rate is $\frac{1}{3}$ for all the choices of distribution $P(\cdot)$. For small values of P, the number of collision are not significant so that the throughput increases linearly with P. For large values of P, however, the collisions become significant and result in a significant drop in the throughput.

6.2. Limited attempts

In this subsection we consider the special case where $K_{i,s,d} \equiv K$ for all nodes. Because of symmetry it is seen that $y_i = y$ for all the nodes. Also, $P_{i,s,d} = (1-P)^2$ and $L \triangleq L_{i,s,d}$ is given by

$$L = \frac{1 - T^K}{1 - T}, \tag{30}$$

where $T = P(2-P)$ is the probability that a transmission attempt is unsuccessful. The global rate balance becomes

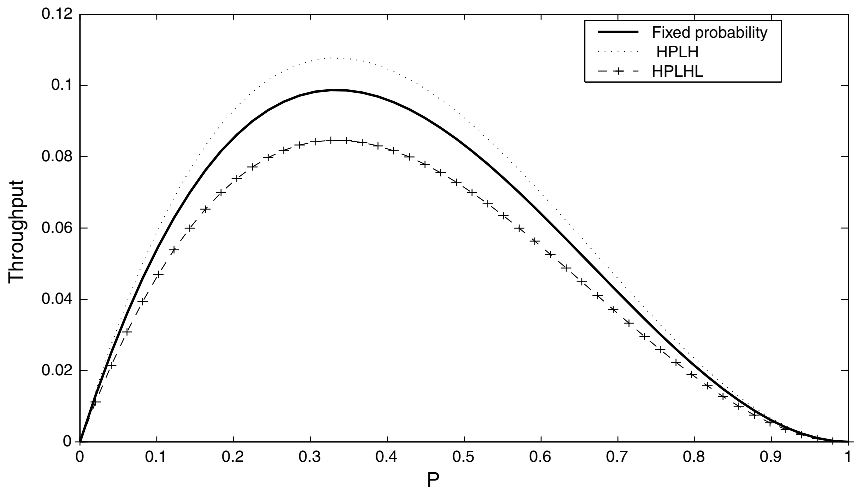


Fig. 16. The throughput of a node versus the channel access probability P.

$$1 - y = 2\sum_{j=1}^{\infty} y(1 - T^K)^j \sum_{h=j+1}^{\infty} P(h),$$
$$y = \frac{1}{1 + 2\sum_{h=2}^{\infty} P(h)(1 - T^K)\frac{1-(1-T^K)^{h-1}}{T^K}}. \quad (31)$$

The long term average rate at which data from node s reach their destinations is given by

$$thp = 2P(1-P)^2 y \sum_{h=1}^{\infty} P(h)(1 - T^K)^{h-1}. \quad (32)$$

For a bound B on the maximum number of hops, we study the effect of parameter B on the stability of the forwarding queues of the nodes.

Lemma 6.1. *If $P(h) \to_{B\to\infty} 0, \forall h$, then we have*

$$y_{\lim} \triangleq \lim_{B\to\infty} y = T^K. \quad (33)$$

Proof

$$y = \frac{1}{1 + 2\sum_{h=2}^{\infty} P(h)(1 - T^K)\frac{1-(1-T^K)^{h-1}}{T^K}} \quad (34)$$
$$= \frac{1}{1 + (1 - T^K)\left[\frac{1-2P(1)}{T^K} - 2\sum_{h=2}^{\infty} P(h)\frac{(1-T^K)^{h-1}}{T^K}\right]}. \quad (35)$$

Since $P(h) \to_{B\to\infty} 0, \forall h$, and since $1 - T^K < 1$, the result follows. □

It is easy to see that the limit $y_{\lim}$ is positive and it is always less than 1. Also, $y_{\lim}$ does not depend on the distribution $P(\cdot)$ as long as the condition of Lemma 6.1 is satisfied. For any finite value of K, the system is always stable irrespective of the value of B. However, when K increases, the value of y decreases, so that the lower bound on the allowed values of f increases, implying smaller stability region.

In Fig. 17, we plot the values of $y_{\lim}$ versus the channel access probability for different values of allowed number of transmission attempts, $K = 1, 4, 20$. We observe that, for a fixed value of channel access probability $P < 1$, $y_{\lim}$ decreases to zero as the allowed number of attempts K increases. Also, for a fixed value of K, $y_{\lim}$ decreases with a decrease in the channel access probability decreases (see Fig. 17). This indicates that the system will be more stable (the allowed values of forwarding probability will be more) when the limit on transmission attempts K decreases or the channel access probability P increases.

