Fluid Model for Cellular Networks

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November 23, 2007

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Preface

In network engineering, and more generally in complex systems, there is often a tradeoff between simplicity of mathematicial tools, formulas or numerical approaches on the one hand, and accuracy on the other hand. We analyze such a situation in cellular radio networks. We propose a spatial **fluid model** network that allows to establish a simple formula of a characteristic of cellular radio networks, the interference factor *f*. This one, useful for network dimensioning and control purposes in the cellular context, simplifies considerably the computation complexity needed to obtain accurate results, without over-simplifying the model which could have resulted in large inaccuracies.

There have been previous approaches that attempted to obtain simple formulas for network dimensioning and control purposes in the cellular context.

One of the main sources of inaccuracies lies in the fact that though the interference factor depends on the position of the mobile in the cell, the computation of f does not take it into account.

The goal of our approach is to propose a model that is more accurate in taking into account the distance of a mobile to the base station, and is still simple enough to lead to close form formulas.

Chapter 1

Introduction

1.1 Motivation

In network engineering one often needs mathematicial tools for dimensionning purposes and for optimisation of resources. In that context, and more generally in complex systems, there is often a tradeoff between simplicity of formulas or of numerical approaches on the one hand, and accuracy on the other hand. In this thesis we analyze such a situation in cellular radio networks.

A mobile (MS) connected to its serving base station (BS) receives signal and interferences. The amount of interferences due to the other BSs of the network depends on many parameters such as their transmitting powers, positions and number, pathloss (including the shadowing an the fast fading effects), topology, environment (rural, urban, other)...

Any analysis of cellular radio systems (dimensioning, quality of service, radio resources management...) has to take into account the interferences effects. This is the reason why it is necessary to evaluate their influence, as precisely as possible. The different kind of studies and developement of models existing in the literature in that objective, based on simulations, approximations, mathematical analyses, empirical approaches ...(see section 1.2), did not allow to derive a simple analytical expression of the downlink interference factor, which takes into account the mobile position.

The analysis is not easy. Since each base station has an influence on each mobile of the network, even in a infinitesimal way, a global analysis has to take into account a great number of constraints or parameters. Let consider for example a dimensioning problem. Let assume a simplified model of the reality, where the only parameters of the analysis are the positions of the transmitters and receivers and the transmitting powers. Since each base station has an influence on each mobile of the system, the system to analyze has to answer to a great number of constraints expressed by a great number of *non independant* equations.

The problem has a high complexity, even with such a simple modelization of the reality. In a cellular radio network, the other-cell interference factor f is an important parameter characterizing these interferences. It represents the 'weigth' of the network on a given cell. The precise knowledge of the interference factor allows the derivation of networks characteristics such as outage probabilities, capacity evaluation and admission control mechanisms. A mobile entering a cell can be seen as **a 'source' or 'generator' of interferences** for the other mobiles already present in the cell. Indeed, in uplink the mobile interfers, as a transmitter, with the other mobiles. And in downlink since the entering mobile needs radio power from its serving BS, this last one has to transmit power and interfers with the other BS. As a consequence a mobile entering a cell increases its load. To analyze the specific overload generated by the new entering mobile, one has to take into account the interference factor. Moreover the BS transmitting power towards the mobile also depends on that factor.

A precise knowledge of the interference factor is necessary to evaluate the influence of a mobile in a cell and a network. It appears interesting to determine with a high accuracy the interference factor, whatever the mobile location. Indeed, the interference factor plays a role in many cases, and in particularly for:

- the dimensioning process

- the quality of service determination
- the system management: mobile scheduling analysis, radio resources management, optimization...

Dimensioning.
The estimation of cellular networks capacity is one of the key points before deployment and mainly depends on the characterization of interference.

Since the interference factor has a strong influence on the transmitting power of a base station, it affects the performance of cellular systems and in particular their cell capacity. That factor plays a fundamental role in dimensioning analysis. In particular, in the CDMA systems case, the coverage of a given cell strongly depends on the traffic and the interferences coming from the neighbors cells. There is a cell breathing [VerSe01]. Thus some margins have to be taken into account during the planning process. One of the main sources of inaccuracies of the existing approaches lies in the fact that though the interference factor depends on the position of the mobile in the cell, the computation of f does not take it into account. An accurate estimation of that parameter would allow a network design avoiding overestimating margins. As a consequence, an investigation of the interference characteristics based on the location of a user seems useful.

For these reasons, a precise knowledge of f whatever the mobile location, seems interesting.

In our analysis, we show that this factor directly depends on the density of base stations, and can be used to determine the number of base stations needed to answer a given traffic. In other terms it allows to analyze the benefit of a densification of a network zone, for example to answer an increasing traffic.

Quality of service.
The probability for a mobile to enter a cell is an important quality of service (QoS) parameter. The outage probability can be calculated using the inter-

ference factor. Since the interference factor depends on the location of the entering mobile, a spatial outage probability can be determined. This last one expresses the probability for a mobile to enter the cell, at a given distance from its serving BS. Other quality of service indicators can be calculated as for example the sojourn time characterizing the total average time lasted in a cell by mobiles using a non real time service.

The calculation of the interference factor, with a high accuracy whatever the mobile location, is interesting in these cases too, and particularly to determine the spatial outage probability.

Mobile management.
As already observed, since the interference factor characterizes the load generated by a mobile, its knowledge allows to determine with a high precision the influence of any mobile in a cell or a network. It thus enables to compare different **admissions control strategies**. As a consequence, it and can help vendors to adopt a given strategy and /or policy according to their specific constraints.

The design of efficient and fair transmission **scheduling policies** constitutes one of the current problems for the third generation cellular networks. In particular, the quality of service (QoS) control related to the flow is not new in the context of the networks integrating several services. The principal problem is to control the throughput in a manner which is effective for the use of the **resources of the network**, and advantageous for the operator by taking into account the impact on the perceived quality of the application, the capacity and the coverage of the network.

The analysis and the **optimisation** of scheduling constitute also a starting point to study and optimize the load of the network and the dimensioning of the networks and cells. An objective can be to design scheduling and access control policies.

A precise evaluation of the influence of any mobile in a cell, which needs a precise knowledge of the interference factor, is necessary in the aim to analyze them and optimize their impact on the capacity, and the quality of service of real-time and best effort applications. And then, tools of optimization theory and optimal control can be used to identify the adequate policies. Stochastic and probabilistic analysis make it possible to quantify the perceived quality corresponding to the suggested policies.

1.2 Previous approaches

There have been previous approaches proposed to model the interference factor f (see for example [Vit01] [Ev99] [FoLi01] [Ata01] [Alm01] [Owe02] for the uplink and [Vit02] [Gil91] [Ela05] [Bac01] [BoP01] [Tjh01] for the downlink). Many among them assumed f as a constant. The interference received at a base station from other cells can be written as fP where P is the power received by the base station from mobiles in its own cell. This approach can be used to approximate the quality of a signal received by a mobile using the ratio 'Signal to Interferences' and then to use it for power and rate control, for call admission decisions etc. That assumption (f is a constant) can be justified in the uplink case, for example when the traffic distribution is considered as uniform. In the downlink, the justification of that assumption is more difficult. Indeed, since the mobiles receive an interfering radio power depending on their location, the interference factor strongly depends on this one. To analyze and determine the interference factor, different methods were used. Though some analytical approaches exist, up to now, the evaluation of the interference factor has been mainly done by using simulations.

Pioneering works on the subject [Vit01] were mainly focusing **on the uplink**. Working on this link, [Ev99] derived the distribution function of a ratio of path-losses, which is essential for the evaluation of external interference. The authors approximate the hexagonal cell with a disk of same area. Based on this result, Liu and Everitt propose in [Liu06] an iterative algorithm for the computation of the interference factor, also on the uplink. [Ata01] proposes an empirical model for the inter-cell interference factor in the uplink of a CDMA cellular system. [Kola1] analyzes the uplink interference factor and compares to upper bounds given by [Vit01].

[StaLe1] and [StaLe02] propose an approximation analysis of the uplink othercell interference distributions using a fix-point based approach and for inhomogeneous UMTS networks. [LiEv01] proposes an analytical characterization of other-cell interference in the uplink of CDMA networks. It is based on an iterative method for solving fixed-point equations employed to determine the mean and variance for lognormal approximation of the other-cell interference.

[KolWi01] uses simulation and modelling to obtain the other-cell interference factor for CDMA systems and proposes comparisons with upper bounds results. In [Alm01], the authors obtain the analytical values of the mean and the standard deviation of the intra-cell and inter-cell interference for the reverse link of CDMA systems.

Some simulation statistics of measured uplink f-factor are given in [Owe01] over different Monte-Carlo snapshots.

[BaR01] deals with numerical average values of f-factor for different environment conditions.

The authors of [BoSt01] introduce a 'frequency reuse factor' F for the reverse link, related to the interference factor f as $F = \frac{1}{1+f}$ which allows to establish a pole capacity formula.

On the **downlink**, [Vit02] and [Vit03] aimed at computing an average interference factor over the cell by numerical integration in hexagonal networks. They did not take into account the distance between the mobile and its serving base station in the calculation of the interference factor.

In [Gil91], Gilhousen et al. show that, particularly for terrestrial network, the interference suppression feature of CDMA systems can result in a capacity increase. They provide Monte Carlo simulations and obtain a distribution of the total interference.

In [Ela03] and [Ela05], other-cell interference is given as a function of the distance to the BS. They however only determined an upper bound of the other cell interference.

Baccelli et al. [Bac01] consider a random network approach, base stations are randomly distributed, to analyze the influence of geometry on the combination of inter-cell and intra-cell interferences in the downlink in the case of large CDMA networks. They use an exact representation of the geometry of the downlink channels to define scalable admission and congestion control schemes, They study the capacity of these schemes when the size of the network tends to infinity using stochastic geometry tools. The authors of [BaB01] consider a random network approach to provide spatial blocking probabilities. The authors rely on an approximated formula for f, the interference factor is nevertheless not their main concern. Their approach is based on an exact representation of the geometry of both the downlink and the uplink channels. The authors show that the associated power allocation problems have solutions under constraints on the maximal power of each station/user. The analysis is implemented in such a way that each base station only has to consider the load brought by its own users to decide on admission. The global feasibility of the power allocation is ensured.

The authors of [BoP01] analyze data transmission on the downlink of cellular networks. Considering a HSDPA/HDR system, the base station transmitting power is time-shared between a dynamic number of mobiles. They derive analytical performances, in terms of blocking probability and data throughput. The impact of interference on cell capacity is studied by considering two types of homogeneous networks: linear networks, where BS are equidistant and placed on a common infinite line, and hexagonal networks, where cells are hexagons of the same size and cover the entire plane. In both cases, all the base stations BS are assumed active and transmitting at the same power. The authors make the choice of considering only the first ring of interferers. They can then express the first ring interference by an approximated formula. Although simple, this approximation is not validated by simulations.

Chan and Hanly [Chan01] precisely approximate the distribution of the other-cell interference. They develop an analysis to approximate and obtain bounds of it. They provide a spatial Poisson point pattern. model of traffic in CDMA network, and show how the theory of Poisson processes can be applied to provide statistical information about interference levels in the network. Considering a large number of interference they use a Gaussian approximation and calculate a bound of the outage probability at a given cell. The formulas they provide are however difficult to handle in practice. The authors of [MasTa] propose an other-cell-interference factor distribution model for the downlink of CDMA systems. They provide a pdf (probability density function) expression first assuming only one interferer, then they generalize the expression they established to a multiple case interferers. They analyze uniform and non-uniform traffic situations and find a bound for the total other-cell interference.

In [SeK01] the authors analyze the performance of the packet data transmission DS/CDMA downlink. They define a downlink other-cell-interference factor as the ratio between the total power received by a mobile coming from the other base stations to the total power received by its own BS.

In the model proposed by the authors of [GeEi01], the definition of other-to-own-cell interference ratio is slightly different from the classical one. The authors include the orthogonality loss factor, which applies to the own-cell interference. This form lacks an explicit physical interpretation, since the value of the orthogonally factor captures an effect that is related to despreading the signal.

[Mat01] compute the other-cell interference factor of a CDMA system for a three-dimensional air-to-ground cellular-like network consisting of a set of uniformly distributed base stations and airborne mobile users. The interference factor is found larger than that for terrestrial propagation models and depends approximately logarithmically upon both the cell height and cell radius. The analysis proposed by the authors of [**?**] provide the mean expression of the downlink other-cell-interference factor, and an experimental variation of this mean value, as a function of the distance by considering it as a sum of random variables. Their analysis does not allow to explicit its expression for each area of the cell with/without shadowing.

The literature does not provide a simple analytical expression of the downlink interference factor, which takes into account the mobile location.

1.3 Our approach: a Fluid Network Model

The goal of our approach is to propose a model that is accurate in taking into account the distance of a mobile to its serving base station, and is still simple enough to lead to close form formulas.

We propose a spatial fluid model that allows to obtain an explicit expression of the **downlink interference factor**. That expression allows to simplify considerably the computation complexity needed to obtain accurate results, without over-simplifying the model which could have resulted in large inaccuracies.

The interference factor is generally defined as the ratio of other-cell interference to inner-cell interference. In our analysis, interference factor is rather defined as the ratio of *total other-cell received power* to *the total inner-cell received power*. Although very close to the first one, this last definition is interesting for three reasons. Firstly, total received power is the metric that mobile stations (MS) are really able to measure on the field. Secondly, f represents now a characteristic of the network and does not depend on the considered MS or service. At last, the definition of f is still valid if we consider cellular radio systems without inner-cell interference. In this last case, the denominator of f is reduced to the useful power.

To establish an analytical expression of f, we need a network model. Classical models mostly consider hexagonal networks [Vit03] [BoP01] or 'random ones' where base stations are randomly distributed [Bac01] [BaB01].

In contrast to previous works in the field, the modelling key of our approach is to consider the *discrete set of BS entities* of a cellular network as a *continuum.*

Continuum approach in physics sciences

In many situations, a great set of discrete entities may be observed as a continuum in physics sciences.

In electromagnetism, the electrical charges are in many cases considered as a continuum with a given density, to analyze macroscopic systems. In mechanics, the analysis of gravity effects is determined by considering a mass density. In thermodynamic systems, many analyzes assume atoms in term of density, to analyze their macroscopic properties.

In fact, the approach considering a great set of discrete entities as a continuum with a given density allows to determine macroscopic properties of the systems, without the complexity of the calculations due to the number of entities. That approach gives results with a sufficient precision to allow analyzing and understanding the reality observed in many situations.

For example, sending satellites around the Earth needs a precise knowledge of its gravitation field. It is however not necessary, in many cases, to calculate the gravitation field generated by each element constituting the Earth (atoms, molecules or stones): an approach considering a mass continuum (with the knowledge of mass density) is sufficient.

In many situations, the determination of an electrical field generated by a great set of electrical charges does not need to consider the field generated by each discrete charge. The knowledge of a continuum charge (with the knowledge of a charge density) is sufficient.

In thermodynamics, the temperature of a gas in a closed system can be determinated by using the Maxwell-Boltzman theory applied to each atom or molecule of that gas. It can also be calulated by applying the equation linking the volume, the pressure and the temperature. Though that last approach is simpler than the first one, the two methods give the same values in most classical cases. Differences appear only when microscopic effects are analyzed.

Continuum approach in radio network analysis

The method used in physics sciences can be applied to radio network analysis. A great number of studies consider a set of discrete 'entities' as a continuum. In particular, mobiles can be described as a mobile density and traffic can be considered as a fluid, to drive macroscopic analysis of radio systems such as dimensioning or optimisation. Indeed, in many situations, a macroscopic analysis is sufficient.

In queueing analysis, performances can also be analyzed by considering a fluid approach. The fluid flow representation proposed by the authors of [KuM01] is an approximation to cell-level behavior in which cell-level details are smoothed into a steady rate of fluid flow. The authors of [MiG01] exploit fluid modeling of the data traffic to present a general methodology for the analysis of a network of routers supporting active queue management with TCP flows.

Recently, the authors of [ToTa01] described a network in terms of macroscopic quantities such as the node density. The authors investigate the spatial distribution of wireless nodes that can transport a given volume of traffic in a sensor network. They assume a *massively dense* network: there are so many nodes, that is 'does not make sense to specify their placement in terms of the positions of individual nodes' (as said by the authors).

Assuming massively dense wireless networks, Toumpis [Tou01] presents a few examples on the optimal design of these ones. He shows that these networks contain such a large number of nodes that a macroscopic view of them emerges. Though not detailed, that one preserves sufficient information to allow a meaningful network optimization.

The same idea is used in [Jac04] for ad hoc networks, to analyze the routing of the information in the case of massively dense ad-hoc networks. In his approach, the author assumes a very high density of nodes and unlimited networks.

Fluid model network

Since in the model we propose the base stations of a network are replaced by an equivalent continuum of transmitters which are spatially distributed in the network, we denote it a *fluid network model*. This also means that the transmitting power is now considered as a continuum field all over the network.

The fluid approach we propose is however quite different than the existing ones in physics and radio analysis. These last ones assume a great set of entities (atoms, massively dense radio nodes) to tend towards a *fluid limit* of the system. This one allows to calculate macroscopic quantities. In contrast, in this thesis we establish the accuracy of the cellular network fluid model we develop *whatever the density* of base stations, even when this one *is very low* and *whatever the network size*, even when this last one is *very limited*. Since the network fluid model is accurate even for very few transmitting base stations in the network, we will rather denote it as a *fluid approximation*.

Since the fluid model takes into account networks characteristics, such as the pathloss parameter, the shadowing and the topology of the system (see chapter 8), it can be used for any kind of environment. Our model takes into account the whole inter-cell interference and gives results close to the ones obtained by planning tools (see remark 7.7.1) which take into account a real environment.

In this thesis, the network access considered is constituted by a set of transmitters (denoted base stations BS) whose transmitting channels share a common radio frequency bandwidth, like in CDMA systems. Since we mainly focus on these last ones, though our approach is still valid for other systems, like OFDMA (WiMAX), TDMA (GSM) or even ad hoc networks, we present some recalls about them.

1.4 Reminder about UMTS system

The UMTS (Universal Mobile Telecommunications System) is based on a W-CDMA (Wideband Code Division Multiple Access) technology. It represents one of the the third generation mobile transmission systems. It was standardised by the ETSI and defined by the ITU. It was developed to deliver multimedia mobiles services (as voice, data) with a high quality, a high rate (for operators/ service providers/users): The range of throughputs offered to connections can reach 2Mbps locally, and up to 384 Kbit/s for greater distances. These high throughputs require a larger frequency bandwidth. A sa consequence, a 5MHz carrier has been chosen for the WCDMA.

The UMTS was developed in a 'tool box' approach, in the aim to describe the mechanisms of the mapping of services (QoS, transport formats, transport channels, codages, physical channels etc...). For each service, the codage and of the physical channel associed is the choice of the vendor. Indeed, the UMTS is developed as a multi-service system. The main idea is to define a certain number of global parameters, sufficient precise to describe a service in term of QoS, and neverthless global to be able to describe any service in this feature even the ones which do not exist yet.

The UMTS has the ability to offer a dynamical resource allocation. The choice of CDMA technology mainly lies on its flexibility (see [OJR01] and [PEH01]). It allows to improve the use of limited radio resource and to offer a quality of service according to the users needs. However, since the performances of CDMA systems are limited by the interference level, it is necessary to develop radio resources managements algorithmes and mobile admission control ones. One of the main challenges of CDMA networks is to maximize network capacity and offer quality of service answering the users needs.

The UMTS offers four classes of services: "conversational", "streaming", "interactive" and "background" which cover real-time and "best-effort" applications. The UMTS allows moreover offering variable throughputs. In particular, for the voice service, the UMTS will use the AMR (Adaptive

Multi-rate) codec which offers eight different transmission rates, from 4.75 kbps to 12.2 kbps, and which can vary in a dynamical way every 20ms.

1.5 Organization

This thesis is organized as follows:

In chapter 2 we present the frame of our analysis. The interference model is introduced and the basic derivations of cellular radio network. We first define the **downlink interference factor** f as the ratio between external received power experienced by a mobile coming from all the base stations of the network and internal received power coming from the base station it belongs to. The definition we propose is quite different than the common one which only considers a ratio of interferences. Since it represents the 'weight' of the network on a given mobile, we show that it characterizes cellular radio systems. Though we mainly focus our analysis on CDMA systems since they are typically systems where the downlink interference factor represents a fundamental parameterfocus, that one can however be applied to other system such as OFDMA, WLAN or GSM ones.

In chapter 3 we develop the fluid network model. Using this approach, we establish an analytical expression of the interference factor. Since in a real network, the base stations are a discret set, and not a continuum, we validate our model by comparing it to a simulated hexagonal network. And we show through Monte Carlo simulations that the obtained formula provides a very good approximation of f. This closed formula accounts for the cell radius, the network size and the path-loss exponent. We establish the accuracy of the fluid model *whatever the density* of base stations, even when it *is very low* and *whatever the network size*, even when it is *very limited*.

In chapters 4, 5 and 6, we propose some possible applications, as an analytical study based on the fluid model, in the case of CDMA networks.

The chapter 4 analyzes the capacity of a cell and a network based on a CDMA frequency division duplex (FDD) technology, in term of mobile number.

As an application of the fluid model, we calculate the capacity of a cell in term of number of mobiles, and analyze the densification as solution to an increasing traffic. The fluid model approach allows to analyse and compare instantaneously different solutions with the aim to adapt the network, or a given zone of the network, to an increasing traffic demand. The sectorisation, and a comparison of these two means to anwer a traffic demand, is developed in chapter 7.

In the chapter 5, we propose a study control policy for a multiservice case, considering two types of services **non real time** ones (NRT) and **real time** ones (RT), in a CDMA network. We develop a simplified mathematical model that allows us to analyze the performance of call admission control combined with GoS control in a WCDMA environment with integrated RT and NRT traffic. Performance measures, as call blocking probabilities and expected transfer times, are then computed by modeling the CDMA system as a quasi-birth-and-death (QBD) process. We establish these performances depend on a parameter, the average interference factor, which is function of the exponential pathloss factor, and show the fluid model can be used to determine analytically this one.

In the chapter 6, the fluid model is used to analyze the global outage probability: we get a simple outage probability approximation by integrating f over a circular cell. In addition, as f is obtained as a function of the distance to the BS, we derive a *spatial outage probability*, which depends on the location of a newly initiated call. As downlink is often the limited link w.r.t. capacity, we focus on this direction, although our framework can easily be extended to the uplink.

Since one of the hypothesis on which the calculation of the interference factor lays is a radial and deterministic pathloss depending on the distance between the transmitter and the receiver, we propose two refinements in the chapters 7 and 8. Moreover an extension to the uplink is developed in the chapter 9, too.

In the chapter 7, we propose a first refinement by considering a pathloss also depending on the antenna gain: this last approach enables to analyze networks with sectored cells. As an application, a comparison between the sectorisation of a network (i.e. to add directional antennas in the existing base station sites) and a densification (i.e. to add new sites) is proposed. Considering a fluid network approach, we first establish the expression of the interference factor for a three-sectored network. We validate this approach comparing it to a numerical computed network. We compare, as solutions to an increasing traffic, the densification to the sectorisation. And we show, this model enables to analyse the mobile admission in CDMA networks. We end by generalizing our model for a *q-sectored* network, with $q \geq 1$.

In chapter 8, we propose a second refinement by taking into account the shadowing effect and more generally the effects of each specific environment (urban, rural, streets buildings)... since in a real network, the pathloss also depends on the local environment (terrain, buildings, trees). As a consequence, the radio link can be modelled by a term which expresses that the power received at any point of the system depends on the distance r from the transmitter (the line-of-sight path), and the environment (terrain, buildings, trees). The first term depends on the type of the global environment, urban or rural, and may moreover depend on the type of cells: macro or micro. The last term, the shadowing, is generally modelled as a lognormally distributed function [Stu01].

Using the fluid network approach, we express the interference factor's mean value and standard deviation, and analyze the influence of different network's parameters: cell radius, exponential pathloss parameter, distance of the mobile to its serving base station.

And in chapter 9, we analyze the uplink in term of fluid model. We show the uplink analysis can follow an analogue way as the downlink one, done in the chapter 3. We develop, and validate, an uplink fluid model of the network, define an uplink interference factor, and establish an explicit formula of this parameter. The key modelling of the approach we develop is to consider the discrete entities of the network, BS and MS, as continuum.

As an application, we propose an analytical admission control study for the two links, which takes into account the whole network around a given cell.

Fluid Model

Chapter 2

Interference factor

The aim of this chapter is to present the frame of our analysis, and to introduce the downlink interference factor parameter which represents a characteristics of cellular radio networks.

2.1 Introduction

In this chapter, the interference model is introduced and the basic derivations of cellular radio network. We first define the **downlink interference factor** f. It represents the ratio between external and internal received power experienced by a mobile. We show that it characterizes cellular radio systems. Though our analysis is focused on CDMA systems, it can be applied to any system where transmitting nodes generate interferences such as OFDMA ones.

2.2 Interference Model and Notations

In this section, we introduce the interference model and give the notations used throughout the chapter.

2.2.1 Network

We consider a cellular radio system with B base stations (BS) and U mobile stations (MS) and we focus on the downlink. Since we assume all the BS transmit power at the same frequency, we focus on CDMA systems. If a mobile u is attached to a base station b (or serving BS), we write $b = \psi(u)$. Soft handover (SHO) situations are not considered here. The position of station u is denoted $x(u)$. Notice that "position" may include location and antenna pointing and gain. To simplify the presentation, position will just refer to a geographical location in a plane, $x = (x_1, x_2) \in \Re^2$. The location of a base station is, as usual, called a site, and we assume omni-directionnal antennas, so that a base station covers a single cell.

2.2.2 Propagation

The propagation path gain $g_{b,u}$ designates the inverse of the pathloss L between stations b and u, $g_{b,u} = 1/L_{b,u}$. If a propagation path is considered between a BS and a particular location x, the corresponding path gain will also be denoted by $g_{b,x}$. In this way, we assimilate $g_{b,x(u)}$ and $g_{b,u}$ by a slight abuse of notation, and more generally we will assimilate throughout the chapter u and $x(u)$.

2.2.3 Power

The following power quantities are considered:

- $P_{b,u}$ is the useful transmitted power from station b towards mobile u (for user's traffic);

- $P_b = P_{cch} + \Sigma_u P_{b,u}$ is the total power transmitted by station b, P_{cch} represents the amount of power used to support broadcast and common control channels.

- $p_{b,u}$ is the power received at mobile u from station b; we can write $p_{b,u} = P_b\, g_{b,u}$;
- $S_{b,u} = P_{b,u}\, g_{b,u}$ is the useful power received at mobile u from station b (for traffic data); since we do not consider SHO, we can write $S_u = S_{\psi(u),u} = S_{b,u}$.

2.2.4 Interferences

The total amount of power experienced by a mobile station u belonging to a cell b in a cellular system can be split up into several terms: useful signal ($S_{b,u}$), interference and noise (N_0). It is common to split the system power into two terms: $p_{b,u} = P_{int,u} + P_{ext,u}$, where $P_{int,u}$ is the *internal* (or own-cell) received power and $P_{ext,u}$ is the *external* power (or other-cell interference). Notice that we made the choice of including the useful signal $S_{b,u}$ in $P_{int,u}$, and, as a consequence, it has to be distinguished from the commonly considered own-cell interference.

With the above notations, we define the interference factor in u, as the ratio of total power received from other BS to the total power received from the serving BS b:

$$f_u = P_{ext,u}/P_{int,u} \tag{2.1}$$

The quantities f_u, $P_{ext,u}$, and $P_{int,u}$ are location dependent and can thus be defined in any location x as long as the serving BS is known.

We can express the interference factor as:

$$f_u = \frac{1}{P_b g_{b,u}} \sum_{i \neq b}^{B} P_j g_{j,u} \tag{2.2}$$

As a special case we notice that for a homogeneous network and traffic (uniformly distributed), all the base stations transmit the same power. The interference factor can thus be expressed as

$$f_u = \frac{1}{g_{b,u}} \sum_{i \neq b}^{B} g_{j,u} \tag{2.3}$$

These expressions (2.2 and 2.3) show that, although very close to the common definition [HoT01] that considers a ratio of interferences, this new definition of f is interesting for the following reasons:

- Firstly, the total radio power received by a mobile $P_{ext,u} + P_{int,u}$ is a metric easy and simple to be measured by that mobile.
- Secondly, using this definition, the parameter f represents a characteristic of the network. It does not depend on any considered MS or service, but only on the number of base stations, their positions and transmitting power and the pathloss. This last one depends on the environment (urban, rural...): it can be considered characterizing a network zone. We moreover observe that in a case of an homogeneous network, the interference factor does no more depend on the base station transmitting power.
- At last, that definition of f is still valid when the considered cellular system has no inner-cell interference. In this case, the denominator of f is reduced to the useful power. So that definition of interference factor can be applied to other systems than CDMA, as for example OFDMA (WiMAX) or TDMA ones (GSM with frequency hopping), and can be extended to ad-hoc networks.

2.2.5 Transmitting channels orthogonality

In downlink, a coefficient α may be introduced to account for the lack of orthogonality between physical channels in the own cell (see for example [NeM01]). Note that α, $0 \leq \alpha \leq 1$, a priori depends on the location, and should be noted α_x. However this case is almost never considered. In the rest of our analysis, we follow the common assumption that α is not location dependent. An intra cell interference expressed as $\alpha(P_{int,u} - S_{b,u})$ can appear due to the transmitting powers towards the other mobiles of the cell.

2.2.6 Signal to Interference Ratio

The signal to interference plus noise ratio (SINR)is denoted :

- γ_u the SINR evaluated at station u;
- γ_u^* the target SINR for the service requested by station u.

2.2.7 Basic Derivations

The signal to interference plus noise ratio will be used as the criteria of radio quality. Assuming mobiles use only one service, γ_u^* is the target SINR for the service requested by MS u. This figure is a priori different from the SINR evaluated at mobile station u. However, we assume perfect power control, so $\gamma_u = \gamma_u^*$ for all users. In the UMTS case, we assume that perfect power control (PC) is performed for all users ([LaWN01][HiB01]), the SINR γ_u experienced by a mobile has to be at least equal to the target value γ_u^*.

Remark

As a consequence of the perfect power control, at each moment the transmitting power $P_{b,u}$ is adapted to the propagations conditions for the mobile to receive the power he needs. It means that $P_{b,u}$ is not a constant. As a consequence the total transmitting power P_b of the base station b should not be constant, even in a homogeneous network. However we can assume that statistically, when some mobiles need a lower power, others need a higher one. And the total transmitting power is about constant. With the introduced notations, the SINR experimented by u can be derived (see e.g. [Lagr05]):

$$\gamma_u = \frac{S_u}{\alpha(P_{int,u} - S_u) + P_{ext,u} + N_0} \tag{2.4}$$

where the term $\alpha(P_{int,u} - S_u)$ represents the intra cell interferences.
For an **UMTS System**, we can write:

$$\gamma_u^* = \frac{S_u}{\alpha(P_{int,u} - S_u) + P_{ext,u} + N_0} \tag{2.5}$$

And for an **OFDMA system**, there is no internal interference, so we can consider that $\alpha(P_{int,u} - S_u) = 0$. From the expression (2.4), and introducing the parameter

$$\beta_u = \frac{\gamma_u}{1 + \alpha\gamma_u} \tag{2.6}$$

we can express S_u as:

$$S_u = \beta_u P_{int,u}(\alpha + P_{ext,u}/P_{int,u} + N_0/P_{int,u}) \tag{2.7}$$

and the transmitted power for MS u, $P_{b,u} = S_u/g_{b,u}$, using relations $P_{int,u} = P_b g_{b,u}$ and $f = P_{ext}/P_{int}$ as:

$$P_{b,u} = \beta_u(\alpha P_b + f_u P_b + \frac{N_0}{g_{b,u}}). \tag{2.8}$$

For an isolated cell $f_u = 0$, the term $f_u P_b$ vanishes. As a consequence, in a multi-cell situation, the interference factor well characterizes, as a network, the system we analyze. Indeed this parameter gives the influence of the network, especially the number, the positions and the power transmissions of the base stations, on a mobile belonging to a given cell b of a CDMA network. Without this term, the network 'disappears'. Since it represents the relative weight of the network on a cell, to analyze such a network, it appears interesting to have an analytical expression of f.

Remark

Though we mainly focused on systems based on a CDMA access technology, the interference factor represents the 'relative weight' of the network on a mobile belonging to a given cell also in the case of OFDMA based systems or TDMA ones.

