Discrete Power Control: Cooperative and Non-Cooperative Optimization

Eitan Altman*, Konstantin Avrachenkov*, Gregory Miller†, and Balakrishna Prabhu‡

*INRIA Sophia Antipolis, 2004 route des Lucioles, 06902 Sophia Antipolis, France
†Moscow Aviation Institute, 4 Volokolamskoe sh., GSP-3, Moscow A-80, Russia
‡CWI, Kruislaan 413, 1098 SJ Amsterdam, The Netherlands

Eitan.Altman@sophia.inria.fr; K.Avrachenkov@sophia.inria.fr; GMiller@ipiran.ru; B.J.Prabhu@cwi.nl

***Abstract*— We consider an uplink power control problem where each mobile wishes to maximize its throughput (which depends on the transmission powers of all mobiles) but has a constraint on the average power consumption. A finite number of power levels are available to each mobile. The decision of a mobile to select a particular power level may depend on its channel state. We consider two frameworks concerning the state information of the channels of other mobiles: (i) the case of full state information and (ii) the case of local state information. In each of the two frameworks, we consider both cooperative as well as non-cooperative power control. We manage to characterize the structure of equilibria policies and, more generally, of best-response policies in the non-cooperative case. We present an algorithm to compute equilibria policies in the case of two non-cooperative players. Finally, we study the case where a malicious mobile, which also has average power constraints, tries to jam the communication of another mobile. Our results are illustrated and validated through various numerical examples.**

I. Introduction

The multiple access nature of wireless networks represents a fundamentally different resource allocation problem as compared to wired networks which provide a dedicated channel for each user. The shared nature of the wireless channel implies that the rate obtained by a user depends not only on its own transmit power level but also on the transmit power levels of the other users. A user who transmits at a relatively high power level, though may increase its own rate, will interfere with the transmissions of the other users and prompt them to increase their own transmission power. Such a situation is undesirable in wireless networks where mobile devices are usually equipped with limited-lifetime batteries which require judicious utilization. It is, therefore, in the interests of the users to control their transmit powers levels so as to increase the information transfer rate and the lifetime of the devices. Power control also has the added benefit of allowing the spatial reuse of channels, i.e., the same channel can be concurrently used by mobiles at locations where interference is sufficiently low.

In this paper, we consider *dynamic* uplink power control in cellular networks: mobiles choose their transmission power level from a discrete set in a *dynamic* way, i.e., the transmission power level is chosen based on the available channel state information. By controlling the power one can improve connectivity and coverage, spend less battery energy of terminals, increase device lifetime, and maximize the throughput. In terms of decision making, we consider two cases:

- **Decentralized case:** Each mobile chooses its own power level based on the condition of its own radio channel to the base station.
- **Centralized case:** The transmission power levels for all the mobiles are chosen by the base station that has full information on all channel states.

We assume that there are upper bound constraints on the average power that a mobile can use. Thus in very bad channel conditions, one can expect a mobile to avoid transmission and save its power for more favorable channel conditions.

Applications that can mostly benefit from our proposed *decentralized* power control are ad-hoc and sensor networks with no predefined base stations. In such networks, mobiles may have to act temporarily as base stations [1]–[3], which can involve a heavy burden in terms of energy. The limited processing capacity and battery lifetime of devices precludes the use of centralised schemes, thereby making decentralized approaches for power control more appropriate in such networks. We note that the design of decentralized power control has for long interested the networking community even before ad-hoc and sensors networks have been introduced (see [4], [5] and references therein).

We obtain results for both the cooperative setting in which the mobiles' objective is to maximize the global throughput, as well as the non-cooperative case in which the objective of each mobile is to maximize its own transmission rate.

We identify the structure of equilibria policies for the decentralized non-cooperative case. We show that the following structure holds for any mobile i, given any set of policies u^{-i} chosen by mobiles other than i. Any best response policy (i.e. an optimal policy for player i for a given policy u^{-i} other mobiles) has the following properties:

(i) It needs randomization between at most two adjacent power levels,
(ii) the optimal power levels are non-decreasing functions of the channel state, and
(iii) if two power levels are both optimal at a given channel

state then they cannot be jointly optimal for another channel state.

We present an algorithm to compute equilibria policies in the case of two non-cooperative players.

For the cooperative centralized problem with two mobiles, we obtain insight on the structure of optimal policies through a numerical study. An interesting property that we obtain is the fact that the optimal policy has a TDMA structure: in each combined state (x_1, x_2) there is only one mobile that will transmit information. This will of course eliminate the interference. We also show that unlike the decentralized case, the average power level constraints may hold with strict inequality when using the optimal policy.

We finally study the case where a malicious mobile, which also has average power constraints, tries to jam the communications of another mobile. Our results are illustrated and validated through various numerical examples.

A. Related work

There has been an intensive research effort on non-cooperative power control in cellular networks [4], [6]–[13]. In all these work, however, the set of available transmission powers has been assumed to be a whole interval or the whole set of nonnegative real numbers. In this paper we consider the case of a discrete set of available power levels, which is in line with standardized cellular technologies. Very little work on power control has been done on discrete power control. Some examples are [14] who considered the problem of minimizing the sum of powers subject to constraints on the signal to noise ratio, [15] who studied joint power and rate control, and [8] (which we describe in more detail below).

The mathematical formulation of the power control problem shows much similarity with a well studied problem of assigning transmission powers to parallel channels between a mobile and a base station with a constraint on the sum of assigned powers, see e.g. [16, p. 161]. This problem is often known as the "water filling" (which is in fact the structure of the optimal policy). The difference between the models is that in our case we split powers over time, whereas in the water filling problem the powers are split over space. Our results are therefore quite relevant to the water filling problem as well. Some work on water filling games can be found in [9] where not only mobiles take decisions, but also the base station does, with the goal of maximizing a weighted sum of the individual rates. In [17], the non-cooperative water filling game is studied in the context of the interference channel; two mobiles and two corresponding base stations.