6.2.1. The FP distribution for route length

We assume that the probability $P(h)$ is constant, i.e, $P(h) = \frac{1}{2B}$ for $h \leqslant B$ and zero otherwise. The quantity y is given by

$$y = \frac{T^{2K} B}{T^K(B+1) + (1 - T^K)^{B+1} - 1}. \quad (36)$$

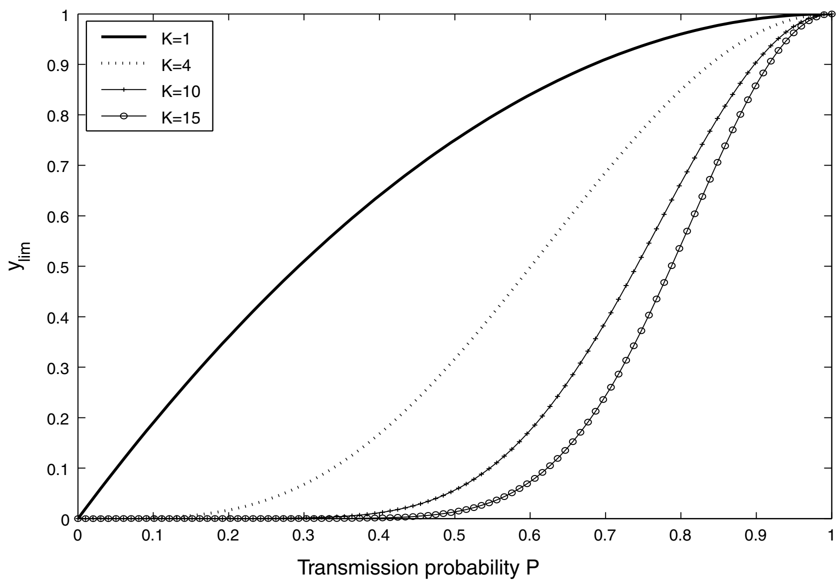


Fig. 17. $y_{\lim}$ as a function of the channel access probability P.

The rate at which data from node s reaches their destinations d is given by

$$thp = \frac{P(1-P)^2 y(1-(1-T^K)^B)}{BT^K}. \quad (37)$$

From the above relations, the stability region and throughput depend on three parameters: transmission probability P, maximum number of hops B and limited attempts K.

In Figs. 18–21, the throughput and the quantity y are plotted versus the transmission probability for the different values of limit attempts $K = 1, 4, 10, 15$ and maximum hops $B = 5, 10$. Both Figs. 18 and 21 show that increasing the bound on attempts K significantly improves the throughput but the region of stability deceases as shown in Figs. 19 and 21. It is, therefore, clear, there is a throughput–stability tradeoff which can be obtained by changing the limit of attempts.

By comparing the throughput and the quantity y for two different values of $B = 5$ and $B = 10$, we observe that the system will be more stable and the throughput will be better when the bound B decreases.

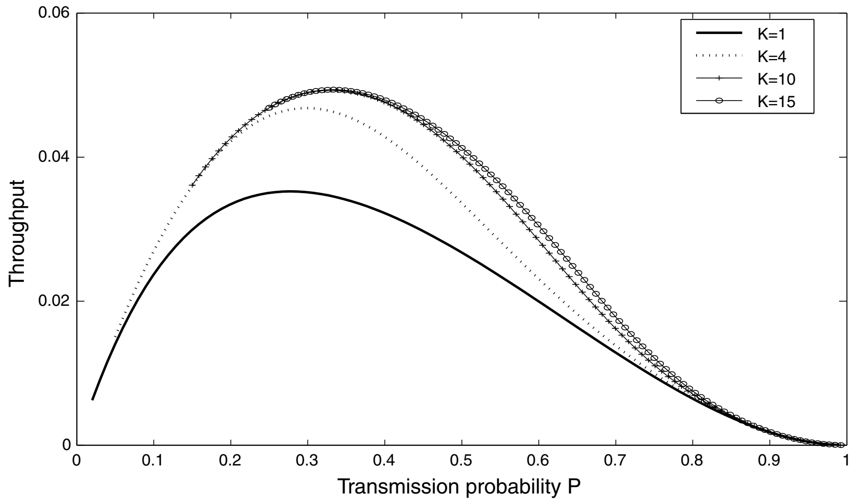


Fig. 18. The throughput of a packet as function of the transmission probability P for $B = 5$ and $K = 1, 4, 10, 15$.