2.3 Base station transmitting power

Assume that there are M mobiles in the cell b, the total output power P_b of BS b is given by:

$$P_b = P_{cch} + \sum_u P_{b,u}, \tag{2.9}$$

where P_{cch} is the power dedicated to the common channels. And so, according to Eq.2.8, we can express the total transmitting power of the base station as:

$$P_b = \frac{P_{cch} + \sum_u \beta_u \frac{N_0}{g_{b,u}}}{1 - \sum_u \beta_u(\alpha + f_u)}. \tag{2.10}$$

The minimum transmitting power towards the mobile u is given by the relation (2.8). It points out the different effects useful power has to overcome to reach the target SINR when PC is activated: internal interference, external interference and noise. Let notice that for an isolated cell, there are no external interferences: the parameter f_u disappears. If moreover the third term $N_0/g_{b,u}$ is low compared to the first one αP_b, the minimum transmitting power towards the mobile u, which can be expressed as

$$P_{b,u} = \beta_u \alpha P_b, \tag{2.11}$$

does no more depend on the pathloss.

2.4 Concluding remarks

In this chapter, we defined the **downlink interference factor** f as the ratio between external and internal received power experienced by a mobile belonging to a cell of a cellular radio network. We moreover showed that it can characterize, as a network, a cellular radio system.

Chapter 3

Fluid Model for Cellular Networks

The aim of this chapter is to develop an analytical cellular network model. Since we consider the base stations of the network as a continuum, we denote it a fluid model. Using this approach, we establish an analytical expression of the interference factor. In a real network, the base stations are a discret set, and not a continuum, so we validate our model by comparing it to a classical hexagonal model network. We show the accuracy of the fluid model even for a very low density of BS and for a very limited size of network.

3.1 Introduction

In this chapter, the 'so called' fluid model leading to the expression of f is presented and an analytical formula approaching the factor f is derived using this model. We indeed show [Kel01] that the ratio between external and internal power can be well approximated by an analytical formula in all points of the cell. This closed formula accounts for the cell radius, the network size and the path-loss exponent. We validate this fluid model comparing it by simulation to a regular hexagonal layout. We show through Monte Carlo simulations that the obtained formula provides a precise approximation of f.

We end by proposing different possible applications of the fluid model as the network densification, quality of service, and mobile management analysis.

3.2 Assumptions

The key modelling step of the model we propose consists in replacing a given fixed finite number of base stations by an equivalent continuum of transmitters which are spatially distributed in the network. This means that the transmitting power is now considered as a continuum field all over the network. As already detailled in section 1.3, in physics sciences, many kind of systems (electromagnetic, mechanics, thermodynamic ones) can be analyzed as a continuum. In radio network analysis too, there are studies [ToTa01] [Jac04] which consider the discrete transmitting nodes of adhoc systems as a continuum, in term of node density.

When a uniform traffic and a uniform BS density are assumed (homogeneous network), and using a model where the pathloss $g_{b,u}$ is only a function of the distance r (between the base station b and the mobile u), f_u only depends on that distance. So the interference factor which was written as a function of u (with an index u) can now be written as a function of r. Since the density of BS is considered as uniform, it also means that mobiles positioned at the same distance from the BS have the same interference factor. In fact, all the mobiles positioned on circles whose centre is the serving BS b have thus the same pathloss and the same interference factor. In this context, the network is characterised by a base station density ρ_{bs} [Kel01]. We assume that mobiles and base stations are uniformely distributed in the network, so that ρ_{bs} is constant. As the network is homogeneous, all base stations have the same output power P_b.

We can notice that classical network models also consider either hexagonal or random base stations distributions. They are considered as homogeneous but locally they are not, since the power received at a point of the cell depends not only on the distance r between the serving BS and the mobile, but also on an angle θ characterizing the relative positions of the base stations.

3.3 Basic Model

We focus on a given cell and consider a round shaped network around this centre cell with radius R_{nw}. Half the distance between two base stations is R_c (see figure 3.1).

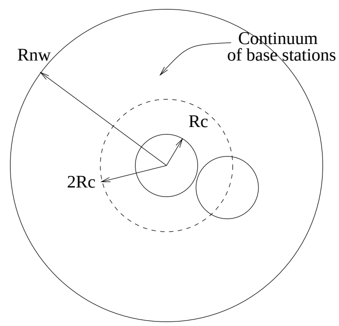

Figure 3.1: Network and cell of interest in the fluid model. The distance between two BS is $2R_c$ and the network is made of a continuum of base stations.

For the assumed omni-directional BS network, we use a propagation model where the path gain, $g_{b,u}$, only depends on the distance r between the BS b and the MS u. The power, $p_{b,u}$, received by a mobile at distance r_u can be written $p_{b,u} = P_b K r_u^{-\eta}$, where K is a constant and $\eta > 2$ is the path-loss exponent.

3.4 Interference Factor

Let's consider a mobile u at a distance r_u from its serving BS $b = \psi(u)$. Each elementary surface $zdzd\theta$ at a distance z from u contains $\rho_{bs} zdzd\theta$ base stations which contribute to $P_{ext,u}$. Their contribution to the external interference is $\rho_{\mathrm{BS}} zdzd\theta P_b K z^{-\eta}$. We approximate (see further remark) the integration surface by a ring with centre u, inner radius $2R_c - r_u$, and outer radius $R_{nw} - r_u$ (see figure 3.2).

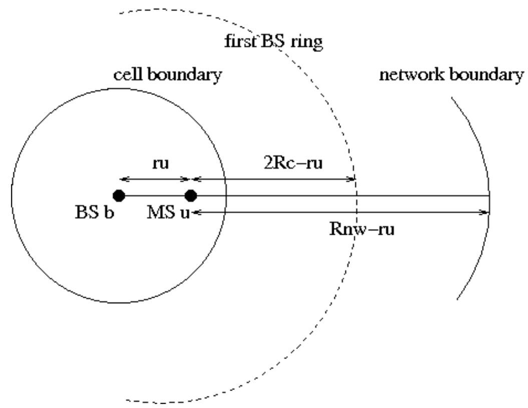

Figure 3.2: Integration limits for external interference computation.

$$\begin{aligned} P_{ext,u} &= \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{bs} P_b K z^{-\eta} z dz d\theta \\ &= \frac{2\pi \rho_{bs} P_b K}{\eta - 2} \left[(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta} \right]. \end{aligned} \tag{3.1}$$

Moreover, MS u receives internal power $P_{int,u}$ from b, which is at distance r_u: $P_{int,u} = P_b K r_u^{-\eta}$. So, the interference factor defined as $f_u = P_{ext,u}/P_{int,u}$ can be expressed by:

$$f_u = \frac{2\pi \rho_{bs} r_u^{\eta}}{\eta - 2} \left[(2R_c - r_u)^{2-\eta} - (R_{nw} - r_u)^{2-\eta} \right]. \tag{3.2}$$

Note first of all that f_u does not depend on the BS output power. This is due to the fact that we assumed an homogeneous network and so all base stations transmit the same power. In our model, f only depends on the distance r to its serving BS and can be defined in each location, so that we can write f as a function of r, $f(r)$.

$$f(r) = \frac{2\pi \rho_{bs} r^{\eta}}{\eta - 2} \left[(2R_c - r)^{2-\eta} - (R_{nw} - r)^{2-\eta} \right]. \tag{3.3}$$

Thus, if the network is large, *i.e.* R_{nw} is big in front of R_c, f_u can be further approximated by:

$$f(r) = \frac{2\pi\rho_{bs}r^{\eta}}{\eta - 2}(2R_c - r)^{2-\eta}. \tag{3.4}$$

The expression (3.3) lays on the assumption of a network uniformly distributed with a constant BS density ρ_{bs}. We can notice that if it is not the case, for example when ρ_{bs} depends on the location and thus has to be written as $\rho_{bs}(r, \theta)$, the approach remains the same, only (3.1) differs. Our approach enabled to establish the expression (3.2) of the interference factor $f(r)$. This one appears very simple and easy to calculate. It depends on

- the density of base stations ρ_{bs},
- the radius of a cell R_c,
- the size of the considered network R_{nw} and
- the pathloss parameter η.

The fluid approach we proposed may be applied to any system with interferences, like for example CDMA systems or OFDMA ones.

Remark about the integration domain

It is important to notice that the expression (3.1) of the interferences due to the other base stations of the network is a combination of calculation and empiric analysis. We denote d the distance between the mobile belonging to the observed cell and the transmitter M located at u. The power received by a mobile located at the distance r from its serving BS should rigorously be expressed as the following exact expression (see figure 3.3):

$$\begin{aligned} P_{ext,u} &= \int_0^{2\pi}\int_{R_c}^{R_{nw}} \rho_{bs}P_bK\left\|\vec{d}\right\|^{-\eta} udud\theta \\ &= \int_0^{2\pi}\int_{R_c}^{R_{nw}} \rho_{bs}P_bK\left\|\vec{u}-\vec{r}\right\|^{-\eta} udud\theta \end{aligned} \tag{3.5}$$

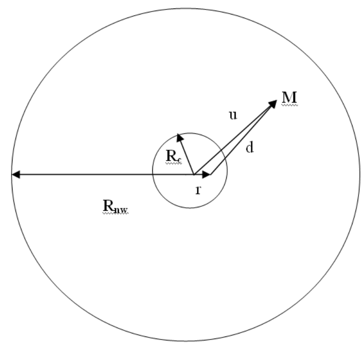

Figure 3.3: Network: continuum of base stations. Influence of a BS located at $M(u,\theta)$ on a mobile located at r

and finally

$$P_{ext,u} = \int_0^{2\pi} \int_{R_c}^{R_{nw}} \rho_{bs} P_b K \left(u^2 + r^2 - 2urcos\theta\right)^{-\eta/2} ududθ \qquad (3.6)$$

That elliptic integral has no simple explicit expression. Our aim is however to obtain an explicit expression of P_{ext} .

So we can notice that:

- For a homogeneous network constituted by a discrete set of base stations regularly distributed in the plane, the closest base stations of the serving base station b are located at a distance of $2R_c$. So it can appear reasonnable to consider that value as the low limit of the integration domain.

- We notice moreover that the base stations of the network can be considered at a great distance from most of the receiver mobiles: So we can write $\left\|\vec{d}\right\|^{-\eta} \approx \|\vec{u}\|^{-\eta}$

- considering this last approximation, it is however needed to adapt the integration domain: For a mobile located at a distance r from its serving base station b, the closest base stations of the network are located at a distance $2R_c - r$.

These three reasons lead us to approximate the integration surface by a ring with centre a mobile located at r, inner radius $2R_c - r_u$, and outer radius $R_{nw} - r_u$. With these approximations we obtain (3.1) as the explicit expression of P_{ext}.

Since the expression (3.2) allows calculating analytically the influence of a network on each point of a given cell, it opens a large number of possibilities of analysis for cellular networks such as CDMA ones: quality of service indicators, planning analysis, or scheduling policies are also depending on this parameter...(detailled in section 1.1). This closed-form formula will allow us to fastly compute performance parameters of a CDMA network.

Before going ahead, it is however necessary to verify that approach and to validate the different approximations we made in this model. The fluid model makes some unusual assumptions: the network's base stations set is considered as a *continuum*. The existing approaches [ToTa01] [Tou01] [Jac04] considering the transmitting nodes as a continuum assume a massively dense network. It means that the distance between two neibors transmitters is very low. In a real network, the base stations do not constitute a continuum, and moreover, the distance between two neighbors base stations can reach several kilometers. As a consequence that assumption, on which we develop our analysis, can seem not justified. That is the reason why we propose hereafter a validation of the fluid model.

3.5 Validation of the Fluid Model

3.5.1 Simulation Methodology

To validate our network approach, we choose to compare it to a hexagonal classical one. We calculate the interference factor values given by the continuum set of base stations of our fluid model network, and the ones obtained with a discrete set of base stations distributed according to a hexagonal pat-

tern.
We will compare the figures obtained with Eq. (3.2) with those obtained by simulations. The simulator assumes an homogeneous hexagonal network made of several rings around a cell we analyze. Figure 3.4 shows an example of such a network with the main parameters involved in the study.
The traditional hexagonal model is widely used, especially for dimensioning purposes. That is the reason why a comparison of our model to a hexagonal one is useful.
The validation of the fluid model is based on Monte Carlo simulations, with snapshots approach.

3.5.2 MS locations

At each snapshot of the Monte Carlo simulation, random locations are drawn for the mobile stations of the network. The number of mobile stations per cell is fixed all along the simulation and their spatial distribution within one cell is uniform. Path-loss model is implemented as described in section 3.2.

3.5.3 Serving base stations

A MS is served by the base station with the smallest path-loss, including shadowing if considered. So, the MS is assumed to choose the best base station and soft-handover is not taken into account.

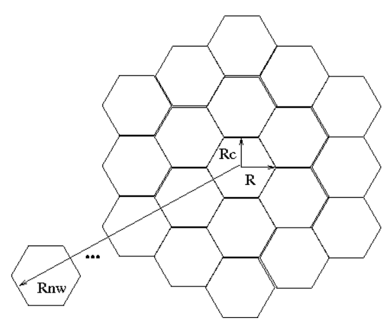

Figure 3.4: Hexagonal network and main parameters of the simulation.

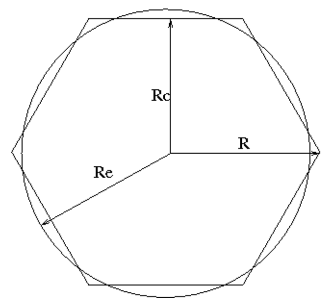

Figure 3.5: Different ranges involved in the analysis: R the hexagonal cell radius, R_c the half distance between two BS, and R_e the radius of the equivalent disk.

The validation is done by computing f in each point of the reference cell and averaging the values at a given distance from the BS. This computation can be done independently of the number of MS in the cell and of the BS output power.

We further analyze the influence of each parameter of Eq.3.2:

- the network size R_{nw},
- the hexagonal cell radius R,
- the path-loss parameter η.

Remark

For the validation process, we choose to consider the hexagonal cell radius R i.e. the distance between the base station and a vertex of the hexagon, to be sure to take into account all the positions of the system. This choice induces an *over coverage* of the zone located close to the cell limits. However, any other choice, for example considering half the distance R_c between two neighbors base stations, or the equivalent cell radius R_e (i.e. the cell disk has the same surface as the hexagonal one) would *under cover* the network.

We define R as the hexagonal cell radius, R_c as the cell radius, and R_e the radius of the equivalent disk.

3.5.4 Simulation Results

Simulation parameters are the following:

- $\alpha = 0.7$,
- $\eta = 2.5$, $\eta = 3$, $\eta = 3.5$ and $\eta = 4$,
- $R = 0.5$ Km, $R = 1$ Km, $R = 1.5$ Km and $R = 2$ Km,
- $R_{nw} = 3R_c$, $R_{nw} = 5R_c$, $R_{nw} = 9R_c$ and $R_{nw} = 21R_c$. These sizes correspond to 1 ring, 2 rings, 4 rings and 10 rings of cells around the observed one.
- $\rho_{bs} = (3\sqrt{3}R^2/2)^{-1}$

Eq.3.2 is also plotted for comparison.

3.5.5 Accuracy of the fluid model

We observe the fluid model matches very well the simulations on an hexagonal for wide ranges of pathloss exponents (figures 3.6 and 3.7), cell radii (figures 3.8 and 3.9) and network dimensions (figures 3.10 and 3.11). Until a distance of $0.9R$, the difference is minute. Only when distance is close to R, a difference appears, which increases with η: around 9% for $\eta = 3$ and 12% for $\eta = 4$. This is due to the fact that the fluid model is basically circular and thus does not capture the extreme parts of the hexagon.

Note that the considered network size can be finite and chosen to characterize each specific local network environment: figure 3.10 and 3.11 show the influence of the network size. As a consequence, this model allows to develop analyses adapted to each zone, taking into account each specific considered network parameters.

We notice moreover that the fluid model can be used even for great distances between the base stations: We validated the model considering a distance reaching 4 Km between the BS (see figure 3.9).

Influence of the pathloss parameter η

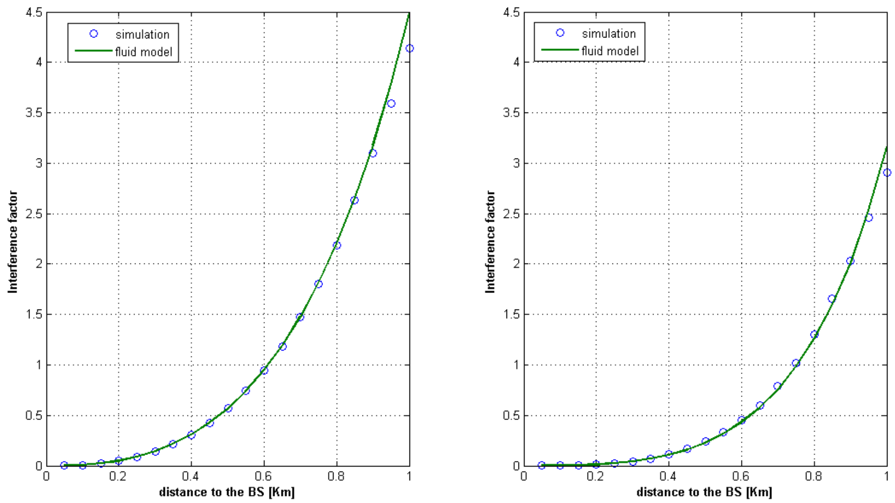

Figure 3.6: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on a ten ring hexagonal network with a hexagonal cell radius $R = 1$ Km for $\eta = 2.5$ (left) and $\eta = 3$ (right).

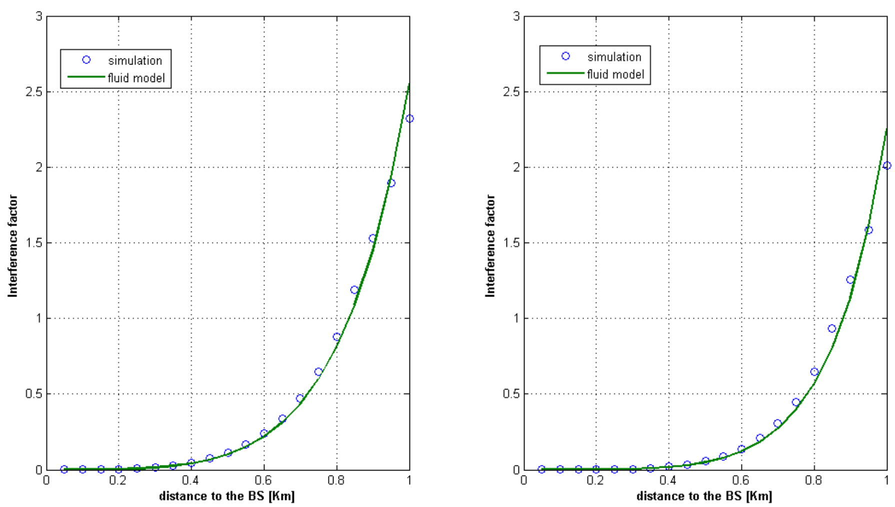

Figure 3.7: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on a ten ring hexagonal network and a hexagonal cell radius $R = 1$ Km for $\eta = 3.5$ (left) and $\eta = 4$ (right).

Influence of the hexagonal cell radius R

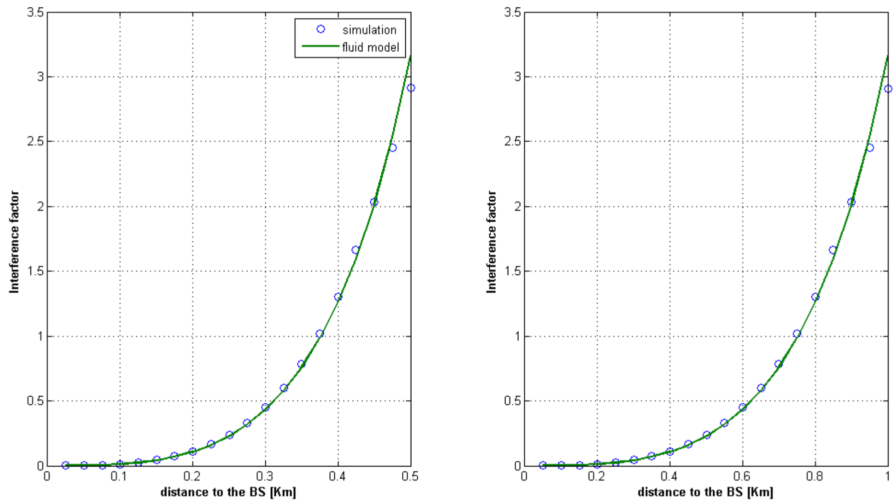

Figure 3.8: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on a ten ring hexagonal network and a pathloss exponent $\eta = 3$ for cell radii $R = 0.5$ Km (left) and $R = 1$ Km (right).

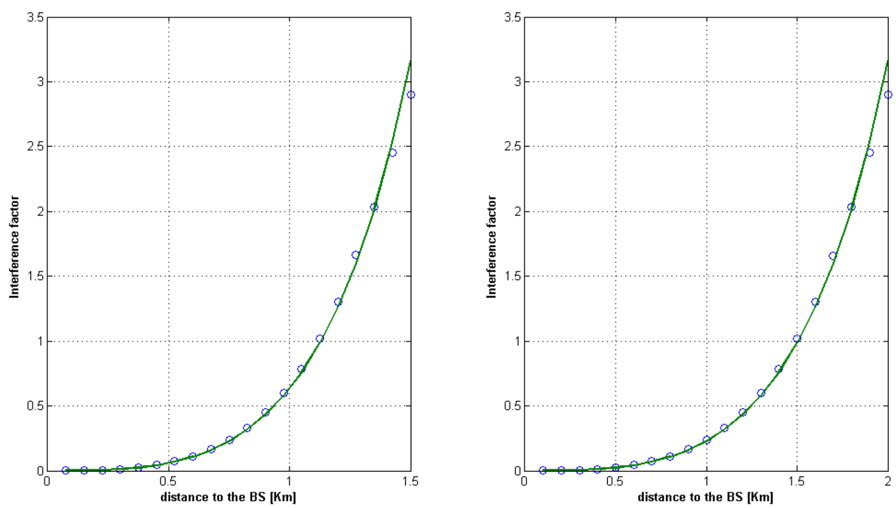

Figure 3.9: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on a ten ring hexagonal network with a pathloss exponent $\eta = 3$ for cell radii $R = 1.5$ Km and$R = 2$ Km (right).

Influence of the network's size R_{nw}

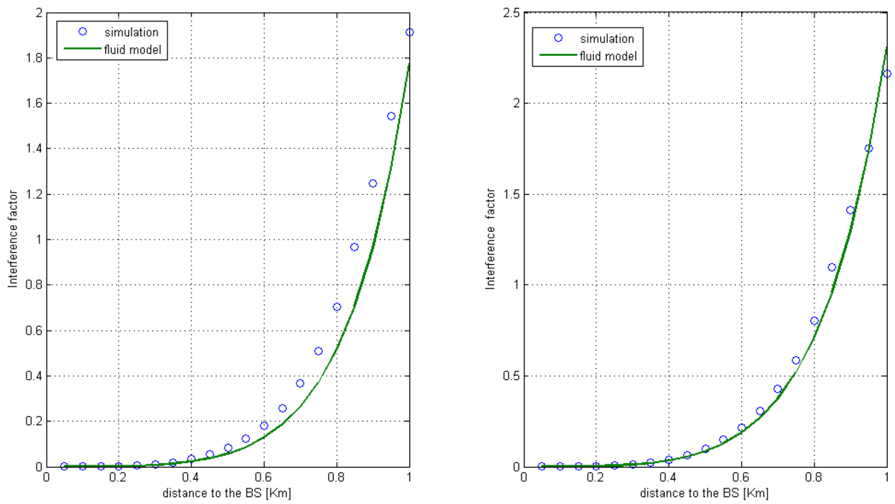

Figure 3.10: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on an one ring (left) and two ring (right) hexagonal network with cell radius $R = 1$ Km and a pathloss exponent $\eta = 3.5$.

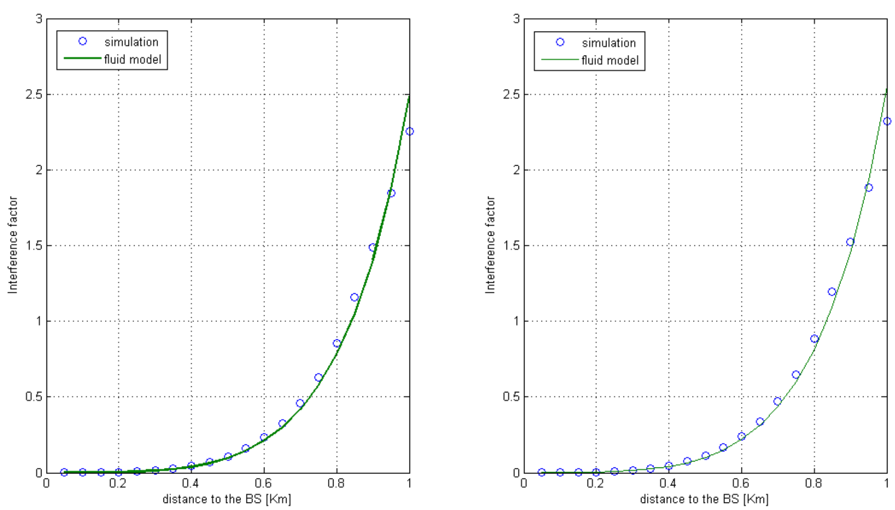

Figure 3.11: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on a four ring (left) and ten ring (right) hexagonal network with cell radius $R = 1$ Km and a pathloss exponent $\eta = 3.5$.

We conclude that the fluid model approach is accurate *even for a very low base station's density*: It allows to calculate the interference factor experienced by a mobile, whatever its position in a cell, and to characterize cellular radio networks.

3.5.6 Limits of the fluid model

Extreme values of the model parameters

We showed the fluid model matches very well the simulations on an hexagonal network, for wide ranges of pathloss exponent, cell radii and network sizes. It appears interesting to analyze until which limits the model matches. figure 3.12 and figure 3.13 show the fluid model is accurate even for very high and very low pathloss, from $\eta = 2.1$ until $\eta = 6.5$, and for very high and very low size of hexagonal cell radius, from $R = 50\ m$ until $R = 10\ km$.

Our model considers a continuum of BS. As a consequence the interference factor only depends on the distance r between a mobile and its serving base station. The figures 3.6 to 3.13 show the accuracy of the fluid model. However, considering an hexagonal network, it seems logical that the interference factor also depends on the angular position of the mobile at a given distance. We analyze this point hereafter.

Radial dependency of the fluid model

The interference factor values obtained with a simulated discrete hexagonal network allow drawing the curves of figure 3.14. The left-hand figure shows the positions in the cell for which the interference factor are constant value: It seems to be circles for distances 400, 600, 800 and 900 m from the serving base station. On the right-hand, the interference factor is drawn as a function of the distance by the fluid model and compared to the values given by the simulated hexagonal network. They are very close to each other. We however observe some differences. Until 0.9 R, these differences are less then 3%, and they reach about 10% at the edge of the cell. It means that the real values

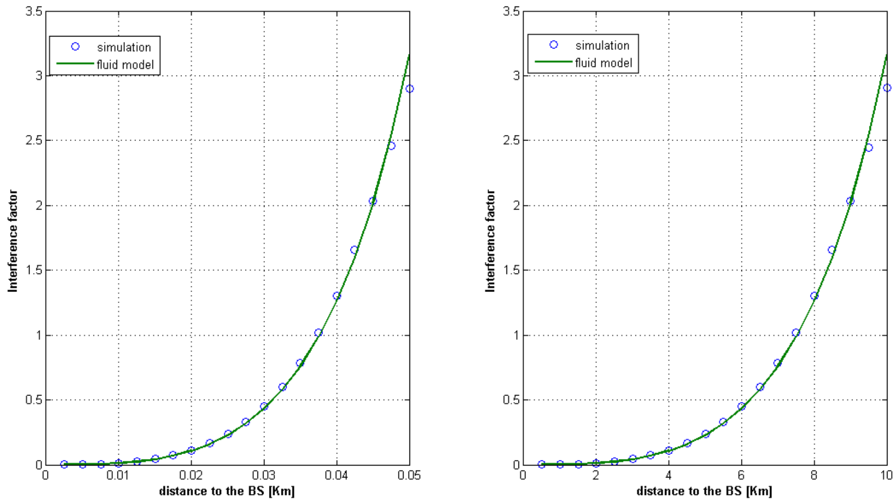

Figure 3.12: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on a ten ring hexagonal network with a pathloss exponent $\eta = 3$ for cell radii $R = 50$ meters (left) and $R = 10$ Km (right).

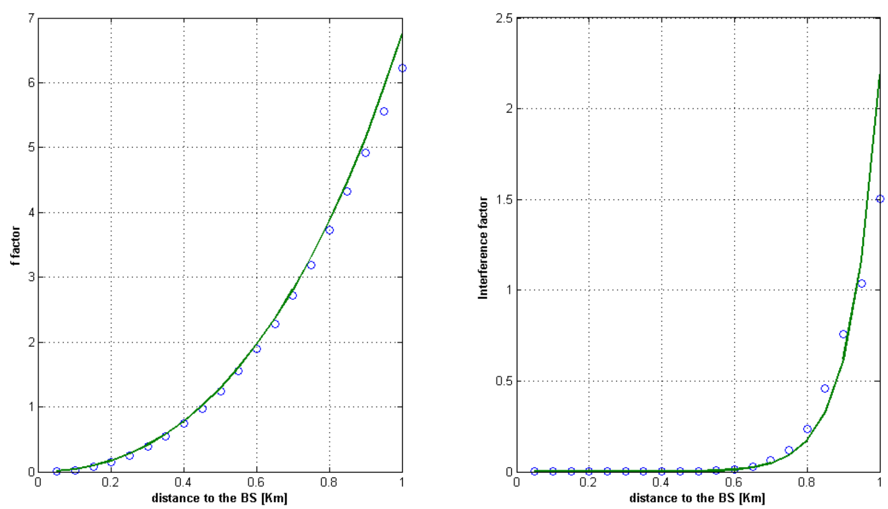

Figure 3.13: Interference factor vs. distance to the BS; comparison of the fluid model with simulations on a ten ring hexagonal network and a hexagonal cell radius $R = 1$ Km for pathloss exponent $\eta = 2.1$ (left) and $\eta = 6.5$ (right).

of f experienced by a mobile located at (r, θ) mainly depend on the distance r and very few on the angle θ of the position.

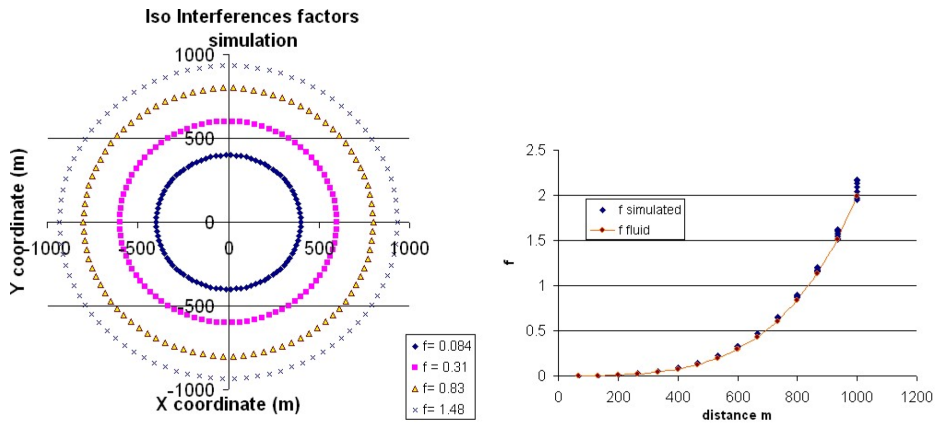

Figure 3.14: Iso-interference factor curves vs. position of the mobile, for an hexagonal network.

Moreover, let us remind that the hexagonal pattern is only a representation of the reality. A real network *is not* hexagonal. We can conclude that our approximation considering a dependence of the interference factor only with the distance and not with the angle (the radial dependence of f) represents a fair approximation.

Fluid and Hexagonal models

We validated the fluid model, comparing it to a simulated hexagonal one. We especially established the accuracy of the fluid model for wide ranges of distance between neighbor base stations, *i.e.* even for very low base stations densities, wide ranges of pathloss exponent and wide ranges of network sizes. Moreover, these expressions take into account the size zones R_{nw} to be considered, which can be chosen characterizing a typical environment (pathloss exponent, urban, rural, macro or micro cells).

The fluid model and the traditional hexagonal model are two simplifications of the reality. Though the latter is widely used, none is a priori better than the other.

3.6 Properties of the interference factor

We established, using (2.8), that the interference factor characterizes a CDMA network. The expression (3.4), which reflects the topological and propagations properties of CDMA network, confirms moreover that result: It depends on the pathloss exponent, the number of base stations, their sizes and their positions. As a consequence, the properties of the interference factor give informations on the properties of 'CDMA-type' networks.