Game theoretic formulations for non cooperative power control with finite actions (power levels) and states (channel attenuations) have been proposed in [8]. An ϵ equlibrium is obtained there for the case of a large number of players. The cost to be minimized by a player i in [8] is the quadratic difference between the desired and the actual SINR (Signal to Interference plus Noise Ratio) of that player. In contrast, in the model we introduce in this paper, the choice of the transmission power is done in the purpose of maximizing its own throughput subject to a limit on the average power. Our setting is different also in the following. In our model, in a given channel state, each mobile can either choose a fixed power level or can make randomized decisions, i.e. it can make the choice of power levels in a state based on some (state dependent) randomization.

B. Organization of the paper

The structure of the paper is as follows. We first present the model (Section II) as well as the mathematical formulation of both the case of centralized information (Section III) as well as the one of decentralized information (Section IV). In Section V we identify the structure of best-response policies and thus of equilibria for the decentralized case. Power control in the presence of a malicious mobile is studied in Section VI. In Section VII we present numerical examples that illustrate the structural properties that we had obtained and which allow us further to obtain insight in cases for which the question about the structure of optimal policies remains open. After a concluding section we present a computation methodology for computing equilibria in the game of two players. The technical proofs can be found in Research Report [18].

II. The model

A. Preliminaries

Consider a set of N mobiles and a single base station. As in several standard wireless networks (e.g., UMTS and IEEE 802.11), we assume that time is slotted. In each time slot t, each mobile i transmits data with power level $A_i(t)$ chosen from a finite set $\mathbf{A}_i = (1, 2, 3, \dots, \alpha_i)$ containing α_i power levels. The actual power corresponding to the ath power level where $a \in \mathbf{A}_i$ is given by $h_i(a)$. Denote $\mathbf{A} = \prod_{i=1}^{N} \mathbf{A}_i$.

The channel state model: We assume that the channel between mobile i and the base station can be modelled as an ergodic finite Markov chain $X_i(t)$ taking values in a set $\mathbf{X}_i = (1, 2, \dots, m_i)$ of m_i states with transition probabilities $\mathbf{P}^i_{xy}$. The Markov chains $X_i(t)$, $i = 1 \dots N$, are assumed to be independent. Let π_i be the row vector of steady state probabilities of Markov chain $X_i(t)$; let $\pi_i(x)$ be its entry corresponding to the state $x \in \mathbf{X}_i$. It is the unique solution of

$$\pi_i \mathbf{P}^i = \pi_i, \qquad \pi_i(x) \geq 0, \ \forall x \in \mathbf{X}_i, \qquad \sum_{x \in \mathbf{X}_i} \pi_i(x) = 1.$$

We also denote by $\pi(\mathbf{x})$ the probability of state $\mathbf{x} = (x_1, \dots, x_N)$. Since the Markov chains that describe the channel states are independent, $\pi(\mathbf{x}) = \prod_{i=1}^{N} \pi_i(x_i)$.

The power received at the base station from mobile i is given by $g_i(t) h_i(A_i(t))$ where the attenuation $g_i(t) = g_i(X_i(t))$ is a function of the channel state $X_i(t)$. We shall denote the global state space of the system by $\mathbf{X} = \prod_{i=1}^{N} \mathbf{X}_i$.

Performance measures: The signal to interference plus noise ratio $SINR_i$ at the base station related to mobile i when the power level choices of the mobiles are $\mathbf{a} = (a_1, \dots, a_N)$ and the channel states are $\mathbf{x} = (x_1, \dots, x_N)$ is given by

$$SINR_i(\mathbf{x}, \mathbf{a}) = \frac{g_i(x_i) h_i(a_i)}{N_o + \sum_{j \neq i} g_j(x_j) h_j(a_j)}.$$

We consider the following instantaneous utility of mobile i:

$$r_i(\mathbf{x},\mathbf{a}) = \log_2\left(1 + SINR_i(\mathbf{x},\mathbf{a})\right). \quad (1)$$

$r_i(\mathbf{x},\mathbf{a})$ is known as the Shannon capacity and can thus be interpreted as the throughput that mobile i can achieve at the uplink when the channel conditions are given by $\mathbf{x}$ and the power levels used by all mobiles are $\mathbf{a}$.

Notation: In the rest of the paper, we shall use the following notation. We shall denote an element of the set $\mathbf{X}$ by $\mathbf{x}$. The ith component of $\mathbf{x}$ will be denoted by x_i, i.e., $\mathbf{x} = (x_1, x_2, \ldots, x_N)$, where $x_i \in \mathbf{X}_i$ for $i = 1, 2, \ldots, N$. We define $\mathbf{a}$ and a_i in a similar manner. Let $\mathbf{X}^{-i}$ and $\mathbf{A}^{-i}$ denote the set of channel states and the set of actions, respectively, corresponding to all the players other than player i. For an element $\mathbf{x}^{-i} \in \mathbf{X}^{-i}$, let x_j^{-i} denote the jth component of $\mathbf{x}^{-i}$. We define $\mathbf{a}^{-i}$ and a_j^{-i} in a similar way.

B. Policy types

A mobile's choice of successive transmission power levels is made based on the information it has. The latter could be local, in which case the policy is said to be distributed. We shall also consider centralized policies in which all decisions are taken at the base station. We have the following definitions.

Centralized policy, $u(\mathbf{a}\,|\,\mathbf{x})$, is the probability that the base station assigns the transmission power levels $\mathbf{a} = (a_1, \ldots, a_N)$ to the mobiles if the current channel's states are given by the vector $\mathbf{x} = (x_1, \ldots, x_N)$. This is equivalent to the situation where all system information is available to all mobiles, and moreover, all mobiles can *coordinate* their actions. This situation describes central decision making by the base station. The class of centralized policies is denoted by U_{ce}.