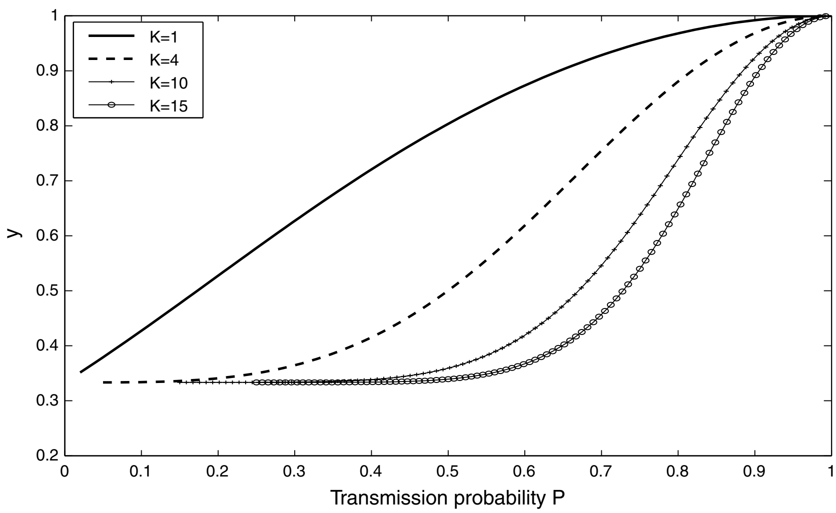


Fig. 19. The region of stability as function of the transmission probability P for $B = 5$ and $K = 1, 4, 10, 15$.

6.2.2. The HPLH distribution for route length

In this scheme, the throughput *thp* is given by

$$thp = 2P(1-P)^2 y \sum_{h=1}^{B} \frac{h}{B(B+1)} (1-T^K)^{h-1}. \qquad (38)$$

In Figs. 22 and 23 we plot the quantity y and throughput versus the channel access probability for different values of allowed number of transmission attempts $K = 1, 4, 10, 15$ and we observe similar results as shown in (FP) case. An interesting feature to note is that the optimal transmission probability in (HPLH) case is more sensitive of limit attempts K comparing with (FP) case. In (FP) the optimal transmission is almost the same for different values of K.

6.2.3. The LPLH distribution for route length

In this scheme, the throughput is given by

$$thp = 2P(1-P)^2 y \sum_{h=1}^{\infty} \frac{B-h+1}{B(B+1)} (1-T^K)^{h-1}.$$

In Figs. 24 and 25, the throughput and region of stability are plotted versus the transmission probability for the different values of limit attempts $K =$

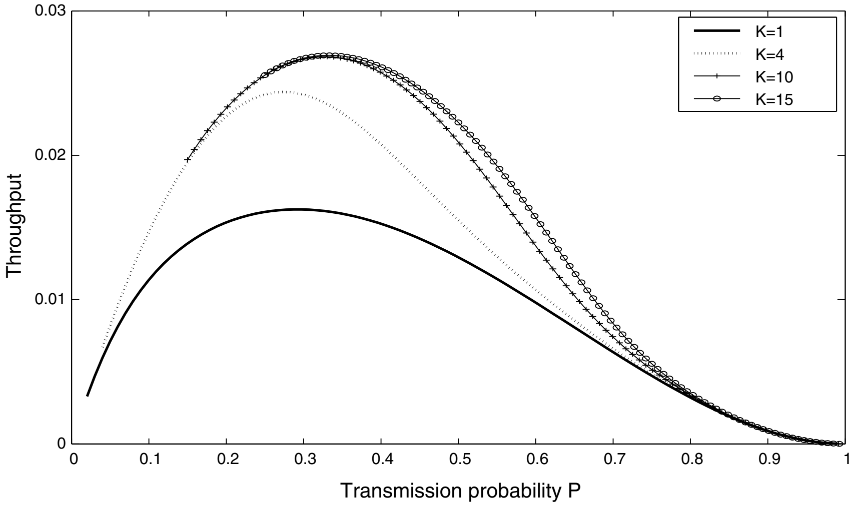


Fig. 20. The throughput of a packet as function of the transmission probability P for $B = 10$ and $K = 1, 4, 10, 15$.

