Random Variables

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1 Simulating Random Variables

- Random variables with different distributions can be created in ns.
- For more background, see `http://www.xycoon.com/`.
- For a random variable (RV) X, we denote $F_x(s) = P(X \leq s)$, $\overline{F}_x(s) = P(X > s)$ and by $f_x(s)$ we denote its density. (We often omit the subscript x.)

1.1 Pareto distribution

- A Pareto RV is defined through

$$\overline{F}(s) = (k/s)^{\beta},$$

where k is the minimum size and $\beta > 0$ is the "shape parameter".

- It is defined on the range $X \geq k$.
- The density is given by

$$f(s) = \frac{\beta k^{\beta}}{s^{\beta+1}}.$$

- The expectation and other moments are

$$\begin{aligned} E[X] &= \frac{\beta k}{\beta - 1}, & 1 < \beta \\ E[X^n] &= \frac{\beta k^n}{\beta - n}, & n < \beta \end{aligned}$$

- The nth moment is infinite if $n \geq \beta$.
- The size of files transferred over the Internet is often characterized with a Pareto distribution with $1 < \beta \leq 2$.
- A typical value is $\beta = 1.2$
- A typical value for the expected size of a file in Internet transfers is 10Kbits.
- In the context of WEB transfers, typical values are $\beta = 1.1$ and $k = 81.5$Kbytes (see [**?**, p.34-35].)

1.2 The exponential Random variable

- An exponentially distributed RV with parameter α is defined through

$$\overline{F}(s) = \exp(-\alpha s), \quad f(s) = \alpha \exp(-\alpha s).$$

- All its moments exist and are given by

$$E[X^n] = \frac{n!}{\alpha^n}.$$

- In a WEB transfer, Pareto distributed transfers are typically separated with Exponentially distributed silence times (''thinking times'') with average duriation of $\alpha^{-1} = 5sec$.

1.3 Normal distribution

- Characterized by two parameters (μ, σ^2). Its probability density is given by

$$f(s) = \frac{1}{2\pi} \exp\left[-\frac{1}{2} \left(\frac{s-\mu}{\sigma} \right)^2 \right]$$

and its first moments by

$$E[X] = \mu, \quad E[X^2] = \mu^2 + \sigma^2.$$

- This distribution is mostly used to discribe thermal noise that should be taken into account when computing the signal to noise ratio in radio links.

1.4 Lognormal distribution.

- Characterized by two parameters (μ, σ^2).
- Its density function is given by

$$f(s) = \frac{\exp\left[-\frac{1}{2}\left(\frac{ln(s)-\mu}{\sigma}\right)^2\right]}{\sqrt{2\pi\sigma^2 s^2}}.$$

- Its moments are given by

$$E[X^n] = \exp\left[j\mu + \frac{1}{2}(j\sigma)^2\right].$$

- X is lognormally distributed with parameters (μ, σ^2) if and only if $\ln(X)$ is normally distributed with the same parameters.
- It can thus be written as $X = \exp(Y)$ where $Y \sim N(\mu, \sigma^2)$.
- In CDMA wireless communications, the received power from power controlled sources with fading channels have lognormal distribution where σ is typically between 0.3 and 3dB.

1.5 Gamma distribution

- A Gamma distributed RV with parameters (α, r) has a probability density of

$$f(s) = \frac{\alpha^r}{\Gamma[r]} s^{r-1} e^{-\alpha r}$$

where Γ is the Γ-function which satisfies $\Gamma(r) = (r-1)!$ for r integers.

- The moments are

$$E[X] = \frac{r}{\alpha}, \qquad E[X^n] = \alpha^{-n} \prod_{i=0}^{n-1} (r+i), n > 1.$$

- The distribution is defined on the range $0 \leq s \leq \infty$, and its parameters are defined for $\alpha > 0$ and $r > 0$.

- In the special case where r is an integer, this distribution is called the Erlang distribution.