Decentralized policy, $u_i(a\,|\,x)$, is the probability that player i chooses the transmission power level $a \in \mathbf{A}_i$ if its channel state is $x \in \mathbf{X}_i$. Thus, only local information is available to each mobile, and there is no coordination in the random actions. This situation describes individual decision making by each mobile without any involvement of the base station. The class of decentralized policies for player i is denoted by U_{dc}^i. Define $U_{dc} = \prod_{i=1}^N U_{dc}^i$.

Along with policies we shall use also the occupation measures. For a given $\mathbf{x} \in \mathbf{X}$ and $\mathbf{a} \in \mathbf{A}$, the global occupation measure, $\rho^u(\mathbf{x},\mathbf{a})$, will be used in the context of a **centralized** policy, $u \in U_{ce}$, it is defined as

$$\rho^u(\mathbf{x},\mathbf{a}) = \prod_{i=1}^N \pi_i(x_i) u(\mathbf{a}\,|\,\mathbf{x}).$$

Note that given a global occupation measure, ρ^u, the corresponding u can be obtained by

$$u(\mathbf{a}\,|\,\mathbf{x}) = \frac{\rho^u(\mathbf{x},\mathbf{a})}{\sum_{\mathbf{b}\in\mathbf{A}} \rho^u(\mathbf{x},\mathbf{b})} \quad (2)$$

(it is chosen arbitrarily if the denominator is zero). For a given $x \in \mathbf{X}_i$ and $a \in \mathbf{A}_i$, the local occupation measure, $\rho_i^{u_i}(x,a)$, is defined with respect to a **decentralized** policy, $u_i \in U_{dc}^i$, and is given by

$$\rho_i^{u_i}(x,a) = \pi_i(x) u_i(a|x).$$

For a given local occupation measure, $\rho_i^{u_i}$, the corresponding u_i can be obtained by

$$u_i(a\,|\,x) = \frac{\rho_i^{u_i}(x,a)}{\sum_{b\in\mathbf{A}_i} \rho_i^{u_i}(x,b)} \quad (3)$$

(it is chosen arbitrarily if the denominator is zero). In case of **decentralized** decision making, for a given $(u_1, u_2, \ldots, u_N)$ we define $\rho^u(\mathbf{x},\mathbf{a})$ as

$$\rho^u(\mathbf{x},\mathbf{a}) = \prod_{i=1}^N \rho_i^{u_i}(x_i, a_i). \quad (4)$$

C. Problem formulation: objectives and constraints

For any given policy[1], u, and the corresponding occupation measure[2], $\rho^u(\mathbf{x},\mathbf{a})$, we now define the utility function, the constraints, and the optimization problem.

The utility functions: We define the utility for player i as

$$R_i(u) := \sum_{\mathbf{x}\in\mathbf{X}} \sum_{\mathbf{a}\in\mathbf{A}} r_i(\mathbf{x},\mathbf{a}) \rho^u(\mathbf{x},\mathbf{a}). \quad (5)$$

Power constraints: In the centralized case, player i is assumed to have the following average power constraint

$$\sum_{\mathbf{x}\in\mathbf{X}} \sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x},\mathbf{a}) h_i(a_i) \le V_i, \quad (6)$$

and in the decentralized case the corresponding constraint is

$$\sum_{x\in\mathbf{X}_i} \sum_{a\in\mathbf{A}_i} \rho_i^{u_i}(x,a) h_i(a) \le V_i. \quad (7)$$

Note that in the decentralized case the state-action frequencies of a particular mobile are independent of decisions of the other mobiles (see equation (4)). Consequently, in the decentralized case, the average power constraint of a mobile does not depend on the decision of the others. However, in the centralized case, the decisions of all the mobiles are interdependent.

1) Cooperative optimization: We consider here the problem of maximizing a *common objective* subject to individual side constraints. Namely, we define for any policy u

$$R_\gamma(u) := \sum_{i=1}^N \gamma_i R_i(u), \quad (8)$$

where γ_i are some nonnegative constants. For an arbitrary set of policies $\overline{U}$ we consider the problem:

$$\textbf{COOP}(\overline{U}): \quad \max_{u\in\overline{U}} R_\gamma(u), \ \text{ s.t. (7)}, \ \forall i = 1, \ldots, N. \quad (9)$$

[1]With slight abuse of notation, we shall denote both centralized and decentralized policies by u. In the **centralized** case, $u(\mathbf{a}|\mathbf{x})$ will denote a probability measure over $\mathbf{a}$ for a given $\mathbf{x}$. In the **decentralized** case, u will denote the vector $u = (u_1, u_2, \ldots, u_N)$, where u_i is the decentralized policy for player i, for $i = 1, 2, \ldots, N$.

[2]For the **decentralized** case, we note that $\rho^u(\mathbf{x},\mathbf{a})$ is given by (4).

2) Non-cooperative optimization: Here each mobile is considered as a selfish individual non-cooperative decision maker, which we then call "player". It is interested in maximizing its own average throughput (5). In the non-cooperative it is natural to consider only decentralized policies U_{dc}.

For a policy $u = (u_1, \ldots, u_N) \in U_{dc}$ we define u^{-i} to be the set of components of u other than the ith component. For a policy $v_i \in U^i_{dc}$ we then define the policy $[v_i, u^{-i}]$ as one in which player $j \neq i$ uses the element u_j of u whereas player i uses v_i.

Definition 1: We say that $u^* \in U_{dc}$ is a constrained Nash equilibrium [19] if it satisfies (7) for all players, and if

$$R_i(u^*) \geq R_i([v_i, (u^*)^{-i}])$$

for any i and any $v_i \in U_{dc}$ such that (7) holds for the policy $[v_i, (u^*)^{-i}]$.

III. Centralized cooperative optimization

When the cooperative optimization is considered over the set of *centralized policies*, then the problem is in fact of a single controller (the base station) which has all the information. Let $r_\gamma(\mathbf{x}, \mathbf{a}) := \sum_{i=1}^N \gamma_i r_i(\mathbf{x}, \mathbf{a}),\ \gamma_i \geq 0,\ i = 1, 2, \ldots, N$, denote the common instantaneous utility when power level $\mathbf{a}$ is chosen in channel state $\mathbf{x}$. The next Theorem states the existence of an optimal strategy if the constraint set is not empty. The optimal strategy can be obtained by means of provided Linear Program (LP).