3.6.1 Insensitivity of the fluid model

We can notice that the interference factor is not *directly dependant* on the size zone and on the size of the cell radius R_c but only on the *relative position* of the mobile in the cell, and on the relative cell's size to the network's. We denote $v = \frac{r}{R_c}$, the relative distance of a mobile to its serving base station, and N_c the number of cell rings around the considered one: we have $R_{nw} = (2N_c + 1)R_c$. We thus can express from (3.2) and (3.4):

$$f(v) = 4\frac{\pi\frac{\sqrt{3}}{3}}{\eta - 2}(\frac{v}{2})^{\eta}\left[(1 - \frac{v}{2})^{2-\eta} - (N_c + \frac{1}{2})^{2-\eta}(1 - \frac{v}{2N_c + 1})^{2-\eta}\right]. \tag{3.7}$$

and when $N_c \to \infty$:

$$f(v) = 4\frac{\pi\frac{\sqrt{3}}{3}}{\eta - 2}(\frac{v}{2})^{\eta}(1 - \frac{v}{2})^{2-\eta}. \tag{3.8}$$

3.6.2 Influence of the system parameters

We observed in the validation section, the interference factor variations with the system parameters R_{nw}, R_c and η. We notice that $R_c = \frac{\sqrt{3}}{2}R = \sqrt{\frac{\pi\sqrt{3}}{6}}R_e$ The interference factor is:

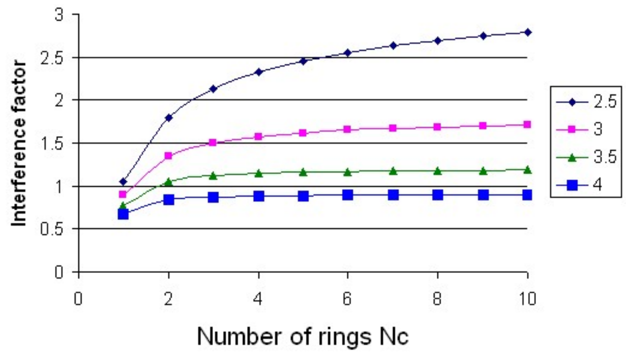

Figure 3.15: Fluid model interference factor vs. number of cell rings, for $\eta = 2.5$, $\eta = 3$, $\eta = 3.5$, $\eta = 4$.

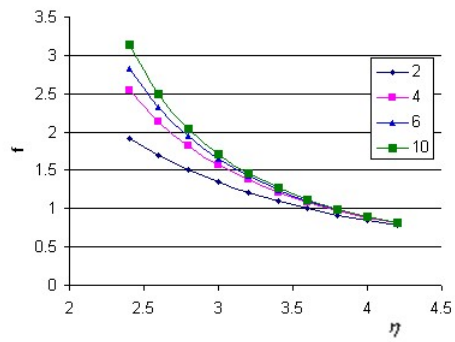

Figure 3.16: Fluid model interference factor vs. pathloss exponent η for different network sizes.

- an increasing function of the network size R_{nw} (figures 3.10 and 3.11),
- an increasing function of R_c, for a given network's size, and the cell radius R_c has *no influence* on the interference factor as long as the number of cell rings remains the same (figures 3.8 and 3.9). That property is a consequence of the insensitivity property.
- an increasing function of the density of base stationsρ_{bs}
- a decreasing function of the pathloss exponent η (figures 3.6 and 3.7).

3.7 Cellular network properties

The important parameters of the interference factor are:

- the pathloss exponent,
- the *relative position* of the mobile in the cell and
- the relative cell's size to the network's one.

Since the interference factor characterizes a cellular network, the cells of an homogeneous network where the base station are regularly distributed and where their transmitting powers are identical, have all the same properties which do not depend on their sizes, but only on the relative position of the mobiles in their cell (most of cellular networks can be considered as infinite). We moreover established the influence of the pathloss exponent, which is characteristic of any specific environment.

3.8 Fluid model limit uses

As observed in Section 1.3, different discrete entities can be considered as a continuum in radio network analysis: mobiles, traffic quantities... Since these quantities allow to analyze macroscopic properties of radio systems, such as dimensioning or optimisation analysis, a macroscopic analysis is sufficient. As a consequence, considering them as a continuum is sufficient. About massively dense wireless networks, Toumpis [Tou01] expresses that a macroscopic view of them emerges, which preserves sufficient information to allow a meaningful network optimization. Following this point of view in the fluid network model case, we can conclude that model is a priori useful to analyze 'macroscopic quantities' of radio networks (dimensioning, blocking probabilities or radio resources management,...).

And what about the microscopic ones? For example is the cellular network's fluid model useful to analyze a given cell of a network in a different way than an other cell, or more precisely, is it useful to analyze networks with non homogeneous traffic or base station distribution? This kind of analysis constitutes one of our future study axes.

A first element of answer is nevertheless given in [Kel01] where a non homogeneous network is considered. In this paper we consider a *high level of non homogeneity* of the base station transmitting power all over the network. That non homogeneity is taken into account by considering a limited development of the BS transmitting power. We show that the fluid model allows to calculate an accurate approximation of the interference factor taking into account that non homogeneity. Moreover in other studies where a *important level of non homogeneity* of the base station density and transmitting power are assumed, we show that the fluid model is still accurate.

A second element of answer consists in comparing the values obtained by the fluid model to the ones obtained by a simulation tool developed by France Telecom, done on a non homogeneous network. The two means give close values of network capacity (see remark in 7.7.1).

However, the useful limits of the fluid model seem to be reached for networks with a given level of non homogeneity. We still need to determine precisely these limits.

3.9 Concluding remarks

The goal of our approach was to propose an analytical model characterizing a cellular radio network. This model had to be accurate in taking into account the distance of a mobile to the base station, and still simple enough to lead to closed form formulas. We moreover needed a not over-simplifying model, otherwise it could have resulted large inaccuracies.

We proposed a spatial fluid model that allows to simplify considerably the computation complexity needed to obtain accurate results. We first defined a parameter, the interference factor f, which well characterizes cellular networks. Considering the base stations as a continuum, we established an analytical expression of f, simple and easy to calculate. The network size R_{nw} to be considered can be chosen characterizing a typical environment (pathloss exponent, urban, rural, macro or micro cells). It moreover allows calculating the precise influence of a mobile on a given cell, whatever its position.

We validated the fluid model approach comparing it to a hexagonal simulated network. We especially established the accuracy of the fluid model for wide ranges of distance between neighbor base stations, *i.e.* even for very low base stations densities, wide ranges of pathloss and wide ranges of size networks. Though the simplicity of formula we established, we showed its high accuracy whatever the network parameters we considered. We particularly showed its accuracy even for very low base stations densities, and very limited zones.

Applications

In this part, we propose some possible applications of the fluid model in the case of CDMA networks.

The chapter 4 analyzes the capacity of a cell and a network, in term of mobile number. Since the fluid model allows to determine with a high accuracy the power needed by a mobile, and since the total BS transmitting power is limited, the capacity calculation becomes easy to determine. Afterwards, the effect of a densification (by adding new base stations) is studied. At last we show how the fluid model can be used for *admission control mechanisms*.

In the chapter 5, we propose a study control policy for a multiservice case. The fluid model is used to determine analytically, an average interference factor depending on the exponential pathloss factor. Thanks to this parameter, performance measures, as call blocking probabilities and expected transfer times, are then computed by modeling the CDMA system as a quasi-birth-and-death (QBD) process.

In the chapter 6, the fluid model is used to analyze the global outage probability of a mobile entering a cell in a CDMA system. Since the interference factor expression takes into account the location of a mobile in a cell, we are able to derive a formula of a *spatial outage probability*. It expresses the outage probability for a mobile entering the cell at a given distance from the serving BS. That one gives a more precise knowledge of the quality of service than the global one. It can allow a provider to do a better management of the traffic, for example to improve the quality of service offered.

An other possible application: the Mobile Scheduling.
In the article [KeA02], the following problem of spatial downlink prioritization is analyzed. Mobiles enter a cell at locations that are determined according to some probability distribution. Various priority policies can be analyzed, where the assigned priority is given in terms of the distance of the mobiles from the base station. This gives rise to a whole continuum of

priority levels. The influence that the combined location density and priority policy have on the quality of service of the mobiles and on the network overall performance is studied. Applying the model to a HSDPA system, different quality of service indicators can be calculated as the sojourn time, using a priority scheduling strategy, a processor sharing one and a first come first served one. Three types of arrival flow, a uniform one, a non uniform one and a flow which generates a constant load in the cell are assumed. A numerical study based on the fluid model shows that the expected sojourn time can be improved by a hybrid policy that defines two zones in the cell and uses maximum SIR priority in one and minimum SIR priority in the other.

Chapter 4

Capacity of a CDMA Network

The aim of this chapter is to use the fluid model to evaluate the capacity, in term of number of mobiles, of a cell and a network based on a CDMA technology. We analyze the densification as solution to an increasing traffic. We finally show the fluid model can drive us to analyze admission control policies.

4.1 Introduction

One of the aims of the planning and the dimensioning process of a network consists to evaluate the number of base stations the network will need to the forecast traffic. Indeed, it is important for a telecom provider to minimize the cost of its network, and particularly the cost due to the base station number it deploys. One of the elements which allows that evaluation is the calculation of the number of mobiles a cell, a zone, and also the whole network, will be able to handle. Moreover, an already existing radio network does not necessarily answer the traffic demand. Either because the traffic evaluation was not known with a sufficient accuracy during the dimensioning process, or because the traffic demand increased since that one was built. Among the solutions to answer an increasing traffic in a already existing CDMA network, a telecom provider can choose to densify the network, *i.e.*, to install new base stations. The analysis of this solution generally requires simulations.

For dimensioning a network or to analyze the advantages of a densification as answer to an increasing traffic, the provider develops simulations tools. These last ones need a preparation: an environment has to be created and the network's parameters have to be set. Moreover, they do not give instantaneous results, may last an important time, and moreover, a great number of simulations are generally required.

The fluid model approach allows to analyse and compare instantaneously different solutions with the aim to adapt the network, or a given zone of the network, to an increasing traffic demand. In spite of their simplicity, the classical CDMA networks models which mostly consider hexagonal networks do not give explicit and simple analytical expressions due to the complexity of the analysis. Indeed, for the downlink, the interferences received by a mobile are due to all the base stations of the network. They depend on their transmitting powers, positions and numbers. With our approach, no complex and time consuming computation are needed to obtain explicit expressions of some important characteristics of the network such as the possibility for a mobile to be admitted in the network.

Many studies were done to analyze the capacity of a cell and a network (see for example [ChGo01] [CoM01] [WuC01] [VeS01] [TaS01] [ZaSol]). For the uplink, the authors of [RaP01] give an analytical expression of a cell pole capacity. The authors of [HiB01] give an analytical expression of a cell capacity. They however need to develop a simulation approach to evaluate the average interference factor F, which allows them to determine the pole capacity. The authors of [AkPa01] calculate per-user interference and analyze the effect of user-distribution on the capacity of a CDMA network. The authors of [Ela03] proposed a calculation of the capacity of a multi cell UMTS system. They however only determined an upper bound of the other cell interference. Our model takes into account the whole inter cell interference and gives results close to the ones obtained by planning tools (see remark Sectio 7.7.1) which take into account a real environment. For the uplink, the authors of [RaP01] give an analytical expression of a cell pole capacity.

As an application of this model, we calculate the capacity of a cell in term of number of mobiles, and analyze the densification as solution to an increasing traffic. The sectorisation, and a comparison of these two means to anwer a traffic demand, will be developed in chapter (7).

We finally show the fluid model can drive us to analyze admission control policies.

Remark

Since the cell capacity of an UMTS network also depends on functions such as power control, handover... that system is known as a softcapacity one.

4.2 Base station transmitting power

Let $P_{u,b}$ be the power transmitted to mobile u from base station b. Assume that there are M mobiles in cell b; the base station of that cell transmits at a total power P_b given by (see also remark 2.3)

$$P_b = \sum_{j=1}^{M} P_{j,b} + P_{CCH}, \tag{4.1}$$

where P_{CCH} designates the power transmitted on common channels (CCH) [HoTo]. Note that this last term is not power controlled, and so it can be modeled by adding a constant power. Also it is assumed not to depend on b.

The equation (2.8) gives the minimum transmission power of a traffic channel from a base station to a mobile. Basic algebra yields the following:

$$P_b = \frac{P_{CCH} + \sum_u \beta_u \frac{N_0}{g_{b,u}}}{1 - \sum_u \beta_u(\alpha + f_u)}. \tag{4.2}$$

In downlink CDMA dimensioning, since the total amount of base station power required limits the capacity, it is important to estimate it. In order to calculate the total base station power in our problem, we have to calculate the transmitted power $P_{j,b}$ for each mobile separately, and substitute them in (4.1). In order to obtain a simplified formula, we could replace $g_{u,b}$, $f_{u,b}$, α_u by single parameters A, F, α. These values can initially be chosen by taking the simple average for $g_{u,b}$, $f_{u,b}$, α over all $u = 1, \ldots, M$. Their accuracy can then be improved based on actual measurements for the mean total base station output power. We will refer to this, albeit somewhat imprecisely, as an "average approximation". Such an approximation has been used in many downlink dimensioning models for CDMA, see e.g. [HiBe, HoTo, SiH01], as it provides an easy way to estimate the pole capacity.

Since the fluid model gives a *precise value* of these parameters for *each individual* connection, we can evaluate *analytically* the pole capacity with a better precision. It can result a more useful dimensioning of the link.

4.3 Cell load and capacity

4.3.1 Base station power limitation

In most cases, the maximum base station output power determines the maximum loading supported by the system. We now further assume that the common channels transmitted power is a fraction φ of the maximum base station's output power P_{max}, i.e.,

$$P_{cch} = \varphi P_{max}. \tag{4.3}$$

Then, according to the transmitting power limitation of base stations, we express (from 4.2) that the power P_b is limited by its maximum value P_{max} as

$$\sum_u \beta_u(\alpha + f_u) + \frac{\sum_u \beta_u \frac{N_0}{g_{b,u}}}{P_{max}} \leq 1 - \varphi. \tag{4.4}$$

Consequently, we can define the system's *nominal* cell's capacity as $\Theta_\varphi = 1 - \varphi$. and the *nominal* capacity required by a connection to be

$$L_u = \beta_u(\alpha + f_u) + \frac{\beta_u \frac{N_0}{g_{b,u}}}{P_{max}}. \tag{4.5}$$

This last expression means that each mobile u generates some kind of *load* L_u in the cell which corresponds to the nominal capacity required by a connection, expressed by the right hand of (4.5).

In other terms, the cell's load defined as

$$L_{DL} = \sum_u L_u \tag{4.6}$$

represents the ratio between the total power required by mobiles and the maximum BS transmitting power. This load is due to an interference term L_{interf} expressed as

$$L_{interf} = \sum_u \beta_u(\alpha + f_u), \tag{4.7}$$

and a thermal Noise term L_{N_0}.

$$L_{N_0} = \frac{\sum_u \beta_u \frac{N_0}{g_{b,u}}}{P_{max}} \tag{4.8}$$

Remark: OFDMA system

In OFDMA, the data is multiplexed over a great number of subcarriers. The expression (2.4) can be written

$$\gamma_u = \frac{P_{b,u} g_{b,u}}{\sum_{i \neq b}^{B} P_j g_{j,u} + N_0} \tag{4.9}$$

and using (2.2)

$$\gamma_u = \frac{1}{f_u + \frac{N_0}{P_{b,u} g_{b,u}}} \tag{4.10}$$

Since $\frac{N_0}{P_{b,u} g_{b,u}} << f_u$ when the cell radius is less than 1 km, we can negligt this term and write

$$\gamma_u = \frac{1}{f_u} \tag{4.11}$$

The OFDMA frequency bandwidth is shared between guard subcarriers and useful subcarriers allocated to mobiles. We define φ_{OFDMA} as the fraction of the bandwidth dedicated to guard subcarriers.

For an OFDMA system, we can define a load. It represents the fraction of the bandwidth used by mobiles at each time, and can be written

$$L_{\varphi_{\text{OFDMA}}} = \sum_{u=1}^{M} \gamma_u f_u \tag{4.12}$$

4.3.2 Fluid Model Analysis

We assume a mobile distribution density ρ_{ms}, a cell's area S_{cell}, and a pahtloss model given by $p_u = P_b K r^{-\eta}$. Using the fluid model approach, we can rewrite (4.2) as:

$$P_b = \frac{P_{CCH} + \int_0^R \int_0^{2\pi} \rho_{ms} \beta N_0 K.r^{1+\eta} dr d\theta}{1 - \beta(\int_0^R \int_0^{2\pi} \rho_{ms}(\alpha + f(r)) r dr d\theta)}. \tag{4.13}$$

Considering a uniform mobile distribution in the cell, we have:

$$P_b = \frac{P_{CCH} + \rho_{ms} \beta N_0 K \int_0^R \int_0^{2\pi} r^{1+\eta} dr d\theta}{1 - \beta \rho_{ms} S_{cell}(\alpha + \frac{1}{S_{cell}} \int_0^R \int_0^{2\pi} f(r) r dr d\theta)}. \tag{4.14}$$

We denote F the downlink interference factor mean in the cell:

$$F = \frac{1}{S_{cell}} \int_0^{R_e} \int_0^{2\pi} f(r) dr d\theta \tag{4.15}$$

and we introduce the parameter A

$$A = K \frac{1}{S_{cell}} \int_0^{R_e} \int_0^{2\pi} r^{1+\eta} dr d\theta \tag{4.16}$$

after calculations, since we have $S_{cell} = \pi R_e^2$:

$$A = \frac{2}{5} K R_e^{\eta} \tag{4.17}$$

We denote $n_{ms} = \rho_{ms} S_{cell}$. The expression 4.4 can be written:

$$n_{MS}\beta(\alpha + F) + n_{MS}(A\frac{N_0}{P_{max}}) \leq 1 - \varphi \tag{4.18}$$

The maximum number of mobiles a cell can handle (pole capacity) is thus given by:

$$n_{MS} = \frac{1 - \varphi}{\beta(\alpha + F)(1 + A\frac{N_0}{P_{max}(\alpha+F)})} \tag{4.19}$$

We moreover can write, as long as long as the Noise is negligible (which is a reasonnable assumption for a cell radius less than 1 km):

$$n_{MS} = \frac{1 - \varphi}{\beta(\alpha + F)} \tag{4.20}$$

Using the expression of the interference factor f given by the fluid model 3.3 and denoting $u = \frac{r}{R_c}$ and $a = \frac{R_{nw}}{R_c}$, we can write

$$F = 2^{4-\eta}(\frac{R_c}{R_e})^4 \frac{1}{\eta - 2} \int_0^{\frac{R_e}{R_c}} u^{\eta+1}[(1 - \frac{u}{2})^{2-\eta} - a^{2-\eta}(1 - \frac{u}{2a})^{2-\eta}]du \tag{4.21}$$

The relation $(\frac{R_c}{R_e})^2 = \frac{\pi\sqrt{3}}{6}$ drives us to finally write:

$$F = 2^{2-\eta}\frac{\pi^2}{3}\frac{1}{\eta - 2} \int_0^{\frac{R_e}{R_c}} u^{\eta+1}[(1 - \frac{u}{2})^{2-\eta} - a^{2-\eta}(1 - \frac{u}{2a})^{2-\eta}]du \tag{4.22}$$

The expression of F only depends on the pathloss parameter η and on the relative network's to cell size ratio a. For large networks, F does no more depend on the network's size, since the mean interference factor tends to an asymptotic value F_{lim} when $a \to \infty$. We have

$$F_{lim} = 2^{4-\eta}(\frac{R_c}{R_e})^4 \frac{1}{\eta - 2} \int_0^{\frac{R_e}{R_c}} u^{\eta+1}[(1 - \frac{u}{2})^{2-\eta}]du \tag{4.23}$$

Considering the expression 4.19, we observe the pole capacity of a cell depends on:

- the environment characterized by the pathloss factor η,
- the cell's size,
- the thermal Noise,
- the maximum transmitting power,
- the target SINR,
- the orthogonality factor,
- the power ratio dedicated to the common channels.

We can conclude from (4.19) that in a homogeneous network, a provider can increase the capacity of a cell by *decreasing* its radius or by *increasing* the maximum base station's transmitting power.

This result is particularly important during the deployment process of radio CDMA networks, to determine the zone covered by a BS. It also plays a role when the provider chooses to add base stations as a solution to an increasing traffic. Since a densification process increases the number of base stations in a given zone, their cell radius decreases and consequently the capacity of each cell is less sensitive to the thermal noise.

We notice that the fluid model allows to calculate the capacity of a cell analytically by using the equation (4.4). And when we consider a uniform mobile distribution, the calculation is based on (4.14). In this case, the fluid model allows to calculate an simple average value of f and g. However, even when that distribution is not uniform, the expression (4.13) enables a calculation of the capacity, as long as we can determine the analytical expression of the mobile distribution $\rho_{ms}(r, \theta)$ which depends on the mobile location.

4.4 Numerical Analysis

We present hereafter a numerical analysis, with $\alpha = 0.7$, $\varphi = 0.2$ and $\gamma = -16dB$ (voice service) a thermal Noise $N_0 = -104dBm$.

4.4.1 Mean interference factor

Using (4.22), we calculate the mean values of F as a function of η (Table 4.4.1) for an infinite network's size: F is a decreasing function of η. This result shows that the relative influence of the network on mobiles in a given cell is lower when the pathloss increases.

η	2.5	2.7	3	3.3	3.5	3.7	4	4.5
$F(\eta)$	1.72	1.16	0.76	0.55	0.46	0.39	0.31	0.24

Table 4.1: Influence of the pathloss on the interference factor F

4.4.2 Mean Cell Capacity

The mean cell capacity depends on the cell radius, the thermal Noise and the total transmitting power of the base station.

The figure 4.1 shows the influence of the cell radius (X axis R_c in Km) and the BS transmitting power limitation on the cell capacity (curves drawn for $5W < P_{max} < \infty$). Since the transmitting power is limited, the relative importance of the thermal Noise increases (see also Table 4.2) when the cell radius increases. As a consequence, the cell capacity decreases. This figure also shows that for an unlimited base station transmitting power ($P_{max} \rightarrow \infty$), the cell capacity is not limited by its radius: it stays constant whatever the cell radius. Furthermore, the influence of the thermal Noise appears only for radii larger than 800 m, and becomes really important for $R_e > 1$ Km. Considering the typical value of P_{max} is $20W$, we also observe that until a radius of 800 meters, the maximum transmitting power can be divided by 4 (5 W) without generating a high degradation of the cell capacity. This one decreases from 34 (for $P_{max} = 20W$) to 31 mobiles per cell (for $P_{max} = 5W$): the cell loss represents less than 10%.

The relative importance of the thermal Noise is highlighted on Table 4.2, done for $P_{max} = 20W$. That table shows that for a cell radius R_c of 1 Km the relative importance of the load due to the thermal Noise term represents only 5% of the total load of the cell. And the decrease of mobile number is only 6%($= \frac{34-32}{32}$). But that influence increases rapidly when the cell radius increases. It reaches 58% of the total load of the cell and the cell capacity decrease reaches about 40% (from 34 to 21 mobiles), when the cell radius R_c is 2 Km.

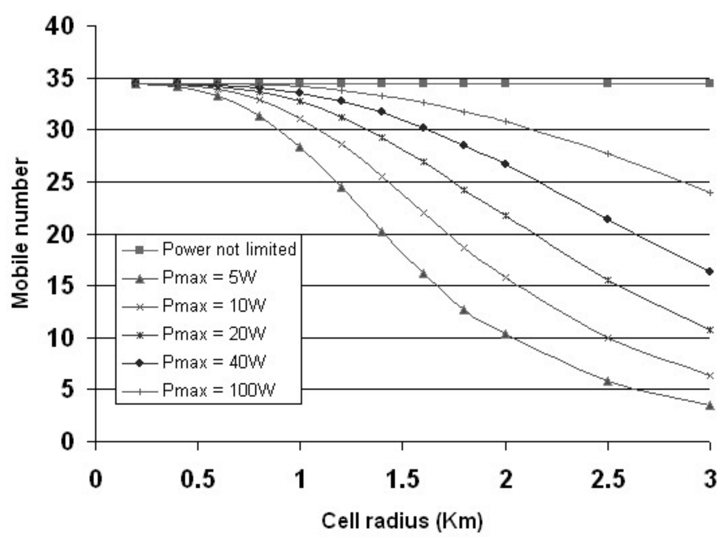

Figure 4.1: Cell Capacity (mobile number) vs Cell Radius with a BS transmitting power varying from $5W$ to $100W$.

4.5 Densification of a network zone

We first consider a CDMA network's zone with a radius of 10 Km, and a *initial* cell radius $R_c = 3$ Km. We observe (figure 4.2) that the number of mobiles handled by that zone linearly increases when the number of base stations increases (R_c decreases). We also observe that increase depends on the maximum transmitting power of the base stations: since the cell radius remains relatively high (about 1 km, the thermal Noise plays a role in the capacity (see expression 4.19).

Now, considering a CDMA network's zone with a radius of 3 Km, and a initial cell radius $R_c = 1$ Km, the mobiles number handled by that zone linearly increases when the number of base stations increases (figure 4.3). However, that increase does not depend on the maximum transmitting power of the base stations: the thermal noise is negligible.

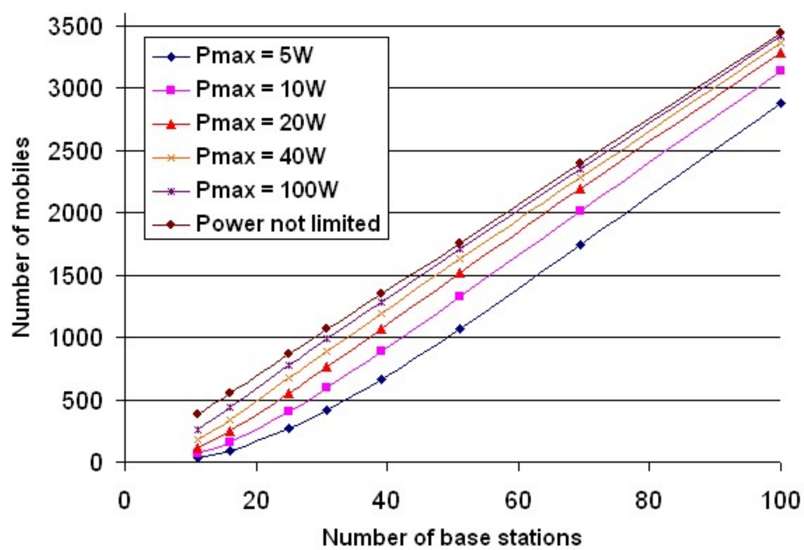

Figure 4.2: Capacity of a network's zone of radius 10 Km: Number of mobiles vs number of BS in that zone .

4.6 Application to Call Admission Control

In a CDMA system, the number of mobiles is limited by the interferences or by the transmitting power of the base stations. The telecom providers need to apply an admission control, to be able to offer the subscribers a quality of

$R_e(km)$	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3
$A\frac{N_0}{P_{max}}\frac{1}{\alpha+F}(\%)$	0.02	0.2	0.9	2.5	5	10	17	28	42	58	120	220
n^{th}	34	34	34	33	32	31	29	27	24	21	15	10

Table 4.2: Thermal Noise Relative Influence, for $\eta = 3.5$ and $P_{max} = 20W$

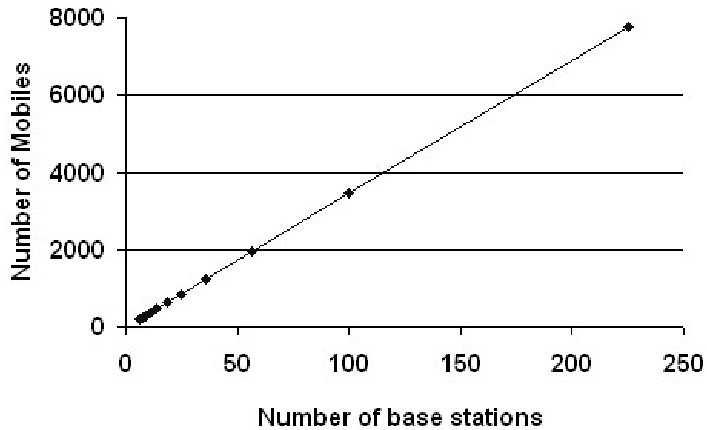

Figure 4.3: Capacity of a network's zone of radius 3 Km: Mobile number vs number of BS in that zone .

service as close as possible to the one they ask for. For the downlink, any admission control has to take into account the base station transmitting power's limitation. To illustrate our analytical model, we proposed an analysis of the capacity of the system in term of number of mobiles. Any kind of admission control policy can be derived using our analytical model. Considering that the thermal Noise is negligible, we established from the expression (4.4), that a mobile entering the cell increases the load by a factor $\beta_u(\alpha+f_u)$ which corresponds to the ratio $P_{b,u}/P_b$. It means that mobiles with high quality of service demand (high β_u), and far from the BS (high f_u) induce a more important overload than the ones close the BS and with low QoS demand. This result is well known. However with our analysis, it becomes possible to *quantify* precisely the overload induced by each mobile and to choose a policy according to the needs of the providers. As an example, the operator can choose, in some given configurations, to refuse the admission of mobiles which induce a too high load. Only the mobiles inducing a load less than a given threshold value would be admitted. Or he can impose a decrease of the throughput of mobiles which are far from the BS in the aim to decrease the load they induce. So its knowledge opens a large set of possible analyses.

4.7 Concluding remarks

The fluid model allowed us to analyze the capacity of a cell (and thus can be used in the planning and dimensioning process) and the solution based on a network's densification to answer an increasing traffic. We instantly obtained explicit expressions of the capacity without any computation, and focused on the influence of the thermal Noise and the BS maximum transmitting power. We particularly established that for high density networks, it is not necessary to increase base stations transmitting power. Since our analytical model enables to know the influence of a mobile on a given zone of a network whatever its position, it can also be used to analyze admission control policies.

Chapter 5

Admission in Multiservice CDMA

We consider in this chapter a WCDMA system with two types of calls: real time (RT) calls that have dedicated resources, and data non-real time (NRT) calls that are treated using a time-shared channel (such as the HDR or the HSDPA). We consider reservation of some resources for the NRT traffic and assume that this traffic is further assigned the resources left over from the RT traffic. The grade of service (GoS) of RT traffic is also controlled in order to allow for handling more RT calls during congestion periods, at the cost of degraded transmission rates. We consider both the downlink (with and without macrodiversity) as well as the uplink and study the blocking probabilities of RT traffic as well as the expected sojourn time of NRT traffic. We further study the conditional expected sojourn time of a data connection given its size and the state of the system. Finally, we extend our framework to handle handover calls.

5.1 Introduction

Important performance measures of call admission control policies in systems with heterogeneous service classes are the probability of rejection of calls of different classes as well as the sojourn time of non-real time transfers. In order to be able to compute these and to design the call admission control policies, a dynamic stochastic approach should be used based on statistical assumptions regarding the call arrival processes and durations, as well as data transfer sizes.

In this context, a classical approach widely used in cellular networks is based on adaptively deciding how many channels (or resources) to allocate to calls of a given service class, see e.g. [FaZ01, LeZ01, LiC01]. Then one can evaluate the performance as a function of some parameters (thresholds) that characterize the admission policy, using Markov chain analysis. This allows to optimize and to evaluate tradeoffs between QoS parameters of the different classes of mobiles. This approach, natural to adopt in TDMA or FDMA systems, can also be followed in the case of a CDMA system, even though the notion of capacity is much more complex to define. For the uplink case in CDMA, the capacity required by a call has been studied in the context of call admission, see e.g. [TaGO01, KoL01, **?**].

We focus here on two types of calls, real-time (RT) and non-real time (NRT) data transfers. Whereas all calls use CDMA, we assume that NRT calls are further time-multiplexed (which diminishes the amount of interference, thus increasing the available average throughputs). This combination of time multiplexing over CDMA is typical for high speed downlink data channels, such as the High Speed Downlink Packet Access (HSDPA) [PaD01] and the High Data Rate (HDR) in CDMA-2000 systems [BeB01].

Similarly to the uplink analysis [NiA01], we propose a simple model that allows us to define in the downlink case the capacity required by a call of a given class when it uses a given grade of service (transmission rate). In particular, we also consider the case of macrodiversity. We then propose a control policy that combines admission control together with a control of the grade of service (GoS) of real-time traffic. Key performance measures are then computed by modeling the CDMA system as a quasi-birth-and-death (QBD) process. We obtain the call blocking probabilities and expected transfer times, already available for the uplink case in [NiA01]. We further obtain (both for the uplink and downlink) another important performance measure: the expected transfer time of a file conditioned on its size. We study the influence of the control parameters on these performance measures. We finally extend the model to handle handover calls.

The structure of the chapter is as follows. We begin by introducing in Sections 5.2, 5.3 and 5.4 the frameworks corresponding to the downlink, with and without macrodiversity, as well as the uplink of a CDMA system. Using power control arguments, we obtain for all three cases the transmission rates for various classes of calls which are compatible with given signal to noise and interference ratios. We then introduce in Section 5.5 the basic control actions: call admission and control of GoS. The statistical modeling of the system is presented in Section 5.6. It is then used in Section 5.7 for an extensive numerical investigation. The extension of the model and the analysis to handover traffic is given in Section 5.8, and we conclude the chapter in Section 5.9.