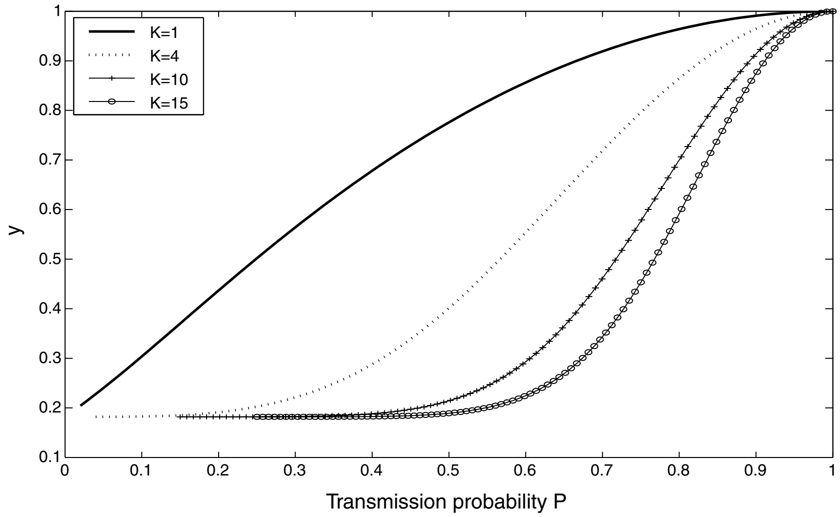


Fig. 21. The region of stability as function of the transmission probability P for $B = 10$ and $K = 1, 4, 10, 15$.

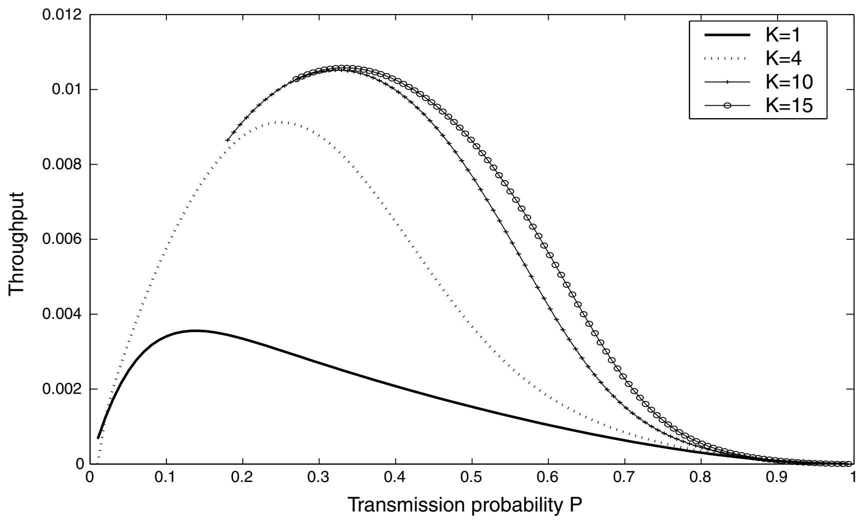


Fig. 22. The throughput of a packet as function of the transmission probability P for $B = 10$ and $K = 1, 4, 10, 15$.

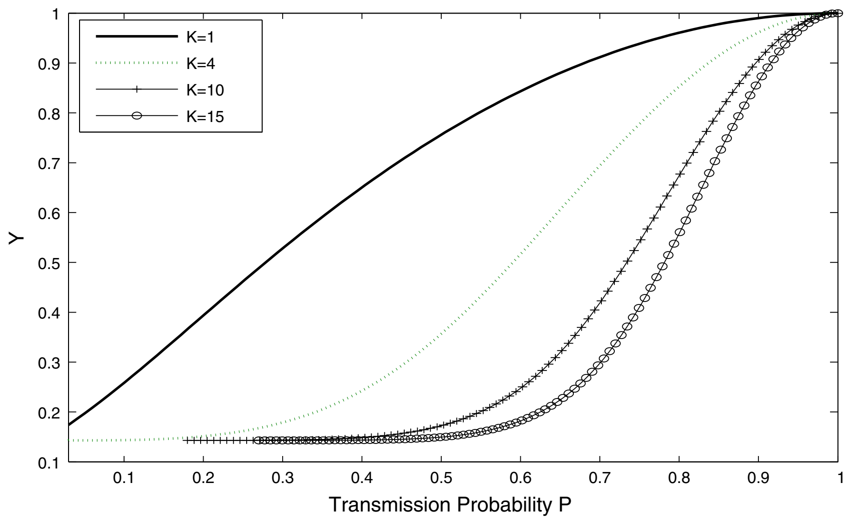


Fig. 23. The region of stability as function of the transmission probability P for $B = 10$ and $K = 1, 4, 10, 15$.