Theorem 1: Consider the cooperative optimization problem $\mathbf{COOP}(U_{ce})$ over the set of centralized policies. Assume that there exists a policy u under which the power constraints (7) hold for all the mobiles. Then,

(i) there exists an optimal centralized policy $u^* \in U_{ce}$. The policy u^* can be obtained from the solution of the following LP by formula (2)

$$\max_\rho R_\gamma(u) := \sum_{\mathbf{x}\in\mathbf{X}} \sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x}, \mathbf{a}) r_\gamma(\mathbf{x}, \mathbf{a}) \tag{10}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{\mathbf{x}\in\mathbf{X}} \sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x}, \mathbf{a}) h_i(a_i) \leq V_i, \quad i = 1, \ldots, N; \\ & \sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x}, \mathbf{a}) = \pi(\mathbf{x}) = \prod_{i=1}^N \pi_i(x_i), \quad \forall \mathbf{x} \in \mathbf{X}; \\ & \rho(\mathbf{x}, \mathbf{a}) \geq 0, \quad \forall \mathbf{x} \in \mathbf{X},\ \forall \mathbf{a} \in \mathbf{A}; \\ & \sum_{\mathbf{x}\in\mathbf{X}} \sum_{\mathbf{a}\in\mathbf{A}} \rho(\mathbf{x}, \mathbf{a}) = 1. \end{aligned} \tag{11}$$

(ii) An optimal policy u^* can be chosen with no more than N randomizations.

Note that in the centralized framework it does not make sense to speak about a non-cooperative game, since there is a single decision maker.

IV. Decentralized Information

A. Non-cooperative equilibrium

Here we consider the case when the players optimize their own objective (5) subject to the constraints (7) given the local information only. For this case we show the existence of the constrained Nash equilibrium.

Theorem 2: Under the assumptions on the objective functions $R_i(u)$, constraints (7), and the set of decentralized policies U_{dc} made above, there exists a policy $u^* \in U_{dc}$ satisfying Definition 1.

B. The cooperative case

Here we discuss the situation where, even though there is a common goal that is optimized, the power level choices are not done by the base station but by the mobiles themselves who have only their local information available to take decisions. Coordination is thus not possible.

Considering the decentralized framework, we make the following observation concerning the relation between the cooperative and the non-cooperative cases.

Theorem 3: Any policy u that maximizes the common objective $R_\gamma(u)$ while satisfying the constraints is necessarily a constrained Nash equilibrium in the game where each mobile maximizes the common objective $R_\gamma(u)$.

Now we show in Theorem 4 that there exists an optimal decentralized policy.

Theorem 4: Let all the players have the common objective function $R_\gamma(u)$ defined by (8). Under the assumptions on constraints (7) and the set of decentralized policies U_{dc} made above, there exists a solution $u^* \in U_{dc}$ to the problem $\mathbf{COOP}(U_{dc})$ (9).

V. Structure of non-cooperative equilibrium

In this section we identify the structure of equilibria policies for the decentralized non-cooperative case. To that end we first study the structure of best response policies of any given user when the policies of the other users are fixed. Using the results on the structure of the best response we then establish the structure of the equilibrium policies.

We fix throughout the policy v^{-i} of players other than player i, where

$$v^{-i}(\mathbf{a}^{-i} \,|\, \mathbf{x}^{-i}) = \prod_{j\neq i} v_j(\mathbf{a}_j^{-i} \,|\, \mathbf{x}_j^{-i})$$

is the probability that each mobile $j \neq i$ chooses a_j when its local state is x_j. The product form here is due to the decentralized nature of the problem and to the fact that there is no coordination between the mobiles is possible.

We shall make the following assumption on the properties of g_i, h_i, and π_i.

Assumption 1: (i) The function g_i has an increasing interpolation in x.

(ii) The function h_i has a strictly convex and increasing interpolation in a.

(iii) The probability measure $\pi_i(x)$ has a non-decreasing interpolation in x.

Let us discuss why the above assumptions are non-restrictive. Assumption 1.i can be satisfied by enumerating the states so that the quality of the associated channel state increases with the index of the state. In $3G$ wireless networks the mobile terminals typically select transmit power levels in steps of 1 or 2 dB [20], [21]. This linear interpolation in the logarithmic scale translates to a strictly convex interpolation in the absolute scale, which is used in our formulation. Thus, Assumption 1.*ii* is naturally satisfied. Assumption 1.*iii* means that we expect the channel to be more often in better states.

We shall establish the following main result on the structure of any best response policy:

Theorem 5: Consider the decentralized non-cooperative case. Under Assumption 1, the following holds:

(i) In each channel state x_i, the best response policy consists of either the choice of a single action, or in a randomized choice between at most two adjacent power levels.

(ii) The optimal power levels are non-decreasing functions of the channel state.

(iii) If two power levels are jointly optimal for a given channel state then they cannot be jointly optimal for another channel state.

Now, using Theorem 5 we can establish the structure of the constrained Nash equilibria.

Corollary 1: Consider the decentralized non-cooperative case. For each mobile i, assume that h_i, g_i, and π_i satisfy Assumption 1. Then there exists at least one equilibrium. Moreover, at any equilibrium u_i^* the following hold for each mobile i:

(i) In each channel state $x \in \mathbf{X}_i$, $u_i^*(\cdot|x)$ consists of either a choice of a single power level, or in a randomized choice between at most two adjacent power levels.

(ii) The power levels used in u_i^* are non-decreasing functions of the channel state.

(iii) If two power levels are used at a state x by mobile i with positive probability (i.e. $u_i^*(a_j|x) > 0$ and $u_i^*(a_k|x) > 0$ for $a_k \neq a_j$) then under u_i^*, not more than one of them is used with positive probability at any other channel state.