5.2 Downlink

We use a model similar to the one presented in [HiBe]. Let there be B base stations. The minimum power received at a mobile u from its base station b is determined by a condition concerning the signal to interference ratio, which should be larger than some constant

$$\gamma := \frac{E_s}{N_0}\frac{R_s}{W}\Gamma, \tag{5.1}$$

where E_s/N_0 is the energy per transmitted bit (of type s) to interference density, W is the WCDMA modulation bandwidth, and R_s is transmission rate of the type s call. The constant Γ accounts for the random behavior of a signal due to shadow fading and imperfect power control; more specifically, to account for this randomness we study a probabilistic condition

$$Pr(\gamma_u > \gamma) > 1 - \chi, \tag{5.2}$$

where χ is a small constant that represents a desired bound on outage probability.

Remark

The transmission rate R_s depends on the signal to noise ratio. Specifically, we consider the model proposed in [NiA01]. We thus modelize the transmission rate to be linear in the signal to noise ratio. This model can describe the low SNR regime of Shannon capacity [Shan01]:

$$R_s \propto \ln_2(1+\gamma) \tag{5.3}$$

Considering a log-normal distribution of the SINR, $SINR_u = 10^{\xi_u/10}$, where $\xi_u \sim N(\mu_\xi, \sigma_\xi)$, it can be derived that the largest Γ that satisfies the above probabilistic condition is given by [KoL01, NiA01]:

$$\log \Gamma = \frac{\sigma_\xi^2}{20h} - \frac{Q^{-1}(1-\chi)\sigma_\xi}{10}, \tag{5.4}$$

where $h = 10/ln10$ and $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$.

Any other causes of randomness, most notably fast fading, can be taken into account the same way by considering a different distribution, e.g. a Rayleigh fading distribution [Sklar97].

We next consider two service classes, denoted by $s = \{1, 2\}$ (that will correspond to RT and NRT traffic, respectively). Let γ_s be the target SINR ratio for mobiles of service class s, α the orthogonally factor, and let

$$\beta_s = \frac{\gamma_s}{1 + \alpha\gamma_s}. \tag{5.5}$$

We now focus on a given cell, and assume that it contains M_s mobiles of class s.

Using the average approximation (which considers that *all the mobiles have the same* average interference factor F see Section 4.2), we finally get for the total output power of base station b:

$$P_b = \frac{P_{CCH} + N_0 A \sum_s \beta_s M_s}{1 - (\alpha + F) \sum_s \beta_s M_s}. \tag{5.6}$$

Fluid model
In particular for a uniform mobile distribution, the parameters F and A can be evaluated with the fluid model (see the expressions 4.22 and 4.16).

Considering a negligible thermal Noise, we define the downlink loading (4.4) as: $L_{DL} = \sum_s (\alpha + F)\beta_s M_s$, this gives

$$P_{max} = \frac{N_0 A \sum_s \beta_s M_s}{Z_2}, \text{ where } Z_2 = (1 - \varphi) - L_{DL}. \tag{5.7}$$

In most cases, the maximum base station output power determines the maximum loading supported by the system. We can define the system's *nominal* capacity as $\Theta_\varphi = 1 - \varphi$, and the *nominal* capacity required by a connection (which was denoted L_u in expression (4.5)) to be

$$\Delta(s) := (\alpha + F)\beta_s. \tag{5.8}$$

We note that β_s will allow to depend on M_s, $s = 1, 2$. Combining this with (5.1) and with (5.5) we get the throughput of a connection s, that "uses a capacity $\Delta(s)$":

$$R_s = \frac{\Delta(s)}{\alpha + F - \alpha\Delta(s)} \times \frac{N_0 W}{E_s \Gamma}. \tag{5.9}$$

5.3 Downlink with macrodiversity

In this section, we extend our analysis by considering the downlink macrodiversity case. Our approach is inspired by [HiBe] who considered the single service case[1]. A mobile i in macrodiversity is connected to two base stations,

[1] We extend the context here to refer more generally to macrodiversity, and not only the soft handover procedure.

b and l. b is defined to be the station with larger SINR. Following [HiBe] we assume that the Maximum Ratio Combining is used and hence the power control tries to maintain

$$\gamma_i = \gamma_{i,b} + \gamma_{i,l}. \tag{5.10}$$

where γ_i is given by the constant in (5.1). We have $\Omega_i \leq 1$ where

$$\Omega_i = \frac{\gamma_{i,l}}{\gamma_{i,b}}. \tag{5.11}$$

This gives for the combined γ_i [HiBe]:

$$\gamma_i = \frac{(1+\Omega_i)P_{b,i}/g_{b,i}}{\alpha(P_b - P_{b,i})/g_{i,b} + f_{i,b}P_b/g_{b,i} + N_0}. \tag{5.12}$$

The transmission power becomes

$$P_{b,i} = \kappa_i(\alpha P_b + f_{b,i}P_b + g_{b,}N_0),$$

where

$$\kappa_i = \frac{\gamma_i}{1 + \Omega_i + \alpha\gamma_i}. \tag{5.13}$$

Let there be M mobiles in a cell b (we shall omit this index) of which a fraction τ is in macrodiversity. We assume that by symmetry, the base station of that cell transmits also to a number τM of mobiles that are geographically situated in neighboring cells. Then the total base station output power can be calculated as

$$P_b = \sum_{i=1}^{(1-\tau)M} P_{b,i} + \sum_{j=1}^{2\tau M} P_{b,j} + P_{CCH},$$

where the notations i,j in the sums should be understood to refer to single link and macrodiversity mobiles, respectively. The power for a single link user should be calculated the same way as in 5.2.

We now consider two classes of services $s = \{1, 2\}$ corresponding to RT and NRT mobiles. We make the following "average approximations", similarly to the previous section: For a given service class $s = \{1, 2\}$, Ω_i is replaced by a constant Ω_s (its average over all mobiles of the same service as i); we also replace $f_{b,i}$ by one of two constants F^{NMD} and F^{MD}, where F^{NMD} (resp. F^{MD}) corresponds to an average value of $f_{b,i}$ over mobiles which are not in macrodiversity (and which are in macrodiversity, resp.). Likewise, we replace $g_{i,b}$ by one of the two constants A^{NMD} and A^{MD}. This gives the total power of a base station b:

$$P_b = \frac{Z_1}{Z_2}$$

as long as Z_2 is strictly positive, where

$$Z_1 := (1-\tau)\sum_{s=1,2} M_s\beta_s A^{NMD} N_0 + 2\tau \sum_{s=1,2} M_s\kappa_s A^{MD} N_0$$

and

$$Z_2 := (1-\varphi)-(1-\tau)\sum_{s=1,2} M_s\beta_s(\alpha+F^{NMD})-2\tau\sum_{s=1,2} M_s\kappa_s(\alpha+F^{MD}). \quad (5.14)$$

Again, we can define the system's *nominal* capacity as $\Theta_\epsilon = 1-\varphi$, and the capacity required by a connection of type $s = 1, 2$ to be $\Delta(s) = (1-\tau)\beta_s(\alpha + F^{NMD}) + 2\tau\kappa_s(\alpha + F^{MD})$. Combining this with (5.1) and (5.13), we get

$$\Delta(s) = (1-\tau)\cdot\frac{R_s\cdot\zeta_s}{1+\alpha R_s\zeta_s}(\alpha+F^{NMD})+2\tau\cdot\frac{R_s\cdot\zeta_s}{1+\Omega_s+\alpha R_s\zeta_s}(\alpha+F^{MD}). \quad (5.15)$$

Here, $\zeta_s = \frac{E_s\Gamma}{N_0 W}$ and we have considered the rate R_s of a connection equal, irrespective if a mobile is in macrodiversity or not. Solving for R_s, this leads to a quadratic equation giving two values, of which we retain the positive.

5.4 Uplink

We briefly recall the capacity notions from the case of uplink from [NiA01]. Define for $s = 1, 2$,

$$\tilde{\Delta}_s = \frac{E_s}{N_0}\frac{R_s}{W}\Gamma, \text{ and } \Delta'(s) = \frac{\tilde{\Delta}(s)}{1+\tilde{\Delta}(s)}. \quad (5.16)$$

The power that should be received at a base station originating from a type s service mobile in order to meet the QoS constraints is given by Z_1/Z_2 [NiA01] where

$$Z_1 = N_0 \Delta'(s)$$

and

$$Z_2 = 1 - (1 + f^{UL}) \sum_{s=1,2} M_s \Delta'(s)$$

(N_0 is the thermal noise power at the base station, f^{UL} is some constant describing the average ratio between inter and intra cell interference, and M_s is the number of mobiles of type s in the cell). Also in this case $Z_2 \geq \epsilon$ for some $\epsilon > 0$. We can thus define the system's *nominal* capacity as $\Theta_\epsilon = 1 - \epsilon$, and the capacity required by a connection of type $s = 1, 2$ to be $\Delta(s) = (1 + f^{UL})\Delta'(s)$. Combining this with (5.16) we get

$$R_s = \frac{\Delta(s)}{1 + f^{UL} - \Delta(s)} \times \frac{N_o W}{E_s \Gamma}. \tag{5.17}$$

5.5 Admission and rate control

In the design of an admission and rate control scheme for heterogeneous services we will consider that RT calls, which have more stringent QoS requirements, have priority over system resources. NRT traffic, on the other hand, has no guaranteed bit rate and can be served in a processor-sharing fashion. However, to prevent RT calls from overwhelming the link we will also assume that a portion of the system resources is reserved for NRT traffic. Further, to also achieve a multiplexing gain for RT calls, we will allow a limited rate degradation for such traffic.

We consider here a fair transmission rate scheme, such that mobiles which belong to the same service class (NRT or RT) transmit or receive at the same rate. For NRT traffic, for which fast-time multiplexing will be considered, this can be viewed as a fair implementation of an HSDPA or HDR scheme where transmission to each mobile takes place at the same *average* rate. For this, an underlying scheduler is also assumed that allocates time slots in proportion to the peak feasible rates of mobiles, in order to achieve the same average rate.

These basic principles of admission and rate control are made more explicit in the following. One must also have in mind that either the uplink or the downlink can be the bottleneck of a CDMA system at one time or another; so from an engineering perspective one should focus only on the more restrictive direction when accepting calls. In our paper, all the notations will be understood to relate to that direction.

5.5.1 Capacity reservation

We assume that there exists a capacity L_{NRT} reserved for NRT traffic. The RT traffic can use up to a capacity of $L_{RT} := \Theta_{\varphi} - L_{NRT}$.

5.5.2 GoS control of RT traffic

UMTS will use the Adaptive Multi-Rate (AMR) codec that offers eight different transmission rates of voice that vary between 4.75 kb/s to 12.2 kb/s, and that can be dynamically changed every 20 ms. The lower the rate is, the larger the amount of compression is, and we say that the grade of service (GoS) is lower. For simplicity we shall assume that the set of available transmission rates of RT traffic has the form $[R^{\min}, R^{\max}]$. We note that $\Delta(RT)$ is increasing with the transmission rate. Hence the achievable capacity set per RT mobile has the form $[\Delta_{RT}^{\min}, \Delta_{RT}^{\max}]$. Note that the maximum number of RT calls that can be accepted is $M_{RT}^{\max} = \lfloor L_{RT}/\Delta_{RT}^{\min} \rfloor$. We assign full rate $R_{RT}^{\max}$ (and thus the maximum capacity $\Delta_{RT}^{\max}$) for each RT mobile as long as $M_{RT} \leq N_{RT}$ where $N_{RT} = \lfloor L_{RT}/\Delta_{RT}^{\max} \rfloor$. For $N_{RT} < M_{RT} \leq M_{RT}^{\max}$ the capacity of each present RT connection is reduced to $\Delta_{MR} = L_{RT}/M_{RT}$ and the rate is reduced accordingly (e.g. by combining (5.1), (5.5) and (5.8) for the case of downlink).

5.5.3 Fast time multiplexing for NRT traffic

The capacity $C(M_{RT})$ unused by the RT traffic (which dynamically changes as a function of the number of RT connections present) is fully assigned to one single NRT mobile, and the mobile to which it is assigned is time multiplexed rapidly so that the throughput is shared equally between the present NRT mobiles. The available capacity for NRT mobiles is thus

$$C(M_{RT}) = \begin{cases} \Theta_\epsilon - M_{RT}\Delta_{RT}^{\max}, & \text{if } M_{RT} \leq N_{RT}, \\ L_{NRT}, & \text{otherwise.} \end{cases}$$

The total transmission rate R_{NRT}^{tot} of NRT traffic for the downlink and uplink is then given respectively by

$$R_{NRT}^{tot}(M_{RT}) = \begin{cases} \frac{C(M_{RT})}{\alpha+F-\alpha C(M_{RT})} \times \frac{N_o W}{E_s \Gamma} & \text{for DL,} \\ \\ \frac{C(M_{RT})}{1+f^{UL}-C(M_{RT})} \times \frac{N_o W}{E_s \Gamma}, & \text{for UL} \end{cases}$$

$$DL: \quad R_{NRT}^{tot}(M_{RT}) = \frac{C(M_{RT})}{\alpha + F - \alpha C(M_{RT})} \times \frac{N_o W}{E_s \Gamma}, \tag{5.18}$$

$$UL: \quad R_{NRT}^{tot}(M_{RT}) = \frac{C(M_{RT})}{1 + f^{UL} - C(M_{RT})} \times \frac{N_o W}{E_s \Gamma}. \tag{5.19}$$

The expression for downlink with macrodiversity is derived similarly, albeit being more complex.

Remark 1. The expressions that we have obtained for the total throughput available for NRT traffic may be in practice non-accurate due to the many approximations we use, such as using the averaged values f and F in the above equations. Since these expressions are used later in a dynamic context, the price of changing the expressions to complex ones can render the later Markovian analysis infeasible. To be able to have better precision, we need to sacrifice the generality of the model.

5.5.4 Remark

Using a Monte Carlo simulator tool developed by *France Telecom R&D*, we ran a simulation to test some of the simplifications that we used in this paper concerning the downlink to check the value of R_{tot}.

Some mobiles using a voice service are randomly generated in a cell located in a UMTS network environment. For each random generation, the simulation calculates the local downlink interference factor parameter f_i for each mobile. The number of mobiles of a cell N_{MS}^{cell} is limited by the total load of the cell which has to be inferior to 100%. We calculate the total throughput of the cell as follows:

$$R_{tot} = 12.2 N_{MS}^{cell} kb/s$$

We obtain an average number of mobiles for the cell, as the ratio between the total number of mobiles generated N_{ms}^{tot} considering all the generations, to the number of generations N_{gen}.

$$N_{MS}^{cell} = \frac{N_{MS}^{tot}}{N_{gen}} = 27.6\ mobiles.$$

We obtain a total throughput of

$$R_{tot} = 336 kb/s.$$

We obtain an average interference factor per mobile as follows:

$$F = \frac{\sum_{i=1}^{N_{MS}^{tot}} f_i}{N_{MS}^{tot}} = 1.13.$$

We then use the following analytical calculation:

$$R_{tot} = N_{MS}^{cell} \frac{1-\varphi}{N_{MS}^{cell}(\alpha + F) - \alpha(1-\varphi)}$$

φ is the fraction of the BS power dedicated to common channels. 1-ϕ is the capacity of the cell. In our simulations we have $\varphi = 0.14$ and $\alpha = 0.79$.

With the analytical method we obtain $R_{tot} = 379kb/s$. The difference between the two values is

$$\frac{379 - 336}{379} = 11\%$$

The total throughput is thus close the simulated one.

5.6 Stochastic model and the QBD approach

In this section we proceed to study a stochastic traffic model and examine steady-state performance measures of the system. We consider the total nominal capacity to remain fixed throughout the system lifetime. This is true in case capacity is limited by base station (DL) output power or interference (UL) and the channel environment conditions do not change very extremely, so that given the rate and power adaptation the same maximum loading is achieved at any time instant. We also assume that for the time-multiplexing of NRT calls an appropriate scheduling scheme is feasible such that each mobile transmits or receives *instantaneously* at a rate given by (5.18), and they all obtain the same average rate.

Model. We assume that RT and NRT calls arrive according to independent Poisson processes with rates λ_{RT} and λ_{NRT}, respectively. The duration of a RT call is exponentially distributed with parameter μ_{RT}. The size of a NRT file is exponentially distributed with parameter μ_{NRT}. Interarrival times, RT call durations and NRT file sizes are all independent.

The departure rate of NRT calls depends on the current number of RT calls:

$$\nu(M_{RT}) = \mu_{NRT} R_{NRT}^{tot}(M_{RT}).$$

QBD approach. Under these assumptions, the number of active sessions in all three models (downlink, with and without macrodiversity and uplink) can be described as a QBD (quasi-birth-and-death) process, and we denote by Q its generator. We shall assume that the system is stable. The stationary distribution of this system, π, is calculated by solving:

$$\pi Q = 0, \tag{5.20}$$

with the normalization condition $\pi e = 1$ where e is a vector of ones of proper dimension. The vector π represents the steady-state probability of the two-dimensional process. We partition π as $[\pi(0), \pi(1), \ldots]$ with the vector $\pi(i)$ for level i, where the levels correspond to the number of NRT calls in the system. We may further partition each level into the number of RT calls, $\pi(i) = [\pi(i,0), \pi(i,1), \ldots, \pi(i, M_{RT}^{\max})]$, for $i \geq 0$. The entries of π are ordered lexicographically, i.e. $\pi(k,i)$ precedes $\pi(l,j)$ if $k < l$, or if $k = l$ *and* $i < j$. The generator matrix Q is given by

$$Q = \begin{bmatrix} B & A_0 & 0 & 0 & \cdots \\ A_2 & A_1 & A_0 & 0 & \cdots \\ 0 & A_2 & A_1 & A_0 & \cdots \\ 0 & 0 & \ddots & \ddots & \ddots \end{bmatrix}, \tag{5.21}$$

where the matrices B, A_0, A_1, and A_2 are square matrices of size $(M_{RT}^{\max}+1)$. The matrix A_0 corresponds to a NRT connection arrival, given by $A_0 = \mathrm{diag}(\lambda_{NRT})$. The matrix A_2 corresponds to a departure of a NRT call and is given by $A_2 = \mathrm{diag}(\nu(j); 0 \leq j \leq M_{RT}^{\max})$. The matrix A_1 corresponds to the arrival and departure processes of RT calls. A_1 is tri-diagonal as follows:

$$\begin{aligned} A_1[j, j+1] &= \lambda_{RT} \\ A_1[j, j-1] &= j\mu_{RT} \\ A_1[j, j] &= -\lambda_{RT} - j\mu_{RT} - \lambda_{NRT} - \nu(j) \end{aligned}$$

. We also have $B = A_1 + A_2$. We follow a matrix-geometric approach for the solution of the QBD process. Assuming a steady-state solution exists, π is given by [neuts]:

$$\pi(i) = \pi(0)\mathbf{R}^i. \tag{5.22}$$

where $\mathbf{R}$ is the minimal non-negative solution to the equation:

$$A_0 + \mathbf{R}A_1 + \mathbf{R}^2 A_2 = 0. \tag{5.23}$$

The vector $\pi(0)$ is obtained from the normalization condition, which in matrix notation writes as: $\pi(0)(I - \mathbf{R})^{-1}e = 1$.

Note that the evolution of number of RT calls is not affected by the process of NRT calls and the Erlang formula can be used to compute their steady state probability, and in particular, the probability of blocking of a RT call:

$$P_B^{RT} = \frac{(\rho_{RT})^{M_{RT}^{\max}}/M_{RT}^{\max}!}{\sum_{j=1}^{M_{RT}^{\max}}(\rho_{RT})^j/j!},$$

where $\rho_{RT} = \lambda_{RT}/\mu_{RT}$. This is the main performance measure for the RT traffic. For NRT calls the important performance measure is expected sojourn time which is given by Little's law as

$$T_{NRT} = E[M_{NRT}]/\lambda_{NRT}.$$

Conditional expected sojourn times. The performance measures so far are similar to those already obtained in the uplink case in [NiA01]. We wish however to present more refined performance measures concerning NRT calls: the expected sojourn times conditioned on the file size and the state upon the arrival of the call. We follow [Nun01] and introduce a non-homogeneous QBD process with the following generator Q^* [Nun01] and the corresponding steady state probabilities π^*:

$$Q^* = \begin{bmatrix} B & A_0 & 0 & 0 & \cdots & \\ (1/2)A_2 & A_1^{(2)} & A_0 & 0 & 0 & \cdots \\ 0 & (2/3)A_2 & A_1^{(3)} & A_0 & 0 & \cdots \\ 0 & 0 & (3/4)A_2 & A_1^{(4)} & A_0 & \cdots \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots \end{bmatrix}, \tag{5.24}$$

where the matrices A_0, A_2, B are the same as introduced before, and $A_1^{(k)}$, $k \geq 2$ is the same as A_1 defined before except that the diagonal element is chosen to be minus the sum of the off-diagonal elements of Q^*, i.e.

$$A_1^{(k)}[i,i] = -\lambda_{RT} - i\mu_{RT} - \lambda_{NRT} - \frac{k-1}{k}\nu(i).$$

The conditional expected sojourn time of a NRT mobile given that its size is v, that there are i RT mobiles and $k-1$ NRT mobiles upon it's arrival, is obtained from [Nun01, Corollary 3.3 and remarks in § 8.3]:

$$T_{k,i}(v) = \frac{v}{R^* - \rho^*} + \overline{1_{k,i}} \left[I - \exp\left(v\mathcal{R}^{-1}Q^* \right) \right] \overline{w}, \tag{5.25}$$

where

$$R^* := \sum_{k,i} \pi^*(k,i) R^{tot}_{NRT}(i), \qquad \mathcal{R} = diag\left[\frac{1}{k} R^{tot}_{NRT}(i) \right],$$

$$\rho^* := \frac{\lambda_{NRT}}{\mu_{NRT}},$$

$\overline{1_{k,i}}$ is a vector of proper dimension whose entries are all zero except for the (k,i)-th entry whose value is 1, and $\overline{w}$ is the solution of

$$Q^* \overline{w} = \frac{1}{R^* - \rho^*} \mathcal{R} \cdot \overline{1_{k,i}} - \overline{1_{k,i}}.$$

The entries of $\mathcal{R}$ along the diagonal are ordered lexicographically in (k,i).

Remark 2. Expression (5.25) simplifies considerably in case the capacity allocated to NRT calls is fixed. Suppose that the number of RT sessions stays fixed to i throughout the system lifetime (this can be used as an approximation when the average duration of RT sessions is very large). The service rate is constant, $R^{tot}_{NRT}(i)$. Then we can study the system as an M/M/1 queue with processor sharing, for which we can easily derive from [Coffman70]:

$$T_k(v) = \frac{v/R^{tot}_{NRT}(i)}{1-\rho'} + [k(1-\rho') - \rho'] \frac{1 - e^{-(1-\rho')\mu_{NRT} \cdot v}}{\mu_{NRT} R^{tot}_{NRT}(i)(1-\rho')^2} \tag{5.26}$$

provided that $\rho' := \dfrac{\lambda_{NRT}}{\mu_{NRT} R^{tot}_{NRT}(i)} < 1$ (ergodicity condition).

The equation is obtained by translating to time units: A job of v size units requires a service time $v/R^{tot}_{NRT}(i)$, if it were served alone in the system. Furthermore, the distribution of service time requirement is also exponential with mean $1/(\mu_{NRT} R^{tot}_{NRT}(i))$.

To illustrate the role of the conditional sojourn time, we use (5.26) to compute the maximum number k of NRT calls present at the arrival instant of an NRT call (we include in this number the arriving call) such that the expected sojourn time of the connection, conditional on its size and on k, is below 1 sec. This is depicted in figure 5.1. For example, if the mean size

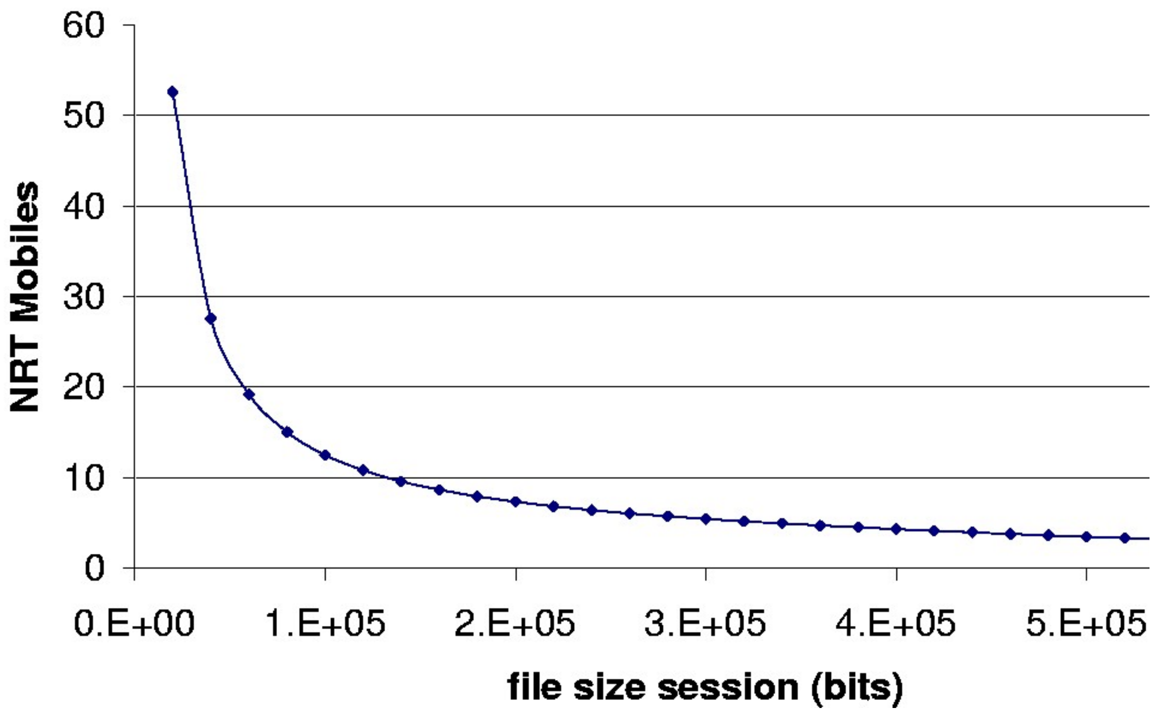

Figure 5.1: Maximum number of NRT connections upon arrival such that the conditional expected sojourn time is below 1 sec, as a function of the mean size of the file

of the file is 100 Kbits then its conditional expected sojourn time will be smaller than 1 sec as long as the number of mobiles upon arrival (including itself) does not exceed 12. This figure is obtained with $R^{tot}_{NRT}(0) = 1000$ kbps, $\lambda_{NRT} = 1$ (we took no RT calls, i.e. $i = 0$).

5.7 Numerical results

In this section, we examine basic performance parameters when RT and NRT traffic is integrated in the link, according to our transmission and rate control scheme. We consider the following setting illustrated in Table 5.7, based on standard WCDMA parameter values (cf. [HoTo]). Unless stated otherwise, the data are for both the downlink (DL) and uplink (UL).

Transmission rate of RT mobiles	*min* 4.75 kbps	*max* 12.2 kbps
E_{RT}/N_0	7.9 dB (12.2 kbps, *UL*)	
	11.0 dB (12.2 kbps, *DL*)	
E_{NRT}/N_0	4.5 dB (144 kbps, *UL*)	
	4.8 dB (384 kbps, *DL*)	
Mean NRT session size	$1/\mu_{NRT} = 160$ kbits	
Mean RT call duration	$1/\mu_{RT} = 125$ s	
Call arrival rates	$\lambda_{RT} = \lambda_{NRT} = 0.4$	
Intercell interference factors	$UL\ : f = 0.73$	$DL\ : F = 0.55$
Non-orthogonality factor (DL)	$a = 0.64$	
Chip rate	$W = 3.84$ Mcps	
Fraction of power for SCH, CCH channels	$\varphi = 0.2$	

Table 5.1: Numerical Values.

Moreover, in our numerical investigation we have chosen a very small value of ϵ ($\epsilon = 10^{-5}$), such that with negligible thermal noise an average mobile is located a few hundred meters from the base station[2]. Of course,

[2] We have considered a thermal noise level of -100 dBm, and a path loss exponent 4 in an urban environment [HoTo]. This roughly yields a power of about 20 Watt in the downlink (transmitted output power of a base station), and 1 Watt in the uplink (transmitted power from a mobile).

in a more realistic application the value of ϵ must be selected more carefully and separately for the uplink and downlink. We assumed here that the target E_s/N_0 depends only on the class of traffic s but not on the number of mobiles.

The constant Γ is computed so as to guarantee that the probability that the target C/I is satisfied is 0.99. It corresponds to a standard deviation constant $\sigma = 0.5$ (see [NiA01]).

Influence of NRT reservation on RT traffic.
In figure 5.2 we depict the average cell capacity in terms of the average number of RT mobiles for both uplink and downlink as a function of the reservation threshold for NRT traffic. We see that it remains almost constant (50 mobiles per cell) for up to 50% of the load.

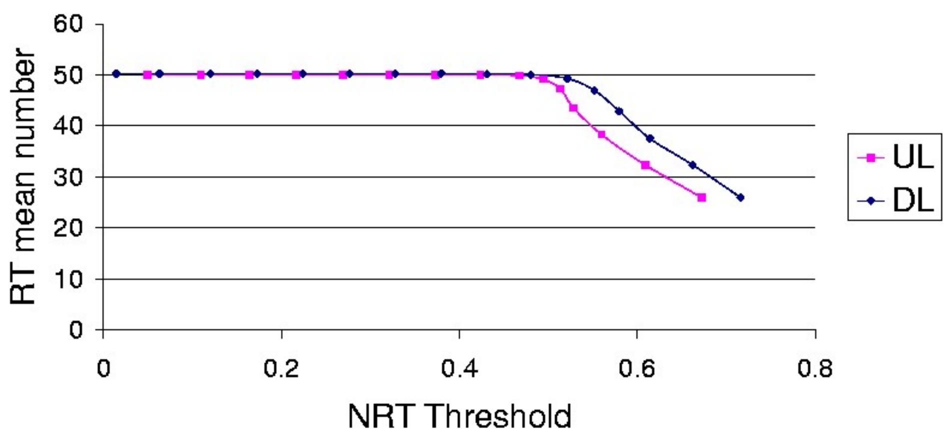

Figure 5.2: Mean number of RT calls in a cell as a function of the reservation level for NRT traffic.

In figure 5.3 we present the blocking rate of RT traffic. At a reservation L_{NRT} of 50% of the maximum load, the dropping rate is still lower than 1%.

Influence of NRT reservation on NRT traffic.
The figure 5.4 shows the impact of the reservation threshold L_{NRT} on the expected sojourn time of NRT calls both on uplink and downlink. We see that

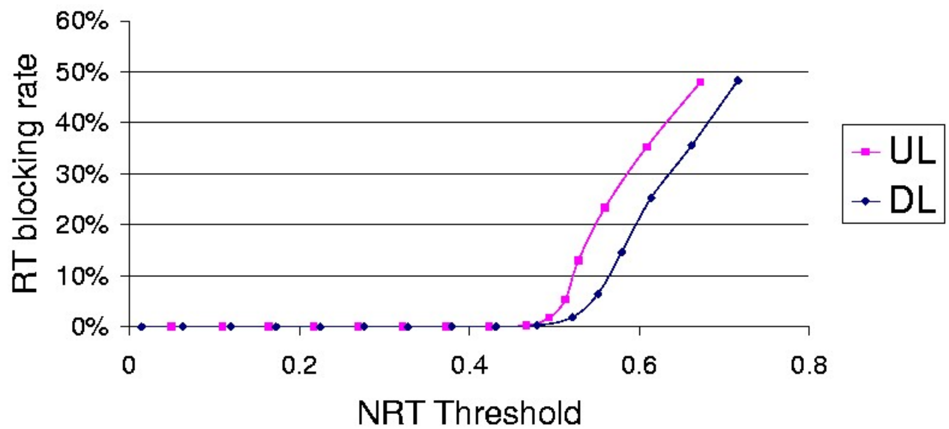

Figure 5.3: Blocking rate for RT calls as a function of the reservation level for NRT traffic

the expected sojourn times become very large as we decrease L_{NRT} below 0.15% of the load. This demonstrates well the need for such a reservation. In the whole region of loads between 0.16 to 0.5 the NRT expected sojourn time is low and at the same time, as we saw before, the rejection rate of RT calls is very small. Thus, this is a good operating region for both RT and NRT traffic.

Conditional expected sojourn time.

The reservation limit L_{NRT} is taken to be 0.27 in $Figure$ 5.5, 5.6. In Figure 5.5 we depict the expected sojourn time conditioned on the number of NRT and RT calls found upon the arrival of the call both being k and on the file being of the size of 100 kbits. The number k is varied in this figure.