$1, 4, 10, 15$ and maximum hops $B = 10$. Similar trends are obtained in the (LPLH) case as comparing to other cases (FP) and (HPLH).

By comparing these three cases, we observe that the LPLH has larger region as compared to other scenarii. And the throughput obtained in (LPHL) is better than that of other cases.

7. Conclusion

Considering a simple random access wireless network we obtained important insights into various tradeoffs that can be achieved by varying certain network parameters.

Some of the important results are that

1. As long as the intermediate queues in the network are stable, the end-to-end throughput of a connection does not depend on the load on the intermediate nodes.
2. Routing can be crucial in determining the stability properties of the network nodes. We showed that if the weight of a link originating from a node is set to the number of neighbors of this node, then shortest-path routing maximizes the minimum probability of end-to-end packet delivery.

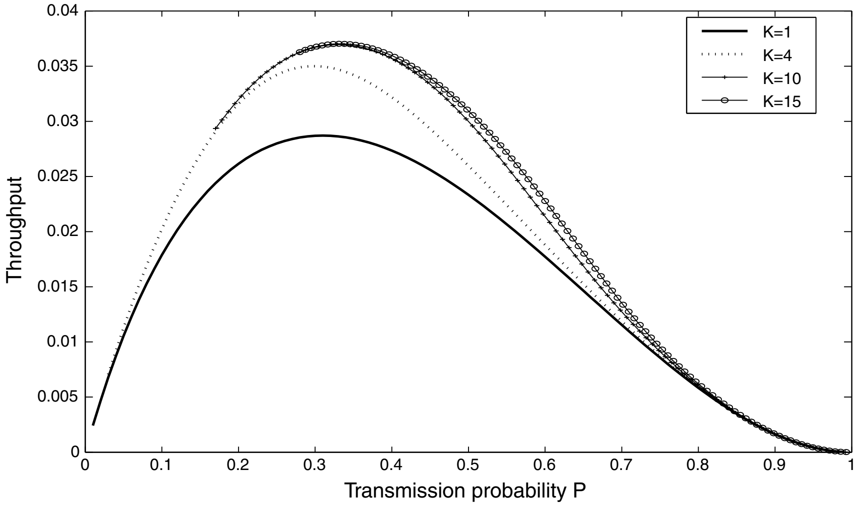


Fig. 24. The throughput of a packet as function of the transmission probability P for $B = 10$ and $K = 1, 4, 10, 15$.

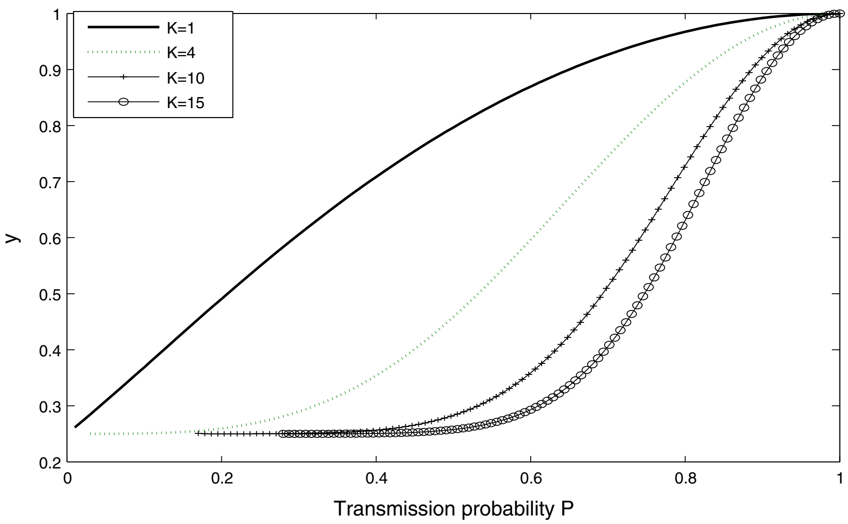


Fig. 25. The region of stability as function of the transmission probability P for $B = 10$ and $K = 1, 4, 10, 15$.

3. Providing a framework for cross-layer study of stability–throughput performance of ad-hoc networks. It has the flexibility for managing at each node forwarded packets and its own packets differently.
4. The results of this paper extended in a straightforward manner to systems of weighted fair queues with coupled servers.
5. The context of the stability that we study is new as it takes into account the possibility of a limited number of transmissions of a packet at each node after which it is dropped.

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