VI. Power control in the presence of a malicious mobile

In recent years, there has been a growing interest in identifying and studying the behavior of potential intruders to networks or of malicious users, and in studying how to best detect these or to best protect the network from their actions (see e.g. [22]–[24] and references therein).

We consider in this section a scenario where a malicious player attempts to jam the communications of a mobile to the base station. We consider the distributed case and restrict for simplicity to two mobiles and a base station.

The first mobile (player 1) seeks to maximize the rate of information that it transmits to the base station. In other words it wishes to *maximize* $R_1(u)$ defined in (5) with r_1 given by (1).

The second mobile (player 2) has an antagonistic objective: to prevent or to jam the transmissions of the first mobile, with the objective of minimizing the throughput of information that mobile 1 transmits to the base station. It thus seeks to *minimize* $R_1(u)$. We assume that the interference of the second mobile is presented as a Gaussian white noise.

Except for the objective of the jamming mobile, the model, including the average power constraints, defined in Section II holds. In particular, we conclude that Theorem 5 applies to player 1 at equilibrium.

We now specify the objective of the players and some properties of the equilibrium. Denote U_c^i the set of policies for player i, (where i takes the values 1 and 2) that satisfy player i's power constraints, i.e., $u_i \in U_c^i$ if it satisfies (7). Player 1 seeks to obtain an optimal policy, i.e. a policy $u_1^* \in U_c^1$ such that for any other $u_1 \in U_c^1$,

$$\inf_{u_2 \in U_c^2} R_1(u_1^*, u_2) \geq \inf_{u_2 \in U_c^2} R_1(u_1, u_2).$$

We call this the jamming problem. It consists of identifying a policy for player 1 that guarantees the largest throughput under the worst possible strategy of player 2. In fact, we shall be able not only to identify the optimal policy for player 1 but also the "optimal" policy for player 2 (which is the worst for player 1).

A policy $u^* = (u_1^*, u_2^*)$ is said to be a saddle point if

$$\begin{aligned} \sup_{u_1 \in U_1^c} \inf_{u_2 \in U_2^c} R_1(u_1, u_2) &= \inf_{u_2 \in U_2^c} R_1(u_1^*, u_2) = R_1(u_1^*, u_2^*) \\ &= \sup_{u_1 \in U_1^c} R_1(u_1, u_2^*) = \inf_{u_2 \in U_2^c} \sup_{u_1 \in U_1^c} R_1(u_1, u_2), \end{aligned}$$

and u_1^* and u_2^* are called saddle point policies or optimal policies.

Unlike all the decentralized problems we considered previously, deriving both u_1^* as well as u_2^* is possible using a linear program. The computation is not included here, but it can be found in [25]. Below we derive the properties of the optimal policies.

Theorem 6: (i) There exists a saddle point policy u^* in the above game.

(ii) Under Assumption 1 any optimal policy for player 1 (transmitter) has the structure identified in Theorem 5.

We now identify a structural property of the optimal policy of player 2, i.e., of the jammer. Let h_2 have a convex interpolation in a, and g_2 have an increasing interpolation in x then

(i) there is only one action, say a, which has a non-zero probability to be used by any optimal policy, or

(ii) except for two adjacent actions, say a and $a+1$, all other actions are not used by any policy which is optimal.

We finally note that the monotonicity property enjoyed by the saddle point policy of mobile 1, *need not hold* for mobile 2. This will be illustrated in Section VII-C (see Figure 5).

VII. Numerical Examples

In this section we provide examples of power control problem for two mobiles that interact with the same base station. The decentralized policies are provided both for the cooperative and non-cooperative cases. Moreover, the single controller problem for centralized cooperative framework is also solved. All three problems are considered in the same settings, so one has an opportunity to compare the obtained strategies and the objective value functions for different approaches.

We assume, that the radio channel between mobile $i = 1, 2$ and the base station is characterized by a Markov chain X_i with states $x_i \in \mathbf{X}_i = \{1, \ldots, M\}$, $M = 11$, and a uniform vector of steady state probabilities. One of the transition probability matrices which has a uniform steady state probability vector is given by $\mathbf{P}^i_{xy} = \frac{1}{M}$.

The power attenuation for each state of the Markov chain X_i is defined by the following:

$$\begin{array}{cccccc} x_i & 1 & 2 & 3 & \ldots & 11 \\ g_i(x_i) & 0.0 & 0.1 & 0.2 & \ldots & 1.0. \end{array}$$

Let mobile i's action set $\mathbf{A}_i$ be given by $\mathbf{A}_i = (0, \ldots, 11)$. The actual power corresponding to the a_ith power level, where $a_i \in \mathbf{A}_i$, is

$$\begin{array}{cccccc} a_i & 0 & 1 & 2 & \ldots & 11 \\ h_i(a_i) & 0 & 0 \text{ dB} & 1 \text{ dB} & \ldots & 10 \text{ dB} \end{array}$$

where the level of 0 dB corresponds to some base value of power W_0. We assume that the background noise power at the base station, N_0, is equal to 0 dB. Since (1) depends only on the ratio between the power of signal received from a certain mobile and the total power received from other mobiles and the thermal noise power at the receiver, we do not specify the exact value of the base power W_0.

We note that, with the above definitions, g_i, h_i and π_i satisfy the properties in Assumption 1.

The constraints for mobiles are given by (6) for the centralized case and by (7) for the decentralized case with the following power consumption bounds:

$$V_1 = 2.7\mathrm{W}_0, \quad V_2 = 5.1\mathrm{W}_0.$$

Note, that both right and left hand sides of (6) and (7) have the multiplier W_0, which can be canceled.

The proposed model is quite simple, we chose it so as to avoid technical difficulties related to Markov chains with infinite state space. Thus we assume that a finite Markov chain can approximate well randomness due to fading, shadowing, mobility, as well as time correlation phenomena which are often ignored. Nevertheless, the main goal of the example is to validate the structure that we obtain rather than to propose a reliable model that could include mobility, handovers, shadowing, fading, interference from other cells etc. Further research including these features is planned.