Figure 5.6 depicts for various file sizes, the maximum number k such that the conditional expected sojourn time of that file with the given size is below 1 sec. k is defined to be the total number of RT calls as well as the total number of NRT calls (including the call we consider) in the cell. We thus assume that the number of NRT and of RT calls is the same, and seek for the largest such number satisfying the limit on the expected sojourn time.

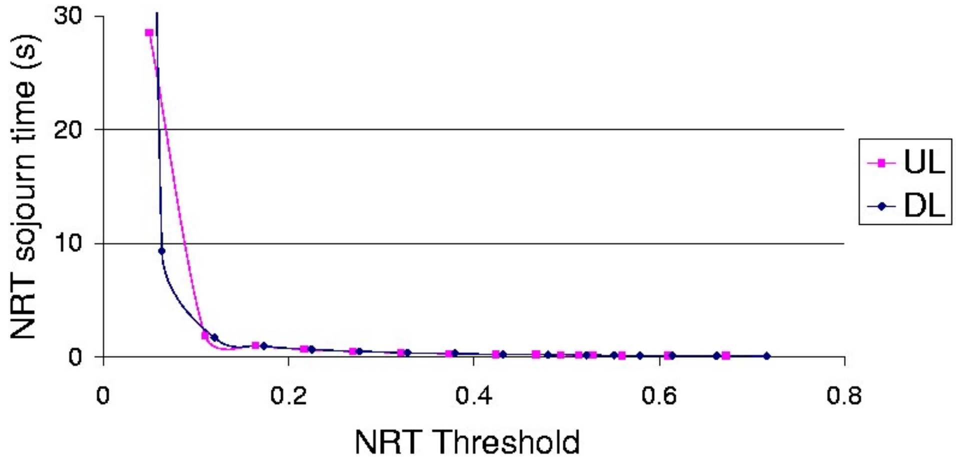

Figure 5.4: Expected sojourn times of NRT traffic as a function of the NRT reservation

Note, in comparison to $figure$ 5.1, the decrease in the maximum number of mobiles, since RT calls now exist in the system.

5.8 Extension to handover calls

So far in the paper, arrivals of new and handover calls in the CDMA cell had been succinctly incorporated in a single rate and thus not treated differently. We now wish to differentiate between these calls. We assume that RT new calls (resp. NRT new calls) arrive with a rate of λ_{RT}^{New} (resp. λ_{NRT}^{New}) where as the handover calls arrive at rate λ_{RT}^{HO} (resp. λ_{NRT}^{HO}). We assume that RT calls remain at a cell during an exponentially distributed duration with parameter μ_{RT}.

From a QoS perspective, avoiding blocking of handover calls is considered more important than avoiding blocking of new ones. So we define a new threshold $\overline{M}_{RT}^{New} < M_{RT}^{\max}$. New RT calls are accepted as long as $M_{RT} \leq \overline{M}_{RT}^{New}$, whereas handover RT calls are accepted as long as $M_{RT} \leq M_{RT}^{max}$. The behavior of NRT calls is as before. Define $\rho_{RT} = \lambda_{RT}/\mu_{RT}$ (the same

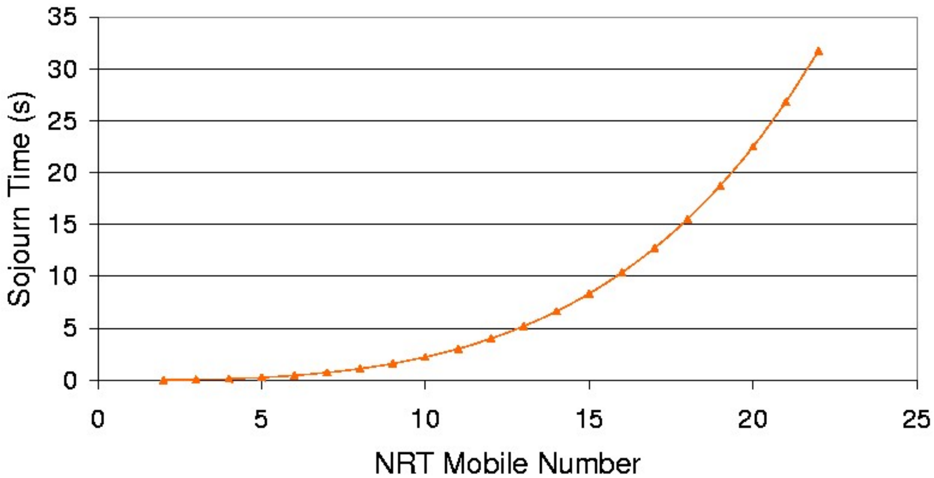

Figure 5.5: Conditional expected sojourn time of an NRT mobile as a function of the number of mobiles in the cell

as before, corresponding to the total arrival rate) and $\rho_{RT}^{HO} = \lambda_{RT}^{HO}/\mu_{RT}$. Let $p_{RT}(i)$ denote the number of RT mobiles in steady state. It is given by

$$p_{RT}(i) = \begin{cases} \frac{(\rho_{RT})^i}{i!} p_{RT}(0), & \text{if } 0 \le i \le \overline{M}_{RT}^{New} \\ \\ \frac{(\rho_{RT})^{\overline{M}_{RT}^{New}} (\rho_{RT}^{HO})^{i-\overline{M}_{RT}^{New}}}{i!} p_{RT}(0), & \text{if } \overline{M}_{RT}^{New} \le i \le M_{RT}^{\max} \end{cases}$$

where $p_{RT}(0)$ is a normalizing constant given by

$$\left(\sum_{i=0}^{\overline{M}_{RT}^{New}} \frac{(\rho_{RT})^i}{i!} + \sum_{i=\overline{M}_{RT}^{New}}^{M_{RT}^{\max}} \frac{(\rho_{RT})^{\overline{M}_{RT}^{New}} (\rho_{RT}^{HO})^{i-\overline{M}_{RT}^{New}}}{i!} \right)^{-1}.$$

The QBD approach introduced before can be directly applied again to compute the joint distribution of RT and NRT calls, and in particular, the expected sojourn time of NRT calls.

A numerical example.
We consider the uplink case of the CDMA system. The input data are the same as before, except that now a fraction of 30% of arriving RT calls are due

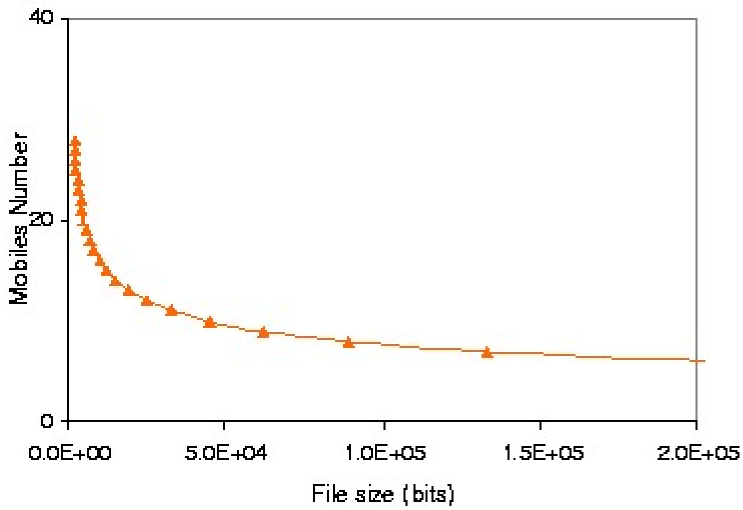

Figure 5.6: Maximum number of NRT connections upon arrival such that the conditional expected sojourn time is below 1 sec, as a function of the size of the file

to handovers. In figure 5.7 we present the impact of the choice of the NRT threshold on the blocking rate of RT mobiles. We also illustrate the impact of the differentiation between new and handover calls. The middle curve is obtained with no differentiation. The total dropping rate of the model with handover differentiation is larger, but the dropping rate of calls already in the system (that arrive through a handover) is drastically diminished (the curve called "Dropping").

5.9 Summary and conclusions

We have developed a simplified mathematical model that allowed us to analyze the performance of call admission control combined with GoS control in a WCDMA environment with integrated RT and NRT traffic. RT traffic has limited adaptive rate functionalities and priority over resources whereas NRT traffic obtains by time sharing the capacity left over by the RT traffic.

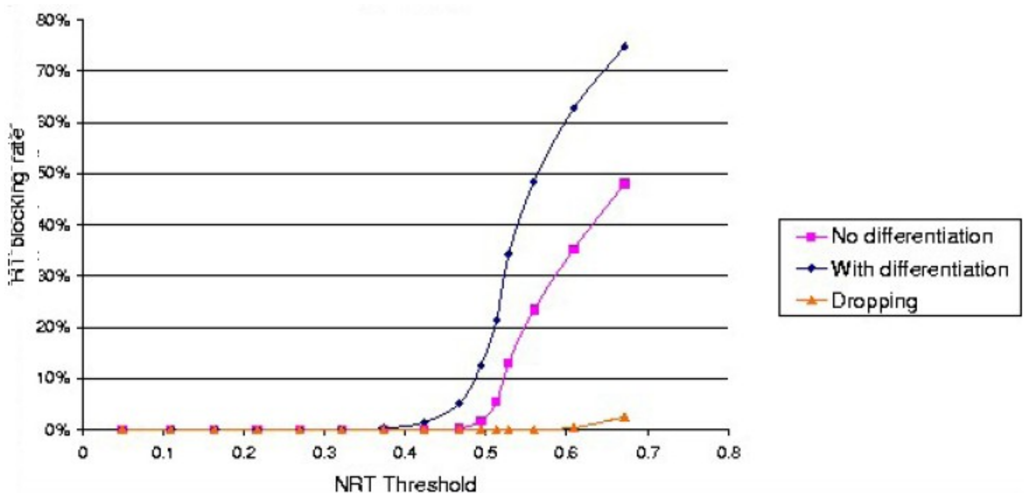

Figure 5.7: RT dropping probabilities

As performance measures we studied the blocking rate of RT traffic and the sojourn times of NRT traffic. We illustrated through numerical examples the importance of adding reserved capacity L_{NRT} for NRT traffic and demonstrated that this reservation can be done in a way not to significantly affect RT traffic. More specifically, we saw that the blocking rate of RT traffic was small and quite robust to the choice of L_{NRT}, over a large interval of values. For NRT traffic, we investigated not only the average sojourn time but also the conditional expected sojourn time given the file size and the number of RT and NRT mobiles present at the cell upon arrival.

Finally, we provided an extension of the multiservice system model to handle handover RT calls. It was shown that differentiating the admission control policy for such calls can greatly reduce their blocking probability, and therefore provide better QoS.

Chapter 6

Outage Probability

In this chapter, using the analytical formula of the interference factor f given by the fluid model, we derive the global outage probability, and the spatial outage probability (see [KeCoGo1]) which depends on the location of a mobile station (MS) initiating a new call.

6.1 Introduction

The estimation of cellular networks capacity is one of the key points before deployment and mainly depends on the characterization of interference. As downlink is often the limited link w.r.t. capacity, we focus on this direction, although the proposed framework can easily be extended to the uplink. As established in chapter 2, in Code Division Multiple Access (CDMA) systems, an important parameter for this characterization is the other-cell interference factor f. The precise knowledge of the interference factor allows the derivation of outage probabilities, capacity evaluation and then, the definition of Call Admission Control mechanisms.

We show that it is possible to get a simple outage probability approximation by integrating f over a circular cell. In addition, as f is obtained as a function of the distance to the BS, it is possible to derive a spatial outage probability, which depends on the location of a newly initiated call.

Remark: blocking vs. outage

Quality of service in cellular networks is characterized for circuit switched services by two main parameters: the blocking probability and the outage probability. The former is evaluated at the steady state of a dynamical system considering call arrivals and departures. It is related to a call admission control (CAC) that accepts or rejects new calls. The outage probability is evaluated in a semi-static system [BaB01], where the number of MS is fixed and their location is random. This approach is often (see e.g. [Bon05]) used to model mobility in a simple way: MS jump from one location to another independently. For a given number n of MS per cell, outage probability is thus the proportion of configurations, where the needed BS output power exceeds the maximum output power: $P_b > P_{max}$.

6.2 Outage probabilities

In this section, we compute the global outage probability and the spatial outage probability with the Gaussian approximation. Using the fluid model, closed-form formulas for the mean and standard deviation of f over a cell are provided.

6.2.1 Global outage probability

For a given number of MS per cell, n, outage probability, $P_{out}^{(n)}$, is the proportion of configurations, for which the needed BS output power exceeds the maximum output power: $P_b > P_{max}$. We assume a single service in the network. We deduce from (4.4)($\beta_u = \beta$ for all u):

$$P_{out}^{(n)} = Pr\left[\sum_{u=0}^{n-1}(\alpha + f_u) > \frac{1-\varphi}{\beta} - \frac{N_0}{P_{max}}\frac{\sum_{u=0}^{n-1} 1}{g_{b,u}}\right], \tag{6.1}$$

where $\varphi = P_{cch}/P_{max}$ and $\beta = \gamma^*/(1 + \alpha\gamma^*)$.

In most of cases the thermal noise may be neglected, we deduce:

$$P_{out}^{(n)} = Pr\left[\sum_{u=0}^{n-1}(\alpha + f_u) > \frac{1-\varphi}{\beta}\right], \tag{6.2}$$

6.2.2 Spatial Outage Probability

For a given number n of MS per cell, a spatial outage probability can also be defined. In this case, it is assumed that n MS have already been accepted by the system, i.e., the output power needed to serve them does not exceed the maximum allowed power. The spatial outage probability at location r_u is the probability that maximum power is exceeded if a new MS is accepted in r_u.

$$P_{sout}^{(n)}(u) = Pr\left[(\alpha + f_u) + \sum_{v=0}^{n-1}(\alpha + f_v) > \frac{1-\varphi}{\beta} | \sum_{v=0}^{n-1}(\alpha + f_v) \leq \frac{1-\varphi}{\beta}\right] \tag{6.3}$$

and so we have

$$P_{sout}^{(n)}(u) = \frac{Pr\left[\frac{1-\varphi}{\beta} - (\alpha + f_u) < \sum_{v=0}^{n-1}(\alpha + f_v) \leq \frac{1-\varphi}{\beta}\right]}{Pr\left[\sum_{v=0}^{n-1}(\alpha + f_v) \leq \frac{1-\varphi}{\beta}\right]} \tag{6.4}$$

6.2.3 Fluid Analysis

To calculate the expressions (6.2) and (6.4) we use the fluid model approach. We make the approximation that the spatial outage, $P_{sout}^{(n)}(u)$, depends on the distance to the BS. Denoting $f(r_u)$ the interference factor of a mobile u at a distance r from its serving BS we can write (6.2) and (6.4) as:

$$P_{out}^{(n)} = Pr\left[\sum_{u=0}^{n-1}(\alpha + f(r_u)) > \frac{1-\varphi}{\beta}\right], \tag{6.5}$$

and

$$P_{sout}^{(n)}(r_u) = \frac{Pr\left[\frac{1-\varphi}{\beta} - (\alpha + f(r_u)) < \sum_{v=0}^{n-1}(\alpha + f(r_v)) \le \frac{1-\varphi}{\beta}\right]}{Pr\left[\sum_{v=0}^{n-1}(\alpha + f(r_v)) \le \frac{1-\varphi}{\beta}\right]} \tag{6.6}$$

The expressions of the outage probability (6.5) and (6.6) are based on the fact that the locations of mobiles in the cell are random. Thus the interference factor value is random too. Since mobiles are assumed to be uniformly distributed over the equivalent disk of the cell, we can express their location as a probability density function (pdf) of r: $p_r(t) = \frac{2t}{R_e^2}$.

6.2.4 Gaussian Approximation

In order to compute these probabilities, we rely on the Central Limit theorem and use a Gaussian approximation. As a consequence, we need to compute the spatial mean and standard deviation of $f(r_u)$. The central limit theorem expresses that considering the following sequence of random variables $X_1, X_2, X_3, ...$ defined on the same probability space, share the same probability distribution D and are independent. Assume that both the expected value μ and the standard deviation σ of D exist and are finite. Consider the sum $Z_n = (X_1 + ... + X_n - n\mu)/\sigma n^{1/2}$, where the expected value of Z_n is $n\mu$ and its standard error is $\sigma n^{1/2}$. Then the distribution of Z_n converges towards the standard normal distribution $N(0,1)$ as n approaches infinity. To apply the gaussian approximation to the calculation of the outage probability, we first have to calculate the expected value μ_f and the standard deviation σ_f of the interference factor. So, we integrate $f(r)$ on an equivalent disk of radius R_e (see figure 3.5). Since the area of a cell is $1/\rho_{BS} = \pi R_e^2$, we have $R_e = R_c\sqrt{2\sqrt{3}/\pi}$.

As mobiles are uniformly distributed over the equivalent disk, the probability density function (pdf) of r is: $p_r(t) = \frac{2t}{R_e^2}$. Let μ_f and σ_f be respectively the mean and standard deviation of $f(r)$, when r is uniformly distributed over the disk of radius R_e.

$$\begin{aligned}\mu_f &= \frac{2\pi\rho_{BS}}{\eta-2}\int_0^{R_e} t^\eta(2R_c-t)^{2-\eta}\frac{2t}{R_e^2}dt \\ &= \frac{2^{4-\eta}\pi\rho_{BS}R_c^2}{\eta-2}\left(\frac{R_e}{R_c}\right)^\eta\int_0^1 x^{\eta+1}\left(1-\frac{R_e x}{2R_c}\right)^{2-\eta}dx \\ &= \frac{2^{4-\eta}\pi\rho_{BS}R_c^2}{\eta^2-4}\left(\frac{R_e}{R_c}\right)^\eta\times \\ &\quad {}_2F_1(\eta-2,\eta+2,\eta+3,R_e/2R_c), \end{aligned} \tag{6.7}$$

where ${}_2F_1(a,b,c,z)$ is the hypergeometric function, whose integral form is given by:

$${}_2F_1(a,b,c,z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 \frac{t^{b-1}(1-t)^{c-b-1}}{(1-tz)^a}dt,$$

and Γ is the gamma function defined as $\Gamma(x) = \int_0^\infty t^{z-1}e^{-t}dt$.
Since the mobile distribution is uniform, μ_f corresponds to an other way to express F_{lim} (see 4.23).

We can notice that for a pathloss parameter $\eta = 3$, we obtain the following closed formula:

$$\mu_f = -2\pi\rho_{BS}R_c^2\left(\frac{\ln(1-\nu/2)}{\nu^2}+\frac{16}{\nu}+4+\frac{4\nu}{3}+\frac{\nu^2}{2}\right),$$

where $\nu = R_e/R_c$. In the same way, the variance of $f(r)$ is given by:

$$\sigma_f^2 = E\left[f^2\right]-\mu_f^2 \tag{6.8}$$

$$\begin{aligned} E\left[f^2\right] &= \frac{2^{4-2\eta}(2\pi\rho_{BS}R_c^2)^2}{(\eta+1)(\eta-2)^2}\left(\frac{R_e}{R_c}\right)^{2\eta}\times \\ &\quad {}_2F_1(2\eta-4,2\eta+2,2\eta+3,\frac{R_e}{2R_c}). \end{aligned}$$

As a conclusion of this section, the outage probability can be approximated by:

$$P_{out}^{(n)} = Q\left(\frac{\frac{1-\varphi}{\beta}-n\mu_f-n\alpha}{\sqrt{n}\sigma_f}\right), \tag{6.9}$$

where Q is the error function defined as $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2} dt$. And the spatial outage probability can be approximated by:

$$P_{sout}^{(n)}(r_u) = \frac{Q\left(\frac{\frac{1-\varphi}{\beta} - n\mu_f - (n+1)\alpha - f(r_u)}{\sqrt{n}\sigma_f}\right) - Q\left(\frac{\frac{1-\varphi}{\beta} - n\mu_f - n\alpha}{\sqrt{n}\sigma_f}\right)}{1 - Q\left(\frac{\frac{1-\varphi}{\beta} - n\mu_f - n\alpha}{\sqrt{n}\sigma_f}\right)}, \tag{6.10}$$

where $f(r_u)$ is given by (3.3).

6.2.5 Validity of the Gaussian Approximation

The question arises of the validity of the Gaussian approximation. The number of users per WCDMA cell is indeed usually not greater than few tens. Figure 6.1 compares the pdf of a gaussian variable with mean μ_f and standard deviation $\sigma_f/\sqrt{n}$ with the pdf of $\frac{1}{n}\sum_u f(r_u)$ for different values for n. The latter pdf has been obtained by Monte Carlo simulations done on a single cell, assuming fluid model formula for f. We observe that gaussian approximation matches better and better when the number of mobiles increases. That one matches well even for very few mobiles in the cell ($n = 10$). So we can use it to calculate the outage probability.

6.2.6 Results

Figures 6.2 and 6.3 show the kind of results we are able to obtain instantaneously thanks to the simple formulas derived in this paper for voice service ($\gamma_u^* = -16$ dB). Figure 6.2 shows the global outage probabilities as a function of the number of MS per cell for various values of the path-loss exponent η. It allows us to easily find the capacity of the network at any given maximum percentage of outage. For example, the outage probability when there are 22 users per cell is about 8% with $\eta = 3.5$. Since the amount of interferences increases when the exponential factor η decreases, we observe that for a given number of mobiles, the outage probability increases when η decreases.

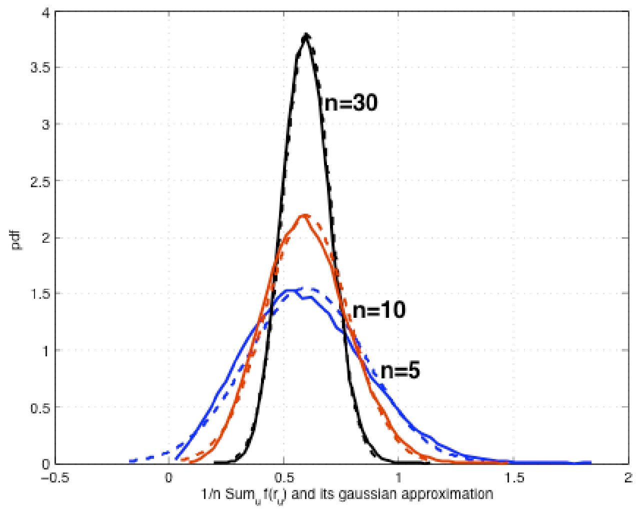

Figure 6.1: Probability density function of $\frac{1}{n}\sum_u f(r_u)$ (solid line) and its Gaussian approximation (dotted line).

Figure 6.3 shows the spatial outage probability as a function of the distance to the BS for $\eta = 3$ and for various number of MS per cell. Given that there are already n, these curves give the probability that a new user, initiating a new call at a given distance, implies an outage. As an example, a new user in a cell with already 16 on-going calls, will cause outage with probability 0.17 at 900 m from the BS and with probability 0.05 at 650 m from the BS.

With this result, an operator would be able to admit or reject new connexions according to the location of the entering MS. Thus, this allows a finer admission control than with the global outage probability.

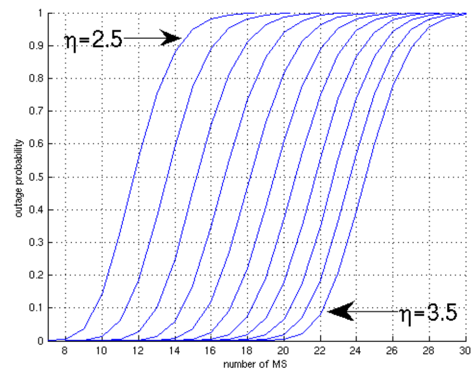

Figure 6.2: Global outage probability as a function of the number of MS per cell and for various values of the path-loss exponent (from $\eta = 2.5$ to 3.5 by steps of 0.1).

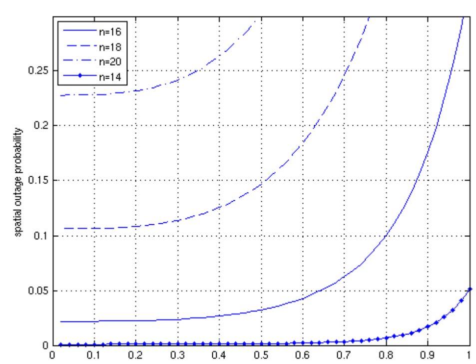

Figure 6.3: Spatial outage probability as a function of the distance to the BS for various number of users per cell and for $\eta = 3$.

6.3 Monte Carlo Simulations Comparisons

6.3.1 Simulation Methodology

The traditional hexagonal model is widely used, especially for dimensioning purposes. That is the reason why a comparison of our model to a hexagonal one is useful.

We compare the outage probability obtained with our fluid model to the ones obtained by simulations done with a hexagonal classical one.

We determine the outage probability with Monte Carlo simulations done with a discrete set of base stations distributed according to a hexagonal pattern. The simulator assumes an homogeneous hexagonal network made of several rings around a cell we analyze. Figure 3.4 shows an example of such a network with the main parameters involved in the study.

At each snapshot of the Monte Carlo simulation, random locations are drawn for the mobile stations of the network. The number of mobile stations per cell is fixed all along the simulation and their spatial distribution within one cell is uniform. Path-loss model is implemented as described in section 3.2.

For a given number of MS per cell, n, outage probability is thus the proportion of configurations, where the needed BS output power exceeds the maximum output power: $P_b > P_{max}$.

6.3.2 Results

Figures (6.4) and (6.5) give comparison of the curves obtained with (6.9) with those obtained by simulations.

There are some differences. These differences are due to the high sensitivy of the error function Q to the mean and standard deviation of f: analysis and Monte Carlo simulations can lead to quite different outage probabilities even if analytical average and variance of the underlying Gaussian distribution are very close to simulated figures. The mean and the standard deviation

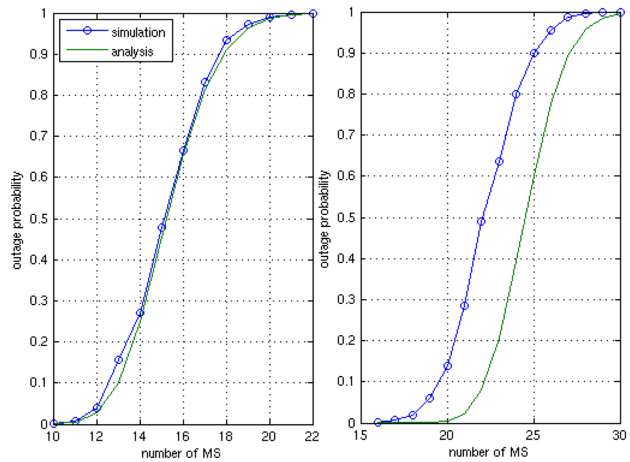

Figure 6.4: Global outage probability as a function of the number of MS per cell and for path-loss exponents $\eta = 2.7$ and 3.5, fluid analysis compared to simulations.

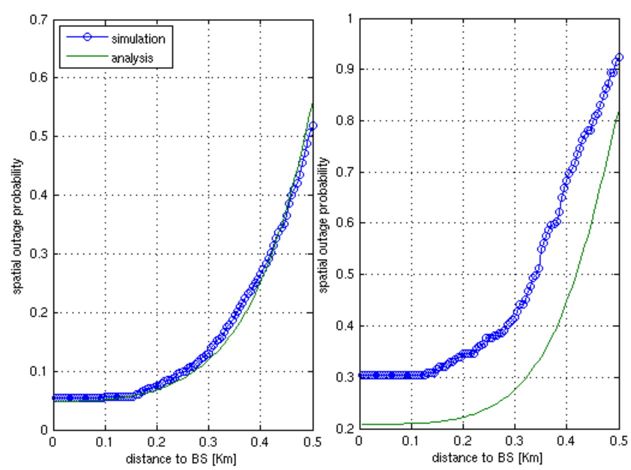

Figure 6.5: Spatial outage probability as a function of the distance to the BS for various number of users per cell and for $\eta = 2.7$ and $\eta = 3.5$.

obtained with the fluid expression of f are slightly different than the ones obtained by simulations. We can say that the results obtained depend on the model used. If we use a hexagonal model, we obtain a result and if we use the fluid model we observe an other result. As no approach is better to modelize the reality of a network, we can say that the observed differences are inherent to the modelization process.

We can also say that we want to have a fluid model as close as possible to a hexagonal one. In that case, we can *fit* the fluid model to the hexagonal one. And we obtain very close results. We however observe the differences between them depend on the value of the pathloss parameter. We thus do a fitting taking into account the value η.

6.3.3 Interference factor formula for hexagonal networks

Two frameworks for the study of cellular networks are considered: the traditional hexagonal model and the fluid model. Both models leads to comparable results for the interference factor as a function of the distance to the BS. If we want to go further in the comparison of both models, in particular with the computation of outage probabilities, we need however to be more accurate.

The aim of this section is to provide an alternative formula for f that better matches the simulated figures in an hexagonal network. Note that this result is not needed if network designers use the new framework proposed in this thesis. An accurate fitting of analytical and simulated curves shows that f should simply be multiplied by an affine function of η to match with Monte Carlo simulations in an hexagonal network. The expression (3.3) can then be re-written as follows:

$$f_{hexa}(r) = (1 + A_{hexa}(\eta)) \frac{2\pi\rho_{BS}r^{\eta}}{\eta - 2}(2R_c - r)^{2-\eta}, \tag{6.11}$$

where $A_{hexa}(\eta) = 0.15\eta + 0.68$ is a corrective term obtained by least-square fitting. Note that for an accurate fitting of the analytical formulas presented in this section to the Monte Carlo simulations performed in an hexagonal network, μ_f should be multiplied by $(1 + A_{hexa}(\eta))$, σ_f by $(1 + A_{hexa}(\eta))^2$ and the expressions (3.3) replaced by (6.11).

6.3.4 Results

Figures 6.2 and 6.3 show the kind of results we are able to obtain instantaneously thanks to the simple formulas derived in this paper for voice service ($\gamma_u^* = -16$ dB). Analytical formulas are compared to Monte Carlo simulations in an hexagonal cellular network. As a matter of fact, Eq.6.11 is used. Figure 6.6 shows the global outage probabilities as a function of the number of MS per cell for various values of the path-loss exponent η. It allows us to easily find the capacity of the network at any given maximum percentage of outage. For example, the outage probability when there are 12 users per cell is about 10% with $\eta = 3.5$. Figure 6.7 shows, as an example, the capacity with 2% outage as a function of η.

Figure 6.8 shows the spatial outage probability as a function of the distance to the BS for $\eta = 3$ and for various number of MS per cell. Given that there are already n, these curves give the probability that a new user, initiating a new call at a given distance, implies an outage. As an example, a new user in a cell with already 16 on-going calls, will cause outage with probability 0.17 at 900 m from the BS and with probability 0.05 at 650 m from the BS.

With this result, an operator would be able to admit or reject new connexions according to the location of the entering MS. Thus, this allows a finer admission control than with the global outage probability.

6.4 Concluding remarks

In this chapter, we showed the simplicity of the fluid model allows a spatial integration of f leading to closed-form formula for the global outage proba-

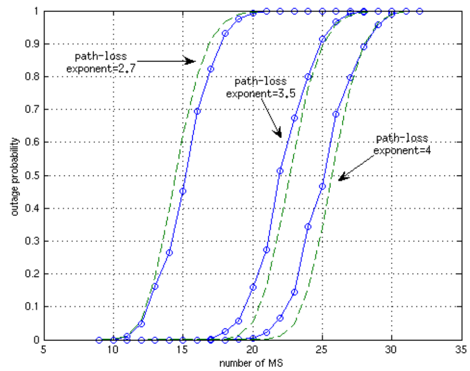

Figure 6.6: Global outage probability as a function of the number of MS per cell and for path-loss exponents $\eta = 2.7$, 3.5 and 4, simulation (solid lines) and analysis (dotted lines).

bility and for the spatial outage probability. Monte Carlo simulations done with a hexagonal network show some differences with the fluid model results, due to the high sensititivy of the error function Q to the mean and standard deviation of f. As no approach is better to modelize the reality of a network, we can say that the observed differences are inherent to the modelization process. However, to have a fluid model as close as possible to a hexagonal one, we *fitted* the fluid model to the hexagonal one as a function of it. The proposed framework is a powerful tool to study admission control in CDMA networks and design fine algorithms taking into account the distance to the BS.

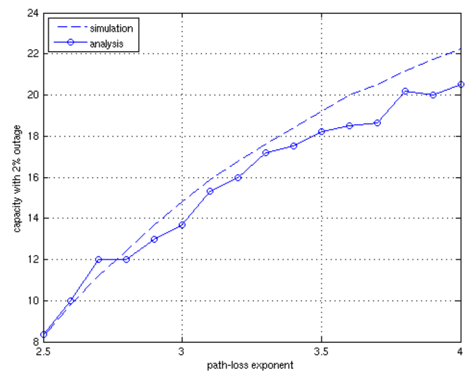

Figure 6.7: Capacity with 2% outage as a function of the path-loss exponent η, simulations (solid lines) and analysis (dotted lines) are compared.