A. Decentralized policies

First we consider the decentralized problems that arise in cooperative and non-cooperative case. Both problems are formulated in terms of occupation measures $\rho_i(x_i, a_i)$. In order to compute the strategies one can use (3).

1) Cooperative optimization: Let $\mathbf{x} = (x_1, x_2)$ and $\mathbf{a} = (a_1, a_2)$. Here we consider the following cost function

$$r(\mathbf{x}, \mathbf{a}) = r_1(\mathbf{x}, \mathbf{a}) + r_2(\mathbf{x}, \mathbf{a}), \tag{12}$$

where $r_i(\mathbf{x}, \mathbf{a})$ are defined by (1).

Consider the following bilinear problem

$$\max_{\rho_1, \rho_2} \sum_{\substack{x_1 \in \mathbf{X}_1 \\ a_1 \in \mathbf{A}_1}} \sum_{\substack{x_2 \in \mathbf{X}_2 \\ a_2 \in \mathbf{A}_2}} \rho_1(x_1, a_1) r(\mathbf{x}, \mathbf{a}) \rho_2(x_2, a_2),$$

where

$$\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i) h_i(a_i) \le V_i,$$

$$\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i)(\delta(x_i, y_i) - \mathbf{P}^i_{x_i y_i}) = 0, \quad \forall y_i \in \mathbf{X}_i,$$

$$\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i) = 1,$$

$$\rho_i(x_i, a_i) \ge 0, \quad \forall x_i \in \mathbf{X}_i, a_i \in \mathbf{A}_i.$$

Here $\mathbf{P}^i$ is the transition matrix of the Markov chain, which describes the radio channel between the mobile i and the base station, and $\delta(x, y)$ is equal to one if $x = y$ and is zero otherwise.

This problem could be solved using the quadratic programming technique. In Fig. 1, the supports of the optimal policies for both players are shown as a function of the channel state.

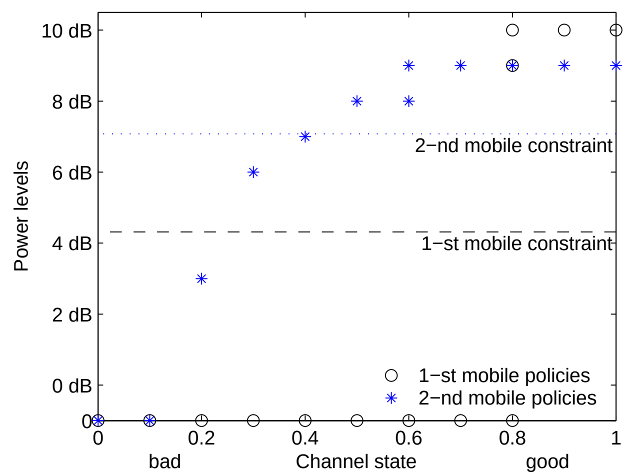

Fig. 1. Supports of the optimal policies in cooperative case.

As one can see[3], the mobile 1 has a pure strategy at all the points but one, where $g_1(x_1) = 0.8$. The mobile 2 also has only one randomization point $g_2(x_2) = 0.6$. The value of the objective function in this problem is $R(u^*) = 1.9225$.

2) Non-cooperative equilibrium: Now, in the same setting as in the cooperative case, we consider an example of non-cooperative optimization. Each mobile needs to maximize its own objective function

$$\max_{\rho_1, \rho_2} \sum_{\substack{x_1 \in \mathbf{X}_1 \\ a_1 \in \mathbf{A}_1}} \sum_{\substack{x_2 \in \mathbf{X}_2 \\ a_2 \in \mathbf{A}_2}} \rho_1(x_1, a_1) r_i(\mathbf{x}, \mathbf{a}) \rho_2(x_2, a_2), \tag{13}$$

[3]Here and in all the rest examples we provide just the structure of the optimal policies. The exact values one can find in Research Report [18]

where $i = 1, 2$, subject to the constraints

$$\begin{aligned} &\rho_i(x_i, a_i) \geq 0, \qquad \forall x_i \in \mathbf{X}_i, a_i \in \mathbf{A}_i, \\ &\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i) = 1, \\ &\sum_{a_i \in \mathbf{A}_i} \rho_i(x_i, a_i) = \pi_i, \quad \forall x_i \in \mathbf{X}_i, \end{aligned} \tag{14}$$

$$\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i) h_i(a_i) \leq V_i, \quad i = 1, 2. \tag{15}$$

By means of the LCP (20) one can obtain the optimal strategies depicted on Fig. 2. We note that the structure obtained in Theorem 5 holds for both the players.

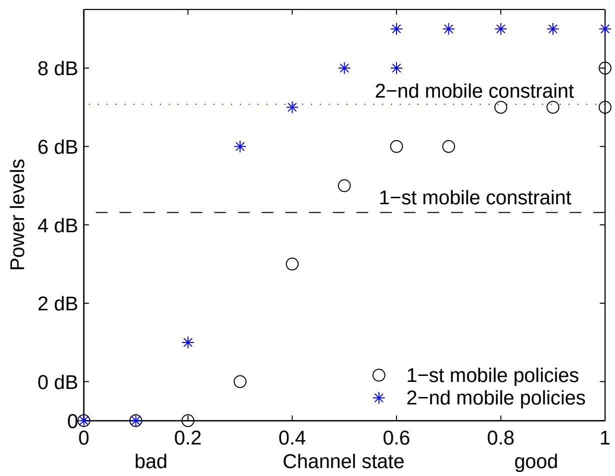

Fig. 2. Supports of the optimal policies in non-cooperative case.

The values of the objective functions in this problem are $R_1(u^*) = 0.6484$, $R_2(u^*) = 1.1584$. As it was expected, the total throughput value $R(u^*) = R_1(u^*) + R_2(u^*) = 1.8067$ is smaller than in cooperative case.