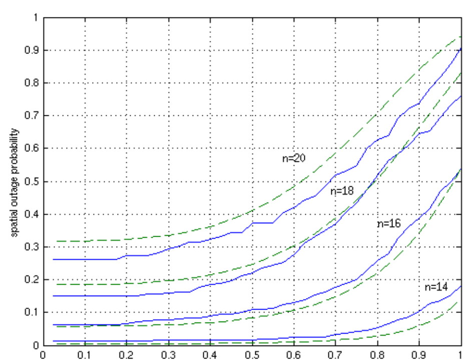

Figure 6.8: Spatial outage probability as a function of the distance to the BS for various number of users per cell and for $\eta = 3$.

Refinement of the fluid model

We developed a fluid model considering a radio network as a continnum of base stations. One of the hypothesis on which the calculation of the interference factor lays is a radial and deterministic pathloss which only depends on the distance between the transmitter and the receiver.
In the chapter 7, we propose a first refinement by considering a pathgain also depending on the antenna gain. This last approach enables to analyze networks with sectored cells. As an application, a comparison between sectorisation and densification is proposed.
In chapter 8, we propose a second refinement by considering the shadowing, in our analysis.
And in chapter 9, we analyze the uplink in term of fluid model.

Chapter 7

Fluid Model for Sectored Networks

This chapter proposes a refinement of the fluid model by considering sectored networks. As an application, we analyze the densification and the sectorization of CDMA networks, in mono and multi services case.

7.1 Introduction

Among the solutions to answer an increasing traffic in a CDMA network, a provider has the possibilities to install base stations in new sites (see section 4.5) or to sectorize it i.e to replace the existing BS with directional antenna BS in the same place as the existing ones. To analyse the advantages and drawbacks of each kind of solution, a provider generates simulations with simulators tools. These last ones need to create an environment and to set the network's parameters. They do not give instantaneous results, may last an important time, and moreover, a great number of simulations are generally required.

We generalize the fluid approach developed for omni-directional base stations networks to sectored ones. We first establish the expression of the interference factor for a three-sectored network. We validate this approach comparing it to a numerical computed network. It becomes possible to analyse and compare instantaneously different solutions with the aim to adapt the network, or a given zone of the network, to an increasing traffic demand. As an application of this model, we analyze the densification and the sectorisation and propose a comparison of these means as solutions to an increasing traffic. We show, this model enables to analyse the mobile admission in CDMA networks. We end by generalizing our model for a *q-sectored* network, with $q \geq 1$.

Our fluid model gives results close to the ones obtained by planning tools (see remark Section 7.7.1) which take into account a real environment.

7.2 Notations

7.2.1 Definitions

We define a *site* as a geographical place where the base stations are located. A cell is the area covered by a BS. An omnidirectional site has *one* BS and one cell. A *q-sectored* site has q BS with directional antenna and q cells. In a sectored site, a cell can be denoted a sector. We focus on *q=3*. Each number of the figure 7.2 represents a site, and each arrow represents a directional antenna.

7.2.2 Propagation

For an omni-directional BS network (see section 2.2.2), we considered a pathgain $g_{b,u}$ only depending on the distance r between the BS b and the MS u. The power, $p_{b,u}$, received by a mobile at distance r_u could thus be written $p_{b,u} = P_b K r_u^{-\eta}$, where K is a constant and $\eta > 2$ is the pathgain exponent. In order to adapt the model in the case of sectored sites, i.e. each geographical site manages several base stations with directional antennas, an other term has to be considered to take into account the directional antenna gain: a mobile positioned at the coordinate (r, θ) of a sector receives a power $p_{b,u} = P_b K r_u^{-\eta} G(\theta)$. We choose the axis origin such as θ is the angle between the azimuth of the sector and the direction BS-MS (see figure 7.2 central zone: sector S1), $0 \leq G(\theta) \leq 1$ is the normalized antenna gain, representing the effect of directional antennas in sectored BS. It is equal to 1 if $\theta = 0$, to a value between 0 and 1 otherwise.

7.3 Sectored Network Fluid Model

7.3.1 Assumptions

The fluid model we developed for a network constituted by omnidirectional base stations can be enhanced in a case of a sectored network. When a uniform traffic and a uniform BS density are assumed, and using now a model where the pathgain $g_{b,u}$ is a function of the position (r, θ) (between the serving base station b and the mobile u), the parameter f_u depends on that position. So the interference factor which was written as a function of r (3.3) has now to be written as a function of (r, θ). Moreover in this context, since a site manages three base stations, the network is characterised by a specific base station density $\rho_{bs,3}$ [Kel02]. We assume that mobiles and base stations are uniformely distributed in the network, so that $\rho_{bs,3}$ is constant. As the network is homogeneous, all base stations have the same output power P_b.

We focus on a given cell and consider a round shaped network around this centre cell with radius R_{nw}. The half distance between two sites is $2R_c$ (see Figure 7.1).

7.3.2 Interference Factor

The figure 7.2 represents a three-sectored network. Each arrow represents an antenna direction. The great circles represent the zone covered by a site, and

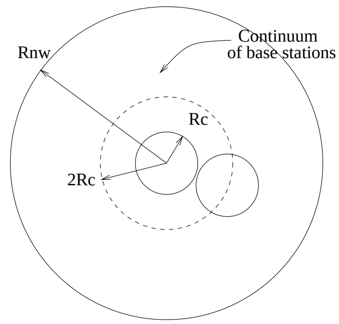

Figure 7.1: Network and cell of interest in the fluid model; the distance between two BS is $2R_c$ and the network is made of a continuum of base stations.

the little ones (central site) represent the zones covered by an antenna. We consider a homogeneous sectored network with a BS (transmitter) density:

$$\rho_{bs,3} = \frac{S_{omni}}{S_{tri}} \rho_{bs}. \tag{7.1}$$

where ρ_{bs} is the omni-directional network BS density , S_{omni} and S_{tri} are the omni-directional and sector cells surfaces. At any point of the network, the power received by a mobile is calculated as follows.

Let's consider a mobile u at the position (r, θ) from its serving BS $b = \psi(u)$. The interference factor is defined as the ratio of the total power received by a mobile coming from all the other base stations of the network on the total power received by its own base station (see 3.4). In the case of a sectored network, the other base stations belonging to the same site contribute to the interferences, too. So to calculate the interference factor in a given sector (sector 1, figure 7.2) we have to consider the total radio power coming from the other sites of the network (denoted 1 to 18 in this case) and the power coming from the other base stations belonging to the same site (sectors S_2 and S_3).

Considering the other sites, each elementary surface $zdzd\theta$ at a distance z from u contains $\rho_{bs,3}zdzd\theta$ base stations which contribute to $P_{ext,u}$. Their contribution to the external interference is $\rho_{bs,3}zdzd\theta P_b K z^{-\eta} G(\theta)$. Like in the omni-directional case, we approximate the integration surface by a ring with centre u, inner radius $2R_c - r_u$, and outer radius $R_{nw} - r_u$ (see figure 3.2). We denote $G_s(\theta)$ the antenna gain for the sector s. For the three-sectored sites, $G(\theta)$ has an angular $\frac{2\pi}{3}$ symmetry. We can write $G_s(\theta) = G(\theta + \frac{(s-1)2\pi}{3})$

We consider a mobile u belonging to the sector S_1. We finally can write the contribution of the other base stations as:

$$P_{ext,u} = \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{bs_3} P_b K z^{-\eta} G(\theta) z dz d\theta + \sum_{j=2}^{3} P_b K r_u^{-\eta} G_j(\theta). \qquad (7.2)$$

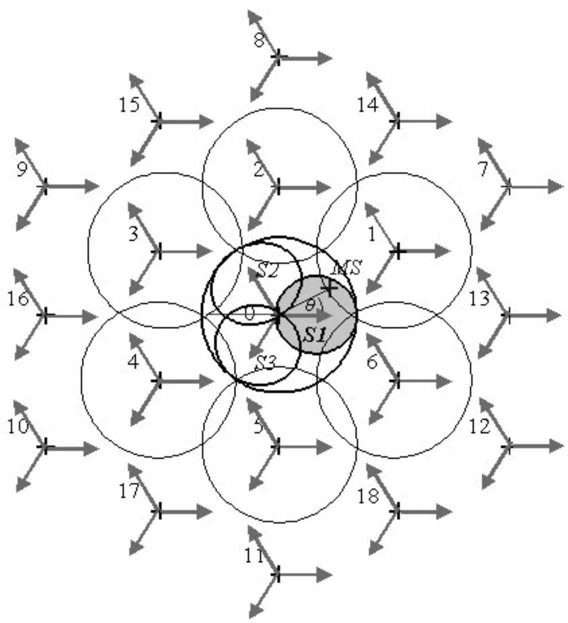

Figure 7.2: Integration limits for external interference computation.

Moreover, a mobile station (MS) u belonging to the sector S_1 at the position (r, θ) receives internal power from b: $P_{int,u} = P_b K r_u^{-\eta} G_1(\theta)$. In this case, the factor f_u depends on the position of the mobile. We denote it $f(r, \theta)$. So considering the whole network, the interference factor defined as $f_u = \frac{P_{ext,u}}{P_{int,u}}$ can be expressed by:

$$f_{sect}(r, \theta) \quad = \quad \frac{1}{P_b K z^{-\eta} G_1(\theta)} \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{bs_3} P_b K z^{-\eta} G(\theta) z dz d\theta$$

$$+ \quad \frac{1}{P_b K z^{-\eta} G_1(\theta)} \sum_{j=2}^{3} P_b K r_u^{-\eta} G_j(\theta). \tag{7.3}$$

We notice that $\frac{\int_0^{2\pi}\int_{2R_c-r_u}^{Rnw-r_u} \rho_{bs} P_b K z^{-\eta} z dz d\theta}{P_b K z^{-\eta}} = f(r)$ (expression 3.3). Since that expression assumes omni directional sites, it can be written $f_{omni}(r)$. Denoting moreover

$$a(\theta) = \frac{1}{2\pi} \frac{S_{omni}}{S_{sect}} \frac{\int_0^{2\pi} G(\theta) d\theta}{G_1(\theta)} \tag{7.4}$$

and

$$b(\theta) = \frac{\sum_{j=2}^{3} G_j(\theta)}{G_1(\theta)} \tag{7.5}$$

So the expression (7.3) can be written as:

$$f_{sect}(r,\theta) = a(\theta) f_{omni}(r) + b(\theta). \tag{7.6}$$

7.3.3 Comments

Linear dependency

We established a very interesting result: The interference factor in a given sector of a three-sectored network can be expressed as a *linear function of the omni-directional one*. The parameters $a(\theta)$ and $b(\theta)$ only depend on the angle, the number of sectors and the normalized antenna gain $G(\theta)$. That linear dependency comes from the analytical expression of the pathloss: the dependency with the distance is decoupled to the one with the angle.

Position dependency

Since we assumed an homogeneous network, and thus all base stations transmit the same power, the interference factor f_u does not depend on the BS output power. The position dependency of f is due to the *pathloss analytical* *expression*. In sectored network, since the pathloss depends on the position of the mobile, f also depends on that one, so that we can write f as a function of (r, θ), $f(r, \theta)$.

We also notice that when $G(\theta) = 1$ whatever the angle θ, i.e. for an omni-directional network, $a(\theta) = 1$ and $b(\theta) = 0$, the expression (7.6) becomes $f_{sect}(r, \theta) = f_{omni}(r)$.

Fluid model general expression

Replacing a discrete set of sites consituted by three base stations by a continuous one, we established the expressions $f_{omni}(r)$ (3.3) and $f_{sect}(r, \theta)$ (7.6): they represent our fluid sector model. The model we propose depends on the exponential pathloss factor, the density of base stations and the cell's radius. It allows calculating the influence of a mobile on a given cell, whatever its position. It moreover takes into account the size zone R to be considered. This last one can be chosen characterizing a typical environment (urban or rural, macro or micro cells). Like in the omni-directional case, it appears important to validate that approach.

7.4 Validation of the Sectored Fluid Model

7.4.1 Simulation Methodology

To validate our network's approach, using a simulator developed for sectored network, we follow an analogue method as the one used for an omni-directional network (chapter 1 section 1.4). We choose to compare our model to a sectored hexagonal classical one. We calculate the interference factor values given by the continuum set of base stations of our fluid model network, and the ones obtained with a discrete set of base stations distributed according to a hexagonal pattern. Since the mobiles are uniformly distributed, all the BS stations have the same transmitting power.

We will compare the figures obtained with Eq. (7.6) to the ones obtained by simulations. The simulator assumes an homogeneous hexagonal three-sectored network made of ten rings of sites around the cell we analyze (figure 7.2).

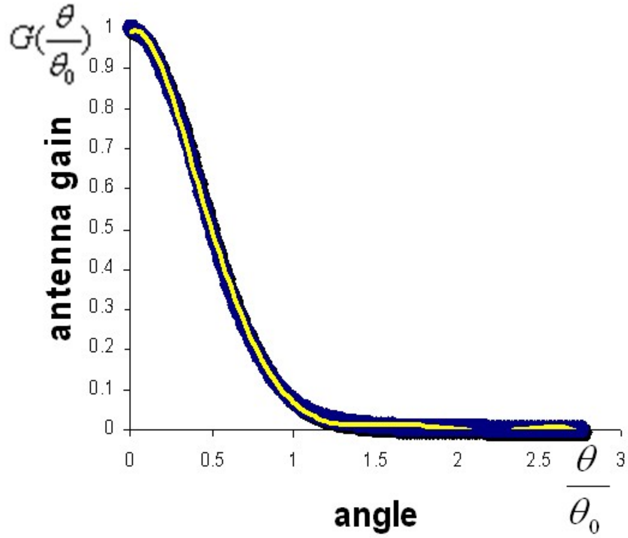

Figure 7.3: Normalized antenna gain.

The validation is done by computing f in each point of the cell S_1 (Figure 7.2). This computation can be done independently of the number of MS in the cell and of the BS output power. Factor f indeed depends only on the path-losses to the BS of the network.

Typical patterns $G(\frac{\theta}{\theta_0})$ for antennas of beam width 60^o and 65^o are used. Figure (7.3) shows the empirical patterns (black curve) and the polynomial regression fitted approximation function (white curve inside the black one) in a linear scale. The following analytical approximation function is used. Denoting $x = \frac{\theta}{\theta_0}$ and $y = G(\frac{\theta}{\theta_0})$ with $\theta_0 = 65$, we have

$$y = -0.1284x^6 + 1.2138x^5 - 4.4096x^4 + 7.5012x^3 - 5.4348x^2 - 0.3349x + 0.9907 \tag{7.7}$$

To validate the model, we need to verify the dependency of the interference factor with the distance r and the angle θ, *i.e.* to compare the values obtained with the two methods: analytical fluid one simulation based one.

7.4.2 Simulation Results

Simulation parameters are the following:

- $\eta = 3$,
- $R = 1$ Km,
- $R_{nw} = 5R$.

The expression (7.6) is also plotted for comparison.

We observe the fluid model matches very well the simulations on an hexagonal sectored network (figures 7.4 and 7.5)

The figure (7.4) compares the interferences factor values obtained with the two methods. The X coordinate represents the interference factor values given by the simulation method and the Y coordinate represents the ones obtained with the fluid model. The simulations and fluid values obtained (orange squares) are fitted according to a linear regression (black dotted line). Its analytical expression, Y=1.05X, shows that the two methods give very close values, with a great precision (correlation coefficient 0.99).

The figure (7.5) shows moreover that the simulated values of the interference factor (X) are close to the fluid analytical ones (points), for each angle between -60^o to 60^o (beam width) and whatever the distances r (from 0 to 1000 m).

We conclude that the fluid model approach is accurate: it allows to calculate the interference factor experienced by a mobile, whatever its position in a cell.

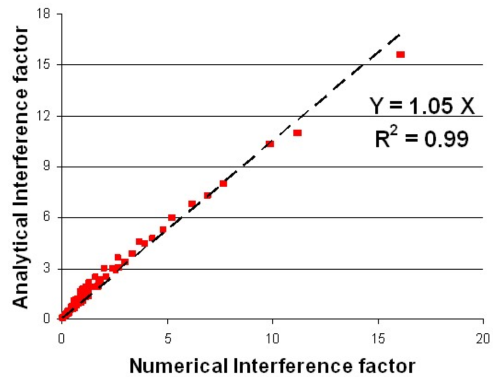

Figure 7.4: Interference factor fluid vs computed.

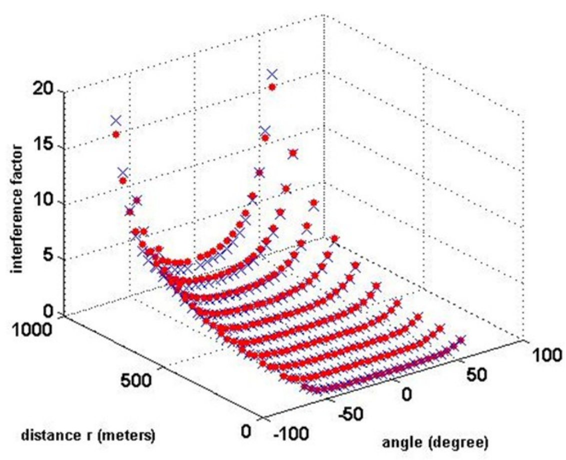

Figure 7.5: Interference factor vs r and θ: comparaison of the fluid method to the computed one.

7.4.3 Conclusion of the validation

Replacing a discrete set of base stations by a continuous one, we established the interference factor expressions (7.6) and (3.2). They depend on the pathloss exponent, the pattern antenna, the density of base stations and the cell's radius. We validated the fluid model, comparing it to a simulated hexagonal one: we showed the interference factor values given by the fluid model are very close to the ones obtained by simulations. As already observed, these expressions take into account the size zones R_{nw} to be considered, which can be chosen characterizing a typical environment (pathloss parameter, urban, rural, macro or micro cells). And they allow calculating the precise influence of a mobile on a given cell, whatever its position.

We can conclude that our model allows us to develop analyses, adapted to each network's zone, taking into account each specific considered zone's parameters.

7.5 Sectored Network: General Case

The analytical model we developed allows the telecom provider to know very easily the advantages and drawbacks to densify or to three-sectorize a CDMA network. However, the provider can also need to know how many sectors to install in a site. In this aim, the model has to be generalized. We consider a homogeneous q-sectored network, $q > 0$, with a BS density

$$\rho_{bs,q} = \frac{S_{omni}}{S_q} \rho_{bs}. \tag{7.8}$$

where ρ_{bs} represents the omni-directional network's BS density and S_{omni} and S_q are the omni-directional and *q-sector* cells surfaces. We aim to adapt the model in the case of *q-sectored* sites, i.e. each geographical site manages q base stations with directional antennas. Using the pathloss model $p_{b,u} = P_b K r_u^{-\eta} G(\theta)$, the interference factor in a given sector denoted S_1 calculated in section (see 3.4) can thus be generalized as follows.

We consider a mobile u belonging to the sector S_1. We can write the contribution of the other base stations as:

$$P_{ext,u} = \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{bs_3} P_b K z^{-\eta} G(\theta) z dz d\theta + \sum_{j=2}^{q} P_b K r_u^{-\eta} G_j(\theta). \qquad (7.9)$$

where $G_q(\theta)$ stands for the sectors s=1, 2 q. For the q-sectored sites, $G(\theta)$ has an angular $2\pi/q$ symmetry: $G_q(\theta) = G_q(\theta + (s-1)2\pi/q)$.

Moreover, a mobile station (MS) u belonging to the sector S_1 at the position (r, θ) receives internal power from b: $P_{int,u} = P_b K r_u^{-\eta} G_1(\theta)$. Since the factor f_u depends on the position of the mobile, we denote it $f(r, \theta)$. So considering the whole network, the interference factor $f_u = \frac{P_{ext,u}}{P_{int,u}}$ can be expressed by:

$$\begin{aligned} f_{sect}(r, \theta) &= \frac{1}{P_b K z^{-\eta} G_1(\theta)} \int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{bs_3} P_b K z^{-\eta} G(\theta) z dz d\theta \\ &+ \frac{1}{P_b K z^{-\eta} G_1(\theta)} \sum_{j=2}^{q} P_b K r_u^{-\eta} G_j(\theta). \end{aligned} \qquad (7.10)$$

Since $f_{omni}(r) = \frac{\int_0^{2\pi} \int_{2R_c - r_u}^{R_{nw} - r_u} \rho_{bs} P_b K z^{-\eta} z dz d\theta}{P_b K z^{-\eta}} = f(r)$ (see chapter 1, 3.3). and denoting moreover

$$a_q(\theta) = \frac{1}{2\pi} \frac{S_{omni}}{S_q} \frac{\int_0^{2\pi} G(\theta) d\theta}{G_1(\theta)} \qquad (7.11)$$

and

$$b_q(\theta) = \frac{\sum_{j=2}^{q} G_j(\theta)}{G_1(\theta)} \qquad (7.12)$$

the expression (7.3) can be written as:

$$f_q(r, \theta) = a_q(\theta) f_{omni}(r) + b_q(\theta). \qquad (7.13)$$

For a q-sectored network, the interference factor is expressed as a linear function of the omni-directional one. It is due to the "non correlation" between the distance r and the angle θ in the analytical pathloss model expression. The parameters $a_q(\theta)$ and $b_q(\theta)$ only depend on the angle and the number of sectors.

7.6 Sectorisation and densification

In a CDMA system, the number of mobiles is limited by the interferences or by the transmitting power of the base stations. The telecom providers need to apply an admission control, to be able to offer the subscribers a quality of service as close as possible to the one they ask for. For the downlink, any admission control has to take into account the base station transmitting power's limitation. To illustrate our analytical model, we propose hereafter an analysis of the capacity of the system in term of number of mobiles. Afterwards, we show how any kind of admission control policy can be derived using our analytical model.

7.6.1 Mono-service case

For the downlink, we express that the power of the base station P_b is limited to a maximum value: the call admission control is based on the probability P_{cell}^{DL} to satisfy the following relation:

$$P_{cell}^{DL} = Pr[P_b > P_{max}] \tag{7.14}$$

where $cell = omni$ or $sect$.

The total power of the serving base station b can be calculated from the equation (2.10) and (4.2):

$$P_b = \frac{P_{cch} + \sum_u \beta_u \frac{N_0}{g_{b,u}}}{1 - \sum_u \beta_u(\alpha + f_u)}. \tag{7.15}$$

The power P_{CCH} dedicated to the common channels is assumed as proportional to the power of the base station

$$P_{cch} = \varphi P_b. \tag{7.16}$$

Considering a density mobile distribution ρ_{ms} in the cell, we can rewrite (7.15) as:

$$P_b = \frac{\int_0^R \int_0^{\frac{2\pi}{q}} \rho_{ms} K r^{1-\eta} G(\theta) dr d\theta}{1 - \varphi - \int_0^R \int_0^{\frac{2\pi}{q}} \rho_{ms} \beta_u (\alpha + f_{cell})(r,\theta) r dr d\theta)}. \tag{7.17}$$

We assume a uniform mobile distribution in the cell. We moreover denote F_{cell} the downlink interference factor average value in the cell:

$$F_{cell} = \frac{1}{S_{cell}} \int_0^R \int_0^{\frac{2\pi}{q}} f_{cell}(r,\theta) r dr d\theta \tag{7.18}$$

and we introduce the parameter A_{cell}

$$A_{cell} = \frac{1}{S_{cell}} \int_0^R \int_0^{\frac{2\pi}{q}} r^{1-\eta} G(\theta) dr d\theta \tag{7.19}$$

We recall that $G(\theta) = 1$ and $q = 1$ for omni-directional antennas, $q = 3$ for three-sector ones.

Denoting $n_{ms} = \rho_{ms} S_{cell}$ the number of mobiles in the cell, using the expressions 7.14 and 7.17, we can write:

$$P_{cell}^{DL} = Pr[n_{MS} > \frac{1 - \varphi}{\beta(\alpha + F_{cell} + A_{cell} \frac{N_0}{P_{max}})}] \tag{7.20}$$

The blocking probabilities have similar analytical expressions for sectored and omni-directional networks. We denote

$$n_{omni}^{th} = \frac{1 - \varphi}{\beta(\alpha + F_{omni})} \tag{7.21}$$

and

$$n_{sect}^{th} = \frac{1 - \varphi}{\beta(\alpha + F_{sect})}. \tag{7.22}$$

We can write, as long as long as the Noise is negligible, which is a reasonnable assumption for a radius cell sizes less than 1 km.

$$P_{omni}^{DL} = Pr[n_{ms,omni} > n_{omni}^{th}] \tag{7.23}$$

and

$$P_{sect}^{DL} = Pr[n_{ms,sect} > n_{sect}^{th}] \tag{7.24}$$

The parameters n_{omni}^{th} and n_{sect}^{th} represent the average capacity of an omnidirectional and sectored cell.

Using the expression of f given by the fluid model, we can write:

$$F_{sect} = \frac{1}{S_{sect}} \int_0^R \int_0^{\frac{2\pi}{3}} \left(a(\theta) f_{omni}(r) + b(\theta)\right) r dr d\theta \tag{7.25}$$

Denoting

$$C_1 = \frac{1}{2\pi} \frac{S_{omni}}{S_{sect}} \int_0^{\frac{2\pi}{3}} a(\theta) d\theta \tag{7.26}$$

and

$$C_2 = \frac{1}{2\pi} \frac{S_{omni}}{S_{sect}} \int_0^{\frac{2\pi}{3}} b(\theta) d\theta \tag{7.27}$$

we can express

$$F_{sect} = C_1 F_{omni} + C_2 \tag{7.28}$$

Using a gain antenna expressed by 7.7, and the definitions 7.26 and 7.26, we obtain the expression:

$$F_{sect} \approx 1.12 F_{omni} + 0.14 \tag{7.29}$$

A numerical analysis, with $\eta = 3$, shows the values of F_{omni} and F_{sect} for an infinite network's size (Table 7.1). The average interference factors are limited; they tend to an asymptotic value, when the network's dimension increases. For high size networks, F_{cell} does no more depend on the network's size. As a consequence, for a homogeneous network the number of mobiles per cell does not depend on the size of the cell. It depends on the environment characterized by the pathloss factor, the target SINR, the orthogonality factor and the power ratio dedicated to the common channels. For a sectored cell, it moreover depends on the antenna pattern.

7.6.2 Multiservice case

The expression 7.15 can be written for each service s and each mobile u, as follows:

Assuming that each mobile uses only one service, the total power of the serving base station b can be calculated from the equation (7.15):

$$P_b = \frac{P_{cch} + \sum_s \sum_u \beta_s \frac{N_0}{g_{b,u}}}{1 - \sum_s \sum_u \beta_s(\alpha + f_u)}. \tag{7.30}$$

Considering a density mobile distribution $\rho_{ms,s}$ in the cell using the service s, we can rewrite (7.17) as:

$$P_b = \frac{\int_0^R \sum_s \int_0^{\frac{2\pi}{q}} \beta_s \rho_{ms,s} K r^{1-\eta} G(\theta) dr d\theta}{1 - \varphi - \sum_s \int_0^R \int_0^{\frac{2\pi}{q}} \rho_{ms,s} \beta_s (\alpha + f_{cell})(r,\theta) r dr d\theta)}. \tag{7.31}$$

Considering uniform mobiles distributions and using (7.18) and (7.19), we finally obtain:

$$P_b = \frac{\sum_s \beta_s n_{ms,s} A_{cell} N_0}{1 - \varphi - \sum_s n_{ms,s} \beta_s (\alpha + F_{cell})}. \tag{7.32}$$

The expression (7.20) becomes, in a multiservice case:

$$P_{cell}^{DL} = Pr[\sum_s \beta_s n_{ms,s} > \frac{1-\varphi}{\alpha + F_{cell} + A_{cell} \frac{N_0}{P_{max}}}] \tag{7.33}$$

7.7 Numerical application

The following numerical application compares the sectorisation and the densification for mono-service and multiservice cases, for $\alpha = 0.7$ and $\varphi = 0.2$.

7.7.1 Monoservice case

Table 7.1 shows, for a voice service ($\gamma = -16dBm$), that the average capacity of a sectored cell n_{sect}^{th} is smaller than for an omni-directional one n_{omni}^{th}. However, since the capacity of a three-sector site is given by $n_{sect}^{tot} = 3n_{sect}^{th}$, replacing an omni-directional site by a three sectors one increases its capacity by a factor $D = \frac{n_{sect}^{tot}}{n_{omni}^{th}}$. The analytical approach we developed showed that the use of three-sector sites instead of omni directional ones increases their capacity by a factor $D \approx 2.6$. The term C_2 gives the influence of the other sectors of the same site. Without it, D would be about 2.9.

F_{omni}	F_{sect}	n_{omni}^{th}	n_{sect}^{th}	n_{sect}^{tot}	n_{omni}^{tot}	D
0.65	0.87	24	21	63	72	2.6

Table 7.1: Relative quantities, for $\eta = 3$.

Replacing an omni-directional site by 3 ones in the same zone would increase the capacity of that one by a factor 3 (in a first approximation: the Noise is considered as negligible). So, to answer an increasing traffic, it theoretically appears better to do a *densification* than a *sectorisation*. Considering economical constraints, it could however be difficult to find new sites.

Remark: comparison with planning tools

Radio planning tools developed by *France Telecom* give an average capacity per cell of about 28 mobiles, for an omni-directional CDMA network in a suburban environment. For 3-sector sites, the average capacity is about 67 mobiles (i.e the D factor obtained is about 2.4). These tools use accurate propagations models based on theoretical analysis and on calibrations with field measurements. We observe that the analytical results obtained with our model presented in Table 7.1, columns 3 and 5, are close to these values.

7.7.2 Multi-service case

We consider a site managing two services, voice with a target SINR $\gamma_1 = -16dB$ and data (384 kb/s) with a target SINR $\gamma_2 = -4.6dB$. We observe (figure 7.6) a high decrease of the limit capacity of a site when the data traffic increases from 0 (no data traffic in the site) to 100% (no voice traffic in the site). For each kind of site, omni-directional and sectored one, the total number of mobiles decrease reaches about 50% though the data traffic is relatively low (10%). These curves moreover show that the sectorisation of a site, i.e. replacing one omni-directional BS by 3 sectored ones, increases its limit capacity by a factor $D \approx 2.6$. We moreover observe the factor D does not depend on the service and on the proportion of traffic of each service.

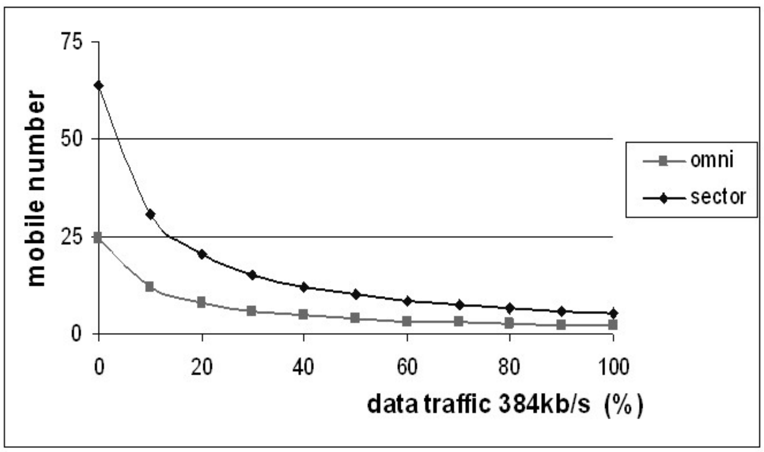

Figure 7.6: Mobile number limit capacity of a site vs 384 kb/s data traffic.

To allow more mobiles to be admitted by a site, the provider can adopt a strategy consisting to decrease the data service throughput offered to mobiles entering the site. For an offered throughput of 144 kb/s, instead of 384 kb/s, the data target SINR becomes $-10dB$. The figure 7.7 shows the new limits of the capacity in this last case. We can analyze, compare and quantify the advantages of a sectorisation and/or a throughput decrease. We observe a first improve of the capacity of an omni-directional site obtained by decreasing the mobiles' throughput. For example, considering a data traffic of 20%, the total number of mobiles increases from 8 ("omni 384" curve) to 14 ("omni 144" curve). A better improve is reached with the sectorisation of the site. For 20% data traffic, the number of mobiles reaches 20 (”sector 384” curve). And the best improve is observed when these two means are used together. In this last case, for a data traffic of 20%, the total number of mobiles admitted in the site reaches 36 (”sector 144” curve). Moreover the decrease of the limit capacity due to the data traffic is lower for 144 kb/s data service than for 384 kb/s data service: considering a data traffic of 10%, it particularly only reaches 30% (dotted curves: 144 kb/s data) instead of 50% (continuous curves: 384 kb/s data). The figure 7.8 focuses on the increase of data number of mobiles: for example when a three sectored site only manages data traffic (100%), the limit number of data mobiles increases from 5 (data ”384”) to 13 (data ”144”).