B. Centralized optimization

Now let us consider the single controller problem, that arises in the case of centralized optimization. As in the decentralized framework, we operate here in terms of occupation measures. Thus, the problem (10) for the case of two players can be rewritten as follows:

$$\max_{\rho} \sum_{\mathbf{x} \in \mathbf{X}} \sum_{\mathbf{a} \in \mathbf{A}} \rho(\mathbf{x}, \mathbf{a}) r(\mathbf{x}, \mathbf{a}), \tag{16}$$

where $r(\mathbf{x}, \mathbf{a})$ is defined by (12). The maximization is performed subject to the following constraints:

$$\begin{aligned} &\sum_{\mathbf{x} \in \mathbf{X}} \sum_{\mathbf{a} \in \mathbf{A}} \rho(\mathbf{x}, \mathbf{a}) h_i(a_i) \leq V_i, \quad i = 1, 2; \\ &\sum_{\mathbf{a} \in \mathbf{A}} \rho(\mathbf{x}, \mathbf{a}) = \pi(\mathbf{x}) = \pi_1(x_1)\pi_2(x_2); \\ &\rho(\mathbf{x}, \mathbf{a}) \geq 0, \quad \forall \mathbf{x} \in \mathbf{X}, \ \forall \mathbf{a} \in \mathbf{A}; \\ &\sum_{\mathbf{x} \in \mathbf{X}} \sum_{\mathbf{a} \in \mathbf{A}} \rho(\mathbf{x}, \mathbf{a}) = 1. \end{aligned} \tag{17}$$

Define the following sets:

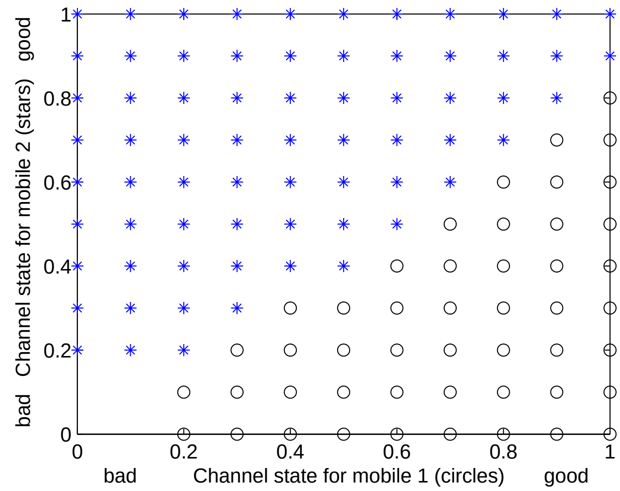

Fig. 3. The sets Ψ_1 and Ψ_2.

- Ψ_1: pairs (x_1, x_2): $\exists a_1^*$ such that $h_1(a_1^*) > 0$ and $u(a_1^*, a_2 \,|\, x_1, x_2) > 0$ for some $a_2 \in \mathbf{A}_2$;
- Ψ_2: pairs (x_1, x_2): $\exists a_2^*$ such that $h_2(a_2^*) > 0$ and $u(a_1, a_2^* \,|\, x_1, x_2) > 0$ for some $a_1 \in \mathbf{A}_1$.

Note, that the set Ψ_i is the set of states in which ith player should transmit with nonzero probability according to the optimal strategy.

In Fig. 3 these sets are provided for the centralized optimization problem (16). The set Ψ_1 is depicted by circles, and the set Ψ_2 — by stars. One can see, that the sets have no mutual points. It means, that the mobiles never transmit at the same time.

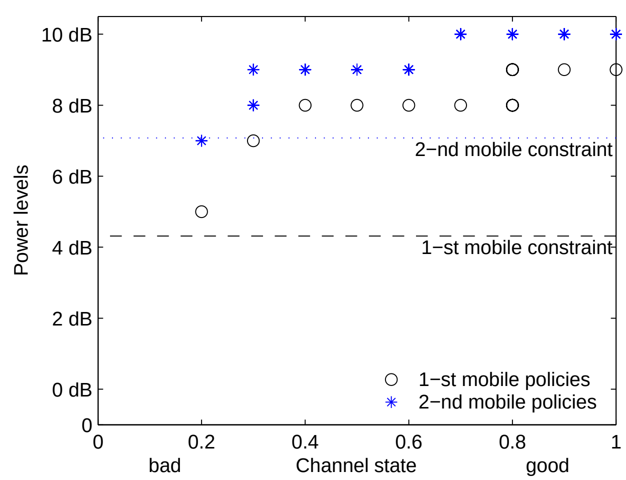

Fig. 4. Supports of the optimal policies in cooperative case.

In Fig. 4 one can see the supports of the optimal strategies.

A circle on the place $(g_1(x_1^*), h_1(a_1^*))$ means that the first mobile should transmit with the power level $h_1(a_1^*)$ with nonzero probability in all states $(x_1^*, x_2) \in \Psi_1$.

A star on the place $(g_2(x_2^*), h_2(a_2^*))$ means that the second mobile should transmit with the power level $h_2(a_2^*)$ with nonzero probability in all states $(x_1, x_2^*) \in \Psi_2$.

If there are two or more power levels $h_i(a_i^*)$ for some particular state $g_i(x_i^*)$, then the player should randomize. In other case (single power level $h_i(a_i^*)$ for the state $g_i(x_i^*)$), the player should always transmit with power level $h_i(a_i^*)$.

Note, that the centralized power management provides better throughput in comparison with other considered controls, the value of the cost function is $R(u^*) = 2.5614$.

Another interesting point that we want to discuss is the attainability of the power constraints.

Consider the problem (16) without power constraints. The optimal policies for this problem are as follows:

- Player 1 should transmit at the top power level if $g_1(x_1) \geq g_2(x_2)$;
- Player 2 should transmit at the top power level if $g_2(x_2) \geq g_1(x_1)$.

The value of the objective function for this policy is $R(u^*) = 2.8560$. The experiments show, that at the optimal point for problem with constraints (17), where the bounds V_i are both greater then 7 dB, the power constraints are not attained, and the optimal strategy and the value of the objective function are the same as in unconstrained case.

C. Jamming

The average power bounds are the same as in previous examples: for the transmitter $V_1 = 2.7$, and for the jammer $V_2 = 5.1$.

The supports of the optimal strategies in this problem are depicted in Fig. 5. We note that the structure obtained in Theorem 5 holds for player 1, whereas the structure obtained in Section VI holds for player 2.

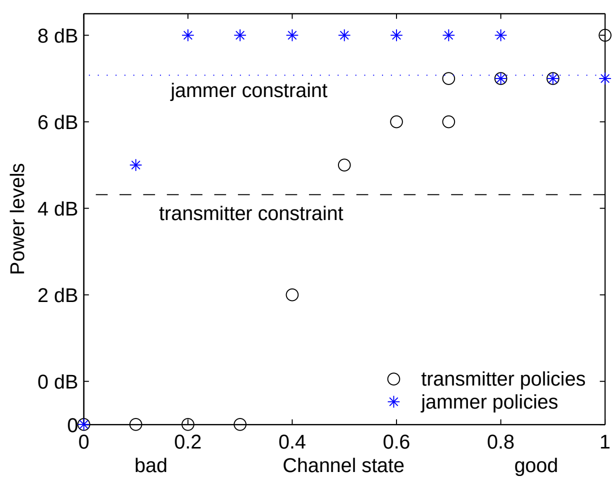

Fig. 5. Supports of the optimal policies in case of jamming.

The value of the objective function is $R_1(u^*) = 0.6237$ which is less then the same value for the decentralized non-cooperative case.

VIII. Conclusion and further work

We have studied power control in both cooperative and non-cooperative setting. Both centralized and decentralized information patterns have been considered. We have derived the structure of optimal decentralized policies of selfish mobiles having discrete power levels. We further studied the structure of power control policies when a malicious mobiles tries to jam the communication of another mobile. We have illustrated these results via several numerical examples, which also allowed us to get insight into the structure in the cooperative framework.

The modelling and results open many exciting research problems. Our setting, which could be viewed as a temporal scheduling problem, is quite similar to the "space scheduling" (i.e. the water-filling) problems discussed in Introduction, for which the context of discrete power levels along with the non-cooperative setting have not yet been explored. It is interesting not only to study the water-filling problem in the discrete noncooperative context but also to study the combined space and temporal scheduling problem, where we can split the transmission power both in time and in space (different parallel channels).

From both a game theoretic point of view as well as from a wireless engineering point of view, it is interesting to study possibilities for coordination between mobiles in the decentralized case (in both cooperative and non-cooperative contexts). This can be done using the concepts from *correlated equilibria* [26]–[29], which is known to allow for better performance even in the selfish non-cooperative cases. We note however, that existing literature on correlated equilibria do not include side constraints, which makes the investigation novel also in terms of fundamentals of game theory.

IX. Appendix

In this section we show how the non-cooperative equilibrium can be obtained in the case of two players by means of linear complementarity problem (LCP). Consider the following problem, where each player wants to maximize his own payoff (13) subject to the constraints (14)-(15).

First, assume, that at the equilibrium point the power consumption constraints (15) are active:

$$\sum_{\substack{x_i \in \mathbf{X}_i \\ a_i \in \mathbf{A}_i}} \rho_i(x_i, a_i) h_i(a_i) = V_i, \quad i = 1, 2. \tag{18}$$

This assumption is not restrictive, because if one or both of these constraints are not active, they can be omitted. So one need first solve the problem without constraints (15), then calculate the values of cost functions in RHS of (15). If both calculated values are less then V_i, then the solution of (13)—(14) is a solution of (13)—(15) as well. If one of the constraints (15) is violated, the problem should be solved again subject to the corresponding constraint of equality type form (18). If in the obtained equilibrium point the second constraint is still violated, then the problem should be solved again subject to both constraints of (18).

Now let ξ be the vector, containing all the $\rho_1(x_1, a_1), \forall x_1 \in \mathbf{X}_1, a_1 \in \mathbf{A}_1$, and ζ — the same vector for $\rho_2(x_2, a_2)$.

Indeed, the problem (13), (14), (18) can be represented in the form of the bimatrix game with linear constraints:

$$\begin{aligned} &\max_{\xi,\zeta} \xi^* A\zeta, \quad \max_{\xi,\zeta} \xi^* B\zeta, \\ \text{s.t.} \quad &C^*\xi = c,\ D^*\zeta = d, \quad \xi \geq 0,\ \zeta \geq 0. \end{aligned} \tag{19}$$

Following [30] we introduce the LCP whose solution characterizes the equilibrium point of (19):

$$\begin{aligned} &z = (\xi, \zeta, z_1, z_2, z_3, z_4)^* \geq 0, \\ &q + Mz \geq 0, \\ &z^*(q + Mz) = 0, \end{aligned} \tag{20}$$

where $q = (0, 0, c^*, -c^*, d^*, -d^*)^*$ and

$$M = \begin{pmatrix} & -A & C^* & -C^* & & \\ -B^* & & & & D^* & -D^* \\ -C & & & & & \\ C & & & & & \\ & -D & & & & \\ & D & & & & \end{pmatrix}.$$

It is also shown in [30], that under the conditions $A \leq 0$ and $B \leq 0$ Lemke's algorithm [31] computes a solution of the LCP (20). It should be noted, that in order to satisfy the conditions $A \leq 0$, $B \leq 0$ we can always replace cost matrices A and B with $A - kE$ and $B - kE$, where E is a matrix of unities, and k is the maximal positive entry of A and B.

Once the solution of LCP (20) (ξ_o, ζ_o) is found, the equilibrium point (ξ', ζ') of the bimatrix game (19) could be computed using the following formulas:

$$\xi' = \frac{\xi_o}{e_1^* \xi_o}, \quad \zeta' = \frac{\zeta_o}{e_2^* \zeta_o}, \tag{21}$$

where e_1 and e_2 are vectors of appropriate dimension, whose components are all ones.

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