We analysed with our model the strategy consisting to decrease the demanded throughput in the aim to increase the admission of mobiles in a site, for omni-directional and sectored sites. We obtained results close to the ones obtained with simulation tools. We showed our model instantly provides, without simulation, an admission control strategy analysis. It can moreover quantify the consequence of a given strategy. Other admission strategies could be analyzed with our model.

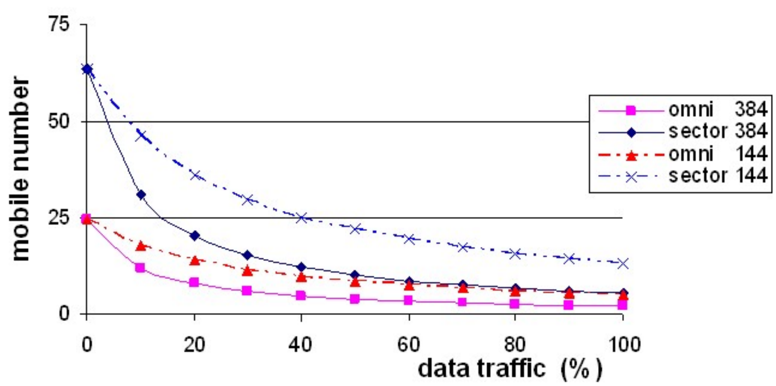

Figure 7.7: Mobile number limit capacity of a site vs data traffic.

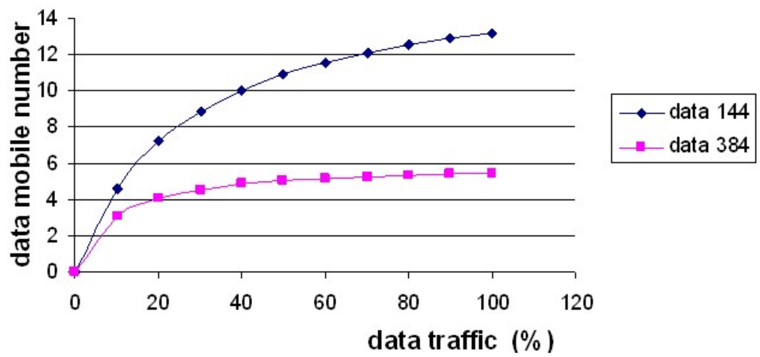

Figure 7.8: Sectored site: data mobile number limit capacity vs data traffic.

7.8 Concluding remarks

We extended and validated the fluid model approach developed for omni-directional networks to sectored ones. Replacing a discrete network of sectorised base stations by a continuum, we established the expressions of the interference factor, $f_{sect}(r, \theta)$. We moreover showed it *linearly depends* on the interference factor of the omni-directional base stations, now denoted $f_{omni}(r)$. We analyzed the solutions based on a densification or a sectorisation to answer an increasing traffic. We instantly obtained explicit expressions of the capacity, and showed that it theoretically appears better to densify a network than to sectorize it. However, considering economical or sociological constraints, it can be difficult to find new sites in a real network. As a consequence, to sectorize a network could be easier than to densify it. A telecom provider has to take into account all these aspects to decide what solution to adopt. A decrease of the demanded throughput in the aim to increase the admission of mobiles in a site, for omni-directional and sectored sites, showed the importance of the choice of an admission control strategy. The admission control analyses generally require the use of simulation tools, and do not give instantaneous results. Our analytical model allows to analyze admission strategies without simulation.

Chapter 8

Shadowing and Environmental Analysis

Since in a real network, the pathloss also depends on the local environment (terrain, urban, rural, streets, trees), we propose in this chapter, a second refinement of the fluid model by taking into account the shadowing effect and more generally the effects of each specific environment.

8.1 Introduction

The radio link model we used only expresses that the power received at any point of the system depends on its distance from the transmitter (the line-of-sight path). In a real network, it however also depends on the local environment (terrain, buildings, trees). Thus, the radio link can be modelled by a term which expresses that the power received at any point of the system depends on the distance r from the transmitter (the line-of-sight path), and the environment (terrain, buildings, trees). The first term depends on the type of the global environment, urban or rural, and may moreover depend on the type of cells: macro or micro. The last term, the shadowing, is generally modelled as a lognormal distributed [Stu01] function. In this chapter, the fluid model takes into account the shadowing and more generally the effects of each specific environment (urban, rural, streets buildings).

Taking into account the shadowing, we first establish the interference factor's analytical expressions, mean value and standard deviation, as a lognormal random variable (RV) according to the Fenton-Wilkinson approximation (denoted FW) for a sum of lognormal RV [Fen01]. To characterize the system topology, we establish the analytical expression of a function depending on the shadowing, the base stations numbers and positions, and the exponential pathloss parameter.

Using the fluid approach, we express the interference factor's mean value and standard deviation, and analyze the influence of different network's parameters: cell radius, exponential pathloss parameter, distance of the mobile to its serving base station.

Since the shadowing depends on the environment around the mobile, in real networks it appears natural to consider some correlations between the signals received by a mobile. We thus establish the mean value and standard deviation expressions for **correlated signals**. We show that the shadowing has a limited influence, for independent and correlated received powers, and we express the interference factor bounds, minimum and maximum.

We finally show that the shadowing has a limited influence, for independent and correlated received signals, and we express the interference factor bounds, minimum and maximum.

8.2 Interferences with shadowing

8.2.1 Propagation

Considering the power P_j transmitted by the BS j, and $r_{j,u}^{-\eta}A$ the pathloss including the shadowing effect, the power $p_{j,u}$ received by a mobile u belonging to j can be written:

$$p_{j,u} = P_j K r_{j,u}^{-\eta} A \tag{8.1}$$

The parameter $A = 10^{\frac{\xi}{10}}$ represents the shadowing effect. It characterizes the random variations of the received power around a mean value. The parameter ξ is a *Normal* distributed random variable *RV*, with mean 0 and standard deviation σ_j comprised between 0 and 10 dB. The term $P_j K r_{j,u}^{-\eta}$ represents the mean value of the signal received by the mobile u at the distance $r_{j,u}$ from the transmitter (BS_j). The probability density function (PDF) of this slowly varying received signal power is given by (since we focus on a given mobile u, we can drop that index):

$$\Psi_j(s) = \frac{1}{a\sigma_j s\sqrt{\pi}} exp - \left(\frac{\ln(s) - m_j}{\sqrt{2} a\sigma_j} \right)^2 \tag{8.2}$$

where

- $a = \frac{\ln 10}{10}$,
- $m_j = \frac{1}{a} \ln(K P_j r_j^{-\eta})$ is the (logarithmic) received mean power expressed in decibels (dB), which is related to the path loss and
- σ_j is the (logarithmic) standard deviation of the received signal due to the shadowing in decibels.

8.2.2 Interference power

Since the interference factor is defined as (2.1)$f_u = P_{ext,u}/P_{int,u}$ we first need to calculate the other cell interference power. The total interference power due to all the BS of the network (except the serving one) $P_{ext,u} = \sum_{i \neq b}^{B} P_j g_{j,u}$ is the sum of B lognormal *RV*. No exact expression for the PDF of the sum of lognormal distributed RV's is known. It is however accepted that such a sum can be approximated by another lognormal distribution [Stu01]. Among the methods developed to find the mean and variance of that last one, the Schwartz-Yeh approximation [SchwY01] is based on a recursive approach.

Some descriptions and comparisons of these methods are available in [Stu01]. We choose the Fenton-Wilkinson one [Fen01] (denoted FW) for its relative simplicity: the logarithmic mean and the logarithmic variance of a sum of lognormal RV can be found by matching the first and second-order moments. This method is especially accurate for standard deviations lower than 4 dB, when the signals components are uncorrelated. For correlated signals, this method is accurate for standard deviations up to 12 dB [AbuBe01]. We aim to calculate the interference factor as a lognormal RV (mean and standard deviation). Assuming the base stations' power are not correlated, we first calculate the mean and the variance of a sum of lognormal RV, according to the FW method. We afterwards apply the result to the sum of B lognormal identically distributed RV.

8.2.3 Sum of lognormal RV

Let X be a lognormal RV. We can write $\ln X \propto N(am, a^2s^2)$. Let $aY = \ln X$ we write $aY \propto N(am, a^2s^2)$. The mean M and the variance S^2 of a lognormal RV are expressed as $M = exp(am + a^2s^2/2)$ and $S^2 = exp(2am + a^2s^2)exp(a^2s^2 - 1)$. So we can write $am = \ln M - a^2s^2/2$ and $a^2s^2 = \ln(\frac{S^2}{M^2} + 1)$. Using a FW approximation [Fen01] [Ala01], the sum of lognormal RV $X_j(M_j, S_j^2)$ is written as a lognormal RV $X(M, S^2)$ where $M = \sum M_j$ and $S^2 = \sum S_j^2$. We can write

$$am = \ln\left(\sum_j exp\left(am_j + \frac{a^2\sigma_j^2}{2}\right)\right) - \frac{a^2s^2}{2} \tag{8.3}$$

and

$$a^2s^2 = \ln\left(\frac{\sum_j exp\left(2am_j + a^2\sigma_j^2\right)\left(exp(a^2\sigma_j^2) - 1\right)}{\sum_j exp\left(2am_j + a^2\sigma_j^2\right)} + 1\right) \tag{8.4}$$

8.2.4 Interference factor

Our aim is to calculate the interference factor for any mobile at the distance r_b from its serving the BS_b. We focus on a mobile belonging to b, so we notice $r_b = r$. Let us consider a network constituted by cells uniformly distributed and a uniform traffic: Each BS_j transmits a power P_j. The power received by a mobile is characterized by a lognormal distribution X_j as $\ln X_j \propto N(am_j, a^2\sigma_j^2)$ and we can write $am_j = \ln(P_j r_j)$, where r_j stands for the distance between the mobile and the BS_j of the network. We consider that all the standard deviations σ_j are identical. We denote $\forall j \sigma_j = \sigma$. The total power received by a mobile is a lognormal RV X characterized by its mean and variance. Expressing the mean interference power received by a mobile, due to all the other base stations of the network (see appendix A) and since the ratio of two lognormal RV's is also expressed as a lognormal RV, the interference factor is also lognormally distributed with the following mean m_f and logarithmic variance σ_f. Assuming that all the base stations have the same transmitting power $P_b = P_j = P$ (uniform traffic), we introduce

$$G(\eta) = \frac{\sum_j r_j^{-2\eta}}{\left(\sum_j r_j^{-\eta}\right)^2} \tag{8.5}$$

$$f(\eta) = \frac{\sum_j r_j^{-\eta}}{r^{-\eta}} \tag{8.6}$$

$$H(\sigma) = exp\left(a^2\sigma^2/2\right)\left(G(\eta)(exp(a^2\sigma^2)-1)+1\right)^{\frac{-1}{2}} \tag{8.7}$$

We can express (see appendix B):

$$m_f = f(\eta)H(\sigma) \tag{8.8}$$

The standard deviation is given by

$$a^2\sigma_f^2 = 2(a^2\sigma^2 - lnH(\sigma)) \tag{8.9}$$

We can deduce the interference factor's limits:

$$f(\eta) \leq m_f \leq \frac{f(\eta)}{G(\eta)^{\frac{1}{2}}} \tag{8.10}$$

and

$$\sigma_f^2 = 2\sigma^2. \tag{8.11}$$

Remark

We notice that $f(\eta)$ corresponds to the donwlink interference factor f *without shadowing*. From (8.5), we can write $G(\eta) < 1$ whatever η

8.3 Topological analysis

8.3.1 Topological characterization

The expression 8.8 means that the effect of the environment of any mobile of a cell, on the interference factor, is characterized by a function $H(\sigma)$. This last one depends on the shadowing of the received signals coming from the base stations, and a G **factor** which depends on the position of the mobile and the characteristics of the network as

- the exponential pathloss parameter η, which can vary with the topography and more generally with the geographical environment as urban or rural, micro or macro cells.
- the base stations positions and number.

It can be interpreted as an environmental **form factor G** of the network. Its analytical calculation may be complex. Indeed, its expression depends on the positions of the considered mobile and the base stations. We notice that the form factor can be rewritten, using (8.6), as:

$$G(\eta) = \frac{f(2\eta)}{f(\eta)^2} \tag{8.12}$$

The shadowing effect consists in **increasing** the mean value and the standard deviation of the interference factor (8.10 and 8.11). This increase is however limited (figure 8.2 and Table 8.2). In a realistic network, σ_j is generally comprised between 6 and 12 dB.

8.3.2 Fluid model approach

In our fluid model approach of the network, the interference factor expression of $f(r)$ (3.3) also depends on the exponential pathloss factor η. Focusing on this dependency with η (forgotting r for a moment), we can denote it $f(\eta)$. Thus, to go further on our analysis, it appears interesting to express $f(\eta)$ and $G(\eta)$ using the fluid approach. Assuming the serving base station is the closest one, we can use the expression (3.3). Since that expression of f also depends on the distance between two neighbors BS $2R_c$ and the dimension of the network R_{nw}, it allows us to explore these network parameters influences. Using the fluid model, we can express the form factor **G limits**, using (8.12). From $f(r)$ (3.3) we can write, dropping the dependency with the distance r

$$f(\eta) = \frac{2\pi\rho_{bs}r^{\eta}}{\eta - 2}\left[(2R_c - r)^{2-\eta} - (R_{nw} - r)^{2-\eta}\right]. \tag{8.13}$$

When the considered zone's radius is great compared to the cell's one, i.e. $R_{nw} >> R_c$, since we have $0 < r < R_c$ we can write:

$$-\frac{(-\eta+2)^2}{16(-\eta+1)} \leq G(\eta) \leq -\frac{(-\eta+2)^2}{4(-\eta+1)} \tag{8.14}$$

Table 8.1 indicates the limits of the form factor G as a function of η. They allow to determine the limits of the interference factors parameters m_f and σ_f.

8.3.3 Interference factor distance dependency

The figure (8.1) shows the influence of the distance between the mobile and its serving BS, for different standard deviations and $\eta = 3$. The mean value

η	3	3.5	4	4.5	5
G_{min}	0.03	0.06	0.08	0.11	0.14
G_{max}	0.12	0.22	0.33	0.45	0.56

Table 8.1: Form factor G limits

of the interference factor increases when the standard deviation increases. Compared to a case without shadowing, and for a mobile located at the edge of the cell (1000m), that increase is about 30% when $\sigma = 4dB$ and reaches about 100% when $\sigma = 12dB$. We observe that the shadowing seems to "increase" the distances from the BS: with a shadowing $\sigma = 12dB$, a mobile at 800m from its serving BS has the same mean interference factor as a mobile at 1000 m without shadowing. This effect explains the importance of considering *shadowing margins* during the planning process.

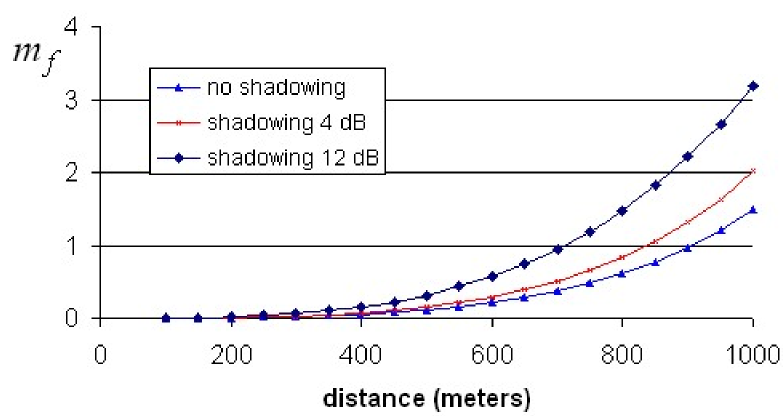

Figure 8.1: Influence of the standard deviation on the mean interference factor m_f for each distance.

8.3.4 Influence of the standard deviation

For low variances, i.e. $a^2\sigma^2 \approx 1$, we can express from (8.7) and (8.8)

$$m_f \approx f(\eta) exp(a^2 \frac{\sigma^2}{2}) \tag{8.15}$$

and

$$a^2\sigma_f^2 \to a^2\sigma^2. \tag{8.16}$$

And for high variances, i.e. $exp(a^2\sigma^2) >> 1$ or$\sigma^2 >> \frac{1}{a^2}$, we can write

$$m_f \approx \frac{f(\eta)}{G^{\frac{1}{2}}} \tag{8.17}$$

and

$$a^2\sigma_f^2 \approx 2a^2\sigma^2 + \ln G. \tag{8.18}$$

For low standard deviations (less then 4 or 5 dB), these expressions show a low dependency of the mean value of f with σ, and the total standard deviation σ_f is very close to σ (Table 8.2).

These expressions show another interesting result: for high variances (higher than 5 dB), the interference factor's mean tends towards a value which does not depend on the variance. Considering the extreme G values (Table 8.1), with $\eta = 3$, the figure (8.2) confirms that the mean value of the interference factor increases until a standard deviation of about 10 dB. For higher values, the interference factor stays constant. We observe the mean interference factor does no more depend on them.

Moreover, the form factor *compensates* the standard deviation influence. It increases with the distance r, and also with the exponential pathloss parameter η. It means that a high value of that parameter, characterizing a given type of cells or environment, may compensate the shadowing effects(see also figure 8.3). In a realistic network σ is generally comprised between 6 and 12 dB, Table 8.2 shows that the standard deviation of the interference factor is close to the BS ones σ (with $\eta = 3$).

8.3.5 Interference factor environmental dependency

The exponential pathloss parameter η can characterize the environment type, urban or rural, and the cell dimensions (pico, micro, macro). The figure (8.3)

σ	0	1	2	3	4	5	6	7	8	9	10	11	12
σ_f	0	1.1	2.2	3.3	4.5	5.8	7.2	8.6	10.1	11.6	13.1	14.6	16.1

Table 8.2: Standard deviation of the interference factor vs σ

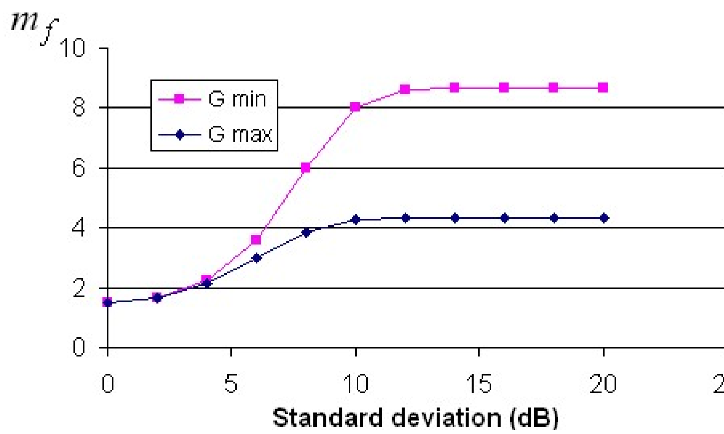

Figure 8.2: Mean interference factor m_f vs deviation σ_j (distance = 1000 m).

confirms that the influence of the shadowing on the mean interference factor decreases when η increases. For $\eta \geq 5$ we observe that the shadowing has almost no influence.

8.3.6 Interference factor cell radius dependency

Figure (8.4) shows the mean interference factor for a mobile at a given relative position r/R_c in the cell from its serving base station. When R_c increases (i.e. the distance between two BS increases) the influence of the shadowing decreases, and becomes very low for a cell's radius higher than 1700 m.

Remark

The analytical results we obtained are based on a Fenton-Wilkinson approach. This one is especially accurate when the variance of lognormal RV is less than 4dB. However, it has been shown in [AbuBe01] the accuracy of

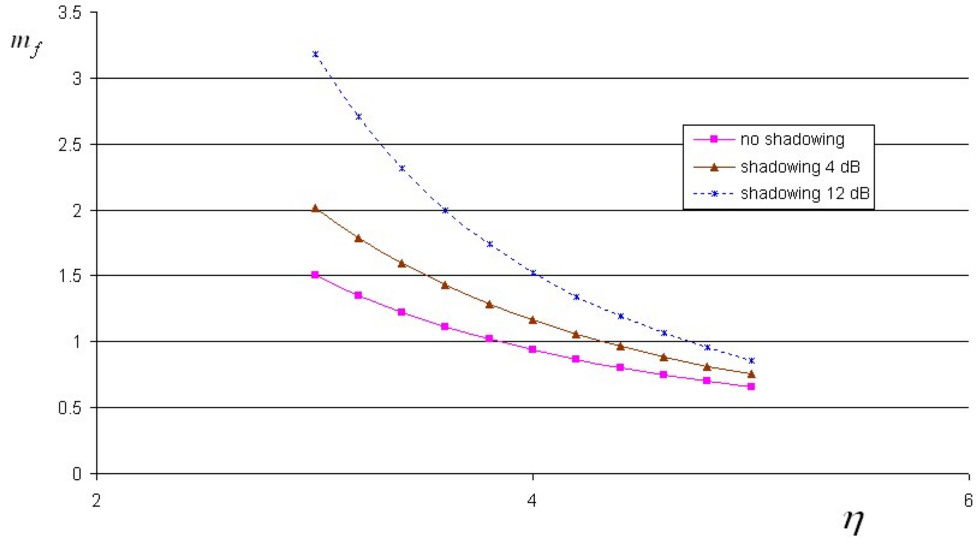

Figure 8.3: Mean interference factor vs η.

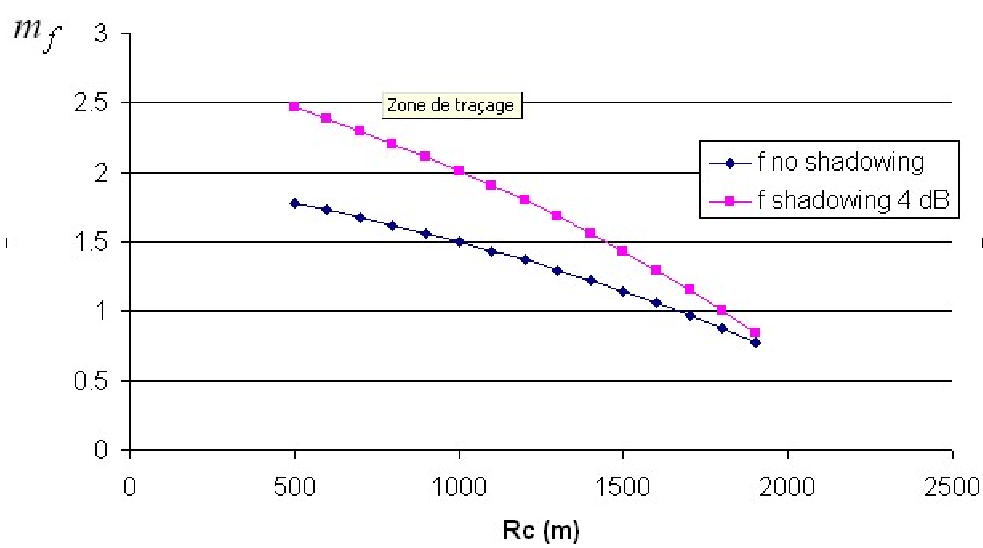

Figure 8.4: Mean Interference factor for Identical relative positions (edge of the cell) and η= 3.

the FW method applied to standard deviations up to 12 dB can be better, as long as there is a correlation between the identical distributed RV. In the next section, we analyze the case of correlated signals.

8.4 Correlation case

8.4.1 Interference factor

Until now we considered non correlated received powers coming from all the base stations of the network. The shadowing depends on the environment and in a real network several BS have analogue environments. Moreover, the powers coming from base stations located at the same relative direction meet the same obstacles. It thus appears natural to consider some correlations between the signals received by a mobile. In [AbuBe01] the coefficient t_{kj} characterizes the correlation between the lognormal RV:

$$t_{kj} = \frac{E\left[(X_k - m_k)(X_j - m_j)\right]}{\sigma_k \sigma_j}, \tag{8.19}$$

where X_i is a RV with mean m_i and variance σ_i^2.

The mean value M_{corr} of the sum of lognormal RV is thus calculated in [AbuBe01] as follows. Denoting

$$u_1 = \sum_{k=1}^{N} exp(m_k + \frac{\sigma_k^2}{2}) \tag{8.20}$$

and

$$u_2 = \sum_{k=1}^{N} exp(2m_k + 2\sigma_k^2) + 2\sum_{k=1}^{N-1}\sum_{j=1}^{N} exp(m_k + m_j) exp\frac{1}{2}(\sigma_k^2 + \sigma_j^2 + 2t_{kj}\sigma_k\sigma_j) \tag{8.21}$$

M_{corr} is thus written as:

$$M_{corr} = 2\ln u_1 - \frac{1}{2} lnu_2 \tag{8.22}$$

That expression can be written as:

$$M_{corr} = \ln u_1 - \frac{1}{2}\frac{\ln u_2}{u_1^2} \tag{8.23}$$

We consider all the BS of the network except the serving one: $N = B - 1$. Let's consider that all the RV have the same variance: $\forall j \sigma_j = \sigma$. We moreover introduce a mean value of the correlation coefficient $t = \bar{t_{kj}}$. Taking into account a correlation between the RV, the expression (8.7) of the mean interference factor becomes (see appendix C):

$$H_{corr}(\sigma) = exp\left(a^2(1-t)\sigma^2/2\right)\left(G(\eta)(exp(a^2(1-t)\sigma^2) - 1) + 1\right)^{\frac{-1}{2}}. \tag{8.24}$$

The mean interference factor is given by (see appendix C):

$$m_{f,corr} = f(\eta)H_{corr}(\sigma) \tag{8.25}$$

and the variance is given by

$$a^2\sigma^2_{f,corr} = a^2\sigma^2(1+t) + \ln\left(G(\eta)(exp(a^2(1-t)\sigma^2) - 1) + 1\right) \tag{8.26}$$

These expressions show that when the powers received by a mobile are completely correlated, $t = 1$, the mean value of the interference factor *does not depend on the shadowing*. Introducing a generalized variance $\sigma_g^2 = \sigma^2(1-t)$ we write (8.24) as:

$$H_{corr}(\sigma_g) = exp\left(a^2\sigma_g^2/2\right)\left(G(\eta)(exp(a^2\sigma_g^2) - 1) + 1\right)^{\frac{-1}{2}} \tag{8.27}$$

The expression (8.25) is analogue to the one (8.8) without correlation, replacing σ by σ_g. The correlation effect is to decrease the standard deviation σ_j. This can explain that the accuracy of the Fenton-Wilkinson (FW) approximation, which is better for low standard deviations, can be extended for standard deviations until 12 dB, as long as they are correlated. As a consequence, the results established in the precedent section stay accurate for high standard deviations as long as the signals are correlated. Figure (8.5) shows that the mean value of the interference factor is comprised between two limits: a high one and a low one corresponding to the two limit cases, i.e. no correlation (t = 0) and total correlation (t= 1) between the powers received from the base stations. That last figure (8.5) shows moreover that for correlations lower than 0.5, the standard deviation has a very low influence on the mean value fo f. Figure (8.6) shows explicitly the correlation coefficient t has almost no influence until 0.6, on the mean value of the interference factor, for a mobile at the edge of the cell (1000m) and $\sigma = 12dB$. For higher values of t the mean interference factor decrease reaches 50%: from 3 to 1.5.

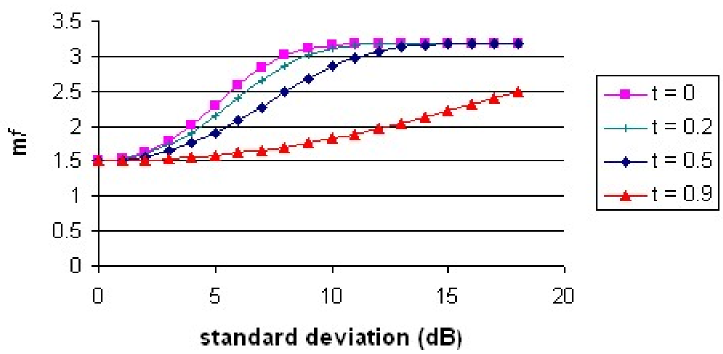

Figure 8.5: Mean interference factor vs σ_j with correlation.

8.4.2 Interference factor extrema

The expressions (8.25) and (8.24) enable to determine analytical extreme values (min and max) for the mean interference factor. Considering two correlations factors t_{min} and t_{max} which characterize the minimum and the maximum correlation between the powers, we can write, introducing $\sigma^2_{g,min} = \sigma^2_g(1 - t_{max})$ and $\sigma^2_{g,max} = \sigma^2_g(1 - t_{min})$

Denoting moreover

$$H_{corr}^{min}(\sigma_g) = exp\left(\frac{a^2\sigma_{g,min}^2}{2}\right)\left(G(\eta)(exp(a^2\sigma_{g,min}^2) - 1) + 1\right)^{\frac{-1}{2}} \tag{8.28}$$

and

$$H_{corr}^{max}(\sigma_g) = exp\left(\frac{a^2\sigma_{g,max}^2}{2}\right)\left(G(\eta)(exp(a^2\sigma_{g,max}^2) - 1) + 1\right)^{\frac{-1}{2}}, \tag{8.29}$$

we can write

$$f(\eta)H_{corr}^{min}(\sigma_g) \leq m_{f,corr} \leq f(\eta)H_{corr}^{max}(\sigma_g). \tag{8.30}$$

When $t_{min} = 0$ (no correlation) and $t_{max} = 1$, we have $\sigma_{g,min}^2 = 0$ and $\sigma_{g,max}^2 = \sigma^2$

And since $0 < \sigma < \infty$ we can write

$$f(\eta) \leq m_{f,corr} \leq \frac{f(\eta)}{G(\eta)^{\frac{1}{2}}} \tag{8.31}$$

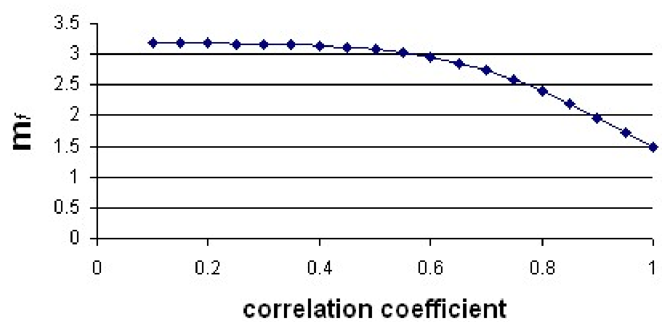

Figure 8.6: Mean Interference factor vs correlation coefficient $\sigma_j = 12dB$.

Remark

We showed (see 4.6) the fluid model approach allows to analyze the admission control of mobiles in a CDMA network using the interference factor expression. It becomes possible to analyze the admission control taking into account the mobile location and each specific environment. Moreover, in our model, the expression of $f(r)$ is characterized by the network size R_{nw}. It thus can be adapted and applied to each part of a network with its own characteristics. Indeed, the parameter η represents the mean exponential pathloss parameter in a given zone and can be specific for each zone of the network. As a consequence, the use of our analytical model enables to establish analytical admissions control expressions related to each specific network zone.

8.5 Concluding remarks

The fluid model of cellular radio networks is now useful to each specific environment characterized by the radio propagation, the shadowing and the network's configuration. We first established the interference factor's analytical expression, mean m_f and standard deviation σ_f, as a lognormal random variable RV, for independent and correlated signals. We expressed them considering the fluid model approach, and showed they depend on a function of the shadowing $H(\sigma_{BS})$ and a form factor G. This last one depends on the mobile's location, the exponential pathloss parameter, the number and the positions of the base stations. We showed a consequence of the shadowing is to increase m_f and f, and these last ones are limited for each specific network. We analyzed different environmental parameters influences, and show some of them may limit the shadowing effects: the form factor, the exponential pathloss parameter, the cell radius. We moreover established the analytical expressions of the mean interference factor bounds, minimum and maximum. We noticed that a mobile admission analysis based on the fluid model depends on the mobile's location and can be adapted to each specific network's zone or environment.

Chapter 9

Fluid Model for Uplink

We proceed in a same way as the one flollowed in the chapters 2 and 3. We show that an uplink analysis can follow an analogue way as the one done for the downlink. We define an uplink interference factor and, considering a fluid model of the network, establish and validate an explicit formula of this parameter. As an application, we propose an analytical admission control study for the two links, which takes into account the whole network around a given cell, and show that it is sufficient to do the analysis only for one link.

9.1 Introduction

The signals and interferences received by the mobiles (MS) and the base stations (BS) of a cellular radio network depend on their transmission powers, positions and numbers. For the downlink, the radio power received by a mobile comes from all the base stations of the network, and for the uplink, the radio power received by a BS comes from all the mobiles of the network. We extend the fluid network model by considering the discrete entities of the network, BS and MS, as continuum. Though focused on CDMA systems, the analysis is still valid for radio networks as OFDMA or TDMA ones.

The chapter is organized as follows. Expressing the target SINR constraints of a CDMA network, we first introduce a specific **uplink interfer-**

ence factor. Assuming the mobiles stations MS as a continuum set, we develop the fluid model, and establish the analytical expression of the uplink interference factor. We validate this approach comparing it to a computed hexagonal network. As an application of this model, we analyze the call admission control, and assuming some hypothesis, we show that it is sufficient to do only one link analysis.

9.2 Interference Model and Notations

In this section, we introduce the interference model and give the notations used throughout the chapter.

9.2.1 Network

We consider a CDMA system with B base stations (BS), each one defining a cell j, and U mobile stations (MS) and we focus on the uplink. If a mobile u is attached to a base station b (or serving BS), we write $b = \psi(u)$. The location of a base station is, as usual, called a site, and we assume omni-directionnal antennas, so that a base station covers a single cell.

9.2.2 Power

The following power quantities are considered:

- $P_{u,b}$ is the useful transmitted power from mobile station u towards base station b;
- $p_{u,b}$ is the power received at base station b from mobile u ; we can write $p_{u,b} = P_{u,b} g_{u,b}$;
- $P_{int,b}$ is the total power received by the base station b due to all the mobiles belonging to the considered cell b;

- $P_{int,b} - p_{u,b}$ represents the *intracellular interferences* due to the other mobiles of the cell b;
- $P_{ext,b}$ is the total power received by the base station b due to all the mobiles belonging to the other cells of the network;
- $P_{i,j}$ is the power transmitted by the mobile i belonging to the cell j defined by the base station j
- M_j is the number of mobiles in the cell j.
- $g_{u,b}$ is the pathloss between the receptor b and the transmitter u.
- N_0^b stands for the level of noise floor at the base station.

9.2.3 Noise and Interferences

The total amount of power experienced by a base station b in a cellular system can be split up into several terms: useful signal ($p_{u,b}$), interference and noise (N_0^b). It is common to split the system power into two terms: $P_{int,b}$ and $P_{ext,b}$, where $P_{int,b}$ is the *internal* (or own-cell) received power and $P_{ext,b}$ is the *external* power (or other-cell interference). Notice that we made the choice of including the useful signal $p_{u,b}$ in $P_{int,b}$, and, as a consequence, it has to be distinguished from the commonly considered own-cell interference.

With the above notations, we define the **uplink interference factor** in b, as the ratio of total power received by b, coming from the mobiles belonging to the other BS of the network, to the total power coming from the mobiles belonging to the cell b.

$$f_b^{UL} = P_{ext,b}/P_{int,b} \tag{9.1}$$

The parameter f_b^{UL} can be written as follows:

$$f_b^{UL} = \frac{\sum_{j=1,j\neq b}^{B} \sum_{i=1}^{M_j} P_{i,j} g_{i,b}}{\sum_{k=1}^{M_b} P_{k,b} g_{k,b}} \tag{9.2}$$

This parameter represents the relative 'weight', on the base station b, of the mobiles belonging to the other base stations of the network, to the ones belonging to the base station b.

9.2.4 Basic Derivations

Let us consider a mobile connected to the base station b of the network. The Signal to Interference plus Noise ratio (SINR) received by a receiver, MS in downlink and BS in uplink, has to be at least equal to a minimum threshold target value, for the two links. Proceeding in an analogue way as the downlink one, we denote $\gamma^*_{u,UL}$ the Uplink target SINR for the service requested by MS u. We express the SINR experimented by b as:

$$\gamma^*_{u,UL} = \frac{p_{u,b}}{P_{int,b} - p_{u,b} + P_{ext,b} + N_0^b}. \tag{9.3}$$

We denote:

$$\delta_u = \frac{\gamma^*_{u,UL}}{1 + \gamma^*_{u,UL}}. \tag{9.4}$$

From the relation (9.3), and using the relation $p_{u,b} = P_{u,b} g_{u,b}$ we can express $p_{u,b}$ as:

$$p_{u,b} = \delta_u (P_{int,b} + f_b^{UL} P_{int,b} + N_0^b). \tag{9.5}$$

For the *Downlink*, we established from (2.8) the useful power received by a mobile u as:

$$P_{b,u} g_{b,u} = \beta_u (\alpha P_b g_{b,u} + f_u P_b g_{b,u} + N_0). \tag{9.6}$$

The analogy of the expressions (9.5) established for the uplink and (9.6) established for the downlink are due to the analogy of the conditions (9.3) and (2.5) concerning the targets SINR in the two directions. As a consequence, it appears natural to develop a **unique analysis for modelling the two links**. We thus extend the fluid model to the uplink. We moreover notice that for an isolated cell the uplink interference factor is equal to zero, and when the Noise N_0^b is low compared to the signal received by the base station, the expression (9.5) becomes $p_{u,b} = \delta_u P_{int,b}$.

The uplink and downlink interference factors show the influence of the network, especially the locations and the power transmissions of the base stations and the mobiles, on a receiver, mobile or base station, on a given cell of the network.

9.3 Uplink Fluid Model Network

9.3.1 Assumptions

Following an analogue approach as the downlink one, the key modelling step of the model we propose consists in replacing a given fixed finite number of transmitters (base stations or mobiles) by an equivalent continuum of transmitters which are distributed according to some distribution function. We consider a traffic characterised by a mobile density ρ_{ms} and a network by a base station density ρ_{bs}.
We consider a network constituted of omni-directional cells uniformly distributed and a uniform traffic: ρ_{bs} and ρ_{ms} are constant. We aim at calculating the uplink interference factor f_b^{UL} at a given BS b of this network. We use the same pathloss model used in Section 3.3: $g_{u,b}$ is only a function of the distance r between the base station b and a mobile u. We consider that the power received by a base station follows the expression $p_{u,b} = P_{u,b} a r^{-\eta}$ where a is a constant.

9.3.2 Uplink fluid interference factor

Considering a given number of mobiles located in the network according to some distribution, the power $P_{int,b}$ and $P_{ext,b}$ received by the base station b are constant. We deduce from (9.5) that the power $p_{u,b}$ received by the base station b coming from the mobile u is a constant whatever u. In uplink, according to the pathloss model only depending on the distance r between a mobile and a base station, we deduce from (9.5) that each mobile u of a cell has a transmission power satisfying: $P(r,\theta) = Pr^{\eta}$, independent of the angle θ, where P is a constant. We also can write it $P(r,\theta) = P(r)$. In the fluid model we develop, we can write the power received by the BS b due to all the mobiles of the network, except the ones located in the cell b, as:

$$P_{ext,b} = \int_{R_c}^{R_{nw}} \int_0^{2\pi} \rho_{ms} P(r) r^{-\eta} r dr d\theta, \tag{9.7}$$

where R_{nw} represents the network size, $P(r)$ the transmitting power of any mobile located at the distance r from its serving BS.

The total power $P_{int,b}$ received by the base station b, emitted by all the mobiles of the cell, can be calculated as:

$$P_{int,b} = \int_0^{R_c} \int_0^{2\pi} \rho_{ms} P r^{\eta} r^{-\eta} r dr d\theta \tag{9.8}$$

We can write

$$P_{int,b} = \rho_{ms} \pi R_c^2 P. \tag{9.9}$$

Since the number of mobiles n_{MS}^{UL} of the cell can be written

$$n_{MS}^{UL} = \rho_{ms} \pi R_c^2, \tag{9.10}$$

so we have

$$P_{int,b} = n_{MS}^{UL} P. \tag{9.11}$$

To calculate the expression (9.7) of the power due to all the mobiles of the other cells of the network, we consider that the network is constituted by rings of interfering cells around the one we consider. The first ring of cells is located between the distances R_c and $3R_c$. The second ring of cells is located between the distances $3R_c$ and $5R_c$, and so one. Expressing the power received by the BS b coming from the mobiles of the cells located at distances r between $(n-1)R_c$ and $(n+1)R_c$, we write the contribution of the n^{th} ring of base stations around the cell b as

$$
\begin{aligned}
P_{n,ext} &= \int_{(2n-1)R_c}^{2nR_c}\int_0^{2\pi}\rho_{ms}P(2nR_c-r)^{\eta}r^{-\eta}rdrd\theta \\
&+ \int_{2nR_c}^{(2n+1)R_c}\int_0^{2\pi}\rho_{ms}P(-2nR_c+r)^{\eta}r^{-\eta}rdrd\theta. \qquad (9.12)
\end{aligned}
$$

For the first integral, we denote $x = 1 - \frac{r}{2nR_c}$ and for the second one we denote $x = -1 + \frac{r}{2nR_c}$ so we obtain

$$
\begin{aligned}
P_{n,ext} &= (2nR_c)^2\rho_{ms}P\int_0^{\frac{1}{2n}}\int_0^{2\pi}x^{\eta}(1-x)^{-\eta+1}dxd\theta \\
&+ (2nR_c)^2\rho_{ms}P\int_0^{\frac{1}{2n}}\int_0^{2\pi}x^{\eta}(1+x)^{-\eta+1}dxd\theta. \qquad (9.13)
\end{aligned}
$$

The network size is expressed as $R_{nw} = (2N_c+1)R_c$, where N_c represents the number of rings of cells. The total power of the network (except the cell b) is thus given by $P_{ext,b} = \sum_{n=1}^{N_c} P_{n,ext}$, so using (9.10) and (9.1) we can deduce:

$$
f^{UL} = 2\sum_{n=1}^{N_c}(2n)^2\int_0^{\frac{1}{2n}}x^{\eta}\left((1+x)^{-\eta+1}+(1-x)^{-\eta+1}\right)dx. \qquad (9.14)
$$

Using a fluid model approach we established an analytical expression of the uplink interference factor. In a homogeneous case, that expression does not depend on the density of mobiles. Moreover the expression of f_b^{UL} does not explicitly take into account the size of the cell: It only depends on the size of the network characterized by the parameter N_c.

9.4 Validation

We build a numerical hexagonal network (figure 9.1: Each point represents a BS). We calculate numerically for a given BS, the uplink interference factor. The base stations are omni-directional.

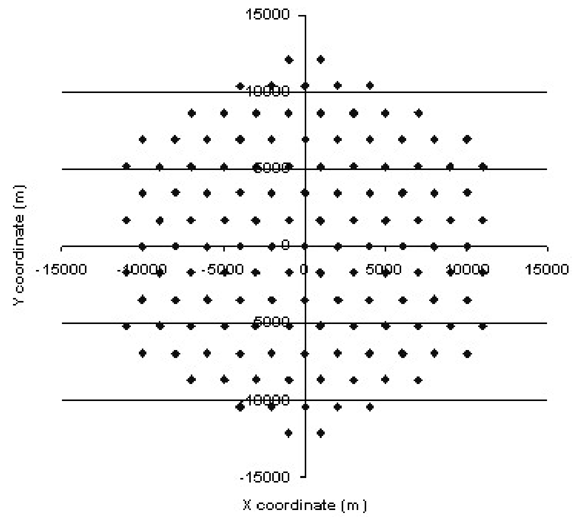

Figure 9.1: Numerical hexagonal network.

The validation of the model will consist in comparing the analytical values of the uplink interference factor f_b^{UL} at a given station b, to the ones obtained numerically with the hexagonal network. We choose the pathloss parameter $\eta = 3$, and a cell radius of 1 km. We allocate to each mobile, a transmitting power depending on its distance from its own serving BS.

We calculate numerically the interference factor at a base station b, using the expression (9.2), considering three network sizes. We compare it to the one obtained analytically with the fluid model using the equation (9.14). Table 9.1 shows that the values obtained with the two methods are very close, whatever the dimensions of the network.

9.5 Application: Admission Control

We consider only one service so that the parameters β_u and δ_u do not depend on the mobile: we can drop the index u. The demand of entry of a mobile in the network is rejected when the uplink target SINR constraint is not satisfied. Considering the uplink power constraint (9.5), and using (9.10)

$\frac{R_c}{R_{nw}}$	$Numerical$	$Fluid$
0.1	0.64	0.61
0.2	0.60	0.55
0.3	0.49	0.48

Table 9.1: Uplink interference factor comparison

and (9.11), the admission is based on the probability P^{UL} to satisfy:

$$P^{UL} = Pr\left(P - \delta n_{MS}(1 + f_b^{UL}) < \delta N_0\right). \tag{9.15}$$

and can be written

$$P^{UL} = Pr\left(n_{MS} > \frac{1}{\delta(1 + f_b^{UL})} - \frac{N_0^b}{P(1 + f_b^{UL})}\right). \tag{9.16}$$

For the downlink, we express that the power P_b of the base station is limited to a maximum value P_{max}. The admission is based on the probability to satisfy the following expression (see 7.15):

$$P^{DL} = Pr\left(P_b > P_{max}\right). \tag{9.17}$$

where

$$P_b = \frac{\sum_u \beta_u \frac{N_0}{g_{b,u}}}{1 - \varphi - \sum_u \beta(\alpha + f_u)}. \tag{9.18}$$

Considering the expression 4.20 we write $F^{DL} = F$, so we denote:

$$n_{MS}^{UL,th} = \frac{1}{\delta(1 + f_b^{UL})} \tag{9.19}$$

and

$$n_{MS}^{DL,th} = \frac{1}{\beta(\alpha + F^{DL})}. \tag{9.20}$$

The admission control for the two links have similar analytical expressions, as long as the Noise (N_0 and N_0^b) can be considered as low:

$$P^{UL} = Pr\left(n_{MS} > n_{MS}^{UL,th}\right), \tag{9.21}$$

and

$$P^{DL} = Pr\left(n_{MS} > n_{MS}^{DL,th}\right). \tag{9.22}$$

A numerical analysis with $\eta = 3$ presented hereafter in the Table 9.2 shows that, although their analytical forms are not exactly identical, f_b^{UL} and F^{DL} have very close values whatever the network size.

N_c	f^{UL}	F^{DL}
1	0.47	0.32
2	0.55	0.50
3	0.58	0.56
4	0.60	0.59
5	0.61	0.62
7	0.62	0.64
10	0.63	0.65
20	0.65	0.67

Table 9.2: Uplink and Downlink interference factor comparison for $\eta = 3$.

There is indeed a priori no reason to do different analysis for the two links, since the constraints have similar analytical expressions for the two directions. In term of interferences, the mobiles play in uplink, a similar role as the base stations in downlink. The fluid model we developed takes into account that analogy. We can also notice that f_b^{UL} can be interpreted as an *average* uplink interference factor of the cell. It is compared to the average downlink interference factor of the cell. If moreover $\varphi = 0$, $\alpha = 1$ and the target SINR is the same in uplink and downlink, we can write $\beta = \delta$.

For $\eta = 3$ and if the noise is negligible, we can deduce from Table 9.2 that the admission control conditions are very close for the two links. These results mean that our fluid model represents a good approach to calculate the interference factors of a CDMA network. The explicit analytical expressions given by this model allow easy analysis. At last, it seems sufficient in this case, to analyze only a unique link, the downlink, since the expression of $n_{MS}^{DL,th}$ takes into account the parameters φ and α.

9.6 Concluding remarks

The analogy of the target SINR constraints, in a CDMA network, for the uplink and the downlink, enabled us to develop a single analysis for modeling the two links. We showed that the uplink and the downlink interference factors, which represent the "weight" of the network on a given cell, are a characterization of CDMA networks. We developed and validated an analytical fluid model, replacing the discrete BS by a base station density and the mobiles by a mobile density. We established the interference factor's analytical expressions for each link. For a homogeneous network, these expressions do not depend on the density of mobiles. As an application, we showed, with some assumptions, that the admission conditions have very close values for the two links. For $\eta = 3$ and a negligible noise, it appeared sufficient to analyze only the downlink.

Chapter 10

Conclusion

10.1 Fluid model

The goal of our approach was to propose an analytical model characterizing a cellular radio network. This model had to be accurate in taking into account the distance of a mobile to its serving base station, and still simple enough to lead to closed form formulas. We moreover needed a not over-simplifying model, otherwise it could have resulted large inaccuracies.

Considering the base stations as a continuum, we proposed a spatial fluid model of cellular networks that allows to simplify considerably their analysis and the computation complexity needed to obtain accurate results. We first defined a parameter, the interference factor f, which well characterizes cellular networks.

Though the interference factor is generally defined as the ratio of other-cell interference to inner-cell interference. we the define interference factor as the ratio of total other-cell received power to the total inner-cell received power. This definition is interesting firstly since total received power is the metric that mobile stations (MS) are really able to measure on the field. Moreover, f represents now a characteristic of the network and does not depend on the considered MS or service. Finally, the definition of f is valid for cellular radio systems without inner-cell interference.

We established an analytical expression of f, simple and easy to calculate. The interference factor depends on the network size R_{nw}, the distance between neighbor base stations, and exponential pathloss parameters.

We validated the fluid model approach comparing it to a hexagonal simulated network. Though the simplicity of formula we established, we showed its high accuracy whatever the network parameters we considered: We especially established the accuracy of the fluid model for wide ranges of distance between neighbor base stations, *i.e.* even for very low base stations densities, wide ranges of exponential pathloss parameters and wide ranges of size networks. Though mainly focused on CDMA networks, our approach is still valid for other systems, like OFDMA (WiMAX), TDMA (GSM) or even ad hoc networks.

As a **first refinement**, we extended our analysis to a sectored network. Denoting $f_{omni}(r)$ the interference factor f for an omni-directional BS network, and $f_{sect}(r, \theta)$ its expression for a sectored BS network, we established their analytical expressions considering a sector fluid model network. We particularly showed that the interference factor of sectored network with q sectors per site can be expressed as a linear function of $f_{omni}(r)$.

As a **second refinement**, we analyzed the impact of the shadowing. We established the analytical expression of f as a lognormal random variable RV, for independent and correlated reeived signals. We expressed the mean m_f and the standard deviation σ_f of f considering the fluid network's approach, and showed they depend on a function of the shadowing $H(\sigma)$ and a form factor $G(\eta)$. This last one depends on the topology of the system: mobile's location, exponential pathloss parameter, the number and the positions of the base stations. We moreover showed a consequence of the shadowing is to increase m_f and σ_f, and these last ones are limited for each specific network. We analyzed different environmental parameters influences, and showed some of them may limit the shadowing effects: the form factor, the exponential pathloss parameter, the cell radius. We moreover established the analytical expressions of the mean interference factor bounds, minimum and maximum. The cellular network fluid model we developed is adapted to each specific local or global environment characterized by the radio propagation, the shadowing and the network's configuration: The network size R_{nw} to be considered can be chosen characterizing a typical environment (pathloss exponent, urban, rural, macro or micro cells). It allows calculating the influence of a mobile on a given cell, whatever its position.

As a **third refinement**, the analogy of the SINR constraints for the uplink and the downlink drived us to develop a single analysis for modeling the two links. We developed and validated an uplink fluid model, replacing the discrete BS by a base station density and the mobiles MS by a mobile density. We established the interference factor's analytical expressions for each link. For an homogeneous network, these expressions do not depend on the density of mobiles.

10.2 Applications

The fluid model opens great number analyses possibilities. We showed the fluid model can be used for different analysis of cellular networks such as quality of service, dimensioning, mobile management, admission control... We particularly focused on three among them: the capacity of a cell and a network, the multiservice admission and the blocking probability

Capacity.
The fluid model allowed us to analyze the capacity of a cell and a network. It thus can be used in the planning and dimensioning process. We instantly obtained explicit expressions of the capacity. We moreover studied the solution based on a network's densification to answer an increasing traffic.

Transmitting power limitation.
Focusing on the influence of the thermal Noise and the BS transmitting power, we particularly established that for high density networks, the increase of base station transmitting powers does not necessarily increase the capacity of a network. as a consesuence, in great number of cases, a provider may

limit the maximum transmitting power of the base stations, .

Admission control policy.
Since our fluid model enables to know the influence of a mobile on a given zone of a network whatever its position, we showed (Section 4.6) it can also be used to analyze admission control policies.

We instantly obtained explicit expressions of the admission conditions in a CDMA network. We analyzed the solutions based on a densification or a sectorisation to answer an increasing traffic. We obtained analytical expressions of the capacity: it theoretically appears better to densify a network than to sectorize it. We moreover showed the importance of the choice of an admission control strategy.

We analyzed the shadowing influence, and more generally, and showed the fluid model can be used to analyze to each specific network's zone or environment.

We concluded that the fluid model is a powerful tool to study admission control in CDMA networks and design fine algorithms taking into account the distance to the BS.

Admission multiservice.
We analyzed the performance of call admission control combined with GoS control in a WCDMA environment with integrated RT and NRT traffic. As performance measures, we studied the blocking rate of RT traffic and the sojourn times of NRT traffic. We illustrated through numerical examples the importance of adding reserved capacity L_{NRT} for NRT traffic and demonstrated that this reservation can be done in a way not to significantly affect RT traffic. More specifically, we saw that the blocking rate of RT traffic was small and quite robust to the choice of L_{NRT}, over a large interval of values. For NRT traffic, we investigated the average sojourn time and the conditional expected sojourn time given the file size and the number of RT and NRT mobiles present at the cell upon arrival.

Outage probability.
We showed the simplicity of the fluid model allows a spatial integration of

f leading to closed-form formula for the global outage probability and for the spatial outage probability. This last one expresses the probability for a mobile to enter the cell, at a given distance from its serving BS.

10.3 Future work

Since the fluid model model allows to know the influence of any mobile entering a radio system, we aim to explore different kind of problems, such as:

- the influence of mobiles mobility on the performances of a radio system
- different kind of scheduling analysis (see for example [KeA02])
- analyses of OFDMA based systems: dimensioning, performances, optimisation...

Appendix A

Interference Power

Each BS transmits a power $P_j = P$ so the power received by a mobile is characterized by a lognormal distribution X_j as $ln(X_j) \propto N(am_j, a^2\sigma_j^2)$ and we can write $m_j = \frac{1}{a}ln(P_j r_j^{-\eta})$ (we aim to finally calulate a ratio of powers so (in appendix B) to simplify we assume $K = 1$). So the total power received by a mobile is a lognormal RV X characterized by its mean and variance $ln(X) \propto N(am, a^2s^2)$ and we can write:

$$am = ln\left(\sum_{j=1,j\neq b}^{B} exp(lnP_j - \eta ln r_j) + \frac{a^2\sigma_j^2}{2}\right) - \frac{a^2\sigma^2}{2} \tag{A.1}$$

so we have since $P_j = P$ whatever the base station j:

$$am = (\ln P + \frac{a^2\sigma_j^2}{2}) + ln(\sum_{j=1,j\neq b}^{B} exp(-\eta\ln(r_j))+) - \frac{a^2s^2}{2} \tag{A.2}$$

So we can express the mean interference power $\overline{P_{ext}}$ received by a mobile coming from all the other base stations of the network as:

$$\ln(\overline{P_{ext}}) = (\ln P + \frac{a^2\sigma_j^2}{2}) + ln(\sum_{j=1,j\neq b}^{B} r_j^{-\eta}) - \frac{a^2s^2}{2} \tag{A.3}$$

and the variance a^2s^2 of the sum of interferences is written as

$$a^2 s^2 = ln\left(\frac{\sum_{j=1,j\neq b}^{B} exp(2am_j + a^2\sigma_j^2)(exp(a^2\sigma_j^2) - 1)}{\sum_{j=1,j\neq b}^{B} exp(am_j + \frac{a^2\sigma_j^2}{2})^2} + 1\right) \quad \text{(A.4)}$$

Introducing

$$G(\eta) = \frac{\sum_{j=1,j\neq b}^{B} r_j^{-2\eta}}{\left(\sum_{j=1,j\neq b}^{B} r_j^{-\eta}\right)^2} \quad \text{(A.5)}$$

the mean value of the total interference received by a mobile is given by (considering identical σ_j denoted σ):

$$\overline{P_{ext}} = P \sum_{j=1,j\neq b}^{B} r_j^{-\eta} exp\left(\frac{a^2\sigma^2}{2}\right)\Big((exp(a^2\sigma^2) - 1)G(\eta) + 1\Big)^{-1/2} \quad \text{(A.6)}$$

and

$$a^2 S^2 = ln\Big((exp(a^2\sigma^2) - 1)G(\eta) + 1\Big) + ln\Big(exp(a^2\sigma^2)\Big) \quad \text{(A.7)}$$

Appendix B

Interference factor

Since the ratio of two lognormal RV's is also a lognormal RV, the interference factor is also lognormally distributed with the following mean and logarithmic variance:

$$m_f = \frac{\overline{P_{ext}}}{\overline{P_{int}}} \tag{B.1}$$

and thus, if we consider that all the base stations have the same transmitting power: $P_b = P$, we can write, dropping the index b:

$$m_f = \frac{\sum_j r_j^{-\eta}}{r^{-\eta}} exp\left(\frac{a^2\sigma_j^2}{2}\right)\Big((exp(a^2\sigma_j^2) - 1)G(\eta) + 1\Big)^{-1/2} \tag{B.2}$$

and finally, considering identical σ_j and denoting:

$$H(\sigma) = exp\left(\frac{a^2\sigma^2}{2}\right)\Big((exp(a^2\sigma^2) - 1)G(\eta) + 1\Big)^{-1/2} \tag{B.3}$$

we have

$$m_f = f(\eta) exp\left(\frac{a^2\sigma^2}{2}\right)\Big((exp(a^2\sigma^2) - 1)G(\eta) + 1\Big)^{-1/2} \tag{B.4}$$

In a analogue analysis, the standard deviation is given by $a^2\sigma_f^2 = a^2S^2 + a^2\sigma^2$ so we have

$$a^2\sigma_f^2 = 2(a^2\sigma^2 - ln(H(\sigma))) \tag{B.5}$$

Appendix C

Interference factor for correlated powers

In [AbuBe01], the coefficient t_{kj} is introduced to take into account the correlation between the lognormal RV:

$$t_{kj} = \frac{E\left[(X_k - m_k)(X_j - m_j)\right]}{\sigma_k \sigma_j}. \tag{C.1}$$

where X_i is a RV with mean m_i and variance σ_i^2.

The mean value M_{corr} of the sum of lognormal RV is thus calculated in [AbuBe01] as follows.

Denoting

$$u_1 = \sum_{k=1}^{N} exp(m_k + \frac{\sigma_k^2}{2}) \tag{C.2}$$

and

$$\begin{aligned} u_2 \quad = \quad & \sum_{k=1}^{N} exp(2m_k + 2\sigma_k^2) \\ & + 2\sum_{k=1}^{N-1} \sum_{j=k+1}^{N} exp(m_k + m_j) exp\frac{1}{2}(\sigma_k^2 + \sigma_j^2 + 2t_{kj}\sigma_k\sigma_j) \end{aligned} \tag{C.3}$$

the mean value M_{corr} of the sum of N lognormal RV is thus calculated in [AbuBe01] as:

$$M_{corr} = 2\ln u_1 - \frac{1}{2}\ln u_2 \tag{C.4}$$

That expression can be written as:

$$M_{corr} = lnu_1 - \frac{1}{2}\frac{lnu_2}{u_1^2} \tag{C.5}$$

Let's consider that all the RV have the same variance σ. We moreover introduce a mean value of the correlation coefficient $t = \overline{t_{kj}}$. We can write

$$\begin{aligned}
&2\sum_{k=1}^{N-1}\sum_{j=k+1}^{N} exp(m_k + m_j)exp\frac{1}{2}(\sigma_k^2 + \sigma_j^2 + 2t_{kj}\sigma_k\sigma_j) \\
&= 2\sum_{k=1}^{N-1}\sum_{j=k+1}^{N} exp(m_k + m_j)exp(\sigma^2 + t\sigma^2) \\
&= \left(\frac{1}{2}\sum_{j=1}^{N} exp(m_j + \sigma^2/2) + \frac{1}{2}\sum_{k=1}^{N} exp(m_k + \sigma^2/2)\right)^2 exp(t\sigma^2) \\
&-\frac{1}{2}\left(\sum_{j=1}^{N} exp(2m_j + \sigma^2) + \sum_{k=1}^{N} exp(2m_k + \sigma^2)\right) exp(t\sigma^2)
\end{aligned} \tag{C.6}$$

We notice that

$\sum_{j=1}^{N} exp(m_j + \sigma^2/2) = \sum_{k=1}^{N} exp(m_k + \sigma^2/2)$ and

$\sum_{k=1}^{N} exp(2m_k + \sigma^2) = \frac{1}{2}\left(\sum_{j=1}^{N} exp(2m_j + \sigma^2) + \sum_{k=1}^{N} exp(2m_k + \sigma^2)\right)$

so we have

$$\begin{aligned}
&2\sum_{k=1}^{N-1}\sum_{j=k+1}^{N} exp(m_k + m_j)exp(\sigma^2 + t\sigma^2) \\
&= \left(\sum_{j=1}^{N} exp(m_j + \sigma^2/2)\right)^2 exp(t\sigma^2) \\
&-\left(\sum_{k=1}^{N} exp(2m_k + \sigma^2)\right) exp(t\sigma^2)
\end{aligned} \tag{C.7}$$

so we have (from C.3)

$$\begin{aligned} u_2 &= \left(\sum_{k=1}^{N} exp(2m_k+\sigma^2)\right) exp(\sigma^2) + \left(\sum_{k=1}^{N} exp(m_k+\sigma^2/2)\right)^2 exp(t\sigma^2) \\ &\quad - \left(\sum_{k=1}^{N} exp(2m_k+\sigma^2)\right) exp(t\sigma^2) \\ &= \left(\sum_{k=1}^{N} exp(m_k+\sigma^2/2)\right)^2 exp(t\sigma^2) \\ &\quad + \left(\sum_{k=1}^{N} exp(2m_k+\sigma^2)\right) \left(exp\sigma^2 - exp(t\sigma^2)\right) \end{aligned} \tag{C.8}$$

so we express

$$\ln\frac{u_2}{u_1^2} = ln\left(\left(exp(\sigma^2)-exp(t\sigma^2)\right)\frac{\sum_{k=1}^{N} exp(2m_k+\sigma^2)}{(\sum_{k=1}^{N} exp(m_k+\sigma^2/2))^2} + exp(t\sigma^2)\right) \tag{C.9}$$

This expression is analogue to the one we established for the variance (A.7) of the sum of interferences. So, taking into account a correlation between the RV, the parameter a, and using the expression of M_{corr} (C.5), the expression of the mean value of the interference factor becomes

$$m_{f,corr} = \frac{\sum_j r_j^{-\eta}}{r^{-\eta}} exp\left(\frac{a^2\sigma^2}{2}\right)\left((exp(a^2\sigma^2) - exp(ta^2\sigma^2))G(\eta) + exp(ta^2\sigma^2\right)^{-1/2} \tag{C.10}$$

and finally denoting

$$H_{corr}(\sigma) = exp\left(\frac{a^2\sigma^2}{2}(1-t)\right)\left((exp(a^2\sigma^2)(1-t)-1)G(\eta)+1\right)^{-1/2} \tag{C.11}$$

we can write

$$m_{f,corr} = f(\eta)H_{corr}(\sigma) \tag{C.12}$$

And we have for the variance:

$$a^2\sigma^2_{f,corr} = a^2\sigma^2(1+t) + ln\left((exp(a^2\sigma^2)(1-t)-1)G(\eta)+1\right) \tag{C.13}$$

Appendix D

List of Publications

[1] Jean-Marc Kelif, Multi-service Admission on Sectored CDMA Networks - An analytical model, WCNC March 2007 Hong Kong

[2] Dinesh Kumar Eitan Altman Jean-Marc Kelif Globally Optimal User-Network Association in an 802.11 WLAN & 3G UMTS Hybrid Cell, ITC20 june 2007, Ottawa

[3] Jean Marc Kelif, Eitan Altman, Ioannis Koukoutsidis Admission and GoS control in a multiservice WCDMA system, Computer Networs, 2007

[4] Dinesh Kumar Eitan Altman, Jean-Marc Kelif, User-Network Association in an 802.11 WLAN & 3G UMTS Hybrid Cell: Individual Optimality, Sarnoff Symposium April 2007, Princeton

[5] Kinda Khawam and Jean-Marc Kelif, A Hierarchical Proportional Fair Scheduler, Euro-NGI, Valencia April 2006

[6] Jean-Marc Kelif and Eitan Altman, Admission and GoS control in multiservice WCDMA system, ECUMN04, Porto, Portugal October 2004

[7] Jean-Marc Kelif, Admission Control for Fluid CDMA Networks, WiOpt06 Boston April 2006

[8] Jean-Marc Kelif and Eitan Altman, Downlink Fluid Model for CDMA Network, VTC05 Stockholm Mai 2005

[9] I Koukoutsidis, E. Altman , J-M Kelif, A non Homogeneous QBD approach for the admission control in mutiservice WCDMA system, IWQoS

Passau Juin 2005
[10] I Koukoutsidis, E. Altman , J-M Kelif, Fair rate sharing models in a CDMA link with multiple classes of elastic traffic ITC19 Beijing Septembre 2005
[11] Jean-Marc Kelif, Marceau Coupechoux and Philippe Godlewski, Spatial Outage Probability for Cellular Networks, Globecom 2007
[12] Jean-Marc Kelif, Marceau Coupechoux and Philippe Godlewski, Spatial Outage Formula for CDMA Networks, VTC Fall 2007
[13] Jean-Marc Kelif and Eitan Altman, Cellular network with continuum priority set, Valuetools 2007 Nantes (France)

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